Fastest-Known Maximum-Likelihood Decoding of Quasi-Orthogonal STBCs with QAM Signals

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***Abstract*—We present a maximum-likelihood (ML) decoder with the lowest computational complexity known to-date for full-diversity, arbitrary size Quasi-Orthogonal Space-Time Block Codes (QO-STBCs) with symbols from square or rectangular quadrature amplitude modulation (QAM) constellations. We start with the formulation of an explicit joint two-complex-symbol decoder for general QO-STBCs with arbitrary complex symbols and then derive the proposed ML decoder for QO-STBCs with QAM symbols. The complexity savings are made possible by a simplified quadratic ML decoding statistic that utilizes algebraically the structure of the signal points of the QAM constellation. Comparative computational complexity analysis with existing ML implementations and simulation studies are included herein for illustration and validation purposes.**

***Index Terms*—Full diversity, maximum-likelihood (ML) detection, multi-input multi-output (MIMO) communications, quadrature amplitude modulation (QAM), space-time block codes (STBC).**

I. Introduction

ORTHOGONAL space-time block codes (O-STBCs) [1] offer full transmit diversity and allow independent single-complex-symbol maximum-likelihood (ML) decoding. Rate-one O-STBCs do not exist, however, for systems with more than two transmit antennas [1], [2]. For systems with four transmit antennas, the rate limitation of O-STBCs was overcome in [3], [4] by Quasi-Orthogonal Space-Time Block Codes (QO-STBCs) of full-diversity via signal constellation rotation for some transmitted symbols in the codeword. Unfortunately, ML decoding of the QO-STBCs in [3], [4] requires joint detection of two complex symbols, which is computationally prohibitive for high-order constellations such as 16-QAM or higher. Single-complex-symbol decodable full-rate full-diversity QO-STBCs for four transmit antennas were presented in [5]-[7], but suffer from performance loss in comparison to [3] due to lower diversity product by design. Low complexity QR decomposition based ML decoding algorithms for full-diversity QO-STBCs have been reported in [8]-[10]. To that respect, Azzam and Ayanoglu [9] presented the lowest-complexity ML decoding algorithm known in past literature, utilizing the QR decomposition of the real channel matrix representation by stacking the real and imaginary components of the original complex channel matrix. In [11], sphere decoding was used to reduce decoding complexity, an approach that is sensitive to the choice of the initial radius and has complexity that increases exponentially with the size of the QO-STBC. In [12], an iterative algorithm that is not provably ML is developed with complexity that has not been analyzed. The works in [13], [14] study code structures amendable to low-complexity decoding.

In this paper, we present the lowest-complexity known to-date ML decoder for full-diversity QO-STBCs of the form in [3], [4] with QAM signal constellations and arbitrary number of receive antennas. The proposed algorithm applies, in general, to any joint two-complex-symbol decodable QO-STBC. The computational hurdle faced in joint ML decoding of two complex symbols is tackled by (i) simplifying the ML expression and partitioning the global minimization problem into equivalent independent multiple minimization problems of quadratic structure and (ii) simultaneously utilizing algebraically the quadratic nature of the minimization terms and the geometric positions of the QAM signal points. The identified algebraic manipulations drastically reduce the complexity of the ML decoding procedure and the decoding complexity stays independent of the operating signal-to-noise ratio (SNR). The general complexity expression is derived and compared with that of conventional ML decoding and the prior state-of-the-art ML implementation in [9]. Simulation results are also presented to demonstrate experimentally the theoretical ML equivalence of the proposed decoding procedure.

II. System Model and Notation

We use the following notation: $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real and imaginary part, respectively, of a complex number; $\mathrm{tr}(\mathbf{B})$, $\|\mathbf{B}\|_F^2$, $\mathbf{B}^T$, and $\mathbf{B}^H$ denote the trace, Frobenius norm, transpose, and Hermitian operator, respectively, on matrix $\mathbf{B}$; $(\cdot)^*$ denotes complex conjugation and $\mathbf{I}_L$, $\mathbf{0}_L$ denote the identity and all-zero matrix, respectively, of size $L \times L$. Let M_t be the number of transmit antennas, M_r the number of receive antennas, and T the number of time slots over which an STBC is transmitted. M_t and T are assumed to be even. An O-STBC of K symbols and size $N \times N$ can be written as [2]

