EE310 – Chapter 2

Operational Amplifiers

Lecture Slides

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Chap. 2 Op Amp

- An Integrated Circuit that possesses a nearly ideal amp property

Terminals

1 = Inverting Input

2 = Noninverting Input

3 = Output

4, 5 = DC power supplies

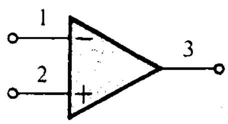


FIGURE 2.1 Circuit symbol for the op amp.

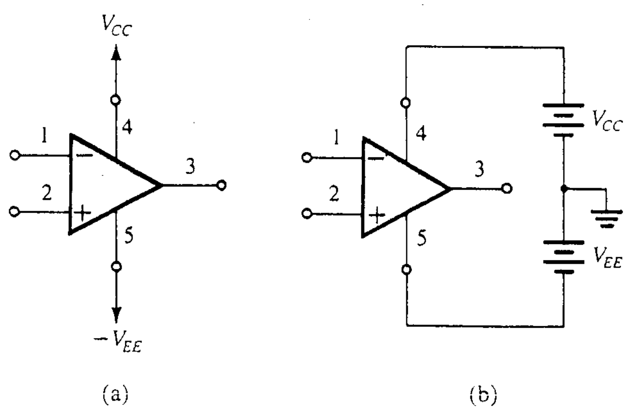


FIGURE 2.2 The op amp shown connected to dc power supplies.

Ideal OpAmp Property

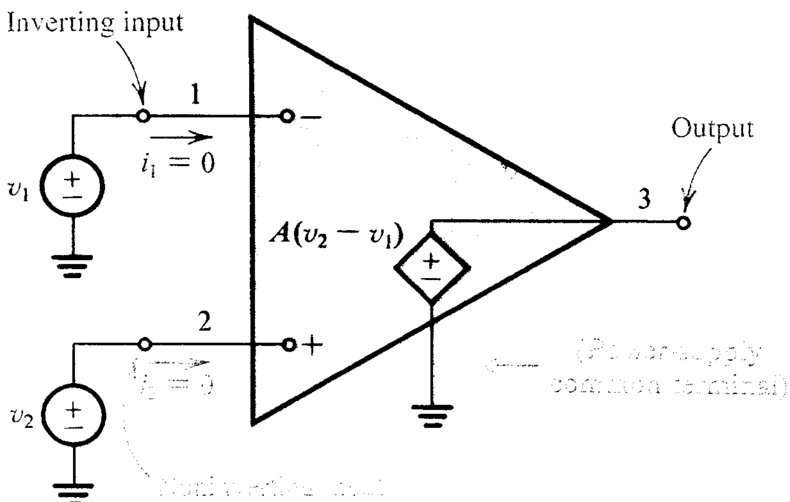
(1) $R_{in} = \infty$

(2) $R_{out} = 0$

(3) $A_{cm} = 0$ Common-mode

(4) $A = \infty$ Differential-mode

(5) Infinite Bandwidth



Equivalent circuit of the ideal op amp.

$$V_{cm} \equiv \frac{1}{2}(V_1 + V_2) \quad \text{Common-mode}$$

$$V_d \equiv V_2 - V_1 \quad \text{Differential-mode}$$

$$\Rightarrow \quad V_1 = V_{cm} - \frac{1}{2} V_d$$

$$V_2 = V_{cm} + \frac{1}{2} V_d$$

$\Rightarrow$ Output

$$V_o = A_{cm} V_{cm} + A V_d$$

2.2 Consider an op amp that is ideal except that its open-loop gain $A = 10^3$. The op amp is used in a feedback circuit, and the voltages appearing at two of its three signal terminals are measured. In each of the following cases, use the measured values to find the expected value of the voltage at the third terminal. Also give the differential and common-mode input signals in each case. (a) $v_2 = 0$ V and $v_3 = 2$ V; (b) $v_2 = +5$ V and $v_3 = -10$ V; (c) $v_1 = 1.002$ V and $v_2 = 0.998$ V; (d) $v_1 = -3.6$ V and $v_3 = -3.6$ V.

ideal Opamp except $A = 1{,}000 \frac{V}{V}$

Find the voltage @ 3rd Terminal

sol) (a) $V_2 = 0 \quad V_3 = 2$

$$V_3 = A(V_2 - V_1) = 1{,}000\,(0 - V_1) = 2 \qquad \therefore V_1 = -0.002\,V$$

