EE310 – Chapter 3

Diodes

Lecture Slides

Instructor:
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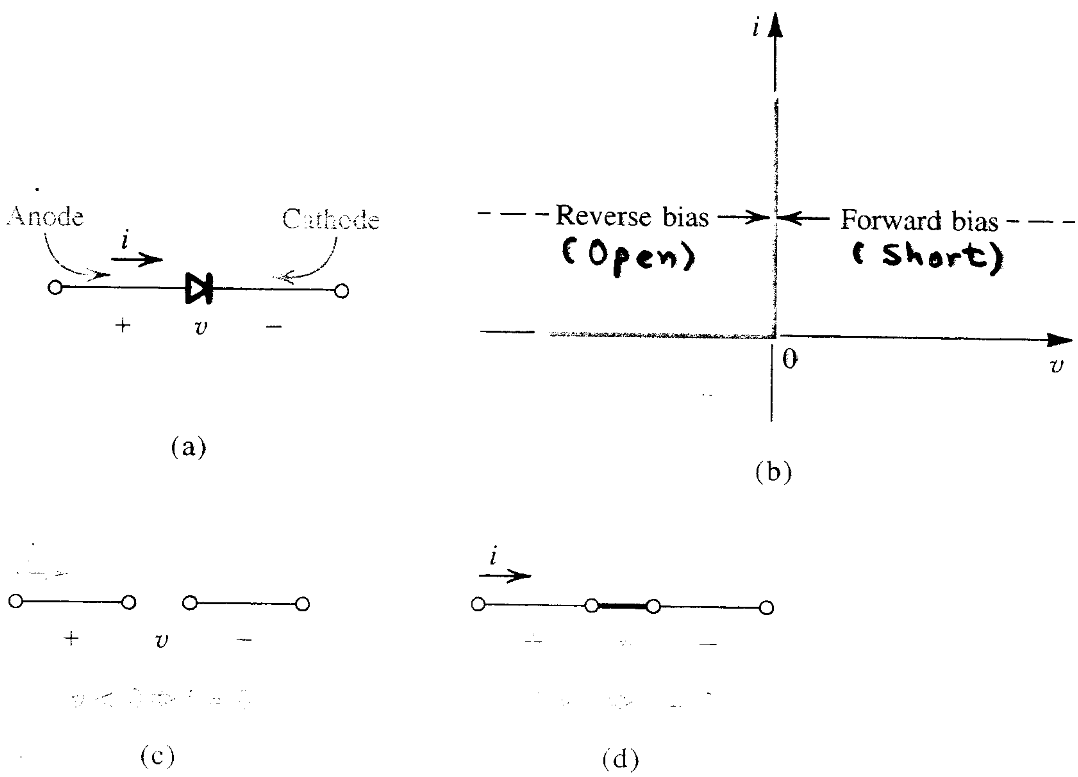
Chap. 3 Diodes

3.1 Ideal Diode

I-V Characteristic



The ideal diode: **(a)** diode circuit symbol; **(b)** i–v characteristic; **(c)** equivalent circuit in the reverse direction; **(d)** equivalent circuit in the forward direction.

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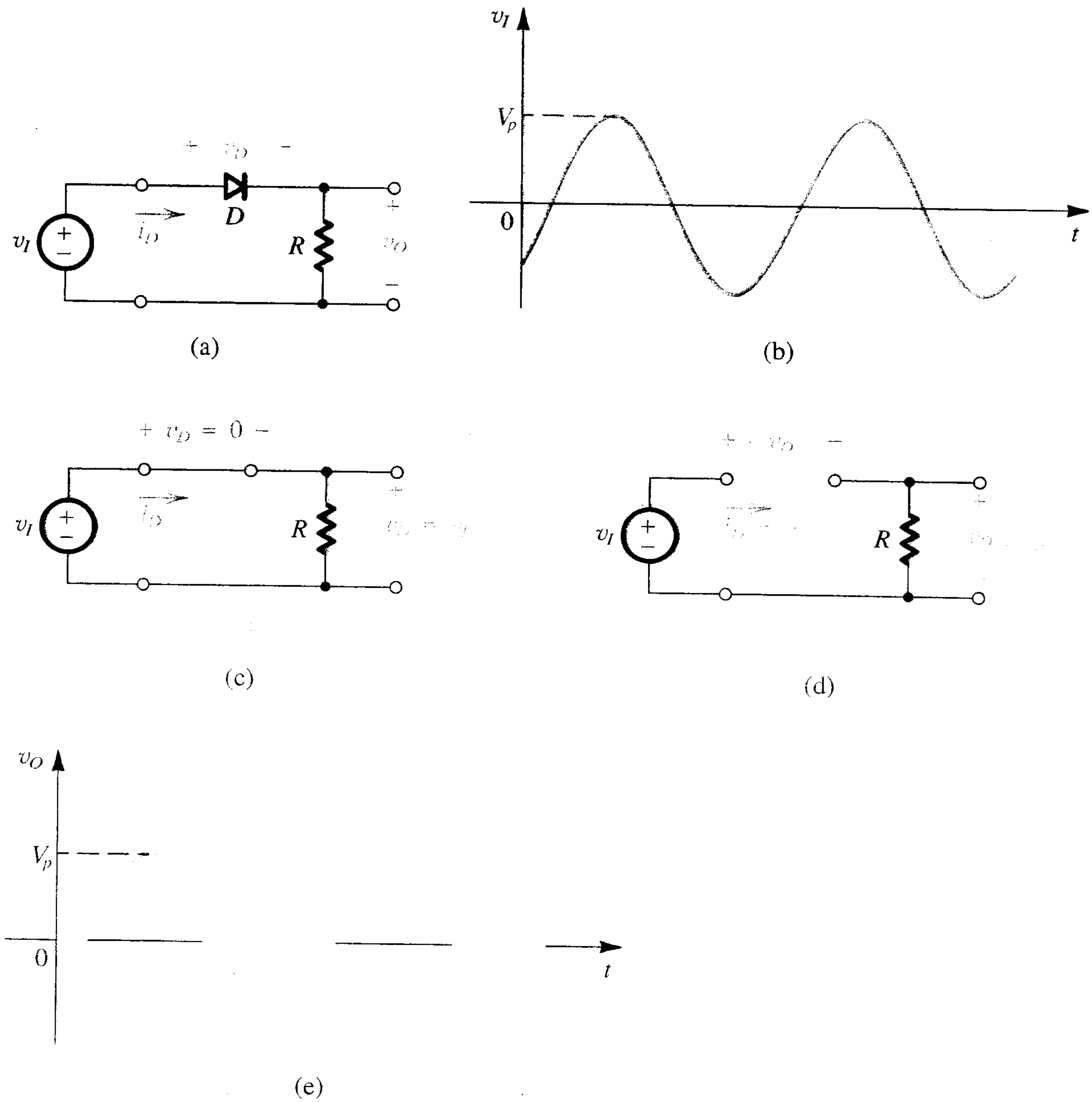
Application – Rectifier



(a) Rectifier circuit. **(b)** Input waveform. **(c)** Equivalent circuit when $v_I \geq 0$. **(d)** Equivalent circuit when $v_I \leq 0$. **(e)** Output waveform.

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3.3 In the circuit of Fig. 3.3(a), let v_I have a peak value of 10 V and $R = 1\ k\Omega$. Find the peak value of i_D and the dc component of v_O.

Ans. 10 mA; 3.18 V

Sol) $\hat{V}_I = 10V \quad R = 1K\Omega \qquad \hat{i}_D = (\quad) \quad \overline{V_o} = (\quad)$

$\hat{i}_D = \frac{\hat{V}_I}{R} = \frac{10V}{1K\Omega} = 10mA$

$\overline{V_o}$ = DC Comp of V_o = $\frac{\int_0^T V_o dt}{T} = \frac{\int_0^{\pi} V_p \sin\theta d\theta}{2\pi} = \frac{2V_p}{2\pi} = \frac{10V}{\pi}$

3.2 Diode Terminal Characteristic

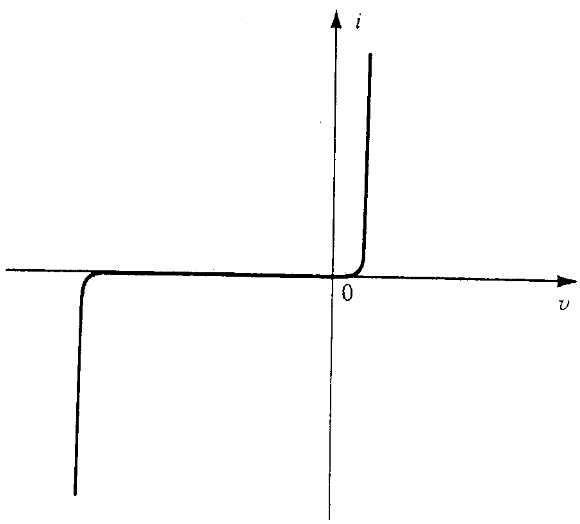


FIGURE 3.7 The i–v characteristic of a silicon junction diode.

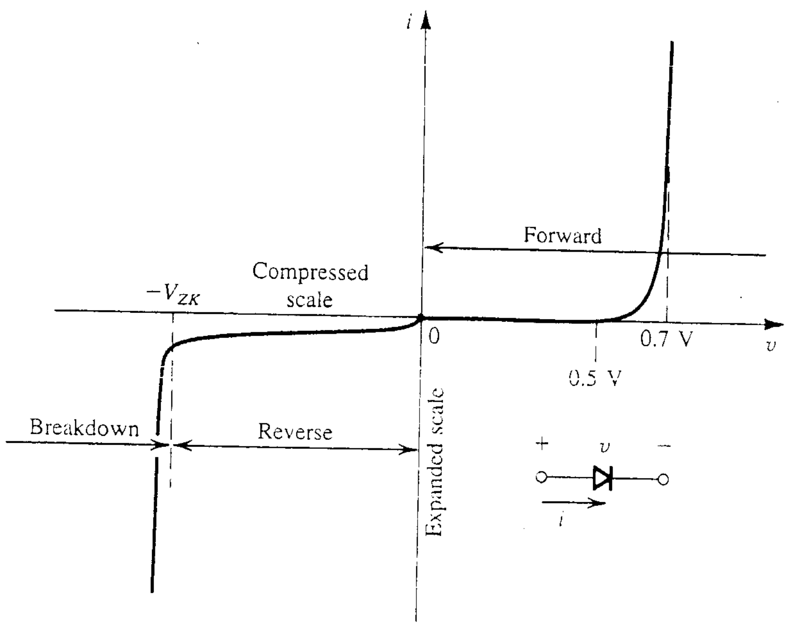


FIGURE 3.8 The diode i–v relationship with some scales expanded and others compressed in order to reveal details.

3.2.1 The Forward-Bias Region

$$i = I_s(e^{v/mV_T} - 1)$$

where, $V_T = \frac{kT}{q} = 25\,mV$ at $T=300K$ (R.T.)

m = ideality Factor = 1 − 2
(IC) (Discrete)

For $i \gg I_s$, then $i \approx I_s e^{v/mV_T}$

$$\Leftrightarrow\ v = mV_T \ln \frac{i}{I_s} = 2.3\, mV_T \log \frac{i}{I_s}$$

ex) $i_1 @ V_1$, $i_2 @ V_2$ then, $V_2 - V_1 = 2.3\, mV_T \log \frac{i_2}{i_1}$

For example, $i_2 = 10 \times i_1$, then $V_2 - V_1 = 2.3\, mV_T$

⇒ Diode voltage increases $2.3\, mV_T$ volts per each <u>decade</u> increase in current.

<u>Note</u> $2.3\, mV_T$ = 57.5 mV ~ 115 mV at RT
(m=1) (m=2)

ex) $i \approx 0$ for $V \le 0.5V$
∴ $V = 0.5V$ is called cut-in voltage

i = fully conducting for $V \simeq 0.6 \sim 0.8\, V$!
∴ $V \simeq 0.7V$ is turn-on voltage

3.6 Consider a silicon diode with $n = 1.5$. Find the change in voltage if the current changes from 0.1 mA to 10 mA.

Ans. 172.5 mV

$n = 1.5 \qquad i_1 = 0.1\,mA \longrightarrow i_2 = 10\,mA$

$V_2 - V_1 = (\qquad)$

Sol) $V_2 - V_1 = 2.3\,nV_T \log \frac{i_2}{i_1} = 2.3\,nV_T \log \frac{10\,mA}{0.1\,mA}$

$= 2.3 \times 1.5 \times 25\,mV \times \log 100 = 172.5\,mV$

3.8 Using the fact that a silicon diode has $I_S = 10^{-14}$ A at 25°C and that I_S increases by 15% per °C rise in temperature, find the value of I_S at 125°C.