$$\mathbf{V}_N(x_1, \cdots, x_K) = \sum_{n=1}^{K} (x_n \mathbf{A}_n + x_n^* \mathbf{B}_n) \tag{1}$$

where $\mathbf{A}_n$ and $\mathbf{B}_n$ are called dispersion matrices [1], [15]. A QO-STBC of $2K$ symbols and size $T \times M_t$ can be designed as [3]

$$\mathbf{X}(x_1, \cdots, x_{2K}) = \begin{bmatrix} \mathbf{V}_{\frac{Mt}{2}}(x_1, \cdots, x_K) & \mathbf{V}_{\frac{Mt}{2}}(x_{K+1}, \cdots, x_{2K}) \\ \mathbf{V}_{\frac{Mt}{2}}(x_{K+1}, \cdots, x_{2K}) & \mathbf{V}_{\frac{Mt}{2}}(x_1, \cdots, x_K) \end{bmatrix}. \tag{2}$$

For the QO-STBC to be full-diversity, if the symbols $x_1, \cdots, x_K$ come from an (L_R, L_I)- QAM constellation $\mathcal{A}$,

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the symbols $x_{K+1}, \cdots, x_{2K}$ will be from $\mathcal{A}_\theta \triangleq e^{j\theta}\mathcal{A}$ where the diversity-product optimal θ for QAM is $\pi/4$ [3]. For square or rectangular QAM, the symbols are of the form

$$\begin{aligned} x_k &= a_{k,R} + ja_{k,I}, \quad k = 1, \cdots, K, \\ x_k &= (a_{k,R} + ja_{k,I})e^{j\theta}, \quad k = K+1, \cdots, 2K, \end{aligned} \tag{3}$$

where $a_{k,R}$ and $a_{k,I}$, $k = 1, \cdots, 2K$, denote the real and imaginary part of the k-th symbol.

The received signal matrix $\mathbf{Y}$ of size $T \times M_r$ is given by

$$\mathbf{Y} = \sqrt{\frac{q\rho}{M_t}}\mathbf{X}\mathbf{H} + \mathbf{N} \tag{4}$$

where $\mathbf{H}$ is the channel matrix of size $M_t \times M_r$ and $\mathbf{N}$ is the additive noise matrix of size $T \times M_r$. The channel is assumed to be quasi-static with Rayleigh fading coefficients and known to the receiver but unknown to the transmitter. The elements of $\mathbf{H}$ and $\mathbf{N}$ are modeled as independent, identically distributed complex Gaussian random variables with zero mean and unit variance. In (4), ρ is the average SNR at each receive antenna and the constant q is chosen such that each codeword $\mathbf{X}$ satisfies the energy constraint $E\{\|\mathbf{X}\|_F^2\} = TM_t$ [3], where $E\{\cdot\}$ denotes statistical expectation.

III. Proposed Decoder

In this section, we present the details of the proposed simplified ML decoding algorithm for full-diversity, maximum-diversity-product QO-STBCs with M_t-transmit antennas. ML decoding of the $2K$ symbols $x_1, \cdots, x_{2K}$ relies on the quasi-orthogonal property of $\mathbf{X}^H\mathbf{X}$ [3], [4]

$$\mathbf{X}^H\mathbf{X} = \begin{bmatrix} \alpha\mathbf{I}_{\frac{M_t}{2}} & \beta\mathbf{I}_{\frac{M_t}{2}} \\ \beta\mathbf{I}_{\frac{M_t}{2}} & \alpha\mathbf{I}_{\frac{M_t}{2}} \end{bmatrix} \tag{5}$$

where
$\alpha = \sum_{k=1}^{2K} |x_k|^2,\ \beta = 2\,\mathrm{Re}\Big\{\sum_{k=1}^{K} x_k x_{K+k}^*\Big\}$.
We note the manifested in β "inter-symbol interference" among the symbol pairs $(x_k,\ x_{K+k})$, $k = 1, \cdots, K$.