$$\Rightarrow V_d = V_2 - V_1 = 0.002\,V$$

$$V_{cm} = \frac{1}{2}(V_1 + V_2) = \frac{1}{2}(-0.002) = -0.001\,V$$

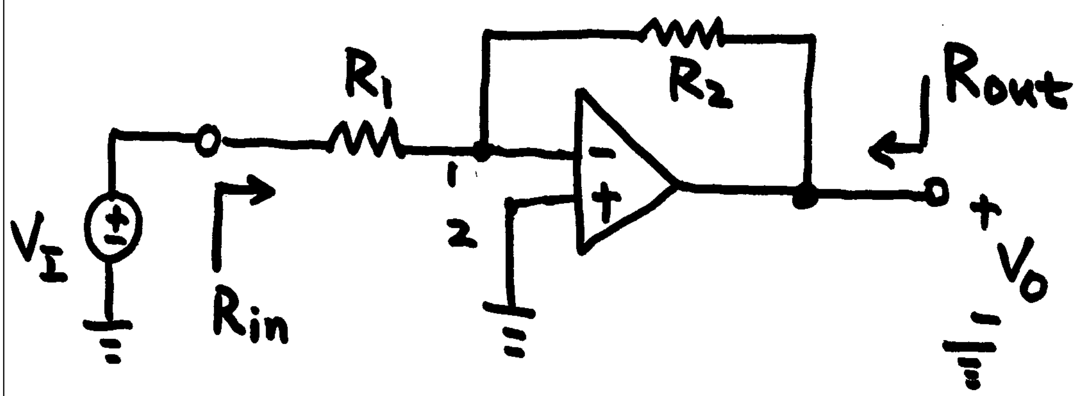
(c) $V_1 = 1.002 \quad V_2 = 0.998$

$$V_3 = A(V_2 - V_1) = 1{,}000\,(0.998 - 1.002) = -4\,V$$

$$\Rightarrow V_d = V_2 - V_1 = 0.998 - 1.002 = -0.004\,V$$

$$V_{cm} = \frac{1}{2}(V_1 + V_2) = \frac{1}{2}(1.002 + 0.998) = 1\,V$$

2.2 Inverting Configuration



(1) Gain $= \dfrac{V_O}{V_I} = ?$

(2) $R_{in} = ?$ $\quad$ $R_{out} = ?$

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(1) Gain $\quad$ For $A = \infty$, $G = \dfrac{V_O}{V_I} = -\dfrac{R_2}{R_1}$

For $A \neq \infty$,

$$V_O = A(V_2 - V_1) = -AV_1 \quad \therefore V_1 = -\frac{V_O}{A}$$

$$V_O = V_1 - i_1 R_2 = V_1 - \frac{V_I - V_1}{R_1} R_2$$

$$= -\frac{V_O}{A} - \left(V_I + \frac{V_O}{A}\right)\frac{R_2}{R_1}$$

$$\rightarrow G = \frac{V_O}{V_I} = \frac{-R_2/R_1}{1 + \dfrac{1 + R_2/R_1}{A}}$$

(2) $R_{in} = \dfrac{V_I}{i_I} = R_1$

$R_{OUT} = \dfrac{V_O}{i_O} = 0 \quad \because V_O = A(V_2 - V_1) =$ indept. of i_O

D2.4 Use the circuit of Fig. 2.5 to design an inverting amplifier having a gain of −10 and an input resistance of 100 kΩ. Give the values of R_1 and R_2.

$G = -10 \qquad R_{in} = 100\,k\Omega$

Sol) $G = -\dfrac{R_2}{R_1} = -10 \qquad R_{in} = R_1 = 100K$

$\therefore R_2 = 10 \times R_1 = 1M$

2.5 The circuit shown in Fig. E2.5(a) can be used to implement a transresistance amplifier (see Table 1.1 in Section 1.5). Find the value of the input resistance R_i, the transresistance R_m, and the output resistance R_o of the transresistance amplifier. If the signal source shown in Fig. E2.5(b) is connected to the input of the transresistance amplifier, find its output voltage.

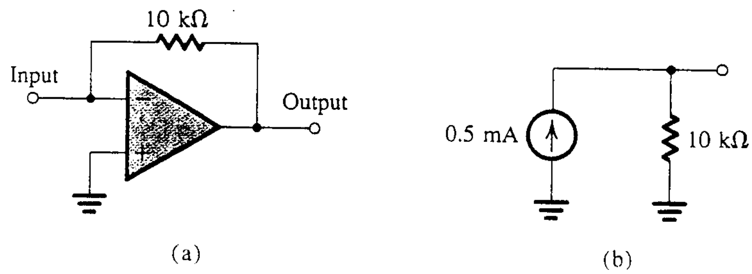
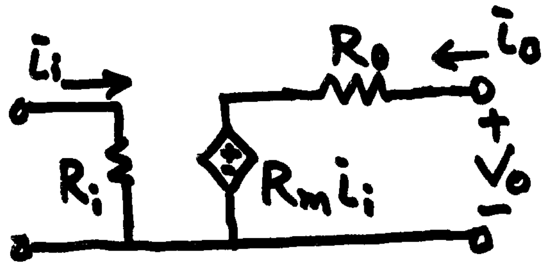


FIGURE E2.5

Sol) Transresistance Amp



$$\begin{cases} R_i = \dfrac{V_i}{i_i} = \dfrac{0}{i_i} = 0 \\ R_o = \dfrac{V_o}{i_o} = 0 \quad \because V_o = \text{indept. of } i_o \end{cases}$$

