Joint Power Optimization for Multi-Source Multi-Destination Relay Networks

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***Abstract*—In this paper, low-complexity joint power assignment algorithms are developed for multi-source multi-destination relay networks where multiple sources share a common relay that forwards all received signals simultaneously to destinations. In particular, we consider the following power optimization strategies: (i) Minimization of the total transmission power of the sources and the relay under the constraint that the signal-to-interference-plus-noise ratio (SINR) requirement of each source-destination pair is satisfied, and (ii) Maximization of the minimum SINR among all source-destination pairs subject to any given total power budget. Both optimization problems involve K power variables, where K is the number of source-destination pairs in the network, and an exhaustive search is prohibitive for large K. In this work, we develop a methodology that allows us to obtain an asymptotically tight approximation of the SINR and reformulate the original optimization problems to single-variable optimization problems, which can be easily solved by numerical search of the single variable. Then, the corresponding optimal transmission power at each source and relay can be calculated directly. The proposed optimization schemes are scalable and lead to power assignment algorithms that exhibit the same optimization complexity for any number (K) of source-destination pairs in the network. Moreover, we apply the methodology that we developed to solve a related max-min SINR based optimization problem in which we determine power assignment for the sources and the relay to maximize the minimum SINR among all source-destination pairs subject to any given total power budget. Extensive numerical studies illustrate and validate our theoretical developments.**

***Index Terms*—Cooperative networks, interference relay channels, max-min SINR optimization, multi-source multi-destination relay networks, optimum power allocation.**

I. Introduction

RECENT work on information-theoretic aspects of cooperative relaying, as well as recent proposals of practical cooperative relaying protocols ([1]–[10] and references therein) suggest that cooperative relaying may lead to significant improvements in detection reliability at destinations and overall system performance. In cooperative relaying, a user/node may serve as a relay and assist others by forwarding their signals to destinations, thus enhancing detection reliability at the destination. Various relaying strategies have been studied in the literature for relay to forward signals. For example, relay may decode the received signal and forward the decoded information to a destination, or it may simply amplify the received signal and forward it to the destination.

More recently, there is increasing interest in investigating the advantages of relaying in multi-source multi-destination networks [11]–[18], [24]–[26], which promise significant achievable rate improvement in shared-spectrum multiple access wireless networks. The simplest multi-source multi-destination relay network is modeled as an interference relay channel (IRC) [11] where a relay helps two independent source-destination pairs by using different relaying strategies such as decode-and-forward, amplify-and-forward, or compress-and-forward. Past literature on multi-source multi-destination relay networks focused primarily on information-theoretic studies including achievable rate regions or bounds of capacity region [11]–[18]. For example, in [11] a rate splitting technique is used to study the problem of achievable rate region for a Gaussian IRC channel, where each message is split into a common message which is decodable at all destinations and a private message which is decodable only at the intended destination [14]. Since the receivers are able to decode part of the interference messages, the effect of interference is reduced and the overall communication rate is therefore increased. The achievable rate region of [11] was further improved in [12] by considering both intended message and interference forwarding at the relay (optimal relay strategies were studied under the assumption that the relay is connected to each source and each destination via orthogonal and finite capacity links). The capacity region of the interference channel with a single-relay was investigated in [15] and [16], where it was shown that forwarding the intended message of just one source, the achievable rates for both source-destination pairs can be improved. By assuming that the relay knows the source message *a priori*, a relaying strategy was proposed in [17] and [18] where generalized beamforming with dirty paper coding was considered for a two-source two-destination relay network to further improve the capacity region of the network.

Power control is important to improve overall performance in multi-source multi-destination relay networks [19]. While past literature offers a significant amount of work on power allocation for single-source single-destination relay networks (see, for

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example [20]–[23] and references therein), there are rather limited studies on power optimization for multi-source multi-destination relay networks. In [24], power allocation was optimized by exhaustive search for a two-source two-destination relay network where a half-duplex decode-and-forward relay was considered. Unfortunately the exhaustive search is not scalable and leads to prohibitive optimization complexity for networks with larger number of source-destination pairs. In [25] and [26], a power allocation scheme was proposed for a multi-source multi-destination relay network based on geometric programming (the scheme assumes that signals from different sources are sent through orthogonal channels and the direct transmission link is not involved in detection at destinations).

In this paper, we analyze and optimize a general multi-source multi-destination relay network with K source-destination pairs (K can be large). The network allows simultaneous multi-source transmissions through nonorthogonal, in general, channels. We examine two power optimization strategies: i) Minimization of the total power consumption of all sources and relay under the constraint that the signal-to-interference-plus-noise ratio (SINR) requirement of each source-destination pair is satisfied; and ii) maximization of the minimum SINR among all source-destination pairs subject to any given total power budget. Thanks to an asymptotically tight approximation of the SINR that we develop, we are able to reformulate the original optimization problems, which involve K power variables, to single-variable optimization problems. Then, the resulting optimization problems can be easily solved by a simple numerical search of the single variable. The proposed optimization schemes are scalable and lead to power assignment algorithms that exhibit the same optimization complexity for any number (K) of source-destination pairs in the network. Moreover, for the special case of transmission over orthogonal channels, we are able to further simplify the single-variable optimizations and obtain analytical solutions for a symmetric system. Extensive numerical studies included in this paper illustrate and validate our theoretical developments, and show that, the proposed power assignment is almost identical to the exhaustive search method, and the optimum power assignment schemes, in general, can significantly improve the performance of multi-source multi-destination relay networks compared to an equal power assignment scheme.

The paper is organized as follows. In Section II, we introduce briefly the system model of a multi-source multi-destination relay network where transmissions occur over nonorthogonal, in general, channels. In Section III, we determine the maximum ratio combining of the received signals at each intended destination and exploit the resulting SINR. In Section IV, we determine the optimum power assignment for the sources and the relay that minimizes the total power consumption under the condition that the SINR requirement of each source-destination pair is satisfied. In Section V, we determine the optimum power assignment that maximizes the minimum SINR among all source-destination pairs subject to any given total power budget. Numerical studies are provided in Section VI, and finally, some conclusions are drawn in Section VII.

The following notation is used in the paper. Bold letters in uppercase and lowercase denote matrices and vectors, respectively. $(\cdot)^*, (\cdot)^T$ and $(\cdot)^H$ represent the conjugate, the transpose and the Hermitian transpose operation, respectively. $|\cdot|$ and $\|\cdot\|$ represent the absolute value of a complex number and the Frobenius norm of a vector/matrix, respectively. $\mathbf{I}_L$ is an $L \times L$ identity matrix. $\mathrm{diag}(h_1, h_2, \ldots, h_L)$ is an $L \times L$ diagonal matrix with diagonal elements $h_1, h_2, \ldots, h_L$. $\mathbf{A}_{\bar{k}}$ denotes a sub-matrix of $\mathbf{A}$ obtained by deleting the kth column and kth row of $\mathbf{A}$. If $\mathbf{a}_k$ represents the kth column of the matrix $\mathbf{A}$, then $\mathbf{a}_{\bar{k}}$ denotes the vector obtained after removing the kth entry from $\mathbf{a}_k$.

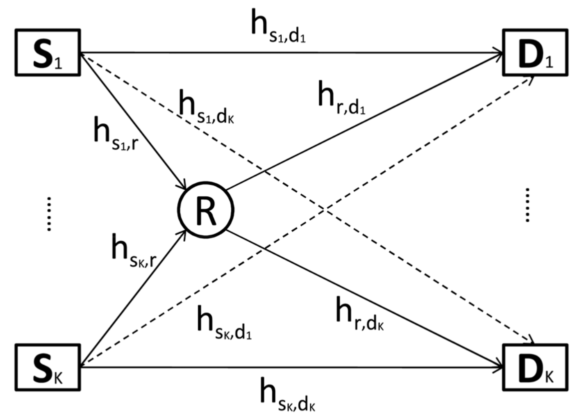

Fig. 1. Multi-source multi-destination relay network.