Ans. 1.17×10^{-8} A

$I_S = 10^{-14}\,A$ @ 25°C $\qquad \frac{\Delta I_S}{\Delta T} = \frac{0.15\,I_S}{°C}$

$I_S = (\qquad)$ @ 125°C

Sol) $I_S(125°C) = (1+0.15)\,I_S(124°C)$

$= (1+0.15) \times (1+0.15)\,I_S(123°C)$

$= (1+0.15)^{100}\,I_S(25°C)$

$= 1.15^{100} \times 10^{-14}A = 1.17 \times 10^{-8}\,A$

I_S doubles every 5°C

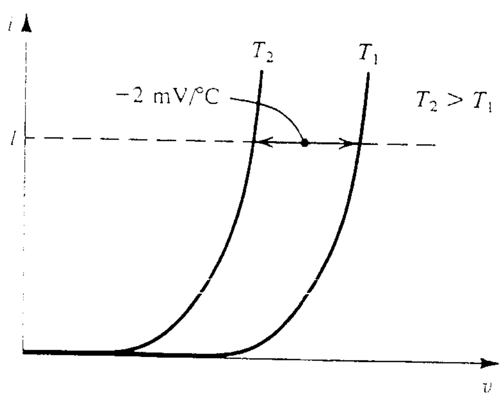


FIGURE 3.9 Illustrating the temperature dependence of the diode forward characteristic. At a constant current, the voltage drop decreases by approximately 2 mV for every 1°C increase in temperature.

3.2.2 The Reverse-Bias Region

$V < 0$

$i = I_S(e^{V/nV_T} - 1) \simeq -I_S \equiv$ reverse saturation current

For real devices, often $i = i_R \gg I_S$

ex) a small-signal diode with $I_S = 10^{-14} - 10^{-15}$ A

can show $i_R = 10^{-9}$ A ! due to Leakage

$i_R \rightarrow$ 2x every 10°C

Note $I_S \rightarrow$ 2x every 5°C

3.9 The diode in the circuit of Fig. E3.9 is a large high-current device whose reverse leakage is reasonably independent of voltage. If $V = 1$ V at 20°C, find the value of V at 40°C and at 0°C.

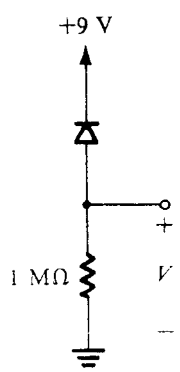


FIGURE E3.9

Ans. 4 V; 0.25 V

i_R = indept. of V

$V = 1$ V @ 20°C

$V = (\quad)$ @ 40°C, 0°C

Sol) $V = i_R \times 1\,\text{M}\Omega$

20°C : $1\,\text{V} = i_R \times 1\,\text{M}$

$\therefore i_R = 1\,\mu\text{A}$

40°C : 20°C → 30°C → 40°C

$i_R =$ 2x 2x

$= 4 \times 1\,\mu\text{A}$ $\therefore V = 4\,\mu\text{A} \times 1\,\text{M}\Omega$

$= 4\,\text{V}$

0°C : 20°C → 10°C → 0°C

$i_R = \frac{1}{2}\times$ $\frac{1}{2}\times$ $= \frac{1}{4} \times 1\,\mu\text{A}$

$V = \frac{1}{4}\,\mu\text{A} \times 1\,\text{M}\Omega = 0.25\,\text{V}$

3.2.3 The Breakdown Region

Fig. 8 V_{ZK} = "Knee" voltage (Breakdown Voltage)

Breakdown is NOT destructive unless the power dissipated exceeds the safe level.

3.3 Forward Characteristics – Models!

1) Exponential Model

– Most accurate
– Most complicated to use.

Fig. 10

$$I_D = I_S e^{V_D/mV_T} \quad \text{... (diode model)}$$

$$I_D = \frac{V_{DD} - V_D}{R} \quad \text{... (circuit)}$$

(First) Graphical Solution = Fig. 11

(Second) Iterative Solution

① Let $V_D = 0.7V$ (diode)

② Calc $I_D = \frac{V_{DD} - 0.7}{R}$ (circuit)

③ Refine $V_D = 2.3\, nV_T \log \frac{I_D}{I_S}$ (diode)

Exponential Model

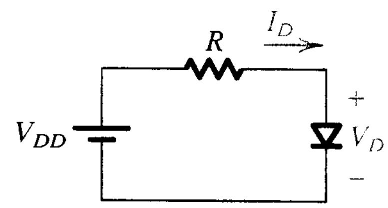
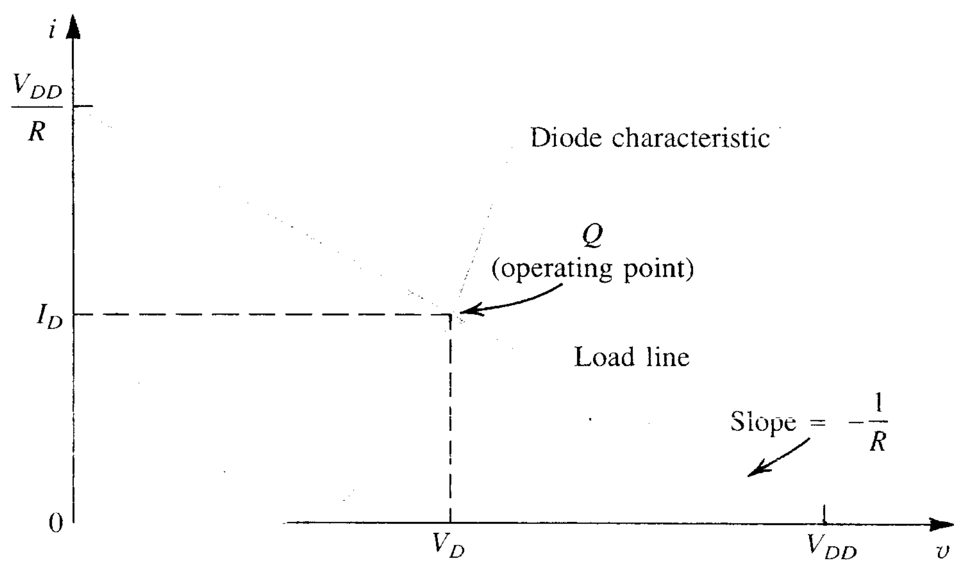


FIGURE 3.10 A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.



Graphical analysis of the circuit in Fig. 3.10 using the exponential diode model.

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3.10 For the circuit in Fig. 3.10, find I_D and V_D for the case $V_{DD} = 5$ V and $R = 10$ kΩ. Assume that the diode has a voltage of 0.7 V at 1-mA current and that the voltage changes by 0.1 V/decade of current change. Use (a) iteration, (b) the piecewise-linear model with $V_{D0} = 0.65$ V and $r_D = 20$ Ω, (c) the constant-voltage-drop model with $V_D = 0.7$ V.

Sol) Circuit : $V_{DD} = 5V \qquad R = 10K$

Diode : $\dfrac{\Delta V_D}{\Delta I_D} = \dfrac{0.1V}{decade} \quad \therefore 2.3\, nV_T = 0.1\,V$

$0.7V$ @ $1mA \rightarrow 1mA = I_s e^{0.7V/nV_T}$

$= I_s e^{0.7 \times 2.3/0.1}$

$\therefore I_s = 1mA\, e^{-16.1}$

Iteration:

Let $V_D = 0.7V \longrightarrow I_D = \dfrac{5-0.7}{10k} = 0.43\,mA$ (circuit)

$\longrightarrow 0.43mA = 1mA\, e^{-16.1}\, e^{23 V_D} \quad \therefore V_D = 0.663$ (diode)

$\longrightarrow I_D = \dfrac{5-0.663}{10k} = 0.434\,mA$ (circuit)

D3.12 Design the circuit in Fig. E3.12 to provide an output voltage of 2.4 V. Assume that the diodes available have 0.7-V drop at 1 mA and that $\Delta V = 0.1$ V/decade change in current.

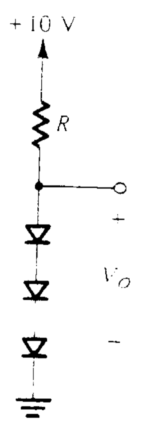


FIGURE E3.12

Ans. $R = 760\ \Omega$

Sol) Diode: 0.1V/decade, 0.7V @ 1mA

Design for $V_o = 2.4V \longrightarrow R = \dfrac{10-2.4}{I_D}$

$I_D = (\quad)$ @ $V_D = \dfrac{2.4}{3} = 0.8V$?

Diode has $I_s = 1mA\, e^{-16.1}$, $2.3\,nV_T = 0.1\,V$

$I_D = I_s e^{V_D/nV_T} = 1mA\, e^{-16.1}\, e^{2.3 V_D/0.1}$

$\therefore I_D = 9.994\,mA$ @ $V_D = 0.8V$

$\therefore R = \dfrac{10-2.4}{9.994} = 0.762\,k\Omega$

2) Piecewise Linear Model

(– 2nd most accurate
– 2nd most involved to use!) Fig.12

$$i_D = \begin{cases} \dfrac{V_D - V_{D0}}{r_D} & \text{for } V_D \geq V_{D0} \\ 0 & \text{for } V_D \leq V_{D0} \end{cases}$$

3) Const. Voltage Drop Model

(– 3rd most accurate
– 3rd most involved!) Fig.15

$$V_D = 0.7\ V$$

4) Ideal Diode Model – most simple!

$$i_D = \begin{cases} \text{short} & (V_D > 0) \\ \text{open} & (V_D < 0) \end{cases}$$

5) Small-signal Model

For small signals, Diode is modeled as

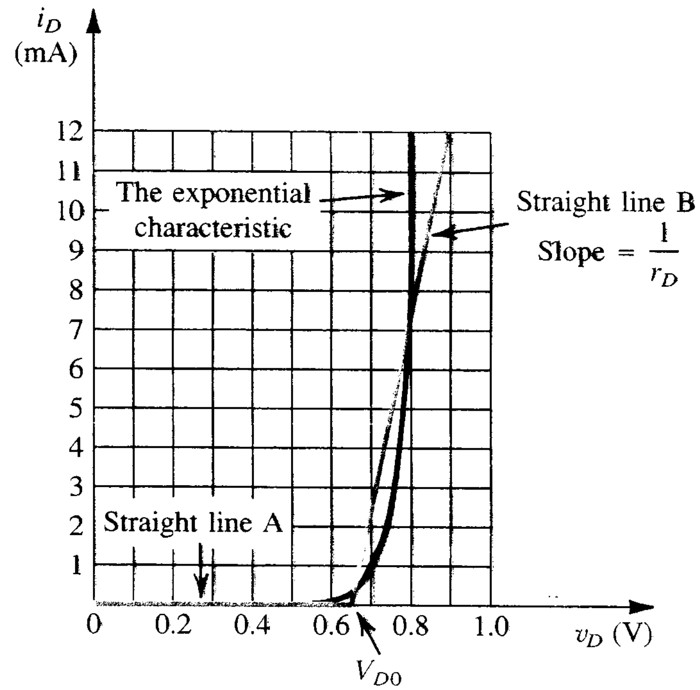
$$i_d = \frac{V_d}{r_d} \quad \text{where } r_d \equiv \frac{nV_T}{I_D}$$

Fig.17

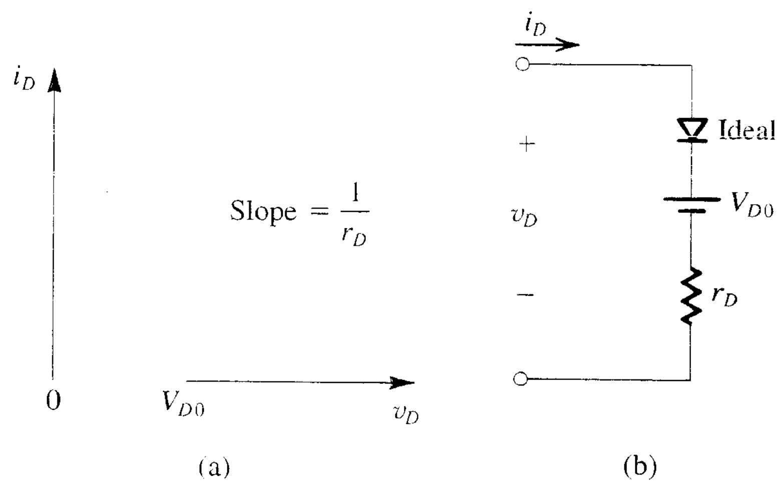
$V_D + v_d$

$I_D + i_d$

Piecewise Linear Model



Approximating the diode forward characteristic with two straight lines: the piecewise-linear model.



Piecewise-linear model of the diode forward characteristic and its equivalent circuit representation.