$R_m = \left.\dfrac{V_o}{i_i}\right|_{i_o=0}$

$R_m = ?$

$V_o = -10K\Omega \cdot i_i$ from circuit

$V_o = R_m i_i$ from model

$\therefore R_m = -10\,K\Omega$

If $i_i = 0.5\,mA$, then

$V_o = -10K\Omega \times 0.5mA = -5V$

2.2.4 Weighted Summer

$$i = i_1 + i_2 + \cdots + i_n$$

$$= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots + \frac{V_n}{R_n}$$

$$V_0 = -i R_f = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots + \frac{V_n}{R_n} \right)$$

D2.8 Use the idea presented in Fig. 2.11 to design a weighted summer that provides

$$v_O = 2v_1 + v_2 - 4v_3$$

sol) $V_0 = \underbrace{2V_1 + V_2}_{\text{invert 2x}} \; \underbrace{-4V_3}_{\text{invert once}}$

Fig 2.11 : $V_0 = -\frac{R_c}{R_3} V_3 - \frac{R_c}{R_4} V_4 - \frac{R_c}{R_b}\left(-\frac{R_a}{R_1} V_1 - \frac{R_a}{R_2} V_2\right)$

$-4V_3$ $\quad$ $+2V_1$ $\quad$ $+V_2$

$\rightarrow \frac{R_c}{R_3} = 4$ $\quad$ $\frac{R_c}{R_b}\frac{R_a}{R_1} = 2$ $\quad$ $\frac{R_c}{R_b}\frac{R_a}{R_2} = 1$

ex) $R_c = 4K$ $\quad$ $R_b = 4K$ so, $\frac{R_c}{R_b} = 1$

$R_3 = 1K$ $\quad$ $R_1 = 1K$ $\quad$ $R_a = 2K$

$R_2 = 2K$

2.3 Noninverting Configuration

Fig.12

If A (open-loop) $= \infty$,

$$V_O - V_I = \frac{V_I}{R_1} \cdot R_2 \qquad \therefore V_O = \left(1 + \frac{R_2}{R_1}\right) V_I$$

$$\boxed{\text{Gain} = 1 + \frac{R_2}{R_1}}$$

If A (open-loop) $=$ finite,

Opamp model:

$$V_O = A(V_2 - V_1) = A(V_I - V_1) \rightarrow V_1 = V_I - \frac{V_O}{A}$$

(feedback) Circuit: $V_O - V_1 = \frac{V_1}{R_1} \cdot R_2 \rightarrow$

$$V_O = \left(1 + \frac{R_2}{R_1}\right) V_1 = \left(1 + \frac{R_2}{R_1}\right)\left(V_I - \frac{V_O}{A}\right)$$

$$\therefore \boxed{\text{Gain} = \frac{V_O}{V_I} = \frac{1 + R_2/R_1}{1 + \dfrac{1 + R_2/R_1}{A}}}$$

2.9 Use the superposition principle to find the output voltage of the circuit shown in Fig. E2.9.

Ans. $v_O = 6v_1 + 4v_2$

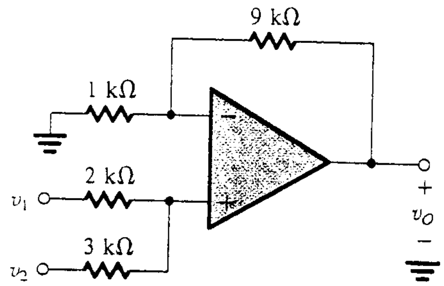
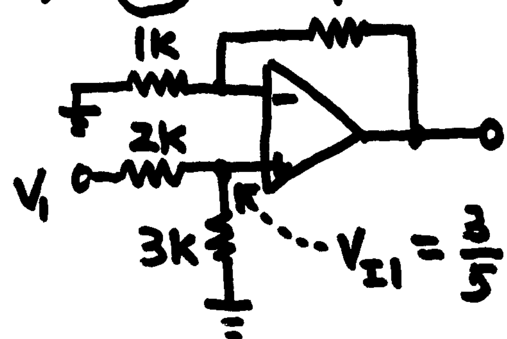


FIGURE E2.9

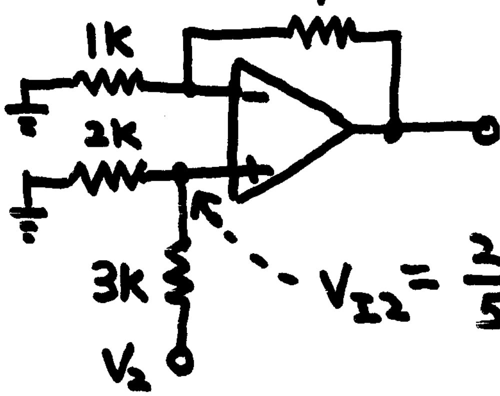
Sol) ⊖



$V_{O1} = (1 + \frac{9}{1})V_{I1} = 10 \times \frac{3}{5} V_1 = 6V_1$

$V_{I1} = \frac{3}{5} V_1$

⊕



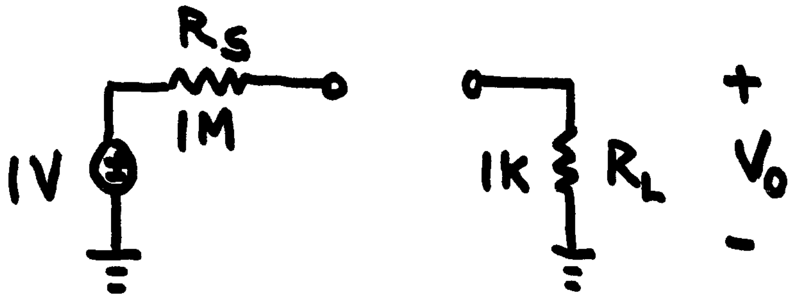
$V_{O2} = (1 + \frac{9}{1})V_{I2} = 10 \times \frac{2}{5} V_2 = 4V_2$