II. System Model

For illustration purposes and simplicity in presentation, we consider a single relay code division multiplexing system with K sources and K destinations as shown in Fig. 1, where transmissions occur over nonorthogonal, in general, channels. Our developments can be generalized to multiple-relay systems and other multiplexing schemes in frequency and/or time. Let S_k denote the kth source and D_k the corresponding destination, $k = 1, 2, \ldots, K$, and let R denote the relay. The relay forwards simultaneously the signals received from all sources. Let b_k denote the transmitted information symbol of the source S_k with unity average energy, i.e., $E\{|b_k|^2\} = 1, \forall k$. The signal sent by the source S_k can be expressed as

$$\mathbf{s}_k = \mathbf{c}_k b_k,$$

where $\mathbf{c}_k = (c_k^{(1)}, c_k^{(2)} \cdots, c_k^{(L)})^T$ is the code/signature of the source S_k, which is a unit-energy column vector with length L. The codes/channels of different sources are, in general, correlated. Let $\rho_{kj} \triangleq \mathbf{c}_k^T \mathbf{c}_j$ denote the cross-correlation between codes/channels k and j, where $\rho_{kj} \in [0, 1)$ for $k \neq j$, and $\rho_{kk} = 1$. Let $\mathbf{R} \triangleq (\rho_{kj})$ denote the $K \times K$ cross-correlation matrix, i.e.

$$\mathbf{R} = (\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K)^T (\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K). \tag{1}$$

We consider the following two-phase amplify-and-forward relay strategy with L time slots in each phase. In Phase 1, each source S_k transmits the signal $\mathbf{s}_k$ with transmitted power P_k. Then, the received signals at the destination D_k and at the relay

R during the ith $(1 \le i \le L)$ time slot can be modeled, respectively, as

$$y_{s,d_k}^{(i)} = \sum_{l=1}^{K} \sqrt{P_l} h_{s_l,d_k}^{(i)} c_l^{(i)} b_l + n_{s,d_k}^{(i)}, 1 \le i \le L \quad (2)$$

$$y_{s,r}^{(i)} = \sum_{l=1}^{K} \sqrt{P_l} h_{s_l,r}^{(i)} c_l^{(i)} b_l + n_{s,r}^{(i)}, 1 \le i \le L. \quad (3)$$

In Phase 2, the relay amplifies the received signals and forwards them to the destination with an amplification factor α and transmission power P_r. The received signal at the destination D_k during the ith $(1 \le i \le L)$ time slot can be written as

$$y_{r,d_k}^{(i)} = \sqrt{P_r} h_{r,d_k}^{(i)} \alpha y_{s,r}^{(i)} + n_{r,d_k}^{(i)}, \quad i = 1, 2, \ldots, L. \quad (4)$$

In (2)–(4), $h_{s_l,d_k}^{(i)}, h_{s_l,r}^{(i)}$, and $h_{r,d_k}^{(i)}, (l, k = 1, \ldots, K)$ are the coefficients of the channels between the source S_l and the destination D_k, between the source S_l and the relay R, and between the relay R and the destination D_k, respectively, during the ith $(1 \le i \le L)$ time slot. $n_{s,d_k}^{(i)}$ and $n_{r,d_k}^{(i)}$ represent noise at the destination D_k during the ith time slot of Phase 1 and Phase 2, correspondingly, while $n_{s,r}^{(i)}$ represents noise at the relay R during the ith time slot. The channels $h_{s_l,d_k}^{(i)}, h_{s_l,r}^{(i)}$, and $h_{r,d_k}^{(i)}$ are assumed to be independent Gaussian random variables with zero-mean and variances $\sigma_{s_l,d_k}^2, \sigma_{s_l,r}^2$, and σ_{r,d_k}^2,, respectively. All noise terms are assumed to be independent Gaussian random variables with zero-mean and variances σ^2. Without loss of generality, we assume $\sigma^2 = 1$.

The channel coefficients in matrix format can be written as $\mathbf{H}_{s_l,d_k} = \mathrm{diag}(h_{s_l,d_k}^{(1)}, \ldots, h_{s_l,d_k}^{(L)})$, $\mathbf{H}_{s_l,r} = \mathrm{diag}(h_{s_l,r}^{(1)}, \ldots, h_{s_l,r}^{(L)})$ and $\mathbf{H}_{r,d_k} = \mathrm{diag}(h_{r,d_k}^{(1)}, \ldots, h_{r,d_k}^{(L)})$. Then, the received signals can be expressed as follows:

$$\mathbf{y}_{s,d_k} = \sum_{l=1}^{K} \sqrt{P_l} \mathbf{H}_{s_l,d_k} \mathbf{c}_l b_l + \mathbf{n}_{s,d_k}, \quad (5)$$

$$\mathbf{y}_{s,r} = \sum_{l=1}^{K} \sqrt{P_l} \mathbf{H}_{s_l,r} \mathbf{c}_l b_l + \mathbf{n}_{s,r}, \quad (6)$$

$$\mathbf{y}_{r,d_k} = \sqrt{P_r} \alpha \mathbf{H}_{r,d_k} \mathbf{y}_{s,r} + \mathbf{n}_{r,d_k}, \quad (7)$$

where $\mathbf{y}_{s,d_k} = (y_{s,d_k}^{(1)}, y_{s,d_k}^{(2)}, \ldots, y_{s,d_k}^{(L)})^T$, $\mathbf{y}_{s,r} = (y_{s,r}^{(1)}, y_{s,r}^{(2)}, \ldots, y_{s,r}^{(L)})^T$ and $\mathbf{y}_{r,d_k} = (y_{r,d_k}^{(1)}, y_{r,d_k}^{(2)}, \ldots, y_{r,d_k}^{(L)})^T$. In (5)–(7), the noise vectors $\mathbf{n}_{s,d_k}, \mathbf{n}_{s,r}$ and $\mathbf{n}_{r,d_k}$ have elements that are independent Gaussian random variables with zero mean and unit variance. The amplification factor in (7) is specified as

$$\alpha^2 = \frac{1}{E\{||\mathbf{y}_{s,r}||^2\}} = \frac{1}{\sum_{l=1}^{K} P_l \beta_{s_l,r} + L} \quad (8)$$

where $\beta_{s_l,r} = \mathbf{c}_l^H E\{\mathbf{H}_{s_l,r}^H \mathbf{H}_{s_l,r}\} \mathbf{c}_l$. By substituting (6) and (8) into (7), we obtain

$$\mathbf{y}_{r,d_k} = \sqrt{P_r} \alpha \mathbf{H}_{r,d_k} \sum_{l=1}^{K} \sqrt{P_l} \mathbf{H}_{s_l,r} \mathbf{c}_l b_l + \sqrt{P_r} \alpha \mathbf{H}_{r,d_k} \mathbf{n}_{s,r} + \mathbf{n}_{r,d_k}. \quad (9)$$

At each destination D_k, it combines the signals received in Phase 1 and the signals received from the relay in Phase 2 to jointly detect the information transmitted by the source S_k. The combined signal from Phase 1 and Phase 2 at destination D_k can be expressed in vector form as follows:

$$\mathbf{y}_k \triangleq \begin{pmatrix} \mathbf{y}_{s,d_k} \\ \mathbf{y}_{r,d_k} \end{pmatrix} = \mathbf{H}_{k,k} \mathbf{c}_k b_k + \sum_{l=1, l \ne k}^{K} \mathbf{H}_{l,k} \mathbf{c}_l b_l + \mathbf{n}_k,$$

where

$$\mathbf{H}_{l,k} = \begin{pmatrix} \sqrt{P_l} \mathbf{H}_{s_l,d_k} \\ \sqrt{P_l P_r} \alpha \mathbf{H}_{s_l,r} \mathbf{H}_{r,d_k} \end{pmatrix}$$

is a $2L \times L$ virtual channel matrix from the source S_l to the destination D_k, and

$$\mathbf{n}_k = \begin{pmatrix} \mathbf{n}_{s,d_k} \\ \sqrt{P_r} \alpha \mathbf{H}_{r,d_k} \mathbf{n}_{s,r} + \mathbf{n}_{r,d_k} \end{pmatrix}$$

is an equivalent noise vector of length $2L$. We note that $\mathbf{H}_{kk}$ is the channel matrix associated with the desired source S_k, while $\mathbf{H}_{lk}, (l \ne k)$ are the channel matrices of the interfering sources. Based on maximum ratio combining (MRC) detection [28], the transmitted signal from the source S_k is detected as

$$\hat{b}_k = \arg \min_{b_k \in \mathcal{A}} \left| \mathbf{w}_{k,o}^H \mathbf{y}_k - b_k \right|^2, \quad (10)$$