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Constant Voltage Drop Model

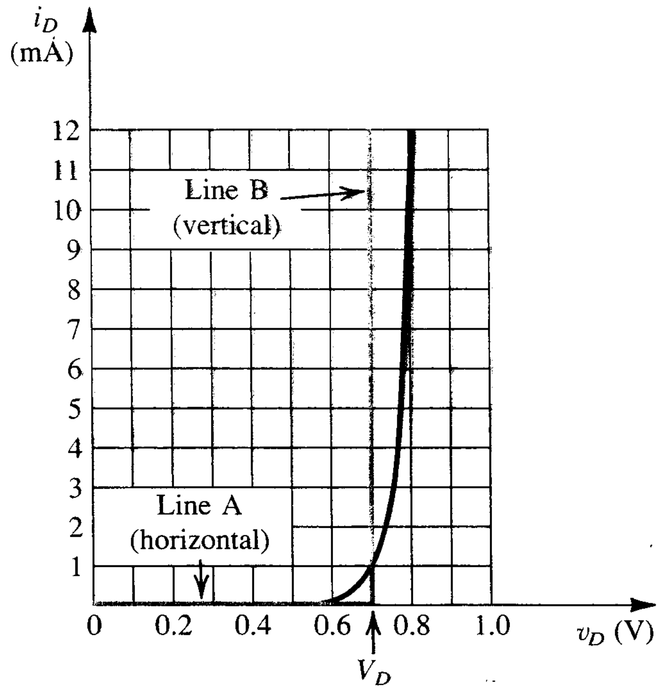
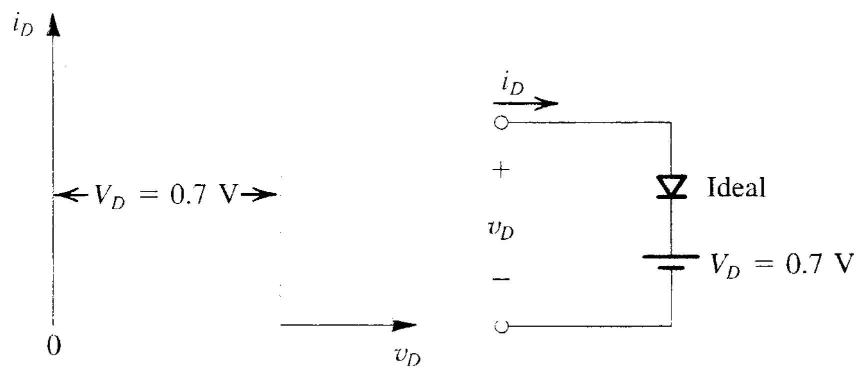


FIGURE 3.15 Development of the constant-voltage-drop model of the diode forward characteristics. A vertical straight line (B) is used to approximate the fast-rising exponential. Observe that this simple model predicts V_D to within ±0.1 V over the current range of 0.1 mA to 10 mA.



(a) (b)

The constant-voltage-drop model of the diode forward characteristics and its equivalent-circuit representation.

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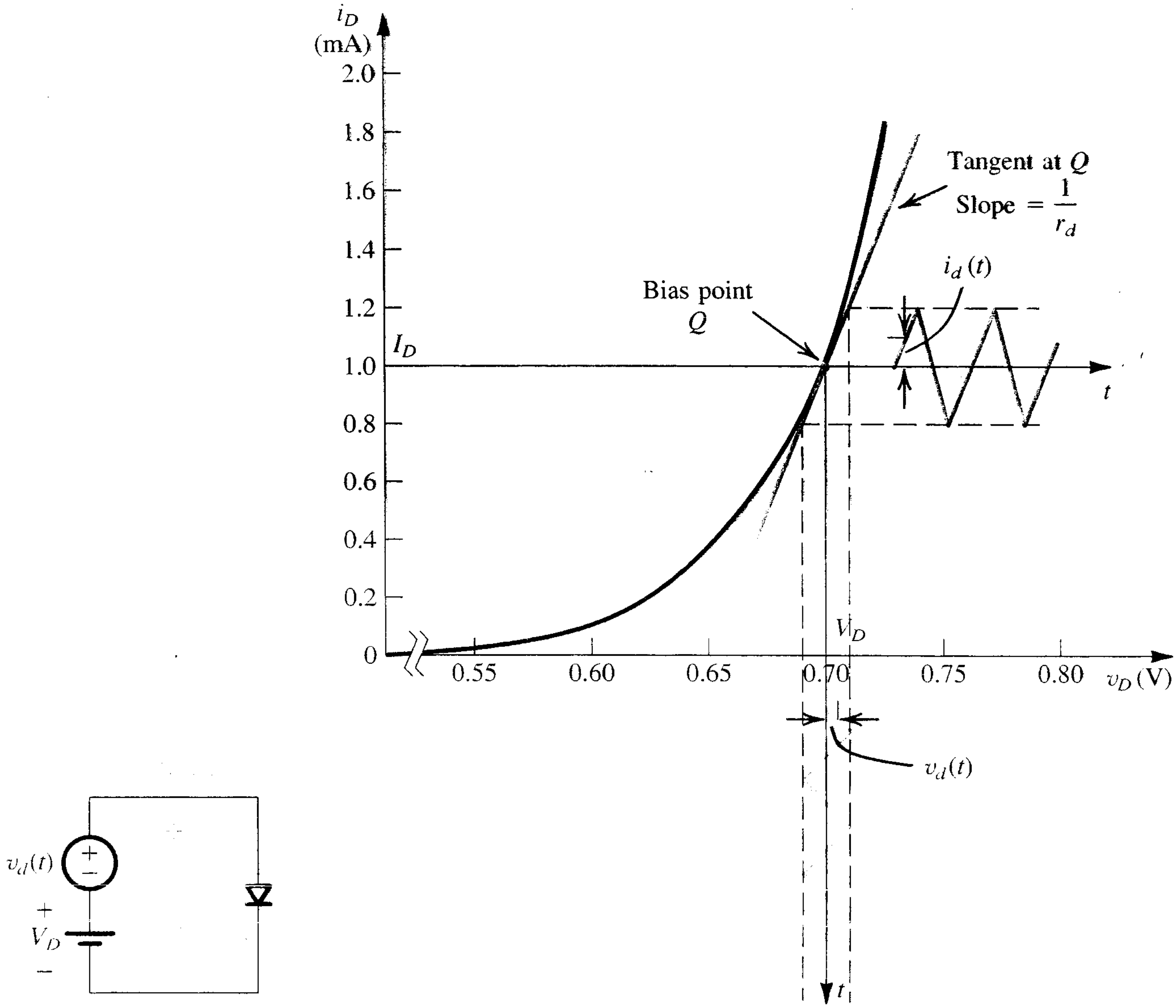
Small-signal Model



(a)

(b)

Development of the diode small-signal model. Note that the numerical values shown are for a diode with $n = 2$.

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3.10 For the circuit in Fig. 3.10, find I_D and V_D for the case $V_{DD} = 5$ V and $R = 10$ kΩ. Assume that the diode has a voltage of 0.7 V at 1-mA current and that the voltage changes by 0.1 V/decade of current change. Use ~~(a) iteration~~, (b) the piecewise-linear model with $V_{D0} = 0.65$ V and $r_D = 20$ Ω, (c) the constant-voltage-drop model with $V_D = 0.7$ V.

Ans. (a) ~~0.434 mA, 0.663 V~~; (b) 0.434 mA, 0.659 V; (c) 0.43 mA, 0.7 V

Sol)

(b) Piecewise Linear : $V_{D0} = 0.65V \quad r_D = 20\Omega$

$$\left.\begin{array}{ll} \text{Diode:} & i_D = \dfrac{V_D - 0.65}{20\Omega} \\ \text{Circuit:} & i_D = \dfrac{5 - V_D}{10k\Omega} \end{array}\right\} \xrightarrow{\text{Solve}} \begin{array}{l} i_D = 0.434mA \\ V_D = 0.659V \end{array}$$

(C) Constant Voltage Drop : $V_D = 0.7V$

Circuit : $i_D = \dfrac{5 - 0.7V}{10k\Omega} = 0.43mA$

3.15 Consider a diode with $n = 2$ biased at 1 mA. Find the change in current as a result of changing the voltage by (a) −20 mV, (b) −10 mV, (c) −5 mV, (d) +5 mV, (e) +10 mV, and (f) +20 mV. In each case, do the calculations (i) using the small-signal model and (ii) using the exponential model.

Ans. (a) −0.40, −0.33 mA; (b) −0.20, −0.18 mA; (c) −0.10, −0.10 mA; (d) +0.10, +0.11 mA; (e) +0.20, +0.22 mA; (f) +0.40, +0.49 mA

Sol) Diode : $n = 2, \ I_D = 1mA \rightarrow r_d = \dfrac{nV_T}{I_D} = \dfrac{2\times 25mV}{1mA} = 50\Omega$

$V_{D2} - V_{D1} = \text{given} \qquad I_{D2} - I_{D1} = ?$

(a) $V_{D2} - V_{D1} = -20mV$

Small-signal : $i_d = \dfrac{-20mV}{50\Omega} = -0.4mA$

Exponential : $\dfrac{I_{D2}}{I_{D1}} = \dfrac{I_S e^{V_{D2}/nV_T}}{I_S e^{V_{D1}/nV_T}} = e^{(V_{D2}-V_{D1})/nV_T}$

$\rightarrow I_{D2} = I_{D1} e^{(V_{D2}-V_{D1})/nV_T} = 1mA\, e^{-20mV/2\times 25mV} = 0.67mA$

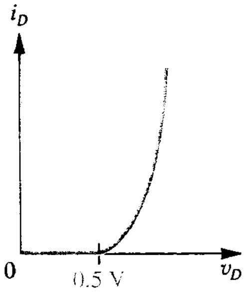
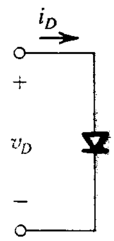
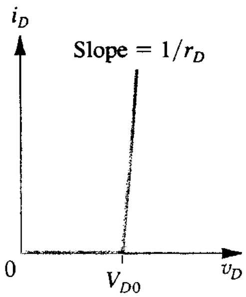
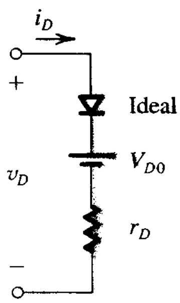
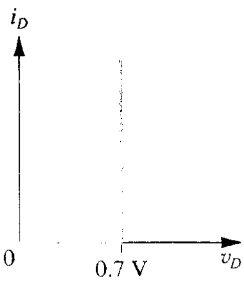
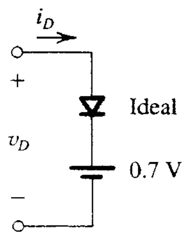
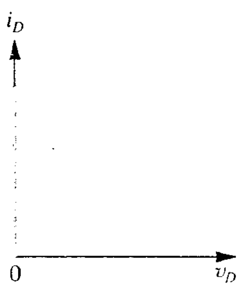
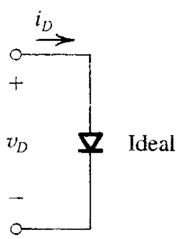
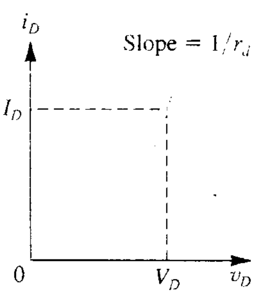
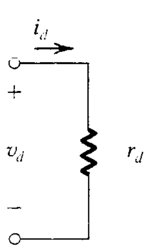
$\therefore I_{D2} - I_{D1} = 0.67 - 1 = -0.33mA$

(C) $V_{D2} - V_{D1} = -5mV \qquad$ Small-signal $i_d = \dfrac{-5mV}{50\Omega} = -0.1mA$

Exponential $I_{D2} = 1mA\, e^{-5mV/2\times 25mV} = 0.9mA \quad \therefore \Delta I_D = -0.1mA$

Summary of Diode Models

TABLE 3.1 **Modeling the Diode Forward Characteristic**

Model	Graph	Equations	Circuit	Comments
Exponential		$i_D = I_S e^{v_D/nV_T}$ $v_D = 2.3nV_T \log\left(\frac{i_D}{I_S}\right)$ $V_{D2} - V_{D1} = 2.3nV_T \log\left(\frac{I_{D2}}{I_{D1}}\right)$ $2.3nV_T = 60$ mV for $n = 1$ $2.3nV_T = 120$ mV for $n = 2$		$I_S = 10^{-12}$ A to 10^{-15} A, depending on junction area $V_T \cong 25$ mV $n = 1$ to 2 Physically based and remarkably accurate model Useful when accurate analysis is needed
Piecewise-linear (battery-plus-resistance)		For $v_D \le V_{D0}$: $i_D = 0$ For $v_D \ge V_{D0}$: $i_D = \frac{1}{r_D}(v_D - V_{D0})$		Choice of V_{D0} and r_D is determined by the current range over which the model is required. For the amount of work involved, not as useful as the constant-voltage-drop model. Used only infrequently.
Constant-voltage-drop (or the "0.7-V model")		For $i_D > 0$: $v_D = 0.7$ V		Easy to use and very popular for the quick, hand analysis that is essential in circuit design.
Ideal-diode		For $i_D > 0$: $v_D = 0$		Good for determining which diodes are conducting and which are cutoff in a multiple-diode circuit. Good for obtaining very approximate values for diode currents, especially when the circuit voltages are much greater than V_D.
Small-signal		For small signals superimposed on V_D and I_D: $i_d = v_d/r_d$ $r_d = nV_T/I_D$ (For $n = 1$, v_d is limited to 5 mV; for $n = 2$, 10 mV)		Useful for finding the signal component of the diode voltage (e.g., in the voltage-regulator application). Serves as the basis for small-signal modeling of transistors (Chapters 4 and 5).