The full-diversity QO-STBC in (2) requires joint two-complex-symbol ML decoding [3]. Specifically, with known channel matrix $\mathbf{H}$ at the receiver, the ML decoder is given by

$$\hat{\mathbf{a}}_{ML} = arg \min_{\substack{x_1,\cdots,x_K \in \mathcal{A} \\ x_{K+1},\cdots,x_{2K} \in \mathcal{A}_\theta}} \Big[-2\,\mathrm{Re}\Big\{\mathrm{tr}\Big(\mathbf{X}\mathbf{H}\mathbf{Y}^H\Big)\Big\} + \sqrt{\frac{q\rho}{M_t}}\mathrm{tr}\Big(\mathbf{X}^H\mathbf{X}\mathbf{H}\mathbf{H}^H\Big)\Big]. \tag{6}$$

By (5), $\mathrm{tr}(\mathbf{X}^H\mathbf{X}\mathbf{H}\mathbf{H}^H) = \alpha\|\mathbf{H}\|_F^2 + \beta c_H$ where

$$c_H = 2\,\mathrm{Re}\left\{\sum_{i=1}^{M_r}\sum_{j=1}^{\frac{M_t}{2}} h_{\frac{M_t}{2}+j,i}h_{j,i}^*\right\}. \tag{7}$$

We define $\mathbf{U} \triangleq \mathbf{H}\mathbf{Y}^H = \begin{bmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} \\ \mathbf{U}_{21} & \mathbf{U}_{22} \end{bmatrix}$ with each submatrix $\mathbf{U}_{ij}$, $i,j = 1,2$, of size $\frac{M_t}{2} \times \frac{M_t}{2}$ (assuming $T = M_t$). Using (1), (5), α, β, c_H, and the trace of $\mathbf{XU}$, the ML minimization in (6) can be written in the form

$$\hat{\mathbf{a}}_{ML} = \sum_{k=1}^{K}\left\{ \min_{\substack{x_k \in \mathcal{A} \\ x_{K+k} \in \mathcal{A}_\theta}} F_k(x_k, x_{K+k})\right\} \tag{8}$$

where

$$\begin{aligned} &F_k(x_k, x_{K+k}) \\ &= \sqrt{\frac{q\rho}{M_t}}\Big\{(|x_k|^2 + |x_{K+k}|^2)\|\mathbf{H}\|_F^2 + 2\,\mathrm{Re}\Big\{x_k x_{K+k}^*\Big\}c_H\Big\} \\ &- 2\,\mathrm{Re}\Big[\Big\{x_k\mathrm{tr}\{(\mathbf{U}_{11}+\mathbf{U}_{22})\mathbf{A}_k\} + x_k^*\mathrm{tr}\{(\mathbf{U}_{11}+\mathbf{U}_{22})\mathbf{B}_k\}\Big\} \\ &+\Big\{x_{K+k}\mathrm{tr}\{(\mathbf{U}_{12}+\mathbf{U}_{21})\mathbf{A}_k\} + x_{K+k}^*\mathrm{tr}\{(\mathbf{U}_{12}+\mathbf{U}_{21})\mathbf{B}_k\}\Big\}\Big]. \end{aligned} \tag{9}$$

The above explicit expression for general joint two-complex-symbol decoding of full-diversity QO-STBC symbols shows that joint $2K$-complex-symbol decoding can be broken down to K pairs of joint two-complex symbol decodings without losing ML optimality. However, joint decoding for each pair of two-complex symbol requires $(L_R L_I)^2$ test trials whose complexity is still unacceptably high for high-order signal constellations. Below, we are able to reduce significantly the complexity of the ML decoder for QAM constellations without sacrificing ML optimality.