where $\mathcal{A}$ is the set of transmitted symbols. For BPSK symbols, the detection is reduced to

$$\hat{b}_k = \mathrm{sign}(\mathrm{Re}\{\mathbf{w}_{k,o}^H \mathbf{y}_k\}),$$

while for 4-QAM symbols, the detection can be elaborated as

$$\hat{b}_k = \mathrm{sign}\left(\mathrm{Re}\{\mathbf{w}_{k,o}^H \mathbf{y}_k\}\right) + \mathrm{sign}\left(\mathrm{Im}\{\mathbf{w}_{k,o}^H \mathbf{y}_k\}\right) j,$$

in which $j = \sqrt{-1}$. The combining weight vector $\mathbf{w}_{k,o}$ in (10) of size $2L$ is chosen to maximize the SINR at the destination D_k which is given by

$$\mathtt{SINR}(\mathbf{w}_k) = \frac{E\left\{\left|\mathbf{w}_k^H \mathbf{H}_{k,k} \mathbf{c}_k b_k\right|^2\right\}}{E\left\{\left|\mathbf{w}_k^H \left(\sum_{l=1, l \ne k}^{K} \mathbf{H}_{l,k} \mathbf{c}_l b_l + \mathbf{n}_k\right)\right|^2\right\}}, \quad (11)$$

i.e.,

$$\mathbf{w}_{k,o} = \arg \max_{\mathbf{w}_k} \mathtt{SINR}(\mathbf{w}_k).$$

Note that in (11), the expectation in the numerator is taken over the random variable b_k, while the expectation in the denominator is taken over the random variables $b_l, l \ne k$ and all independent noise terms in $\mathbf{n}_k$.

III. System Performance Analysis

To determine the maximum SINR weight vector $\mathbf{w}_{k,o}$ and the corresponding SINR at the destination D_k, we first define

$$\mathbf{U}_k \triangleq \sum_{l=1, l \ne k}^{K} \mathbf{H}_{l,k} \mathbf{c}_l \mathbf{c}_l^H \mathbf{H}_{l,k}^H + \mathbf{\Gamma}_k,$$

$$\mathbf{\Gamma}_k \triangleq E\{\mathbf{n}_k \mathbf{n}_k^H\} = \begin{pmatrix} \mathbf{I}_L & 0 \\ 0 & P_r \alpha^2 \mathbf{H}_{r,d_k} \mathbf{H}_{r,d_k}^H + \mathbf{I}_L, \end{pmatrix}.$$

Then the SINR in (11) with any given combining weight vector $\mathbf{w}_k$ can be written as

$$\mathtt{SINR}(\mathbf{w}_k) = \frac{\left|\mathbf{w}_k^H \mathbf{H}_{k,k}\mathbf{c}_k\right|^2}{\mathbf{w}_k^H \mathbf{U}_k \mathbf{w}_k}. \tag{12}$$

It is easy to check that $\mathbf{U}_k$ is Hermitian, so are $\mathbf{U}_k^{(1/2)}$ and $\mathbf{U}_k^{-(1/2)}$.[1] According to Schwartz inequality, we have

$$\begin{aligned}\frac{\left|\mathbf{w}_k^H \mathbf{H}_{k,k}\mathbf{c}_k\right|^2}{\mathbf{w}_k^H \mathbf{U}_k \mathbf{w}_k} &= \frac{\left|\mathbf{w}_k^H \mathbf{U}_k^{\frac{1}{2}}\mathbf{U}_k^{-\frac{1}{2}} \mathbf{H}_{k,k}\mathbf{c}_k\right|^2}{\mathbf{w}_k^H \mathbf{U}_k^{\frac{1}{2}}\mathbf{U}_k^{\frac{1}{2}} \mathbf{w}_k} \\ &\le \frac{\left\|\mathbf{w}_k^H \mathbf{U}_k^{\frac{1}{2}}\right\|^2 \left\|\mathbf{U}_k^{-\frac{1}{2}}\mathbf{H}_{k,k}\mathbf{c}_k\right\|^2}{\left\|\mathbf{w}_k^H \mathbf{U}_k^{\frac{1}{2}}\right\|^2} \\ &= \left\|\mathbf{U}_k^{-\frac{1}{2}}\mathbf{H}_{k,k}\mathbf{c}_k\right\|^2\end{aligned}$$

where the equality holds when $\mathbf{w}_k^H \mathbf{U}_k^{(1/2)} = (\mathbf{U}_k^{-(1/2)}\mathbf{H}_{k,k}\mathbf{c}_k)^H$. Thus, the maximum SINR weight vector $\mathbf{w}_{k,o}$ is given by

$$\mathbf{w}_{k,o} = \mathbf{U}_k^{-1}\mathbf{H}_{k,k}\mathbf{c}_k.$$

The corresponding maximum SINR at the destination D_k with the optimum weight vector $\mathbf{w}_{k,o}$ is equal to

$$\mathtt{SINR}_k = \mathbf{c}_k^H \mathbf{H}_{k,k}^H \mathbf{U}_k^{-1}\mathbf{H}_{k,k}\mathbf{c}_k. \tag{13}$$

In order to optimally allocate power to all sources, we further exploit SINR_k in (13) as follows. We define $\hat{\mathbf{C}}_k \triangleq \mathrm{diag}(\mathbf{c}_1,\ldots,\mathbf{c}_{k-1},\mathbf{c}_{k+1},\ldots,\mathbf{c}_K)$, which is a $L(K-1)\times(K-1)$ block diagonal matrix formed by placing all code vectors except $\mathbf{c}_k$ in the diagonal positions, and $\hat{\mathbf{H}}_k \triangleq [\mathbf{H}_{1,k},\ldots,\mathbf{H}_{k-1,k},\mathbf{H}_{k+1,k},\ldots,\mathbf{H}_{K,k}]$ which is a $2L\times L(K-1)$ interference channel matrix. Using the above notation, SINR_k can be expressed as

$$\mathtt{SINR}_k = \mathbf{c}_k^H \mathbf{H}_{k,k}^H \left(\hat{\mathbf{H}}_k\hat{\mathbf{C}}_k\hat{\mathbf{C}}_k^H\hat{\mathbf{H}}_k^H + \mathbf{\Gamma}_k\right)^{-1}\mathbf{H}_{k,k}\mathbf{c}_k. \tag{14}$$

According to the Woodbury matrix inversion lemma [27], we have $(\hat{\mathbf{H}}_k\hat{\mathbf{C}}_k\hat{\mathbf{C}}_k^H\hat{\mathbf{H}}_k^H + \mathbf{\Gamma}_k)^{-1} = \mathbf{\Gamma}_k^{-1} - \mathbf{\Gamma}_k^{-1}\hat{\mathbf{H}}_k\hat{\mathbf{C}}_k(\mathbf{I}_{K-1} + \hat{\mathbf{C}}_k^H\hat{\mathbf{H}}_k^H\mathbf{\Gamma}_k^{-1}\hat{\mathbf{H}}_k\hat{\mathbf{C}}_k)^{-1}\hat{\mathbf{C}}_k^H\hat{\mathbf{H}}_k^H\mathbf{\Gamma}_k^{-1}$.