$V_{I2} = \frac{2}{5} V_2$

$\therefore\ V_O = V_{O1} + V_{O2} = 6V_1 + 4V_2$

2.14 It is required to connect a transducer having an open-circuit voltage of 1 V and a source resistance of 1 MΩ to a load of 1-kΩ resistance. Find the load voltage if the connection is done (a) directly and (b) through a unity-gain voltage follower.

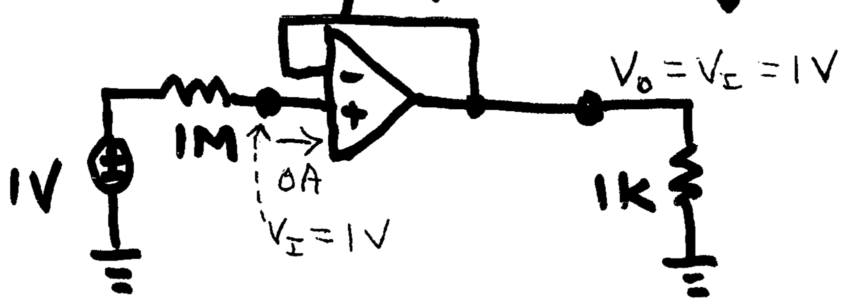
Sol)



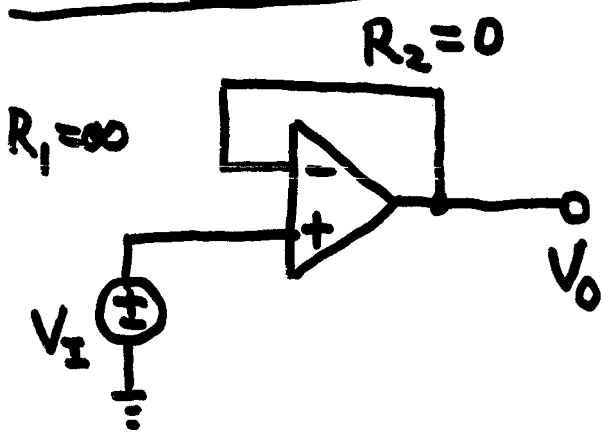
(a) Direct connection:

$$V_o = 1V \frac{1K}{1K + 1M} \simeq 0.001 V$$

(b) thru Unity-Gain Voltage Follower:



Voltage Follower



$$\text{Gain} = 1 + \frac{0}{\infty} = 1$$

$$\therefore V_o = V_I$$

2.4 Difference Amplifier

Let $V_d \equiv V_2 - V_1$ differential mode

$V_{cm} \equiv \frac{1}{2}(V_1 + V_2)$ common mode

$$V_o = A_d V_d + A_{cm} V_{cm}$$

→ Common Mode Rejection Ratio (CMRR)

$$CMRR \equiv 20 \log \frac{|A_d|}{|A_{cm}|}$$

Difference Amp : Fig.16 = Fig.17(a) + Fig.17(b)

Superposition!

(a) V_{I1} only (V_{I2} = GND) $\quad V_{01} = -\frac{R_2}{R_1} V_{I1}$

(b) V_{I2} only (V_{I1} = GND) $\quad V_{02} = \left(1 + \frac{R_2}{R_1}\right) V_{I2} \frac{R_4}{R_4 + R_3}$

$$\Rightarrow \quad V_o = V_{01} + V_{02}$$

Differential Mode $\quad V_d = V_2 - V_1$

Gain: If $\dfrac{R_4}{R_3} = \dfrac{R_2}{R_1}$, then $V_{O2} = \dfrac{R_2}{R_1} V_{I2}$

$$\therefore V_O = V_{O1} + V_{O2} = \frac{R_2}{R_1}(V_{I2} - V_{I1}) = \frac{R_2}{R_1} V_d$$

$$\rightarrow A_d = \frac{V_O}{V_d} = \frac{R_2}{R_1}$$

R_{id} = dibberential input resistance

$$\text{Fig.19}: \quad R_{id} = \frac{V_{Id}}{i_1} = \frac{i_1 (R_1 + R_1)}{i_1} = 2R_1$$

Common Mode $\quad V_{I1} = V_{I2} = V_{cm}$

$$A_{cm} = \frac{V_O}{V_{cm}} = \frac{V_{O1} + V_{O2}}{V_{cm}} = -\frac{R_2}{R_1} + \left(1 + \frac{R_2}{R_1}\right) \frac{R_4}{R_4 + R_3}$$

$$= \frac{1}{1 + \frac{R_4}{R_3}} \left(\frac{R_4}{R_3} - \frac{R_2}{R_1} \right)$$

2.15 Consider the difference-amplifier circuit of Fig. 2.16 for the case $R_1 = R_3 = 2\ \text{k}\Omega$ and $R_2 = R_4 = 200\ \text{k}\Omega$. (a) Find the value of the differential gain A_d. (b) Find the value of the differential input resistance R_{id} and the output resistance R_o. (c) If the resistors have 1% tolerance (i.e., each can be within ±1% of its nominal value), find the worst-case common-mode gain A_{cm} and the corresponding value of CMRR.