Due to the identical algebraic structure of the functionals $F_k(x_k, x_{K+k})$, $k = 1, \cdots, K$, in (9), it suffices to consider the joint ML decoding problem for only one pair of symbols, say x_k and x_{K+k}. The work for all other symbol pairs is identical. We start by defining

$$\begin{aligned} u_{1,R} + ju_{1,I} &\triangleq \mathrm{tr}\{(\mathbf{U}_{11}+\mathbf{U}_{22})\mathbf{A}_k\}, \\ u_{2,R} + ju_{2,I} &\triangleq \mathrm{tr}\{(\mathbf{U}_{11}+\mathbf{U}_{22})\mathbf{B}_k\}, \\ v_{1,R} + jv_{1,I} &\triangleq \mathrm{tr}\{(\mathbf{U}_{12}+\mathbf{U}_{21})\mathbf{A}_k\}, \\ v_{2,R} + jv_{2,I} &\triangleq \mathrm{tr}\{(\mathbf{U}_{12}+\mathbf{U}_{21})\mathbf{B}_k\}. \end{aligned} \tag{10}$$

Expanding $F_k(x_k, x_{K+k})$ in terms of the real and imaginary parts of the symbols x_k and x_{K+k} from (3), we have

$$(\hat{x}_k, \hat{x}_{K+k}) = arg \min_{x_k \in \mathcal{A}, x_{K+k} \in \mathcal{A}_\theta}\Big[f_1(a_{k,R}) + f_2(a_{k,I}) + \sqrt{\frac{q\rho}{4M_t}}\|\mathbf{H}\|_F^2|x_{K+k}|^2\Big] \tag{11}$$

where $f_1(a_{k,R})$ and $f_2(a_{k,I})$ are given by

$$\begin{aligned} f_1(a_{k,R}) &= \sqrt{\frac{q\rho}{4M_t}}\|\mathbf{H}\|_F^2 a_{k,R}^2 + g_1 + \\ &\Big[-\varepsilon + \sqrt{\frac{q\rho}{2M_t}}c_H(a_{(K+k),R} - a_{(K+k),I})\Big]a_{k,R}, \end{aligned} \tag{12}$$

$$\begin{aligned} f_2(a_{k,I}) &= \sqrt{\frac{q\rho}{4M_t}}\|\mathbf{H}\|_F^2 a_{k,I}^2 + g_2 + \\ &\Big[-\gamma + \sqrt{\frac{q\rho}{2M_t}}c_H(a_{(K+k),R} + a_{(K+k),I})\Big]a_{k,I} \end{aligned} \tag{13}$$

with $\varepsilon = u_{1,R} + u_{2,R}$, $\gamma = u_{2,I} - u_{1,I}$, and

$$\begin{aligned} g_1 &= \frac{\sqrt{2}}{2}(a_{(K+k),R} - a_{(K+k),I})(-v_{1,R} - v_{2,R}), \\ g_2 &= \frac{\sqrt{2}}{2}(a_{(K+k),R} + a_{(K+k),I})(v_{1,I} - v_{2,I}). \end{aligned} \tag{14}$$

We observe that $f_1(a_{k,R})$ in (12) depends on $a_{(K+k),R} - a_{(K+k),I}$, while $f_2(a_{k,I})$ in (13) depends on $a_{(K+k),R} + a_{(K+k),I}$. Define

$$b^+ = a_{(K+k),R} + a_{(K+k),I},\ b^- = a_{(K+k),R} - a_{(K+k),I}. \tag{15}$$

Then, $|x_{K+k}|^2 = \frac{|b^+|^2+|b^-|^2}{2}$, $a_{(K+k),R} = \frac{b^+ + b^-}{2}$, and $a_{(K+k),I} = \frac{b^+ - b^-}{2}$. Most importantly, each of b^+ and b^-

has a limited number of possible values, a fact which can be exploited to reduce ML decoding complexity. Specifically, for any given (L_R, L_I)- QAM[1] constellation $\mathcal{A} = \mathcal{A}_R \times \mathcal{A}_I$ with $\mathcal{A}_R = E_s^{-\frac{1}{2}} \{\pm 1, \pm 3, \cdots, \pm(L_R-1)\}$, $\mathcal{A}_I = E_s^{-\frac{1}{2}} \{\pm 1, \pm 3, \cdots, \pm(L_I-1)\}$, and E_s the power normalization factor to set the symbols in $\mathcal{A}$ to unit average power, for all $a_{(K+k),R} \in \mathcal{A}_R$ and $a_{(K+k),I} \in \mathcal{A}_I$ the possible values of b^+ and b^- are in

$$\Omega = \frac{1}{\sqrt{E_s}} \{0, \pm 2, \pm 4, \cdots, \pm(L_R + L_I - 2)\}. \quad (16)$$