Let us now define the following matrix $\mathbf{F}^{(k)} \triangleq (f_{mn}^{(k)})$, where

$$\begin{aligned}f_{mn}^{(k)} = \sqrt{P_mP_n}\,&\big[\mathbf{c}_m^H\mathbf{H}_{s_m,d_k}^H\mathbf{H}_{s_n,d_k}\mathbf{c}_n \\ &+ P_r\alpha^2\mathbf{c}_m^H\mathbf{H}_{s_m,r}^H\mathbf{H}_{r,d_k}^H\left(P_r\alpha^2\mathbf{H}_{r,d_k}\mathbf{H}_{r,d_k}^H + \mathbf{I}_L\right)^{-1} \\ &\times \mathbf{H}_{r,d_k}\mathbf{H}_{s_n,r}\mathbf{c}_n\big]\end{aligned} \tag{15}$$

[1]We may represent $\mathbf{U}_k$ in terms of its eigenvalues $\lambda_1,\lambda_2,\ldots,\lambda_K$ and their corresponding eigenvectors $\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_K$ as: $\mathbf{U}_k = \sum_{l=1}^K \lambda_l\mathbf{q}_l\mathbf{q}_l^H$, where $\lambda_l \ge 0$ for $1\le l\le K$ and $\mathbf{q}_l^H\mathbf{q}_{l'} = 0, \forall l,l', l\ne l'$. Then, $\mathbf{U}_k^{(1/2)} = \sum_{l=1}^K \lambda_l^{(1/2)}\mathbf{q}_l\mathbf{q}_l^H$ and $\mathbf{U}_k^{-(1/2)} = \sum_{l=1}^K \lambda_l^{-(1/2)}\mathbf{q}_l\mathbf{q}_l^H$. It is easy to check that both $\mathbf{U}_k^{(1/2)}$ and $\mathbf{U}_k^{-(1/2)}$ are Hermitian.

and denote by $\mathbf{f}^{(k)}$ the kth column vector of the matrix $\mathbf{F}^{(k)}$. Then, after some algebraic calculations, we can see that

$$\begin{cases} f_{kk}^{(k)} = \mathbf{c}_k^H\mathbf{H}_{k,k}^H\mathbf{\Gamma}_k^{-1}\mathbf{H}_{k,k}\mathbf{c}_k \\ \mathbf{f}_{\bar{k}}^{(k)} = \hat{\mathbf{C}}_k^H\hat{\mathbf{H}}_k^H\mathbf{\Gamma}_k^{-1}\mathbf{H}_{k,k}\mathbf{c}_k \\ \mathbf{F}_{\bar{k}}^{(k)} = \hat{\mathbf{C}}_k^H\hat{\mathbf{H}}_k^H\mathbf{\Gamma}_k^{-1}\hat{\mathbf{H}}_k\hat{\mathbf{C}}_k, \end{cases} \tag{16}$$

where $\mathbf{f}_{\bar{k}}^{(k)}$ contains the channels and the cross-correlation between the intended source S_k and the interfering sources, while $\mathbf{F}_{\bar{k}}^{(k)}$ contains the channels and the cross-correlation among interfering sources. Based on (14)–(16), we can represent SINR_k as

$$\mathtt{SINR}_k = f_{kk}^{(k)} - \mathbf{f}_{\bar{k}}^{(k)H}\left(\mathbf{I}_{K-1} + \mathbf{F}_{\bar{k}}^{(k)}\right)^{-1}\mathbf{f}_{\bar{k}}^{(k)} \tag{17}$$

where the superscript $(\,\cdot\,)^{(k)}$ indicates the corresponding destination D_k.

Note that if the channels are quasi-static (i.e., constant) over each information symbol period, i.e., $h_{s_n,d_k}^{(i)} = h_{s_n,d_k}$, $h_{s_n,r}^{(i)} = h_{s_n,r}$ and $h_{r,d_k}^{(i)} = h_{r,d_k}\ \forall i = 1,2,\ldots,L$ and $\forall n,k = 1,2,\ldots,K$, then $f_{mn}^{(k)}$ in (15) is reduced to

$$\begin{aligned}f_{mn}^{(k)} &= \rho_{mn}\sqrt{P_mP_n} \\ &\quad\times\left(h_{s_m,d_k}^*h_{s_n,d_k} + \frac{\alpha^2P_r|h_{r,d_k}|^2h_{s_m,r}^*h_{s_n,r}}{\alpha^2P_r|h_{r,d_k}|^2+1}\right).\end{aligned} \tag{18}$$

As an illustration example, when there are only two source-destination pairs with one relay in the network, i.e., $K=2$, the SINR_k in (17) with quasi-static channels can be specified as

$$\begin{aligned}\mathtt{SINR}_k &= f_{kk}^{(k)} - \left|f_{kj}^{(k)}\right|^2\left(1+f_{jj}^{(k)}\right)^{-1} \\ &= P_k\left(|h_{s_k,d_k}|^2 + \frac{\alpha^2P_r\left|h_{r,d_k}\right|^2\left|h_{s_k,r}\right|^2}{\alpha^2P_r\left|h_{r,d_k}\right|^2+1}\right) \\ &\quad - \frac{\rho^2P_kP_j\left|h_{s_k,d_k}^*h_{s_j,d_k} + \frac{\alpha^2P_r|h_{r,d_k}|^2h_{s_k,r}^*h_{s_j,r}}{\alpha^2P_r|h_{r,d_k}|^2+1}\right|^2}{1+P_j\left|h_{s_j,d_k}\right|^2 + \frac{\alpha^2P_r\left|h_{r,d_k}\right|^2P_j\left|h_{s_j,r}\right|^2}{\alpha^2P_r\left|h_{r,d_k}\right|^2+1}}\end{aligned}$$

where $\rho = \rho_{12} = \rho_{21}$ and the subscript $j=2$ if $k=1$ while $j=1$ if $k=2$. Furthermore, when the code/channel vectors $\mathbf{c}_k$ are orthogonal to each other, i.e., $\rho_{kj} = \mathbf{c}_k^T\mathbf{c}_j = 0$, for $j\ne k$, then $\mathbf{f}_{\bar{k}}^{(k)} = \mathbf{0}$ for quasi-static channels. Hence the SINR at destination D_k is given by

$$\mathtt{SINR}_k = f_{kk}^{(k)} = P_k\left|h_{s_k,d_k}\right|^2 + \frac{\alpha^2P_r\left|h_{r,d_k}\right|^2P_k\left|h_{s_k,r}\right|^2}{\alpha^2P_r\left|h_{r,d_k}\right|^2+1} \tag{19}$$

which is the sum of the SINRs of the direct link and the relay link.

IV. Optimum Power Assignment Under SINR Constraints for All Source-Destination Pairs

In this section, we determine the optimum power assignment for the sources and the relay that minimizes the total power consumption under the condition that the SINR requirement of each source-destination pair is satisfied. First, we consider the power optimization for the relay network in a general setting where codes/channels of different sources may have arbitrary correlation. Then, we discuss a simplified power optimization scheme for a special case where codes/channels of different sources are orthogonal, and provide an intuitive interpretation for the proposed scheme.

A. Minimization of Total Power Consumption

Let us assume that the SINR requirement for the source-destination pair (S_k, D_k) is γ_k, $k = 1, 2, \ldots, K$. Then, the problem of optimizing power to minimize the total power consumption and satisfy all source-destination SINR requirements can be formulated as

$$\begin{cases} \mathtt{min}_{P_1,\ldots,P_K;\,P_r} & \sum_{k=1}^{K} P_k + P_r \\ \mathtt{s.t.} & \mathtt{SINR}_k \geq \gamma_k,\ 1 \leq k \leq K \end{cases} \tag{20}$$

where the transmission power terms $P_1, P_2, \ldots, P_k$ and P_r are all nonnegative.

Let us define an auxiliary parameter, which can be viewed as a normalized power factor at the relay, as follows:

$$x \triangleq \alpha^2 P_r \tag{21}$$

where α is the amplification factor specified in (8). The auxiliary parameter x will play a key role in the optimization procedure. Let us also denote by $\mathbf{G}^{(k)} = (g_{mn}^{(k)})$ a matrix with elements

$$\begin{aligned} g_{mn}^{(k)} \triangleq\ & \mathbf{c}_m^H \mathbf{H}_{s_m,d_k}^H \mathbf{H}_{s_n,d_k} \mathbf{c}_n + x \mathbf{c}_m^H \mathbf{H}_{s_m,r}^H \mathbf{H}_{r,d_k}^H \\ & \times \left(x \mathbf{H}_{r,d_k} \mathbf{H}_{r,d_k}^H + \mathbf{I}_L \right)^{-1} \mathbf{H}_{r,d_k} \mathbf{H}_{s_n,r} \mathbf{c}_n \end{aligned} \tag{22}$$

for any $m, n = 1, 2, \ldots, K$. Then, from (15), we can represent each entry in $\mathbf{F}^{(k)}$ by

$$f_{mn}^{(k)} = \sqrt{P_m P_n} g_{mn}^{(k)}.$$

It is straightforward to verify that

$$\det\left(\mathbf{F}^{(k)}\right) = \left(\prod_{l=1}^{K} P_l \right) \det\left(\mathbf{G}^{(k)}\right) \tag{23}$$

$$\det\left(\mathbf{F}_{\bar{k}}^{(k)}\right) = \left(\prod_{l=1, l\neq k}^{K} P_l \right) \det\left(\mathbf{G}_{\bar{k}}^{(k)}\right). \tag{24}$$