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3.4 Zener Diode

- Reverse Breakdown Diode

i) Onset of Breakdown (Fig.21)

V_{ZK} = Knee Voltage, I_{ZK} = Knee current

ii) Manufacturer Datasheet gives

V_Z at I_{ZT}

ex) $V_Z = 6.8\,V$ at $I_{ZT} = 10\,mA$

ex) V_Z = a few V ~ a few 100V

ex) A 0.5-W, 6.8-V zener can conduct $I_Z < 70\,mA$

iii) ac

$\Delta V = r_z \Delta I$ around the Q-point

$\rightarrow$ $V_z = V_{z0} + r_z I_z$ (dynamic model)(Fig.22)

iV) Zener diode is used in Shunt Regulator

T-dependence: $\dfrac{dV_z}{dT} = (\quad)\dfrac{mV}{°C}$

3.17 A zener diode whose nominal voltage is 10 V at 10 mA has an incremental resistance of 50 Ω. What voltage do you expect if the diode current is halved? Doubled? What is the value of V_{Z0} in the zener model?

Zener : 10V at 10mA, $r_z = 50\Omega$

$V_z = (\quad)$ at 20mA ; $(\quad)$ at 5mA

Sol) $V_z = V_{z0} + I_z r_z$

$10V = V_{z0} + 50\Omega \times 10mA$

$\therefore V_{z0} = 9.5V$

For $I_z = 20\,mA$

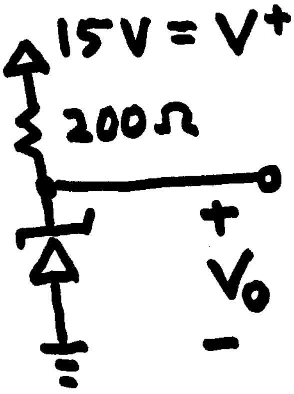
$V_z = 9.5 + 20mA \times 50\Omega = 10.5V$

For $I_z = 5\,mA$

$V_z = 9.5 + 5mA \times 50\Omega = 9.75V$

3.19 A shunt regulator utilizes a zener diode whose voltage is 5.1 V at a current of 50 mA and whose incremental resistance is 7 Ω. The diode is fed from a supply of 15-V nominal voltage through a 200-Ω resistor. What is the output voltage at no load? Find the line regulation and the load regulation.

Shunt Regulator



Zener: 5.1V at 50mA

$r_z = 7\Omega$

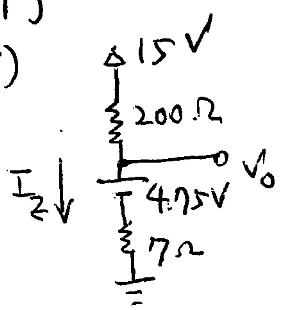
i) $V_o = (\quad)$ with no Load.

ii) Load Regulation $\frac{dV_o}{dI_L} = (\quad) \frac{mV}{mA}$

iii) Line Regulation $\frac{\Delta V_o}{\Delta V^+} = (\quad) \frac{mV}{V}$

Sol)

i)

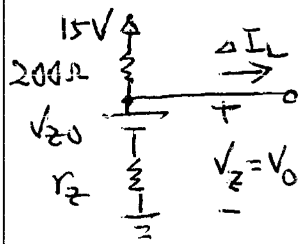


$V_z = 5.1V = V_{z0} + 50mA \times 7\Omega \quad \therefore V_{z0} = 4.75V$

$I_z = \frac{15 - 4.75}{200 + 7} = 49.5mA$

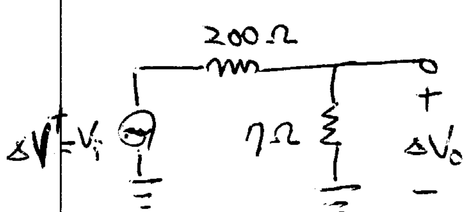
$V_o = 15 - 200\Omega \times 49.5mA = 5.1V$

ii) Load Regulation $= \frac{\Delta V_o}{\Delta I_L}$



$\frac{\Delta V_o}{\Delta I_L} = -\frac{\Delta V_o}{\Delta I_z} = -\frac{1mA \times 7\Omega}{1mA} = -7 \frac{mV}{mA}$

iii) Line Regulation $= \frac{\Delta V_o}{\Delta V^+}$ — change in output voltage / — fluctuation in Line voltage $= V_i$



$\frac{\Delta V_o}{\Delta V^+} = \frac{7}{200 + 7} = 0.0338 = 33.8 \frac{mV}{V}$

3.5 Rectifier Circuit

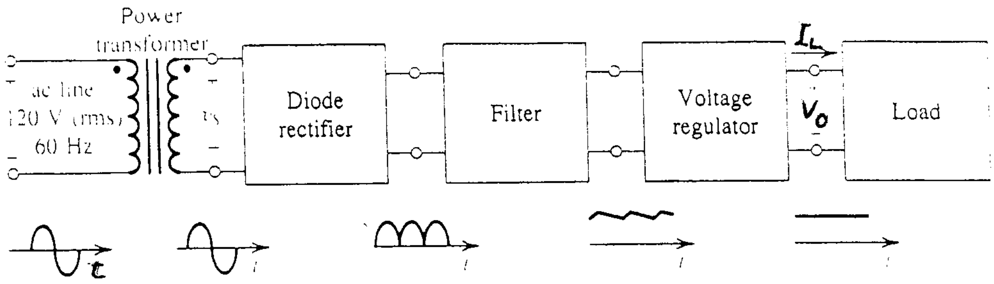


FIGURE 3.24 Block diagram of a dc power supply.

(DC Power Supply)

(1) Half-Wave Rectifier (Fig.25)

$$V_o \cong V_s - V_{DO}$$

PIV = Peak Inverse Voltage

$= V_s$

(2) Full-Wave Rectifier

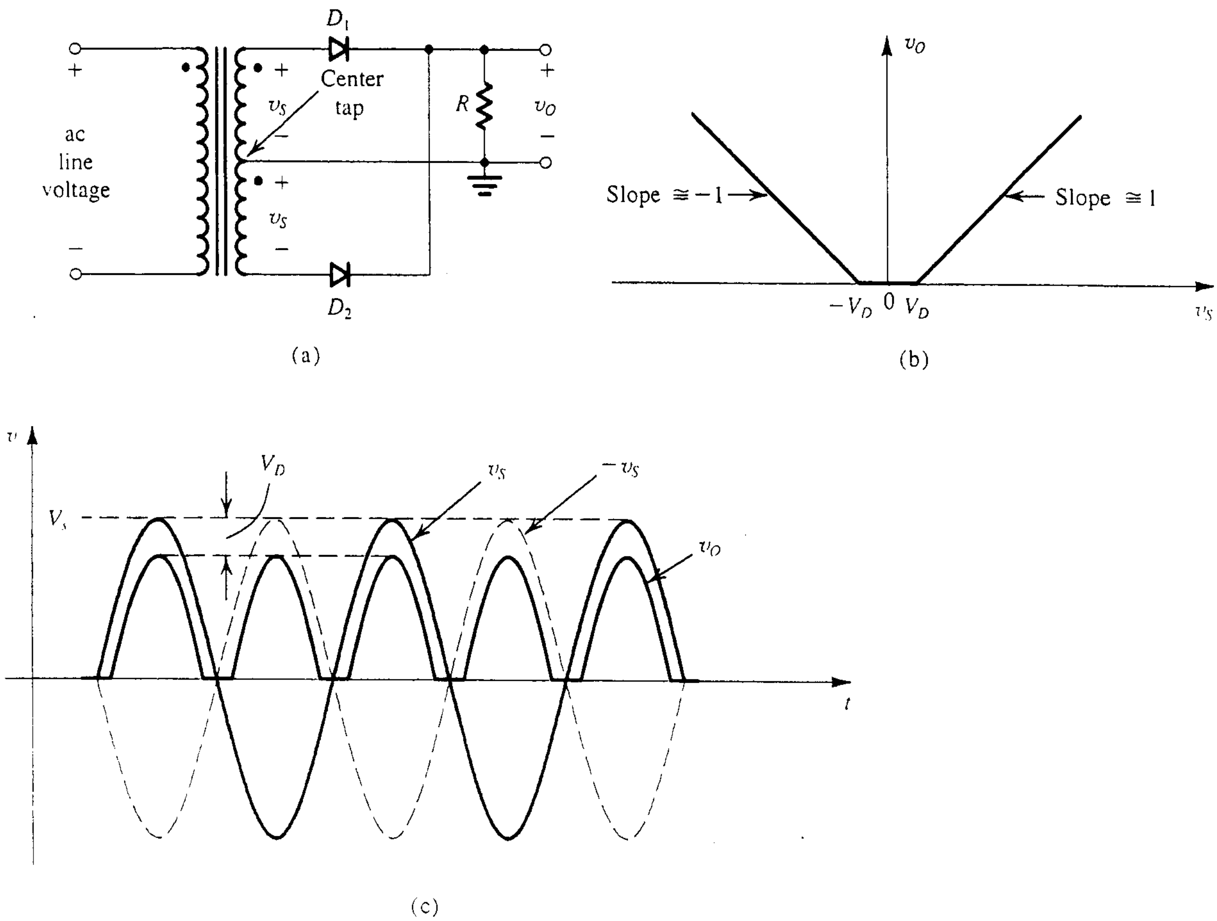


FIGURE 3.26 Full-wave rectifier utilizing a transformer with a center-tapped secondary winding: (a) circuit; (b) transfer characteristic assuming a constant-voltage-drop model for the diodes; (c) input and output waveforms.

(Fig. 26) $V_o = V_s - V_D$

$$PIV = \underset{\substack{\text{(Anode)} \\ D1}}{V_s} - \underset{\substack{\text{(Cathode)} \\ D1}}{(-V_s + V_D)} = 2V_s - V_D$$

Full-Wave Rectifier

3.21 For the full-wave rectifier circuit in Fig. 3.26(a), neglecting the effect of r_D, show the following: (a) The output is zero for an angle of $2\sin^{-1}(V_D/V_s)$ centered around the zero-crossing points of the sine-wave input. (b) The average value (dc component) of v_O is $V_O \simeq (2/\pi)V_s - V_D$. (c) The peak current through each diode is $(V_s - V_D)/R$. Find the fraction (percentage) of each cycle during which $v_O > 0$, the value of V_O, the peak diode current, and the value of PIV, all for the case in which v_S is a 12-V (rms) sinusoid, $V_D \simeq 0.7$ V, and $R = 100\ \Omega$.

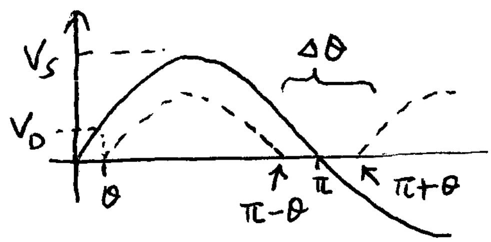
Ans. 97.4%; 10.1 V; 163 mA; 33.2 V

$V_s = 12\text{-V(rms)}$, $V_D = 0.7V$, neglect r_D, $R = 100\Omega$

(a) $V_o = 0$ for $\Delta\theta = 2\sin^{-1}\left(\frac{V_D}{V_s}\right)$ (b) $V_{o,avg} = \frac{2}{\pi}V_s - V_D$

(c) Find $I_{D,max} = \frac{V_s - V_D}{R}$, angle for $V_o > 0$, $V_{o,avg}$, PIV

Sol) (a)



$V_s \sin\theta = V_D \quad \therefore \theta = \sin^{-1}\left(\frac{V_D}{V_s}\right)$

$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{V_D}{V_s}\right)$

(b) $$V_{o,avg} = \frac{1}{\pi}\int_{\pi} (V_s \sin\phi - V_D)\,d\phi = \frac{1}{\pi}\left[-V_s\cos\phi - V_D\phi\right]_{\theta}^{\pi-\theta}$$

$$= \frac{2}{\pi}V_s\cos\theta - \frac{(\pi - 2\theta)}{\pi}V_D \simeq \frac{2}{\pi}V_s - V_D$$

(c)

i) $I_{D,max} = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100} = 163\text{ mA}$

ii) $V_o > 0$ for angle $= 2(\pi - 2\theta) = 2\left(\pi - 2\sin^{-1}\left(\frac{V_D}{V_s}\right)\right)$

angle fraction $= \frac{\text{angle}}{2\pi} = 1 - \frac{2}{\pi}\sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right) = 97.4\%$

iii) $V_{o,avg} = \frac{2}{\pi}V_s - V_D = \frac{2}{\pi}12\sqrt{2} - 0.7 = 10.1\text{ V}$

iv) $PIV = (V_s - V_D) + V_s = 2 \times 12\sqrt{2} - 0.7 = 33.2\text{ V}$

(3) The Bridge Rectifier

Fig.27 $\quad V_0 = V_s - V_D(\text{of } D_1) - V_D(\text{of } D_2) = V_s - 2V_D$

$$PIV = V_s - V_D$$

3.22 For the bridge rectifier circuit of Fig. 3.27(a), use the constant-voltage-drop diode model to show that (a) the average (or dc component) of the output voltage is $V_O \simeq (2/\pi)V_s - 2V_D$ and (b) the peak diode current is $(V_s - 2V_D)/R$. Find numerical values for the quantities in (a) and (b) and the PIV for the case in which v_S is a 12-V (rms) sinusoid, $V_D \simeq 0.7$ V, and $R = 100\ \Omega$.