Now we return to the ML optimization problem in (11) and rewrite

$$(\hat{x}_k, \hat{x}_{K+k}) = arg \min_{x_{K+k} \in \mathcal{A}_\theta} \left\{ \min_{a_{k,R} \in \mathcal{A}_R} G_1(a_{k,R}) + \min_{a_{k,I} \in \mathcal{A}_I} G_2(a_{k,I}) \right\} \quad (17)$$

where

$$G_1(a_{k,R}) = \sqrt{\frac{q\rho}{16M_t}} \|\mathbf{H}\|_F^2 |b^-|^2 + f_1(a_{k,R}), \quad (18)$$

$$G_2(a_{k,I}) = \sqrt{\frac{q\rho}{16M_t}} \|\mathbf{H}\|_F^2 |b^+|^2 + f_2(a_{k,I}). \quad (19)$$

From (17), (18), we see that for any given $b^- \in \Omega$, we can determine $a_{k,R}$ by minimizing

$$G_1(a_{k,R}) = \sqrt{\frac{q\rho}{4M_t}} \|\mathbf{H}\|_F^2 a_{k,R}^2 + \left(-\varepsilon + \sqrt{\frac{q\rho}{2M_t}} c_H b^-\right) a_{k,R} + g_1 + \sqrt{\frac{q\rho}{16M_t}} \|\mathbf{H}\|_F^2 |b^-|^2 \quad (20)$$

over the set $\mathcal{A}_R$. Relaxing, for the moment, the constraint $a_{k,R} \in \mathcal{A}_R$ to $a_{k,R} \in \mathbb{R}$, the quadratic function in (20) is minimized by the real number

$$a_R^{min} = \left(\varepsilon - \sqrt{\frac{q\rho}{2M_t}} c_H b^-\right) \left(\sqrt{\frac{q\rho}{M_t}} \|\mathbf{H}\|_F^2\right)^{-1}. \quad (21)$$

The key observation now -and good fortune- is that the function $G_1(a_{k,R})$ is decreasing for $a_{k,R} \in (-\infty, a_R^{min})$ and increasing for $a_{k,R} \in (a_R^{min}, \infty)$. Hence, the point in $\mathcal{A}_R$ nearest to a_R^{min} minimizes $G_1(a_{k,R})$ over $\mathcal{A}_R$. If $|a_R^{min}| \geq \frac{L_R-1}{\sqrt{E_s}}$, the point in $\mathcal{A}_R$ nearest to a_R^{min} is

$$\hat{a}_{k,R} = sign(a_R^{min}) \frac{L_R - 1}{\sqrt{E_s}}; \quad (22)$$

if $|a_R^{min}| < \frac{L_R-1}{\sqrt{E_s}}$, the point in $\mathcal{A}_R$ nearest to a_R^{min} is

$$\hat{a}_{k,R} = \frac{1}{\sqrt{E_s}} \left[2 \times round\left(\frac{\sqrt{E_s} a_R^{min} + 1}{2}\right) - 1\right] \quad (23)$$

where the sign operator $sign(r)$ returns the sign of the real number r and the round operation $round(r)$ finds the integer number nearest to r.

Similarly, from (17), (19), we can see that for any given $b^+ \in \Omega$, we can determine $a_{k,I}$ by minimizing

$$G_2(a_{k,I}) = \sqrt{\frac{q\rho}{4M_t}} \|\mathbf{H}\|_F^2 a_{k,I}^2 + \left(-\gamma + \sqrt{\frac{q\rho}{2M_t}} c_H b^+\right) a_{k,I} + g_2 + \sqrt{\frac{q\rho}{16M_t}} \|\mathbf{H}\|_F^2 |b^+|^2 \quad (24)$$

[1]We assume that both L_R and L_I are even as mostly used in practice.