From (17), we know that $\mathbf{I}_{K-1} + \mathbf{F}_{\bar{k}}^{(k)}$ is invertible. So, based on the Schur complement formula[2], we have

$$\begin{aligned} & \det\left(\mathbf{I}_K + \mathbf{F}^{(k)}\right) \\ & = \det \begin{pmatrix} 1 + f_{kk}^{(k)} & \mathbf{f}_{\bar{k}}^{(k)H} \\ \mathbf{f}_{\bar{k}}^{(k)} & \mathbf{I}_{K-1} + \mathbf{F}_{\bar{k}}^{(k)} \end{pmatrix} \\ & = \det\left(\mathbf{I}_{K-1} + \mathbf{F}_{\bar{k}}^{(k)}\right) \\ & \quad \cdot \left[1 + f_{kk}^{(k)} - \mathbf{f}_{\bar{k}}^{(k)H} \left(\mathbf{I}_{K-1} + \mathbf{F}_{\bar{k}}^{(k)}\right)^{-1} \mathbf{f}_{\bar{k}}^{(k)} \right] \\ & = \det\left(\mathbf{I}_{K-1} + \mathbf{F}_{\bar{k}}^{(k)}\right) \cdot (1 + \mathtt{SINR}_k) \end{aligned} \tag{25}$$

where the last equality follows from the expression of SINR_k in (17). Thus, we have

$$1 + \mathtt{SINR}_k = \frac{\det\left(\mathbf{I}_K + \mathbf{F}^{(k)}\right)}{\det\left(\mathbf{I}_{K-1} + \mathbf{F}_{\bar{k}}^{(k)}\right)}. \tag{26}$$

We note that for moderate or high SINR, we may approximate $1 + \mathtt{SINR}_k \approx \mathtt{SINR}_k$ and $1 + f_{ll}^{(k)} \approx f_{ll}^{(k)}$, $\forall k, l = 1, \ldots, K$. So, from (26) we may approximate SINR_k as follows:

$$\mathtt{SINR}_k \approx \frac{\det\left(\mathbf{F}^{(k)}\right)}{\det\left(\mathbf{F}_{\bar{k}}^{(k)}\right)} = \frac{P_k \det\left(\mathbf{G}^{(k)}\right)}{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}. \tag{27}$$

The above approximation is asymptotically tight for high SINR. Based on this approximation, the optimization problem in (20) can be written as

$$\begin{cases} \mathtt{min}_{P_1,\ldots,P_K;\,P_r} & \sum_{k=1}^{K} P_k + P_r, \\ \mathtt{s.t.} & \frac{P_k \det\left(\mathbf{G}^{(k)}\right)}{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)} \geq \gamma_k, \quad 1 \leq k \leq K. \end{cases} \tag{28}$$

Let V denote the set of feasible solutions for the optimization problem in (28), i.e.

$$V = \{P_1, \ldots, P_K, P_r \,|\, \mathtt{SINR}_k \geq \gamma_k,\ \forall 1 \leq k \leq K\}.$$

We may further partition the set V into disjoint subsets such as

$$V = \bigcup_{x \geq 0} V_x$$

where

$$V_x = \left\{ P_1, \ldots, P_K, P_r \,\middle|\, \begin{matrix} \mathtt{SINR}_k \geq \gamma_k,\ \forall 1 \leq k \leq K, \\ \alpha^2 P_r = x \end{matrix} \right\}$$

for any $x \geq 0$.

We note that for any given value of the auxiliary parameter x in (21), according to (8) the transmission power at the relay P_r can be determined as

$$P_r = \frac{x}{\alpha^2} = x \sum_{k=1}^{K} P_k \beta_{s_k,r} + xL. \tag{29}$$

Thus, for any given $x \geq 0$, the optimization problem in (28) over the feasible set V_x becomes

$$\begin{cases} \mathtt{min}_{P_1,\ldots,P_K} & \sum_{k=1}^{K} (x\beta_{s_k,r} + 1) P_k + xL, \\ \mathtt{s.t.} & P_k \geq \frac{\gamma_k \det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)}, 1 \leq k \leq K. \end{cases} \tag{30}$$

We observe that in (30), for any given $x \geq 0$, $(\gamma_k \det(\mathbf{G}_{\bar{k}}^{(k)}))/(\det(\mathbf{G}^{(k)}))$ is a constant which is independent of P_k $(k = 1, 2, \ldots, K)$. Hence the minimal value in (30) is obtained when all constraints hold with equality, i.e.

$$P_k = \frac{\gamma_k \det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)}, \quad k = 1, 2, \ldots, K. \tag{31}$$

[2]If matrix $\mathbf{D}$ is invertible, then [27]

$$\det \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = \det(\mathbf{D}) \cdot \det(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}).$$

Then, the corresponding minimal total power of (30) is

$$v(x) \triangleq \sum_{k=1}^{K} \gamma_k \left(x\beta_{s_k,r}+1\right) \frac{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)} + xL \tag{32}$$

which is a function of $x \geq 0$. Let us denote v^* as the minimal value of the objective function over (28) in the feasible set V. Then, we can see that

$$v^* = \min_{x\geq 0} v(x). \tag{33}$$

The above discussion shows that we are able to convert the optimization problem in (20) over a multidimension space to the minimization problem in (33), which depends only on one variable $x \geq 0$, i.e., over a one-dimension space. The minimization of $v(x)$ in (32) can be easily solved by a numerical search for the optimal value of the parameter $x \geq 0$. With the optimal value x^* that minimizes the function $v(x)$ in (32), we can obtain the corresponding optimal power P_k^* and P_r^* based on (31) and (29), respectively.

B. Simplified Optimization With Orthogonal Codes/Channels

In the previous subsection, we solved the optimization problem for a general multisource multi-destination relay network with arbitrary correlation among user codes. In this subsection, we are able to further simplify the optimization when the signatures of different sources are orthogonal and the fading channels are quasi-static during the transmission period of a signature code.

In particular, for multi-source multi-destination relay networks with orthogonal transmissions, the cross-correlation matrix $\mathbf{R}$ in (1) is an identity matrix, so both $\mathbf{G}_{\bar{k}}^{(k)}$ and $\mathbf{G}^{(k)}$ in (27) are diagonal matrices, and

$$\frac{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)} = \frac{\prod_{l=1,l\neq k}^{K} g_{ll}^{(k)}}{\prod_{l=1}^{K} g_{ll}^{(k)}} = \frac{1}{g_{kk}^{(k)}} \tag{34}$$

where $g_{kk}^{(k)}$ is specified in (22). With the assumption of quasi-static channels, we can write $h_{s_k,r}^{(i)} = h_{s_k,r}$, $h_{s_k,d_k}^{(i)} = h_{s_k,d_k}$, and $h_{r,d_k}^{(i)} = h_{r,d_k}$, $\forall i = 1,2,\ldots,L$, and $\forall k = 1,2,\ldots,K$. We note that in the minimal total power in (32), $\beta_{s_k,r} = \sigma_{s_k,r}^2$, where $\sigma_{s_k,r}^2$ is the variance of the source-relay channel $h_{s_k,r}$, i.e., $\sigma_{s_k,r}^2 = E\{|h_{s_k,r}|^2\}$. Thus, by substituting (34) into (32), we have

$$\begin{aligned} v(x) &= \sum_{k=1}^{K} \frac{\gamma_k\left(x\sigma_{s_k,r}^2+1\right)}{\left|h_{s_k,d_k}\right|^2 + \frac{x\left|h_{r,d_k}\right|^2\left|h_{s_k,r}\right|^2}{x\left|h_{r,d_k}\right|^2+1}} + xL \\ &= \sum_{k=1}^{K} \gamma_k \left(c_k + b_k x + \frac{a_k}{x+d_k}\right) \end{aligned} \tag{35}$$

where

$$\begin{aligned} a_k &= \frac{\left|h_{s_k,r}\right|^2}{A^2\left|h_{r,d_k}\right|^2}\left(1 - \frac{\left|h_{s_k,d_k}\right|^2\sigma_{s_k,r}^2}{A\left|h_{r,d_k}\right|^2}\right), \\ b_k &= \frac{L}{K\gamma_k} + \frac{\sigma_{s_k,r}^2}{A} \\ c_k &= \frac{1}{A} + \frac{\left|h_{s_k,r}\right|^2\sigma_{s_k,r}^2}{A^2\left|h_{r,d_k}\right|^2}, \\ d_k &= \frac{\left|h_{s_k,d_k}\right|^2}{A\left|h_{r,d_k}\right|^2} \end{aligned} \tag{36}$$

and $A = |h_{s_k,d_k}|^2 + |h_{s_k,r}|^2$.