Sol) (a) $A_d = \frac{R_2}{R_1} = \frac{200K}{2K} = 100\frac{V}{V} = 40\,dB$

(b) $R_{id} = 2R_1 = 4K\Omega$

$R_o = \frac{\Delta V_o}{\Delta i_o} = 0 \quad \because V_o = \text{indep. of } i_o$

(C) For $\frac{\Delta R}{R} = \pm 1\%$, worst case $A_{cm} = (\quad)$ and CMRR = ( )

$$\frac{200K(1-.01)K}{2K(1+.01)} \le \frac{R_4}{R_3} \le \frac{200K(1+0.01)}{2K(1-0.01)} \Rightarrow 98.02 \le \frac{R_4}{R_3} \le 102.02$$

The same way, $98.02 \le \frac{R_2}{R_1} \le 102.02$

Therefore, $-4 \le \frac{R_4}{R_3} - \frac{R_2}{R_1} \le 4$

and

$$A_{cm} = \frac{1}{1 + \frac{200K \times 0.99}{2K \times 1.01}} \times 4 \approx 0.04\frac{V}{V}$$

$$= 20\log 0.04$$

$$CMRR = 20\log\frac{A_d}{A_{cm}} = 20\log\frac{100}{0.04} = 68\,dB$$

4.2 Instrumentation Amplifier

Fig. 20(b)

$$V_0 = \frac{R_4}{R_3}(V_{02} - V_{01})$$

$$V_{02} - V_{01} = i(R_2 + 2R_1 + R_2) = \frac{V_{I2} - V_{I1}}{2R_1}(2R_1 + 2R_2)$$

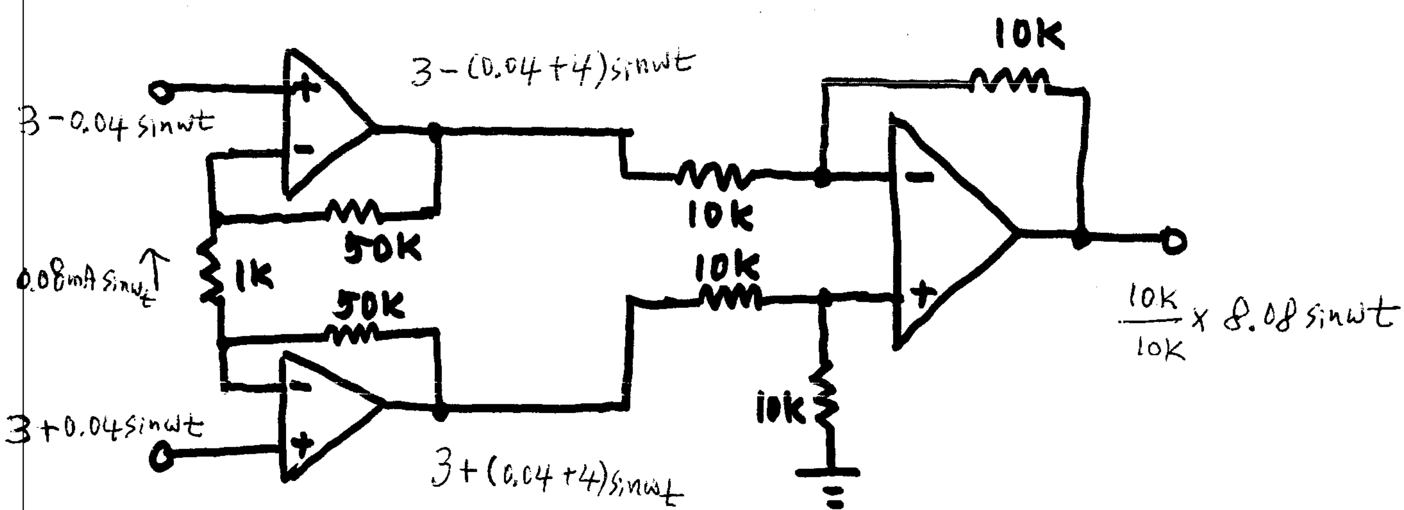
$$\therefore V_0 = \frac{R_4}{R_3}\frac{R_1 + R_2}{R_1}(V_{I2} - V_{I1})$$

2.72 Consider the instrumentation amplifier of Fig. 2.20(b) with a common-mode input voltage of +3 V (dc) and a differential input signal of 80-mV peak sine wave. Let $2R_1 = 1$ kΩ, $R_2 = 50$ kΩ, $R_3 = R_4 = 10$ kΩ. Find the voltage at every node in the circuit.