TABLE I
REAL MULTIPLICATIONS AND ADDITIONS FOR 4 TRANSMIT ANTENNAS

QAM (L_R, L_I)	ML by (8) $\mathbb{R}_M$	ML by (8) $\mathbb{R}_A$	ML [9] $\mathbb{R}_M$	ML [9] $\mathbb{R}_A$	Proposed ML $\mathbb{R}_M$	Proposed ML $\mathbb{R}_A$
16 (4,4)	2569	7103	592	656	471	356
64 (8,8)	31081	108095	1744	2192	695	868
256 (16,16)	467689	1710143	6352	8336	1143	2468

over the set $\mathcal{A}_I$. Changing $a_{k,I} \in \mathcal{A}_I$ to $a_{k,I} \in \mathbb{R}$, the quadratic function in (24) is minimized by the real number

$$a_I^{min} = \left(\gamma - \sqrt{\frac{q\rho}{2M_t}} c_H b^+\right) \left(\sqrt{\frac{q\rho}{M_t}} \|\mathbf{H}\|_F^2\right)^{-1}. \quad (25)$$

Again, we observe that the function $G_2(a_{k,I})$ is decreasing for $a_{k,I} \in (-\infty, a_I^{min})$ and increasing for $a_{k,I} \in (a_I^{min}, \infty)$. Hence, the point in $\mathcal{A}_I$ nearest to a_I^{min} minimizes $G_2(a_{k,I})$ over $\mathcal{A}_I$. If $|a_I^{min}| \geq \frac{L_I-1}{\sqrt{E_s}}$, the point in set $\mathcal{A}_I$ nearest to a_I^{min} is

$$\hat{a}_{k,I} = sign(a_I^{min}) \frac{L_I - 1}{\sqrt{E_s}}; \quad (26)$$

if $|a_I^{min}| < \frac{L_I-1}{\sqrt{E_s}}$, the point in set $\mathcal{A}_I$ nearest to a_I^{min} is

$$\hat{a}_{k,I} = \frac{1}{\sqrt{E_s}} \left[2 \times round\left(\frac{\sqrt{E_s} a_I^{min} + 1}{2}\right) - 1\right]. \quad (27)$$

In view of (22), (23) and (26), (27), to find the optimal pair $\hat{x}_k, \hat{x}_{K+k}$ only $(L_R L_I)$ trials are required instead of $(L_R L_I)^2$ needed by the original statistic in (8). For clarity in presentation, we outline below the algorithm of the proposed simplified ML detector for symbols x_k and x_{K+k}. The same procedure can be followed to decode all other $K-1$ symbol pairs.

Proposed ML Decoder

1. Select a symbol $x_{K+k} \in \mathcal{A}_\theta$, determine $b^+ = a_{(K+k),R} + a_{(K+k),I}$ and $b^- = a_{(K+k),R} - a_{(K+k),I}$.
2. Use b^- to determine $\hat{a}_{k,R}$:
   a. Calculate $a_R^{min} = \frac{\varepsilon - \sqrt{\frac{q\rho}{2M_t}} c_H b^-}{\sqrt{\frac{q\rho}{M_t}} \|\mathbf{H}\|_F^2}$.
   b. Find the closest signal point in set $\mathcal{A}_R$:
      if $|a_R^{min}| \geq \frac{L_R-1}{\sqrt{E_s}}$,
         $\hat{a}_{k,R} = sign(a_R^{min}) \left(\frac{L_R-1}{\sqrt{E_s}}\right)$
      else
         $\hat{a}_{k,R} = \frac{1}{\sqrt{E_s}} \left[2 \times round\left(\frac{\sqrt{E_s} a_R^{min} + 1}{2}\right) - 1\right]$.
3. Use b^+ to determine $\hat{a}_{k,I}$:
   a. Calculate $a_I^{min} = \frac{\gamma - \sqrt{\frac{q\rho}{2M_t}} c_H b^+}{\sqrt{\frac{q\rho}{M_t}} \|\mathbf{H}\|_F^2}$.
   b. Find the closest signal point in set $\mathcal{A}_I$:
      if $|a_I^{min}| \geq \frac{L_I-1}{\sqrt{E_s}}$,
         $\hat{a}_{k,I} = sign(a_I^{min}) \left(\frac{L_I-1}{\sqrt{E_s}}\right)$
      else
         $\hat{a}_{k,I} = \frac{1}{\sqrt{E_s}} \left[2 \times round\left(\frac{\sqrt{E_s} a_I^{min} + 1}{2}\right) - 1\right]$.
4. Repeat (1)-(3) for all other $x_{K+k} \in \mathcal{A}_\theta$.
5. Find the minimum of $G_1(\hat{a}_{k,R}) + G_2(\hat{a}_{k,I})$ and output the corresponding $\hat{x}_k = \hat{a}_{k,R} + j\hat{a}_{k,I}$ and $\hat{x}_{K+k}$.