We note that for any $k = 1,2,\ldots,K$, if $a_k > 0$, each term $c_k + b_k x + (a_k)/(x+d_k)$ in (35) is convex with respect to $x \geq 0$, and it can be minimized by

$$x = \max\left(0, -d_k + \sqrt{\frac{a_k}{b_k}}\right).$$

If $a_k \leq 0$, $c_k + b_k x + (a_k)/(x+d_k)$ is increasing in $[0,+\infty)$, it implies that the minimum point occurs at $x = 0$. Let us denote

$$x_{\min} \triangleq \min(x_1, x_2, \ldots, x_K) \tag{37}$$

$$x_{\max} \triangleq \max(x_1, x_2, \ldots, x_K) \tag{38}$$

where

$$x_k = \begin{cases} \max\left(0, -d_k + \sqrt{\frac{a_k}{b_k}}\right) & \text{if } a_k > 0; \\ 0 & \text{if } a_k \leq 0; \end{cases} \tag{39}$$

for any $k = 1,2,\ldots,K$. Then, each term $c_k + b_k x + (a_k)/(x+d_k)$ in (35) is decreasing in the range $[0, x_{\min}]$ and increasing in the range $[x_{\max}, +\infty)$, which implies that the optimal solution x^* that minimizes the function $v(x)$ in (35) is bounded as

$$x_{\min} \leq x^* \leq x_{\max}. \tag{40}$$

Thus, to find the optimal solution x^*, we only need to search within the range $[x_{\min},\ x_{\max}]$. We note that $x_{\min} = 0$ if there exists $k \in \{1,2,\ldots,K\}$ such that $a_k \leq 0$.

From (39), we can see that the necessary condition for $x_k > 0$ is

$$a_k > b_k d_k^2,$$

which implies that

$$\frac{\left|h_{s_k,r}\right|^2\left|h_{r,d_k}\right|^2}{\sigma_{s_k,r}^2} > \left|h_{s_k,d_k}\right|^2. \tag{41}$$

In a nonfading or *slow-fading* scenario, where the coherence time of the channel is much longer than the delay requirement of the channel, the channel gain remains roughly constant over a period of processing time (for example, transmitting several data packets) [28]. In this case, we may safely assume that $|h_{i,j}|^2 \approx \sigma_{i,j}^2$ during the processing period, where $\sigma_{i,j}^2$ is the variance of the channel $h_{i,j}$. With such an approximation, the necessary condition in (41) is reduced to

$$\sigma_{r,d_k}^2 > \sigma_{s_k,d_k}^2. \tag{42}$$

If the necessary condition in (42) does not hold, i.e., $\sigma_{r,d_k}^2 \leq \sigma_{s_k,d_k}^2$ for all $k = 1,2,\ldots,K$, then $x_{\max} = 0$, which implies that the optimal solution $x^* = 0$. So, the corresponding optimum power at the relay is $P_r^* = 0$, which means that the relay is not needed in this case. In other words, if each relay-destination channel link is weaker than the intended source-destination channel link, then the use of the relay is not helpful.

In addition, if $a_k \leq 0, \forall k = 1, 2, \ldots, K$, then each term $c_k + b_k x + (a_k)/(x + d_k)$ in (35) is an increasing function of $x \in [0, +\infty)$, therefore the optimal solution is $x^* = 0$. Given that $a_k \leq 0$ is equivalent to

$$|h_{s_k,d_k}|^2 \sigma^2_{s_k,r} \geq \left(|h_{s_k,d_k}|^2 + |h_{s_k,r}|^2\right) |h_{r,d_k}|^2 \tag{43}$$

it implies that if all channels $|h_{s_k,d_k}|^2$ $(k = 1, 2, \ldots, K)$ that correspond to direct links are strong enough or all channels of the relay links $|h_{r,d_k}|^2$ $(k = 1, 2, \ldots, K)$ are weak, then avoiding use of the relay can leads to power savings. In other words, if there is at least one relay-destination link that is better than the corresponding source-destination link, the relay should forward its received signal to the destinations in order to help reduce the overall power consumption while maintaining at the same time the target performance level.

Finally, for a symmetric system where $\sigma^2_{s_1,d_1} = \sigma^2_{s_2,d_2} = \cdots = \sigma^2_{s_K,d_K}, \sigma^2_{s_1,r} = \sigma^2_{s_2,r} = \cdots = \sigma^2_{s_K,r}$ and $\sigma^2_{r,d_1} = \sigma^2_{r,d_2} = \cdots = \sigma^2_{r,d_K}$, we are able to obtain an analytical solution for the minimization of $v(x)$. In this case, all d_k are equal, denoted by $d = d_k, \forall k$. Then

$$v(x) = c + bx + \frac{a}{x + d}, \tag{44}$$

where $c = \sum_{k=1}^K \gamma_k c_k, b = \sum_{k=1}^K \gamma_k b_k, a = \sum_{k=1}^K \gamma_k a_k$. Consequently, the value x^* that minimizes $v(x)$ in (44) is

$$x^* = \begin{cases} -d + \sqrt{\frac{a}{b}} & \text{if } a > 0, \\ 0 & \text{if } a \leq 0. \end{cases} \tag{45}$$

Substituting (45) into (31) and (29), we obtain the optimal power P_k^* and P_r^*, respectively.

We note that for a symmetric system, all a_k are equal. When the SINR requirement is high enough at each destination, $L/(K\gamma_k)$ is relatively small compared to $(\sigma^2_{s_k,r})/(A)$, so all b_k's are approximately the same $(k = 1, 2, \ldots, K)$. As a consequence, the ratio a/b in (45) is independent of γ_k. Hence, the optimal value x^* in (45) is independent of the SINR thresholds γ_k in this case.

C. Equal Power Allocation Scheme

In this subsection, for comparison purposes, we discuss the equal power allocation scheme where all the sources and the relay are allocated the same power P. In this case, the corresponding parameter x in (21) is

$$x = \frac{1}{\sum_{k=1}^K \sigma^2_{s_k,r} + \frac{L}{P}}. \tag{46}$$

To find the optimal power P that minimizes the total power of the system under the constraints that the SINR requirements of all source-destination pairs are satisfied, we can follow the procedure described in the previous subsection, i.e.

$$\begin{cases} \texttt{min}_P & P \sum_{k=1}^K \left(x\sigma^2_{s_k,r} + 1\right) + xL, \\ \texttt{s.t.} & P \geq \frac{\gamma_k \det\left(\mathbf{G}^{(k)}_{\bar{k}}\right)}{\det\left(\mathbf{G}^{(k)}\right)}, \quad k = 1, 2, \ldots, K. \end{cases} \tag{47}$$

We note that the objective function in (47) is increasing in terms of increasing $P > 0$. Thus, the optimal solution P^* of (47) is given as

$$P^* = \max_{k=1,\ldots,K} \left\{ \frac{\gamma_k \det\left(\mathbf{G}^{(k)}_{\bar{k}}\right)}{\det\left(\mathbf{G}^{(k)}\right)} \right\}. \tag{48}$$

Since the channel quality is not the same, in general, for all links, it is implied that the terms $(\gamma_k \det(\mathbf{G}^{(k)}_{\bar{k}}))/(\det(\mathbf{G}^{(k)})), k = 1, 2, \ldots, K$ in (47) are not equal in general, which means that the equality in (47) might not hold for all k. As a result, the equal power allocation strategy generally spends more power than what is needed.