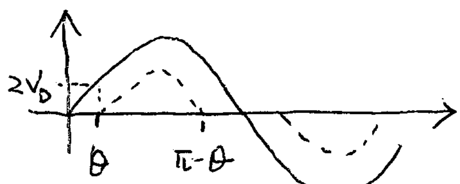
Ans. 9.4 V; 156 mA; 16.3 V

Show (a) $V_{O,avg} = \frac{2}{\pi}V_s - 2V_D$, (b) $I_{D,max} = \dfrac{V_s - 2V_D}{R}$

Calc (c) $V_{O,avg}$, $I_{D,max}$, PIV for $V_s = 12V_{(rms)}$, $V_D = 0.7$, $R = 100$

Sol)

(a)



$\theta = \sin^{-1}\left(\frac{2V_D}{V_s}\right)$

$\because V_s \sin\theta = 2V_D$

$$V_{O,avg} = \frac{\int_{\theta}^{\pi-\theta} (V_s \sin\phi - 2V_D)\, d\phi}{\pi} = \frac{2V_s\cos\theta - 2V_D(\pi - 2\theta)}{\pi} \simeq \frac{1}{\pi}(2V_s - 2V_D)$$

(b) $I_{D,max} = \dfrac{V_{O,peak}}{R} = \dfrac{V_s - 2V_D}{R}$

(c) $V_{O,avg} \simeq \frac{2}{\pi}(12\sqrt{2} - 0.7) = 9.4$ V

$I_{D,max} = \dfrac{12\sqrt{2} - 2\times 0.7}{100\,\Omega} = 156$ mA

$PIV = V_s - V_D = 12\sqrt{2} - 0.7 = 16.3$ V

D3.79 It is required to design a full-wave rectifier circuit using the circuit of Fig. 3.26 to provide an average output voltage of:

(a) 10 V
(b) 100 V

In each case find the required turns ratio of the transformer. Assume that a conducting diode has a voltage drop of 0.7 V. The ac line voltage is 120 V rms.

D3.80 Repeat Problem 3.79 for the bridge rectifier circuit of Fig. 3.27.

Diode $V_D = 0.7V$

ac Line $120\ V_{(rms)}$

Build Bridge Rectifier for (a) $V_{0,avg} = 10V$

(b) $V_{0,avg} = 100V$

⇒ Transformer Turn Ratio?

Sol) $V_{0,avg} = \frac{2}{\pi} V_s - 2V_D$

(a)

$10V = \frac{2}{\pi} V_s - 2\times 0.7 \quad \therefore V_s = 17.914\ V$

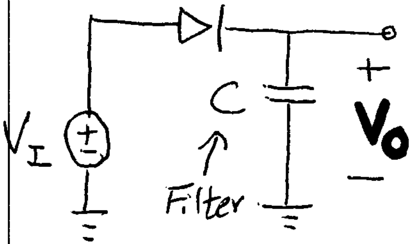
$\Rightarrow$ Turn Ratio $= \frac{120\sqrt{2}}{17.91} = 9.477$ to 1

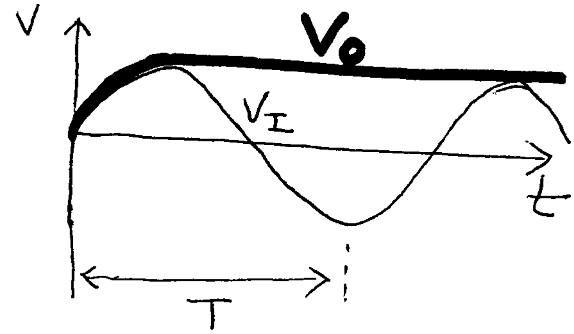
(b)

$100V = \frac{2}{\pi} V_s - 2\times 0.7 \quad \therefore V_s = 159.3\ V$

$\Rightarrow$ Turn Ratio $= \frac{120\sqrt{2}}{159.3} = 1.065$ to 1

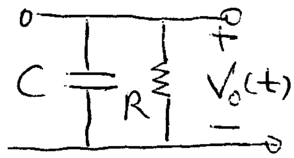
(4) Filtered Rectifier





Real Circuit − Fig. 29
(Leakage from C via R)

Fig. 29: Once $V_o = V_p$ is reached $\Rightarrow$ D = open as $V_I < V_p$

$\Rightarrow$  initial condition $V_o(0) = V_p$

$V_o(t) = V_p e^{-t/CR}$

$\Rightarrow$ For $T \ll CR$, $V_o(t) \simeq V_p\left(1 - \frac{t}{CR}\right)$

Properties of Filtered Rectifier (Fig. 29)

i) Ripple Voltage $\equiv V_r$

ii) DC Output Voltage $\equiv V_{o,avg} = \dfrac{V_p + (V_p - V_r)}{2} = V_p - \frac{1}{2}V_r$

iii) Conduction Angle $= \Delta\theta \equiv \omega \Delta t$

iv) Average Diode Current $= i_{D,avg}$ during Δt

v) Peak 〃 $= i_{D,max}$

Fig. 29

i) Ripple Voltage $= V_r$

$$V_o(T) \equiv V_p - V_r \qquad V_o(T) = V_p\left(1 - \frac{T}{CR}\right)$$

$$\therefore \boxed{V_r = V_p \frac{T}{CR}} \qquad I_L \equiv \frac{V_p}{R} = \text{leakage current}$$

$$f = \frac{1}{T}$$

iii) Conduction Angle $= \Delta\theta = w\Delta t$

$$V_p \cos\Delta\theta = V_p - V_r \qquad \cos\Delta\theta \simeq 1 - \frac{1}{2}\Delta\theta^2$$

$$\therefore \Delta\theta = w\Delta t = \sqrt{\frac{2V_r}{V_p}}$$

iv) $i_{D,avg}$ during conduction : $i_{D,avg} = i_{C,avg} + i_L$

$$i_{C,avg} = \frac{Q}{\Delta t} = \frac{CV_r}{\Delta t} = \pi I_L \sqrt{\frac{2V_p}{V_r}}$$

$$I_L = \frac{V_p}{R}$$

$$\therefore i_{D,avg} = i_{C,avg} + I_L = I_L\left(1 + \pi\sqrt{\frac{2V_p}{V_r}}\right)$$

v) Peak diode current, $i_{D,max}$

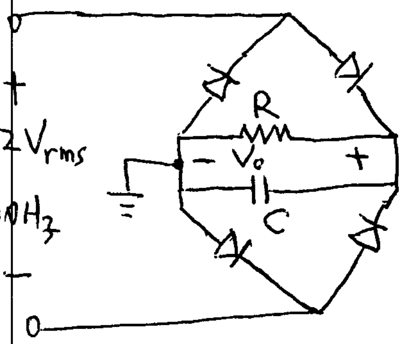
$$i_D = i_C + i_L = C\frac{dV_i}{dt} + i_L$$

$$i_{D,max} = C\left.\frac{dV_i}{dt}\right|_{t=-\Delta t} + i_L \qquad \text{and } V_i(t) = V_p\cos(wt)$$

$$= C w V_p \sin(w\Delta t) + i_L \simeq C w V_p (w\Delta t) + I_L$$

$$i_{D,max} = I_L\left(1 + 2\pi\sqrt{\frac{2V_p}{V_r}}\right)$$

D3.24 Consider a bridge-rectifier circuit with a filter capacitor C placed across the load resistor R for the case in which the transformer secondary delivers a sinusoid of 12 V (rms) having a 60-Hz frequency and assuming $V_D = 0.8$ V and a load resistance $R = 100\ \Omega$. Find the value of C that results in a ripple voltage no larger than 1 V peak-to-peak. What is the dc voltage at the output? Find the load current. Find the diodes' conduction angle. What is the average diode current? What is the peak reverse voltage across each diode? Specify the diode in terms of its peak current and its PIV.



$R = 100\,\Omega \quad V_D = 0.8\,V$

i) $C = (\quad)$ for $V_r \le 1\,V_{p\text{-}p}$

ii) $V_{O,avg} = (\quad)$ iii) $I_L = (\quad)$

iV) $\Delta\theta = \omega\Delta t = (\quad)$

v) $i_{D,avg} = (\quad)$ Vi) $PIV = (\quad)$

Vii) $i_{D,max} = (\quad)$

Sol) $V_o(t) = V_p e^{-t/RC}$, $V_p = V_s - 2V_D$, $V_s = 12\sqrt{2}$

i) $V_o(\frac{T}{2}) = V_p - V_r$, $V_o(\frac{T}{2}) \simeq V_p(1 - \frac{T}{2RC})$ $\therefore V_r = V_p \frac{T}{2RC} \le 1V$

$$C \ge \frac{V_p}{f \cdot 2R} = \frac{12\sqrt{2} - 2\times 0.8}{60\,Hz \times 2 \times 100\,\Omega} = 1.281\times 10^{-3}\,F = 1281\,\mu F$$

ii) DC $V_{O,avg} = V_p - \frac{1}{2}V_r = (12\sqrt{2} - 1.6) - \frac{1}{2}\cdot 1V = 14.87\,V$

iii) $I_L = \dfrac{V_p}{R} = \dfrac{12\sqrt{2} - 2\times 0.8}{100\,\Omega} = 0.15\,A$

IV) $\Delta\theta = \sqrt{\dfrac{2V_r}{V_p}} = \sqrt{\dfrac{2\times 1V}{12\sqrt{2} - 1.6}} = 0.36$ rad or 20.7°

V) $i_{D,avg} = I_L\left(1 + \pi\sqrt{\dfrac{2V_p}{V_r}}\right) = 0.15\left(1 + \pi\sqrt{\dfrac{2(12\sqrt{2}-1.6)}{1}}\right) = 2.0A$

Vi) $i_{D,max} = I_L\left(1 + 2\pi\sqrt{\dfrac{2V_p}{V_r}}\right) = 0.15\left(1 + 2\pi\sqrt{\dfrac{2(12\sqrt{2}-1.6)}{1}}\right) = 2.76A$

Vii) $PIV = V_s - V_D = 12\sqrt{2} - 0.8 = 16.2$

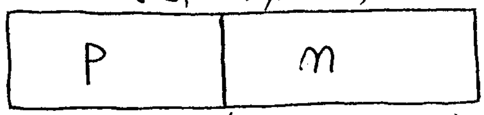
Diode Spec: $\begin{cases} \text{Current Rating} = 3.5 - 4.0\,A \\ PIV = 20\,V \end{cases}$

3.7 Physical Operation of Diodes

(1) Basic Semiconductor Concepts

i) PN Junction

(Si crystal)