$V_{cm} = 3\,V$ $V_d = 0.08 \sin\omega t$

$2R_1 = 1K$ $R_2 = 50K$ $R_3 = R_4 = 10K$

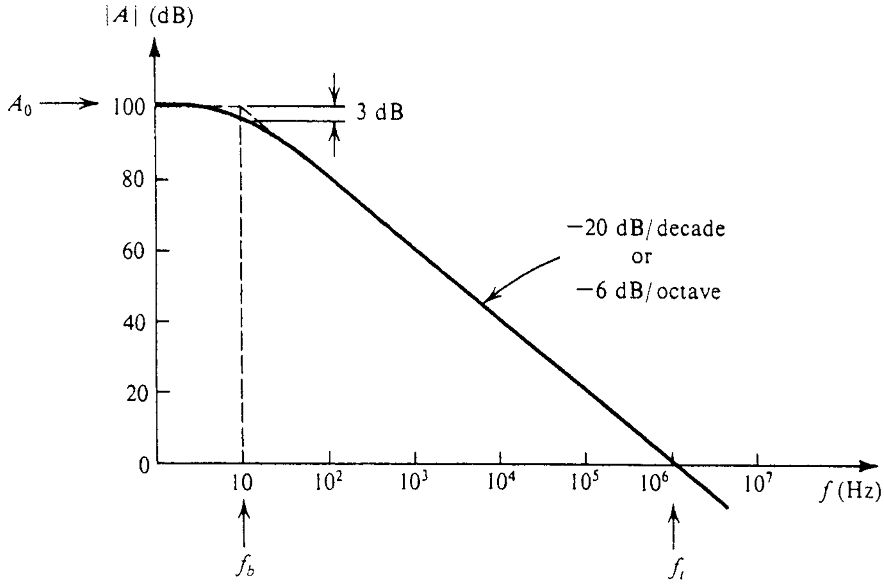
sol)



2.5 Open-Loop $\begin{cases} A = \text{finite} \\ BW = \text{finite} \end{cases}$

Ideal Opamp: $A = \infty$, $BW = \infty$

Real Opamp:



$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_0}} \qquad \omega_t = A_0 \omega_0$$

ex) $f \gg f_b$ or $\omega \gg \omega_0$

$$|A| = \frac{A_0 \omega_0}{\omega} = \frac{\omega_t}{\omega} = \frac{f_t}{f}$$

2.19 An internally compensated op amp has a dc open-loop gain of 10^6 V/V and an ac open-loop gain of 40 dB at 10 kHz. Estimate its 3-dB frequency, its unity-gain frequency, its gain–bandwidth product, and its expected gain at 1 kHz.

sol) $A_0 = 10^6$ V/V, $A = \underset{(40\,dB)}{100\,V/V} = \frac{f_t}{10\,kHz}$ $\therefore f_t = 1\,MHz$

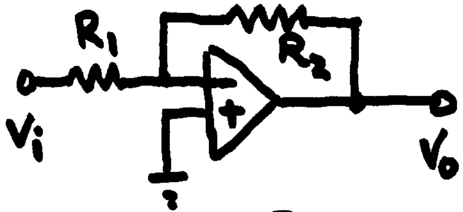
$$\therefore f_b = f_{3dB} = \frac{f_t}{A_0} = \frac{1\,MHz}{10^6} = 1\,Hz$$

$$\therefore GB = A_0 f_b = f_t = 1\,MHz$$

$$\therefore A = \frac{f_t}{f} = \frac{1\,MHz}{1\,KHz} = 1000\,\frac{V}{V} = 60\,dB$$

Closed-Loop Gain

(Inverting)



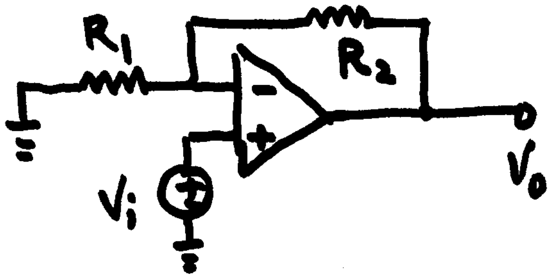
$$G = \frac{V_o}{V_i} = \frac{-R_2/R_1}{1 + \frac{1+R_2/R_1}{A}}$$

$$= \frac{-R_2/R_1}{1 + \frac{1+R_2/R_1}{\left(\frac{A_0}{1+\frac{s}{\omega_0}}\right)}}$$

$$\simeq \frac{-R_2/R_1}{1 + \frac{s}{\omega_t/(1+\frac{R_2}{R_1})}}$$

for $A_0 \gg 1 + R_2/R_1$.