V. Optimum Power Assignment Under a Total Power Budget Constraint

In this section, we apply the methodology that we developed in Section IV to solve a related max-min SINR based power optimization problem for multi-source multi-destination relay networks. Specifically, we design a power assignment scheme that maximizes the minimum SINR among all source-destination pairs subject to any given total power budget. We note that such a scheme introduces a type of fairness among different source-destination pairs. The optimization problem can be formulated as follows:

$$\begin{cases} \texttt{max}_{P_1,\ldots,P_K;P_r} & \texttt{min}_{k=1,\ldots,K}\{\texttt{SINR}_k\} \\ \texttt{s.t.} & \sum_{k=1}^K P_k + P_r \leq P_{\text{total}} \end{cases} \tag{49}$$

where P_{total} is the given power budget of the system.

Let us denote

$$z \triangleq \texttt{min}_{k=1,\ldots,K}\{\texttt{SINR}_k\} \tag{50}$$

then (49) is equivalent to

$$\begin{cases} \texttt{max}_{P_1,\ldots,P_K;P_r} & z, \\ \texttt{s.t.} & \sum_{k=1}^K P_k + P_r \leq P_{\text{total}}, \\ & \texttt{SINR}_k \geq z, k = 1, \ldots, K. \end{cases} \tag{51}$$

According to (27), we may approximate $\texttt{SINR}_k$ as

$$\texttt{SINR}_k \approx \frac{P_k \det\left(\mathbf{G}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}_{\bar{k}}\right)} \tag{52}$$

where the matrices $\mathbf{G}^{(k)}$ and $\mathbf{G}^{(k)}_{\bar{k}}$ are specified in (22). If we define an auxiliary parameter $x \triangleq \alpha^2 P_r$, then for any given $x \geq 0$, the transmission power at the relay P_r is determined as

$$P_r = \frac{x}{\alpha^2} = x \sum_{k=1}^K P_k \beta_{s_k,r} + xL. \tag{53}$$

As a consequence, we can reformulate the problem in (51) as

$$\begin{cases} \texttt{max}_{P_1,\ldots,P_K} & z, \\ \texttt{s.t.} & \sum_{k=1}^K \left(x\beta_{s_k,r} + 1\right) P_k + xL \leq P_{\text{total}} \\ & P_k \geq z \cdot \frac{\det\left(\mathbf{G}^{(k)}_{\bar{k}}\right)}{\det\left(\mathbf{G}^{(k)}\right)}, \quad 1 \leq k \leq K. \end{cases} \tag{54}$$

For any $x \geq 0$, we denote the maximal value of the objective function in (54) as $z(x)$. Then, $z(x)$ must satisfy

$$P_k \geq z(x) \cdot \frac{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)}, \quad \forall k = 1, 2, \ldots, K. \tag{55}$$

Substituting the above constraints into the total power constraint in (54), we have

$$\sum_{k=1}^{K} (x\beta_{s_k,r} + 1)\, z(x) \cdot \frac{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)} + xL \leq P_{\text{total}}. \tag{56}$$

Thus, for any fixed $x \geq 0$, the maximal value of the objective function in (54) is

$$z(x) = \frac{P_{\text{total}} - xL}{\sum_{k=1}^{K} (x\beta_{s_k,r} + 1) \cdot \frac{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)}}. \tag{57}$$

We can see that the maximization of $z(x)$ can be implemented by a numerical search of single (one-dimension) parameter ($x \geq 0$). If we denote the optimal parameter of (57) as x^*, then the optimum power assignment to the sources is

$$P_k^* = z(x^*) \cdot \frac{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)}, \quad k = 1, 2, \ldots, K \tag{58}$$

and the corresponding power assigned to the relay is

$$P_r^* = x^* \left(\sum_{k=1}^{K} P_k^* \beta_{s_k,r} + L \right). \tag{59}$$

We note that $z(x)$ in (57) should be nonnegative for any given x, which implies that $P_{\text{total}} - xL \geq 0$, i.e., $x \leq (P_{\text{total}})/(L)$. Thus, we only need to search the single variable x over the interval $[0, (P_{\text{total}})/(L)]$ to obtain the optimal solution x^* that maximizes the function $z(x)$.

From (52) and (58), we can see that for any $k = 1, 2, \ldots, K$, $\text{SINR}_k = z(x^*)$, which implies that the optimum power assignment achieved through the max-min SINR based optimization in (49) leads to the same SINR values for all source-destination pairs. We recall that for the total power minimization problem in (20), the optimal power assignment ensures that the resulting SINR for each source-destination pair is equal to the SINR requirement of the corresponding source-destination pair, i.e., $\text{SINR}_k = \gamma_k$, $k = 1, \ldots, K$. It is natural to expect that when all source-destination SINR requirements γ_k in (20) are the same, the power assignment strategy for the max-min SINR based optimization and that for the total power minimization based optimization should be the same. This intuitive interpretation can be seen from the derivations in (32) and (57). We note that for reasonably high power budget P_{total}, the term xL in (57) can be ignored, and the maximization of $z(x)$ in (57) is equivalent to the minimization of the dominator which is related to the objective function in (32). If all the source-destination pairs' SINR requirements in (32) are the same, i.e., γ_k, $k = 1, \ldots, K$, then the optimal solutions x^* of $z(x)$ in (57) is also optimal in minimizing $v(x)$ in (32).

For comparison purposes, in the following we illustrate the max-min SINR problem under an equal power allocation. The optimization problem in (49) with an equal power assignment can be expressed as

$$\begin{cases} \mathtt{max}_P & \mathtt{min}_{k=1,\ldots,K}\{\mathtt{SINR}_k\}, \\ \mathtt{s.t.} & (K+1)P \leq P_{\text{total}}, \end{cases} \tag{60}$$

where all the sources and the relay are assigned the same transmission power P. With the SINR approximation in (52) and $z = \min_{1 \leq k \leq K}\{\mathtt{SINR}_k\}$, we have

$$\begin{cases} \mathtt{max}_P & z, \\ \mathtt{s.t.} & P \leq \frac{P_{\text{total}}}{K+1} \\ & P \geq z \cdot \frac{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)}{\det\left(\mathbf{G}^{(k)}\right)}, \quad k = 1, \ldots, K. \end{cases} \tag{61}$$

Thus, the optimal solution is $P = (P_{\text{total}})/(K+1)$ and the maximal value of the worst-case SINR is given by

$$z = \frac{P_{\text{total}}}{K+1} \cdot \max_{1 \leq k \leq K} \left\{ \frac{\det\left(\mathbf{G}^{(k)}\right)}{\det\left(\mathbf{G}_{\bar{k}}^{(k)}\right)} \right\}. \tag{62}$$

We can see that in the equal power assignment scheme, the minimum SINR value among all source-destination pairs is directly determined by the given total power budget value.

VI. Numerical Results

In this section, we perform numerical studies to illustrate the proposed optimum power assignment algorithms. In our studies, we consider a *slow fading* scenario where the coherence time of the channel is much longer than the delay requirement of the channel and then approximate $|h_{i,j}|^2$ by $\sigma_{i,j}^2$ [28]. The channel gain for each channel link is assumed to follow a path loss model, where the variance of channel coefficient is given by $\sigma_{i,j}^2 = \delta_{i,j}^{-\lambda}$ $(i, j \in \{s_k, d_k, r\})$ with $\delta_{i,j}$ as the distance of the channel link and λ as the path-loss exponent ($\lambda = 3$ in our numerical studies).

A. Total Power Minimization

In the first set of numerical studies, we illustrate the optimum power assignment that minimizes the total power consumption under the condition that the SINR requirement of each source-destination pair is satisfied. First, we consider a system with two source-destination pairs and one relay, i.e., $K = 2$, and the cross-correlation of the two source codes is $\rho = 0.25$. We study two cases of SINR requirements for the two source-destination pairs: (i) $[\gamma_1, \gamma_2] = [10, 10]$ dB, and (ii) $[\gamma_1, \gamma_2] = [10, 20]$ dB.