IV. Computational Complexity Comparisons

In Table I, as an example, we explicitly compare numerically the decoding complexity of the proposed ML algorithm,

TABLE II
DECODING COMPLEXITY IN REAL MULTIPLICATIONS AND ADDITIONS

ML by (8)	$\mathbb{R}_M$: $(5+K)\lvert\mathcal{A}\rvert^2 + 2(8K+1)\lvert\mathcal{A}\rvert + 2M_t^2(K+4M_r) + 6M_tM_r + 17$ $\mathbb{R}_A$: $(6+10K)\lvert\mathcal{A}\rvert^2 + 12K\lvert\mathcal{A}\rvert + M_t^2(4M_r-1) + 5M_tM_r - 3 - K$
Proposed ML	$\mathbb{R}_M$: $M_t^2(4K+4M_r) + 6M_tM_r + 28(L_R+L_I-1) + 59$ $\mathbb{R}_A$: $M_t^2(4M_r-1.5) + 2(3M_t+1)M_r + (3\lvert\mathcal{A}\rvert + 14(L_R+L_I-1)+2)K - 6$

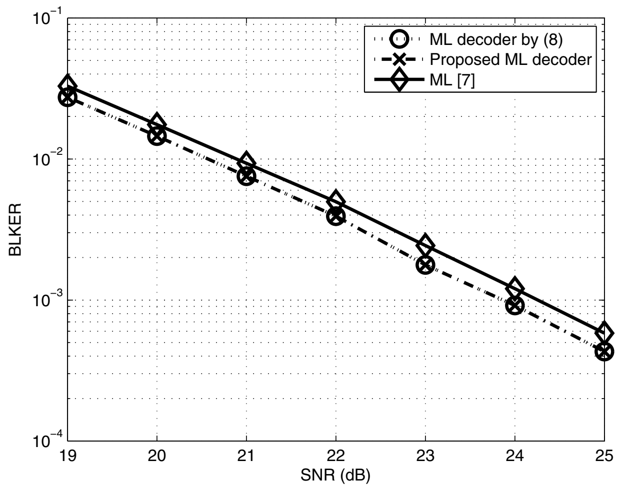

Fig. 1. Block-error-rate (BLKER) versus SNR for $M_t = 4$, $M_r = 1$ and 16-QAM signal constellation.

the original ML decoder by (8), and the current lowest-cost ML implementation in the literature [9], in terms of real multiplications $(\mathbb{R}_M)$ and additions $(\mathbb{R}_A)$ for $M_t = 4$, $M_r = 1$, and $K = 2$ with different size QAM constellations. In Table II, we generalize and present in greatest detail the decoding complexity of ML by (8) and the proposed ML in terms of number real multiplications and additions as a function of M_t, M_r, K, and constellation size $|\mathcal{A}| = L_R L_I$.

We can observe that the proposed algorithm makes 256-QAM directly implementable. [2] Finally, in Fig. 1 we validate experimentally the equivalence (in error-rate) of the ML decoder in (8), the developed simplified ML algorithm for a system with $M_t = 4$, $M_r = 1$ and 16-QAM modulation. For broader reference purposes only, we add the error-rate curve of the codewords in [7] under ML decoding.

V. CONCLUSIONS

We presented explicitly in direct implementation form the lowest-complexity ML algorithm known to us to-date for decoding full-diversity QO-STBCs with square or rectangular QAM signal constellations. As the size of the signal constellation used in the system increases, the relative complexity gap between the developed ML algorithm and the previous state of the art grows rapidly.

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[2] In [11], sphere decoding was seen to offer about 85% computational savings over conventional ML with 4 transmit antennas. The proposed scheme herein shows, under 64-QAM for example, savings of more than 97%.