(Noninverting)



$$G = \frac{V_o}{V_i} = \frac{1+R_2/R_1}{1 + \frac{1+R_2/R_1}{A}}$$

$$\simeq \frac{1+R_2/R_1}{1 + \frac{s}{\omega_t/(1+\frac{R_2}{R_1})}}$$

2.20 An op amp having a 106-dB gain at dc and a single-pole frequency response with $f_t = 2$ MHz is used to design a noninverting amplifier with nominal dc gain of 100. Find the 3-dB frequency of the closed-loop gain.

Sol) $A_0 = 106\,\text{dB} = 2\times10^5\ \text{V/V}$ LP STC, $f_t = 2\text{M}$

Noninverting $G_{DC} = 1 + \frac{R_2}{R_1} = 100\ \text{V/V}$

$$\therefore\ f_b = \frac{f_t}{1+\frac{R_2}{R_1}} = \frac{2\,\text{MHz}}{100\ \text{V/V}} = 20\ \text{kHz}$$

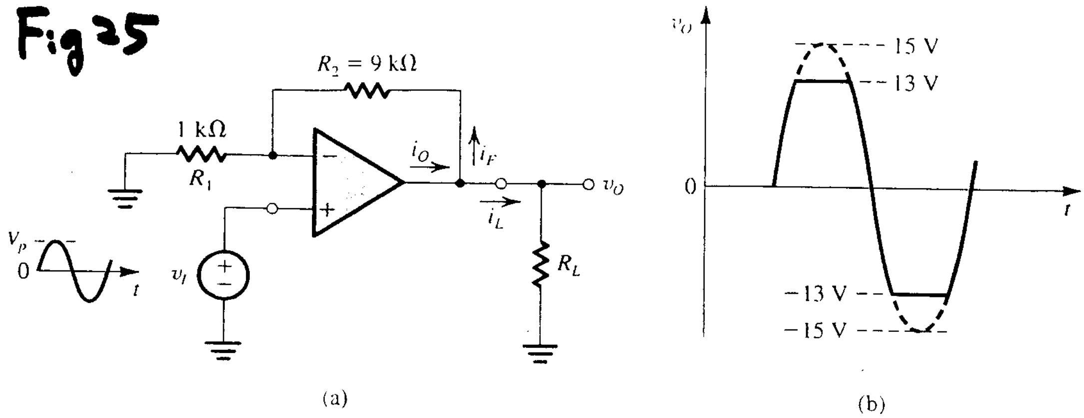
2.6 Large-Signal Operation

(1) V_o saturates

ex) $L_+ \simeq V_+ - 2V$ $\quad L_- \simeq V_- + 2V$

ex) $V_\pm = \pm 15V$ $\quad L_\pm = \pm 13V$, rated output voltage

Fig 25

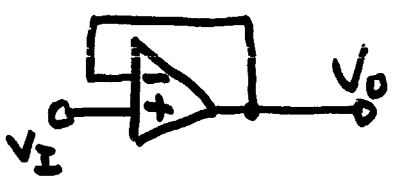


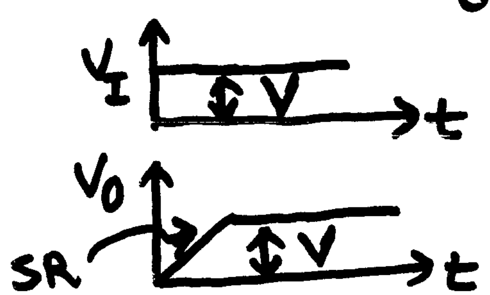
(a) (b)

(2) i_o is limited

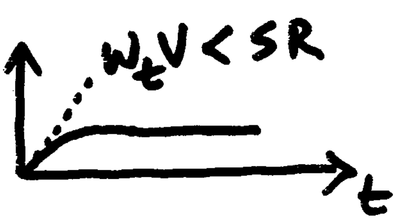
ex) i_o of 741 $= \pm 20mA$

(3) Slew Rate $\quad SR = \max \frac{dV_o}{dt}$



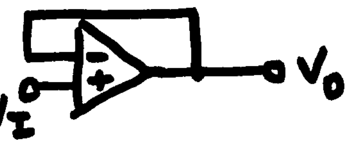


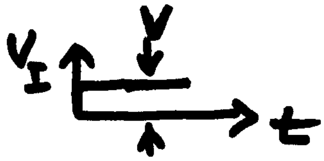
If $W_t V \le SR$, $V_o(t) = V(1 - e^{-W_t t})$



2.21 An op amp that has a slew rate of 1 V/μs and a unity-gain bandwidth f_t of 1 MHz is connected in the unity-gain follower configuration. Find the largest possible input voltage step for which the output waveform will still be given by the exponential ramp of Eq. (2.40). For this input voltage, what is the 10% to 90% rise time of the output waveform? If an input step 10 times as large is applied, find the 10% to 90% rise time of the output waveform.