Large amount of (Positive | negative) charge carriers

holes | electrons

ii) Intrinsic Si = chemically pure Si crystal

Real Space

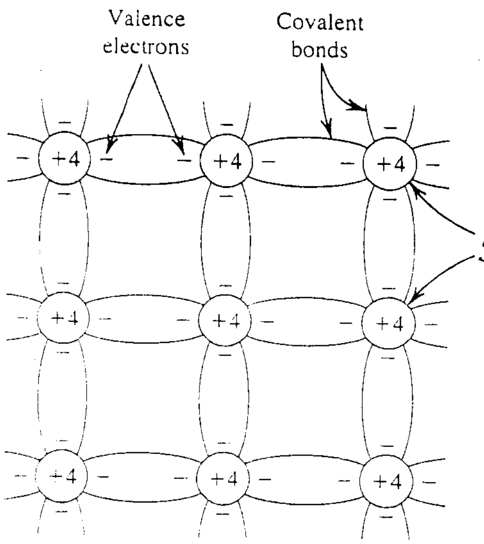


FIGURE 3.40 T = 0K

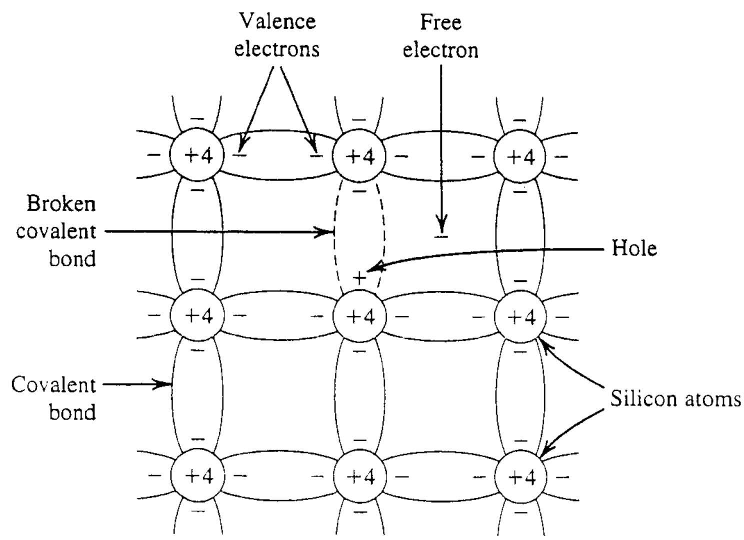
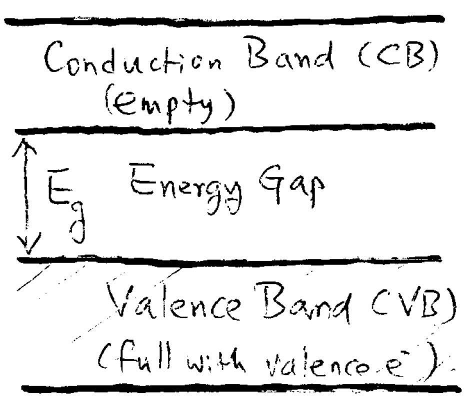
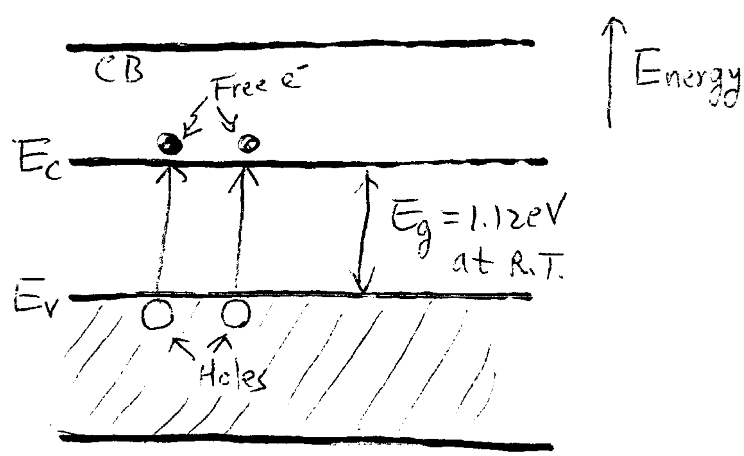


FIGURE 3.41 T > 0K, Broken Bonds

Energy Diagram





iii) Diffusion and Drift

Diffusion

Free particles (electrons, holes) show a net flow if its concentration distribution is not uniform.

ex) if $\frac{dn}{dx} \neq 0$, then electrons diffuse.

Diffusion current density [A/cm^2]

$$J_{n\text{-diff}} = q D_n \frac{dn}{dx}$$

$q = 1.6E{-19}$ Coul

D_n = diffusion const.

$$J_{p\text{-diff}} = -q D_p \frac{dp}{dx}$$

Drift

If E-field is present, then charged particles (e, h) flow.

$$J_{n\text{-drift}} = q n \mu_n E$$

μ_n = electron mobility [$\frac{cm^2}{V \cdot s}$]

$V_{n,drift} = \mu_n E$ = drift speed of electron

$$J_{p\text{-drift}} = q p \mu_p E$$

Total Drift Current:

$$J_{drift} = J_{n\text{-drift}} + J_{p\text{-drift}}$$
$$= q(n\mu_n + p\mu_p)E \equiv \frac{E}{\rho}$$

$$\rho \equiv \frac{1}{q(n\mu_n + p\mu_p)}$$ = Resistivity [Ω-cm]

$$\frac{D_n}{\mu_n} = V_T = \frac{D_p}{\mu_p}$$ (Einstein Relation)

iv) Doped Semiconductor : n-type, p-type

n-type Si

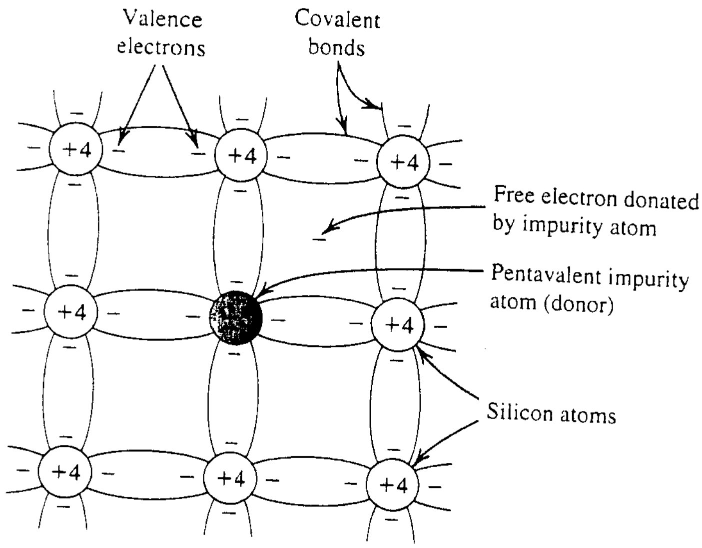


FIGURE 3.43 **n-type Si**

p-type Si

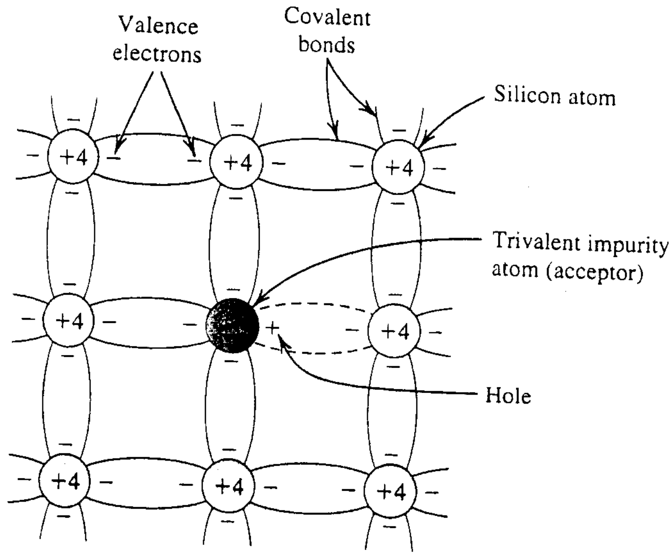
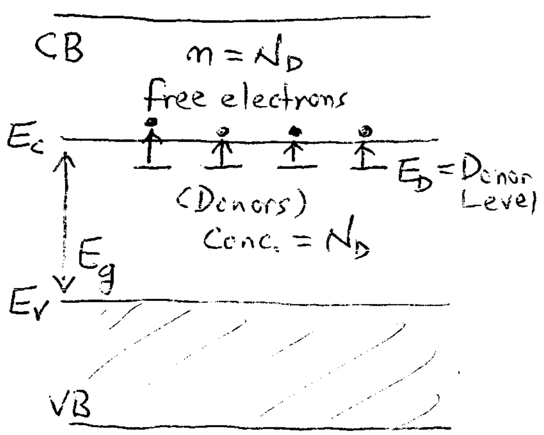
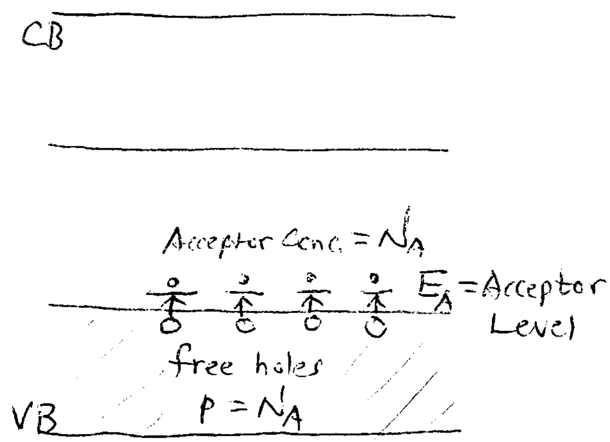


FIGURE 3.44 **p-type Si**





$$np = n_i^2$$

ex) $n_{no} = N_D$, $p_{no} = \dfrac{n_i^2}{N_D}$ (n-type, equil.)

ex) $p_{po} = N_A$ $\quad n_{po} = \dfrac{n_i^2}{N_A}$

3.29 Calculate the intrinsic carrier density n_i at 250 K, 300 K, and 350 K.

Sol) $n_i = \sqrt{BT^3 e^{-E_g/kT}}$

$B = 5.4\times10^{31}$
$E_g = 1.12\text{eV}$ for Si
$kT = 20.8,\ 25,\ 29.2\text{ meV}$
(250K) (300) (350)

T=250: $n_i = \sqrt{5.4E31\times250^3\times e^{-1.12/0.0208}} = 1.5\times10^8\ \text{cm}^{-3}$

300: $n_i = \sqrt{\ \text{''}\ \times300^3\times e^{-1.12/0.025}} = 1.5\times10^{10}$

350: $n_i = \sqrt{\ \text{''}\ \times350^3\times e^{-1.12/0.0292}} = 4.18\times10^{11}\ \text{cm}^{-3}$

3.30 Consider an n-type silicon in which the dopant concentration N_D is $10^{17}/\text{cm}^3$. Find the electron and hole concentrations at 250 K, 300 K, and 350 K. You may use the results of Exercise 3.29.

Sol) $N_D = 10^{17}\ \text{cm}^{-3}$

T=250K: $n = N_D = 10^{17}\ \text{cm}^{-3}$, $p = \frac{n_i^2}{N_D} = \frac{(1.5E8)^2}{1E17} = 0.225\ \text{cm}^{-3}$

T=300K: $n = N_D = 10^{17}\ \text{cm}^{-3}$, $p = \frac{(1.5E10)^2}{1E17} = 2{,}250\ \text{cm}^{-3}$

T=350K: $n = 10^{17}\ \text{cm}^{-3}$, $p = \frac{(4.18E11)^2}{1E17} = 1.75\times10^6\ \text{cm}^{-3}$

3.31 Find the resistivity of (a) intrinsic silicon and (b) p-type silicon with $N_A = 10^{16}/\text{cm}^3$. Use $n_i = 1.5\times10^{10}/\text{cm}^3$, and assume that for intrinsic silicon $\mu_n = 1350\ \text{cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 480\ \text{cm}^2/\text{V}\cdot\text{s}$ and for the doped silicon $\mu_n = 1110\ \text{cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 400\ \text{cm}^2/\text{V}\cdot\text{s}$. (Note that doping results in reduced carrier mobilities.)

Sol)
(a) Intrinsic: $\frac{1}{\rho} = q(n_i\mu_n + n_i\mu_p) = 1.6E{-}19\times1.5E10\times(1350+480)$
$= 4.392\times10^{-6}\ \frac{1}{\Omega\text{-cm}}$

$\therefore \rho = 2.28\times10^5\ \Omega\text{-cm}$

(b) p-type: $N_A = 10^{16}\ \text{cm}^{-3} = p$ $\quad n = \frac{n_i^2}{p} = \frac{(1.5E10)^2}{1E16} = 2.25E4\ \text{cm}^{-3}$

$\frac{1}{\rho} = q(n\mu_n + p\mu_p) = 1.6E{-}19\times(2.25E4\times1100 + 1E16\times400)$
$= 0.64\ \frac{1}{\Omega\text{-cm}}$

$\therefore \rho = \frac{1}{0.64}\ \Omega\text{-cm} = 1.56\ \Omega\text{-cm}$

3.7.2 PN Junction under Open-Circuit Condition

(1) Diffusion Current I_D : $\frac{dp}{dx} \neq 0$, $\frac{dn}{dx} \neq 0$ across the junction

(2) Depletion Region or Space Charge Region

Around the junction, ionized impurity charges become uncovered, or exposed, because the mobile charges (e^- or h^+) leave that region.

The space charge (of impurities) in the depletion region $\xrightarrow{\text{leads to}}$ Electric Field, which $\xrightarrow{\text{leads to}}$ Potential Barrier.

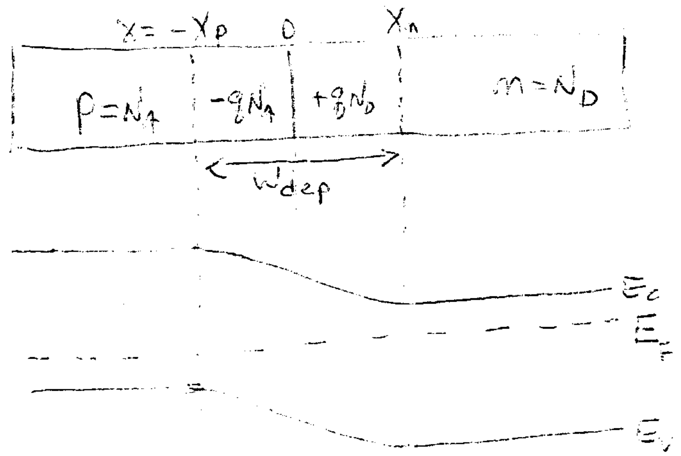
(3) Drift Current I_S: The E-field in the depletion region causes certain carriers to cross the junction.

( ) from p-region to n-region

( ) $n \longrightarrow p$

Equilibrium : $I_D = I_S$

(4) Junction Built-in Potential : $V_0 = V_T \ln \frac{N_A N_D}{n_i^2} = 0.6 - 0.8\,V$



(5) Width of Depletion Region

$$qN_A x_p = qN_D x_n \quad \cdots (1)$$

$$W_{dep} = x_n + x_p = \sqrt{\frac{2\epsilon_{Si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0} \quad \cdots (2)$$

$$= 0.1 - 1.0\,\mu m$$

From (1) and (2),

$$x_n = \frac{N_A}{N_A+N_D} W_{dep}, \qquad x_p = \frac{N_D}{N_A+N_D} W_{dep}$$

3.32 For a pn junction with $N_A = 10^{17}/cm^3$ and $N_D = 10^{16}/cm^3$, find, at $T = 300$ K, the built-in voltage, the width of the depletion region, and the distance it extends in the p side and in the n side of the junction. Use $n_i = 1.5 \times 10^{10}/cm^3$.

Ans. 728 mV; 0.32 μm; 0.03 μm and 0.29 μm

$P = N_A = 10^{17}$	$n = N_D = 10^{16}$

$T = 300K \quad n_i = 1.5\times10^{10}\ cm^{-3}$

$\epsilon_{Si} = 11.07\,\epsilon_0 = 1.04\times10^{-12}\ F/cm$

$V_0 = (\quad), \ W_{dep} = (\quad), \ x_n = (\quad), \ x_p = (\quad)$

Sol)
$$V_0 = V_T \ln\frac{N_A N_D}{n_i^2} = 0.025 \ln\frac{10^{17}\times10^{16}}{2.25\times10^{20}} = 0.728\,V$$

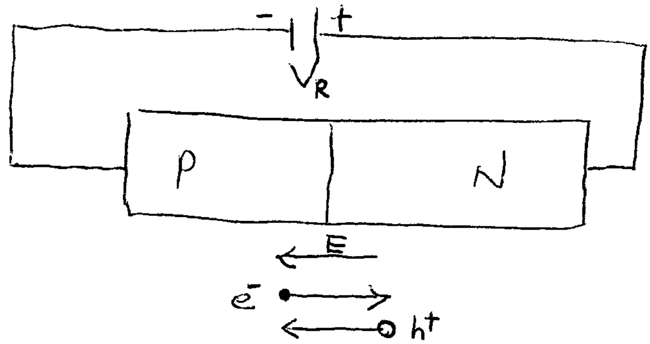
$$W_{dep} = \sqrt{\frac{2\epsilon_{Si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0} = \sqrt{\frac{2\times1.04E\text{-}12}{1.6E\text{-}19}\left(10^{-17}+10^{-16}\right)0.728}$$

$$= 3.2\times10^{-5}\,cm = 0.32\,\mu m$$

$$x_n = \frac{N_A}{N_A+N_D}W_{dep} = \frac{10^{17}}{10^{17}+10^{16}}\,0.32 = \cancel{0.03}\ 0.29\,\mu m$$

$$x_p = \frac{N_D}{N_A+N_D}W_{dep} = W_{dep} - x_n = 0.32 - 0.29 = 0.03\,\mu m$$

3.7.3 PN Junction under Reverse Bias



Drift current across junction $= I_S =$ indept. of V
$= \text{const.}$

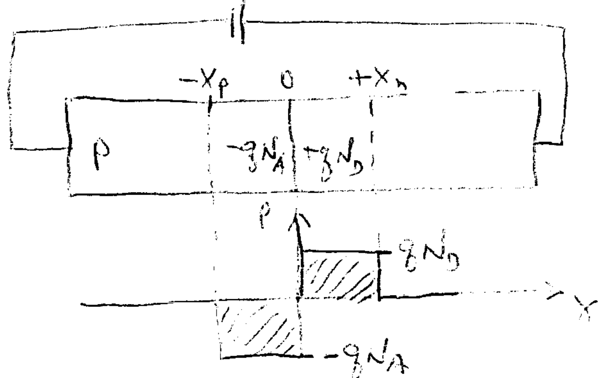
V_R increases the potential barrier from V_0 to $V_0 + V_R$

$\therefore \quad I_S = \text{const.}$

$I_D \downarrow$

$I_{Rev} = I_S - I_D \approx I_S$

Depletion Capacitance



In general,

$$C_J = \frac{C_{J0}}{\left(1 + \frac{V_R}{V_0}\right)^m}$$

m = Junction grading coef

$m = \frac{1}{2}$ for abrupt junction

$m = \frac{1}{3}$ for linear "

Space charge $= Q_J = q N_D x_n A = q N_D \frac{N_A}{N_A + N_D} W_{dep} A$

where $W_{dep} = \sqrt{\frac{2\epsilon_{Si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$

Junction Cap $= C_J = \frac{dQ_J}{dV_R} = q\frac{N_A N_D}{N_A + N_D}\frac{dW_{dep}}{dV_R}A$

$$\boxed{C_J = A\frac{\epsilon_{Si}}{W_{dep}} = \frac{C_{J0}}{\left(1 + \frac{V_R}{V_0}\right)^{1/2}}}$$

where $C_{J0} = A\sqrt{\frac{\epsilon_{Si} q}{2}\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{V_0}}$

-3.29-

3.33 For a pn junction with $N_A = 10^{17}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$, operating at $T = 300$ K, find (a) the value of C_{j0} per unit junction area (μm^2 is a convenient unit here) and (b) the capacitance C_j at a reverse-bias voltage of 2 V, assuming a junction area of 2500 μm^2. Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$, $m = \frac{1}{2}$, and the value of V_0 found in Exercise 3.32 ($V_0 = 0.728$ V).

Ans. (a) 0.32 fF/μm^2; (b) 0.41 pF

abrupt grading, $m = \frac{1}{2}$

$p = N_A = 10^{17}$	$n = N_D = 10^{16}$

← $A = 2500\,\mu m^2$, $V_0 = 0.728\,V$

(a) $\frac{C_{j0}}{A} = (\quad)$, (b) $C_j = (\quad)$ @ $V_R = 2V$

Sol)

(a) $\frac{C_{j0}}{A} = \frac{\epsilon_{Si}}{W_{dep}(V_R=0)}$

$W_{dep}(V_R=0) = \sqrt{\frac{2\epsilon_{Si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0}$

$= \sqrt{\frac{2\times1.04\times10^{-12}}{1.6\times10^{-19}}(10^{-17}+10^{-16})\,0.728}$

$= 0.32\,um$

$\frac{C_{j0}}{A} = \frac{\epsilon_{Si}}{W_{dep}} = \frac{1.04\times10^{-12}\,F/cm}{0.32\times10^{-4}\,cm}$

$= 3.25\times10^{-8}\,F/cm^2 = 3.25\times10^{-16}\,F/um^2$

(b) $C_j = A\frac{C_{j0}/A}{\sqrt{1+\frac{V_R}{V_0}}} = 2500\,um^2 \cdot \frac{3.25\times10^{-16}\,F/um^2}{\sqrt{1+\frac{2}{0.728}}} = 4.1\times10^{-13}\,F$

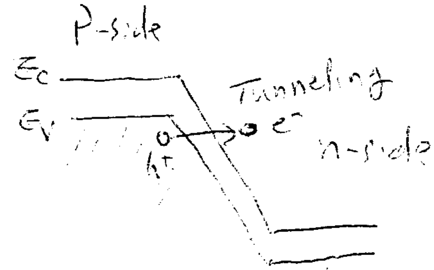
3.7.4 Breakdown Region

Under large reverse bias,

pn junction breaks down via

(i) Zener effect

usually $V_Z < 5V$



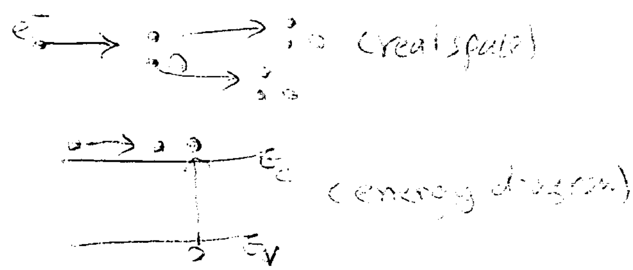
or

(ii) Avalanche effect

usually $V_Z > 7V$

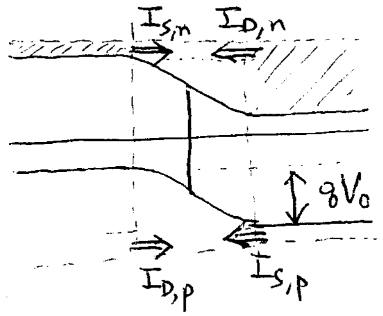
High carrier Kinetic Energy

$\left(\frac{1}{2}mv^2 > E_g\right)$



3.7.5 Forward Biased PN Junction

(1) PN Junction under Zero Bias : Energy Band Diagram



$I_S = I_D$

$\therefore\ I = I_D - I_S = 0$

(2) Forward bias V lowers the pontential Barrier from V_0 to $V_0 - V$

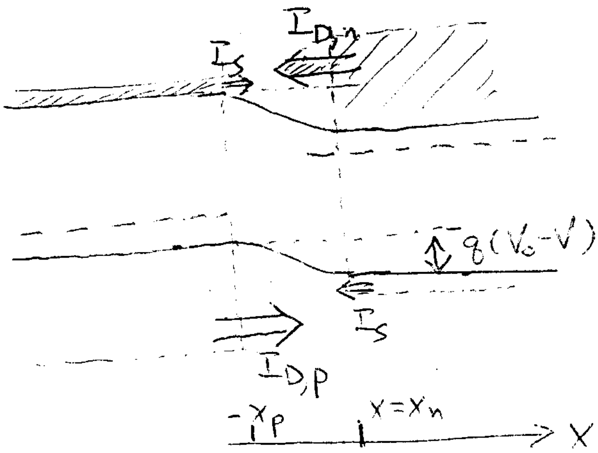


Fig. 50

(3) I-V characteristics

$$p_n(x_n) = p_{no}\, e^{V/V_T}$$

$$p_n(x) - p_{no} = [\,p_n(x_n) - p_{no}\,]\, e^{-(x-x_n)/L_p}$$

$$L_p = \sqrt{\tau_p D_p} = \text{Diffus. Length}$$

$$J_p = -q D_p \left.\frac{dp_n(x)}{dx}\right|_{x=x_n} = q \frac{D_p}{L_p} p_{no} \left(e^{V/V_T} - 1\right)$$

= Hole currents injected from p-side to n-side

By the same way,

J_n = Electron current injected from n-side to p-side

$$= qD_n \left.\frac{dn_p(x)}{dx}\right|_{x=-x_p} = q\frac{D_n}{L_n} n_{po}(e^{V/V_T}-1)$$

Therefore,

$$I = A(J_n + J_p) = Aq\left(\frac{D_n}{L_n}n_{po} + \frac{D_p}{L_p}p_{no}\right)(e^{V/V_T}-1)$$

$$\equiv I_s(e^{V/V_T}-1)$$

where, $I_s = Aqn_i^2\left(\frac{D_n}{L_nN_A} + \frac{D_p}{L_pN_D}\right)$

(4) Diffusion Capacitance

Minority carrier (hole) charge injected and stored in N-side =

$$Q_p = Aq\int_0^\infty [p_n(x) - p_{no}]dx = Aq\,[p_n(x_n) - p_{no}]L_p$$

$$= Aqp_{no}(e^{V/V_T}-1)L_p = \frac{L_p^2}{D_p}I_p = \tau_p I_p$$

The same way, $Q_n = \tau_n I_n$ = minority electron charge stored in p-side

$$Q = \tau_p I_p + \tau_n I_n \equiv \tau_T(I_p + I_n) = \tau_T I$$

$$C_d = \frac{dQ}{dV} = \frac{Q}{V_T} = \frac{\tau_T I}{V_T}$$ because $\frac{dI_p}{dV} = \frac{I_p}{V_T}$ etc

3.120 In a forward-biased *pn* junction show that the ratio of the current component due to hole injection across the junction to the component due to electron injection is given by

$$\frac{I_p}{I_n} = \frac{D_p}{D_n}\frac{L_n}{L_p}\frac{N_A}{N_D}$$

Evaluate this ratio for the case $N_A = 10^{18}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, $L_p = 5\ \mu\text{m}$, $L_n = 10\ \mu\text{m}$, $D_p = 10\ \text{cm}^2/\text{s}$, $D_n = 20\ \text{cm}^2/\text{s}$, and hence find I_p and I_n for the case in which the diode is conducting a forward current $I = 1$ mA.