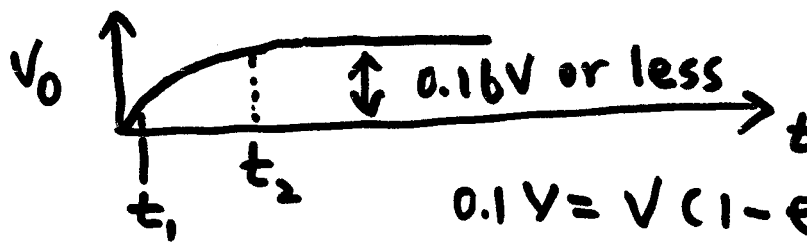
sol) $SR = 1\,V/\mu s \qquad f_t = 1\,MHz$





For a step response of exponential ramp,

$$V \le \frac{SR}{\omega_t} = \frac{1\,V/\mu s}{2\pi \times 1\,MHz} = 0.16\,V$$



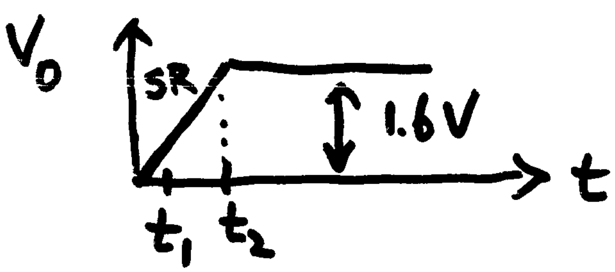
$$0.1\,V = V(1 - e^{-\omega_t t_1})$$

$$0.9\,V = V(1 - e^{-\omega_t t_2})$$

$$\therefore t_2 - t_1 = \frac{\ln(0.9) - \ln(0.1)}{\omega_t}$$

$$= 0.35\,\mu s$$

For $\omega_t V > SR$, say $V = 10 \times 0.16\,V = 1.6\,V$,



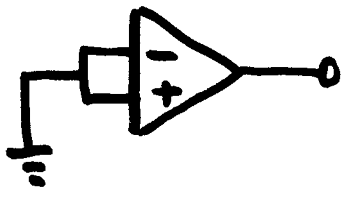
$$t_1 = \frac{0.1 \times 1.6}{SR} = 0.16\,\mu s$$

$$t_2 = \frac{0.9 \times 1.6}{SR} = 1.44\,\mu s$$

$$\therefore t_2 - t_1 = 1.44 - 0.16 = 1.28\,\mu s$$

2.7 DC Imperfections

DC Offset Voltage



$V_o \neq 0$ due to mismatches in IC

$V_{OS} = \frac{V_o}{A}$ = input offset voltage

ex) $V_{OS} = 1 - 5\,mV$ typical.

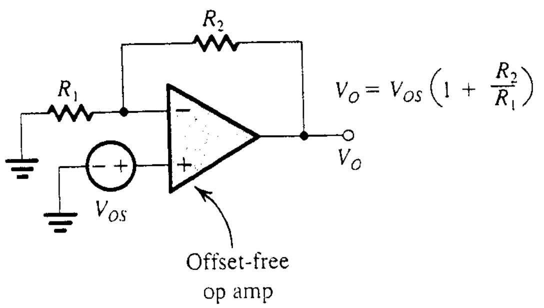


FIGURE 2.29 Evaluating the output dc offset voltage due to V_{OS} in a closed-loop amplifier.

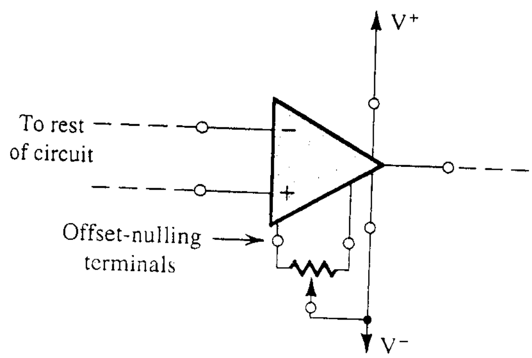


FIGURE 2.30 The output dc offset voltage of an op amp can be trimmed to zero by connecting a potentiometer to the two offset-nulling terminals. The wiper of the potentiometer is connected to the negative supply of the op amp.

2.24 Consider an inverting amplifier with a nominal gain of 1000 constructed from an op amp with an input offset voltage of 3 mV and with output saturation levels of ±10 V. (a) What is (approximately) the peak sine-wave input signal that can be applied without output clipping? (b) If the effect of V_{OS} is nulled at room temperature (25°C), how large an input can one now apply if: (i) the circuit is to operate at a constant temperature? (ii) the circuit is to operate at a temperature in the range 0°C to 75°C and the temperature coefficient of V_{OS} is 10 μV/°C?

sol) invert $G = 1000\ V/V$, $V_{OS} = 3\,mV$, $V_{O,sat} = \pm 10V$

(a) max input: $\frac{V_{O,max}}{1000} = \frac{10V}{1000} = 10mV$, $10 - 3 = 7mV$ = max input

(b) V_{OS} nulled @ 25°C

(i) const. temp. @ 25°C : $\frac{10V}{1000} = 10mV$ max input

(ii) 0°C ↔ 75°C, $\frac{dV_{OS}}{dT} = 10\,\mu V/°C$ 0↔25, 25↔75

$\Delta V_{OS}(75-25°C) = 50°C \times 10\,\mu V/°C = 0.5mV$ ∴ $10 - 0.5 = 9.5m$ max input