$I = 1\,mA$ = Forward Current

$p = N_A = 10^{18}$	$n = N_D = 10^{16}$

$L_n = 10$, $L_p = 5\,um$

$D_n = 20$, $D_p = 10\ cm^2/s$

$I_n = (\quad)$ $I_p = (\quad)$

Sol)

$$\frac{I_p}{I_n} = \frac{AJ_p}{AJ_n} = \frac{q\frac{D_p}{L_p}\frac{n_i^2}{N_D}(e^{V/V_T}-1)}{q\frac{D_n}{L_n}\frac{n_i^2}{N_A}(e^{V/V_T}-1)} = \frac{\frac{D_p}{L_p N_D}}{\frac{D_n}{L_n N_A}} = \frac{D_p}{D_n}\frac{L_n}{L_p}\frac{N_A}{N_D}$$

$$\frac{I_p}{I_n} = \frac{D_p}{D_n}\frac{L_n}{L_p}\frac{N_A}{N_D} = \frac{10}{20}\cdot\frac{10}{5}\cdot\frac{10^{18}}{10^{16}} = 100$$

$\therefore I_p = 100 I_n$

$I = I_p + I_n = 101 I_n = 1\,mA$

$\therefore I_n = 0.01\,mA$

$I_p = 0.99\,mA$

Roughly speaking, $I_p : I_n \approx N_A : N_D$

3.108 Holes are being steadily injected into a region of *n*-type silicon (connected to other devices, the details of which are not important for this question). In the steady state, the excess-hole concentration profile shown in Fig. P3.108 is established in the *n*-type silicon region. Here "excess" means over and above the concentration p_{n0}. If $N_D = 10^{16}/\text{cm}^3$, $n_i = 1.5\times10^{10}/\text{cm}^3$, and $W = 5\ \mu\text{m}$, find the density of the current that will flow in the *x* direction.

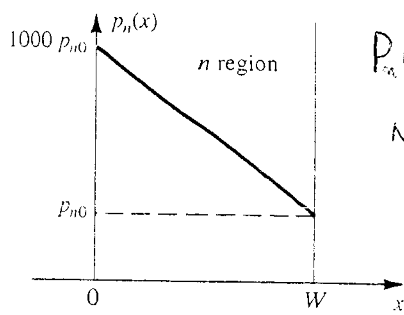


FIGURE P3.108

$P_n(x) - P_{n0} = [P_n(0) - P_{n0}](1 - \frac{x}{W})$

$N_D = 10^{16}\,cm^{-3}$, $W = 5\,um$

$D_p = 12\ cm^2/s$

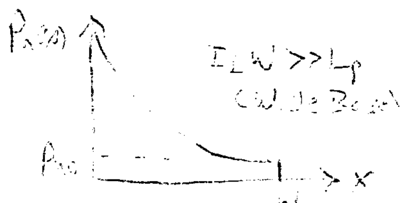
$J_p = (\quad)$

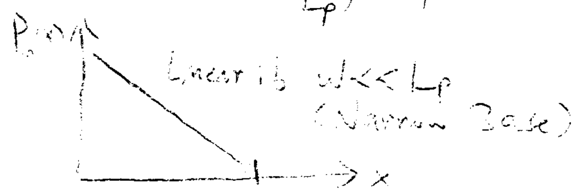
Sol) $$J_p = -qD_n \frac{dP_n(x)}{dx}\bigg|_{x=0}$$

$$= \frac{qD_n}{W}[P_n(0) - P_{n0}] = \frac{qD_n}{W}\,999\,\frac{n_i^2}{N_D} = \frac{1.6\times10^{-19}\times12}{5\times10^{-4}}\times999\times\frac{2.25\times10^{20}}{10^{16}}$$

$$= 8.63\times10^{-8}\ A/cm^2$$

(Note) $P_n(x) - P_{n0} = [P_n(0) - P_{n0}]\,e^{-x/L_p} \approx [P_n(0) - P_{n0}](1 - \frac{x}{L_p})$ if $x \ll L_p$





3.34 A diode has $N_A = 10^{17}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, $n_i = 1.5 \times 10^{10}/\text{cm}^3$, $L_p = 5\ \mu\text{m}$, $L_n = 10\ \mu\text{m}$, $A = 2500\ \mu\text{m}^2$, D_p (in the n region) $= 10\ \text{cm}^2/\text{V}\cdot\text{s}$, and D_n (in the p region) $= 18\ \text{cm}^2/\text{V}\cdot\text{s}$. The diode is forward biased and conducting a current $I = 0.1$ mA. Calculate: (a) I_S; (b) the forward-bias voltage V; (c) the component of the current I due to hole injection and that due to electron injection across the junction; (d) τ_p and τ_n; (e) the excess hole charge in the n region Q_p, and the excess electron charge in the p region Q_n, and hence the total minority stored charge Q, as well as the transit time τ_T; and (f) the diffusion capacitance.

Ans. (a) 2×10^{-15} A; (b) 0.616 V; (c) 91.7 μA, 8.3 μA; (d) 25 ns, 55.6 ns; (e) 2.29 pC, 0.46 pC, 2.75 pC, 27.5 ns; (f) 110 pF

$p = N_A = 10^{17}$	$n = N_D = 10^{16}$
$L_n = 10\,\mu m$	$L_p = 5\,\mu m$
$D_n = 18$,	$D_p = 10\ cm^2/s$

$\leftarrow A = 2500\,\mu m^2$

$n_i = 1.5\times10^{10}\ cm^{-3}$

Forward Current $I = 0.1\,mA$

(a) $I_s = (\quad)$

(b) $V = (\quad)$

(c) $I_n = (\quad)$, $I_p = (\quad)$

(d) $\tau_n = (\quad)$, $\tau_p = (\quad)$

(e) $Q_p = (\quad)$ $Q_n = (\quad)$ $Q = Q_n + Q_p = (\quad)$

$\tau_t = (\quad)$

(f) $C_d = \dfrac{Q}{V_T} = (\quad)$

sol) (a) $$I_s = A q n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right) = (2500\times10^{-8})\,1.6\times10^{-19} \times 2.25\times10^{20}\left(\frac{10}{(5\times10^{-4})\times10^{16}} + \frac{18}{(10\times10^{-4})\times10^{17}}\right)$$
$$= 2\times10^{-15}\,A$$

(b) $$I = I_s e^{V/V_T} \qquad V = V_T \ln\frac{I}{I_s} = 0.025 \ln\frac{0.1\times10^{-3}}{2\times10^{-15}} = 0.616\,V$$

(c) $$\frac{I_p}{I_n} = \frac{D_p}{D_n}\frac{L_n}{L_p}\frac{N_A}{N_D} = \frac{10}{18}\,\frac{10}{5}\,\frac{10^{17}}{10^{16}} = 11.11, \qquad I = I_p + I_n = 12.11\,I_n = 0.1\,mA$$

$$\therefore I_n = \frac{0.1}{12.11}\,mA = 8.3\,\mu A$$

$$I_p = 11.11\,I_n = 91.7\,\mu A$$

(d) $$\tau_n = \frac{L_n^2}{D_n} = \frac{(10\times10^{-4}\,cm)^2}{18\ cm^2/s} = 55.6\,ns$$

$$\tau_p = \frac{L_p^2}{D_p} = \frac{(5\times10^{-4})^2}{10} = 25\,ns$$

(e) $$Q_p = I_p\tau_p = (91.7\times10^{-6})(25\times10^{-9}) = 2.29\times10^{-12}\,C = 2.29\,pC$$

$$Q_n = I_n\tau_n = (8.3\times10^{-6})(55.6\times10^{-9}) = 0.46\,pC$$

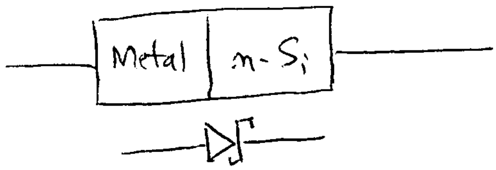
$$Q = Q_n + Q_p = 2.29 + 0.46 = 2.75\,pC$$

$$\tau_t = \frac{Q}{I} = \frac{2.75\times10^{-12}}{0.1\times10^{-3}} = 27.5\,ns$$

(f) $$C_d = \frac{\tau_t I}{V_T} = \frac{Q}{V_T} = \frac{2.75\times10^{-12}}{0.025} = 110\,pF$$

3.8 Special Diode Types

1. Schottky Barrier Diode (SBD or SD)



(i) $\begin{cases} \text{Si SD} & V_0 = 0.3 - 0.5\,V \\ \text{Si PN} & V_0 = 0.6 - 0.8\,V \end{cases}$

GaAs SD $V_0 = 0.7\,V$

(ii) SD = majority carrier device (fast)

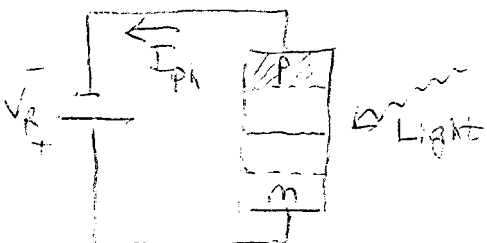
PN = minority carrier injection (slow)

2. Varactor Diode = Reverse-biased PN diode used as Capacitor

$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{V_0}\right)^m}$$

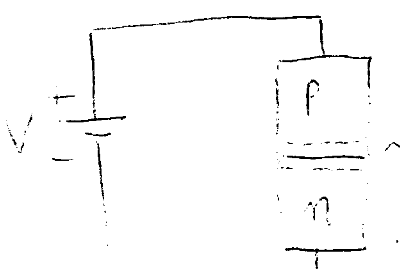
- C_j is variable by V_R
- useful in Radio tuner
- $m = 3$ or 4 ?

3. Photodiode = reverse-biased (GaAs) PN diode or PIN diode



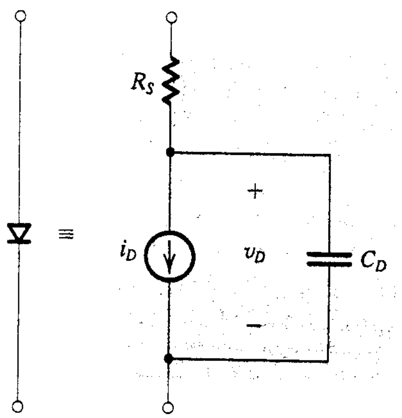
I_{ph} = photocurrent is proportional to Light intensity

4. LED = Forward-biased PN diode



Light is emitted due to e-h recombination

3.9 Spice Diode Model



$$i_D = I_S(e^{v_D/nV_T} - 1)$$

$$C_D = C_d + C_j = \frac{\tau_T}{V_T} I_S e^{v_D/nV_T} + C_{j0}\Big/\left(1 - \frac{v_D}{V_0}\right)^m$$

FIGURE 3.51 The SPICE diode model.

TABLE 3.3 Parameters of the SPICE Diode Model (Partial Listing)

SPICE Parameter	Book Symbol	Description	Units
IS	I_S	Saturation current	A
N	n	Emission coefficient	
RS	R_S	Ohmic resistance	Ω
VJ	V_0	Built-in potential	V
CJ0	C_{j0}	Zero-bias depletion (junction) capacitance	F
M	m	Grading coefficient	
TT	τ_T	Transit time	s
BV	V_{ZK}	Breakdown voltage	V
IBV	I_{ZK}	Reverse current at V_{ZK}	A

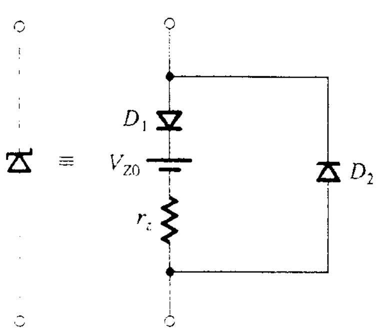


FIGURE 3.52 Equivalent-circuit model used to simulate the zener diode in SPICE. Diode D_1 is ideal and can be approximated in SPICE by using a very small value for n (say $n = 0.01$).