Fig. 2 plots the total power consumption with varying parameter $x \geq 0$ for an *asymmetric* system, where the values of the distance between nodes are set as follows: $\delta_{s_1,d_1} = 2, \delta_{s_1,d_2} = 3, \delta_{s_1,r} = \delta_{s_2,r} = \delta_{s_2,d_1} = \delta_{r,d_1} = 1, \delta_{s_2,d_2} = 3$ and $\delta_{r,d_2} = 2$. The optimal values of the parameter that minimize the total power consumption are $x^* = 0.63$ and $x^* = 1.52$ for $[\gamma_1, \gamma_2] = [10, 10]$ dB and $[10, 20]$ dB, respectively. Based on the optimal value x^*, we obtain the corresponding optimal power assignment P_1, P_2 and P_r according to (31) and (29), as listed in Table I. In this table, we also compare the optimal power values obtained by our proposed approximation method and those obtained by exhaustive search based on the optimization in (20). We observe that the optimal power values obtained by the two methods are almost indistinguishable. In Fig. 3, we plot the total power consumption with varying parameter $x \geq 0$

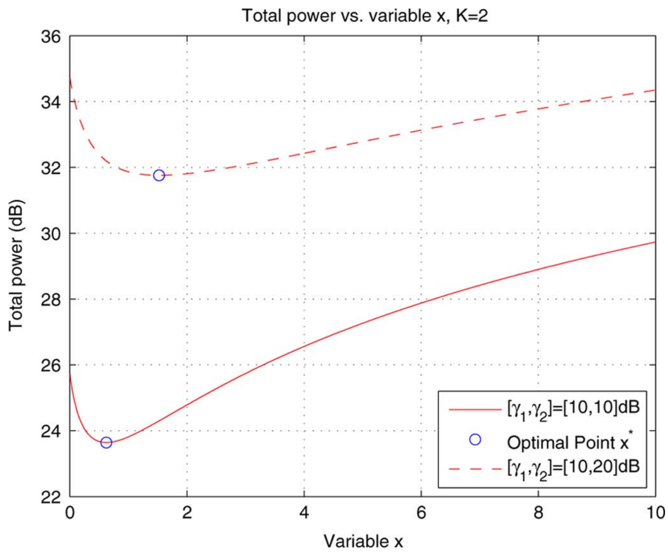

Fig. 2. Minimization of the total power consumption under varying parameter x for an *asymmetric* multi-source multi-destination relay network with given SINR constraints $(K = 2)$.

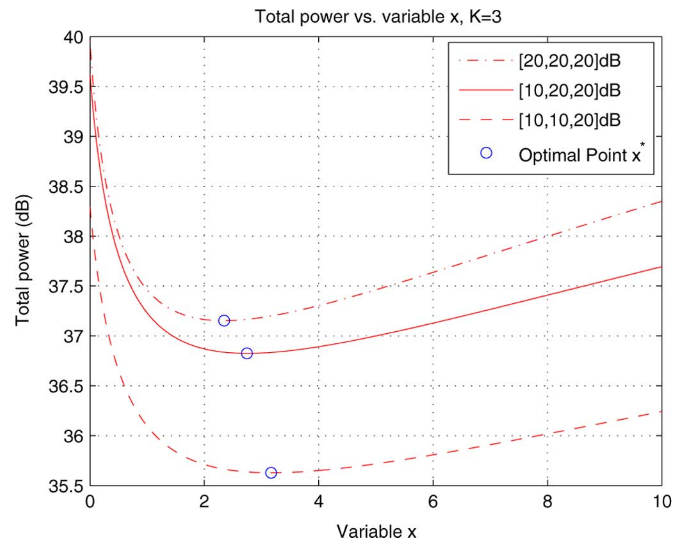

Fig. 4. Minimization of the total power consumption with varying parameter x for an *asymmetric* multi-source multi-destination relay network under given SINR constraints $(K = 3)$.

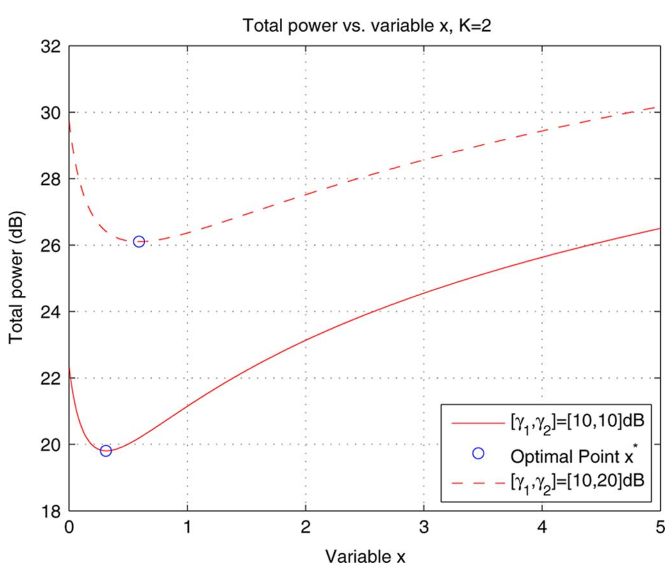

Fig. 3. Minimization of the total power consumption with varying parameter x for a *symmetric* multi-source multi-destination relay network under given SINR constraints $(K = 2)$.

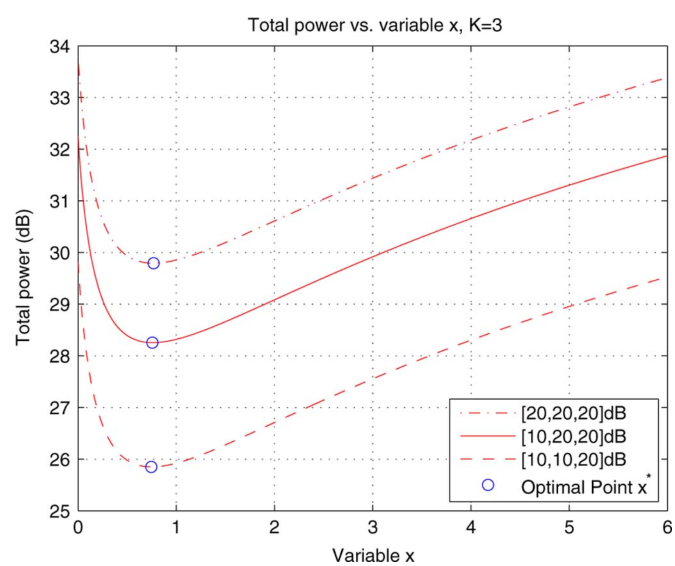

Fig. 5. Minimization of the total power consumption with varying parameter x for a *symmetric* multi-source multi-destination relay network under given SINR constraints $(K = 3)$.

for a *symmetric* system, where the values of the distance between nodes are set as: $\delta_{s_1,d_1} = \delta_{s_2,d_2} = \delta_{s_1,d_2} = \delta_{s_2,d_1} = 2$ and $\delta_{s_1,r} = \delta_{s_2,r} = \delta_{r,d_1} = \delta_{r,d_2} = 1$. The optimal values of the parameter are $x^* = 0.32$ and $x^* = 0.59$ for $[\gamma_1, \gamma_2] = [10, 10]$ dB and $[10, 20]$ dB, respectively. Based on the optimal value x^*, we obtain the corresponding optimal power allocation P_1, P_2 and P_r again according to (31) and (29), as listed in Table II. We observe that when the SINR requirements of the two source-destination pairs are equal, the power assignments to the two sources are the same. However, when the SINR requirements are different ($[\gamma_1, \gamma_2] = [10, 20]$ dB), the equal power assignment is not optimum anymore. We can see in Table II that the optimal power values obtained by our proposed approximation method are indistinguishable from those obtained by the exhaustive search based on the optimization in (20).

We now repeat our studies for a system with three source-destination pairs and one relay, i.e., $K = 3$, where the signatures of the three sources are orthogonal to each other. We examine both an asymmetric case (Fig. 4) and a symmetric case (Fig. 5). In both figures, we consider three sets of SINR requirements for the three source-destination pairs, namely: (i) $[\gamma_1, \gamma_2, \gamma_3] = [20, 20, 20]$ dB, (ii) $[\gamma_1, \gamma_2, \gamma_3] = [10, 20, 20]$ dB, and (iii) $[\gamma_1, \gamma_2, \gamma_3] = [10, 10, 20]$ dB. In Fig. 4, we consider an asymmetric system where the distance values are set as $\delta_{s_k,r} = 1, (k = 1, 2, 3), \delta_{r,d_1} = 1, \delta_{r,d_2} = 2, \delta_{r,d_3} = 3$ and $\delta_{s_1,d_1} = 2, \delta_{s_2,d_2} = 3$ and $\delta_{s_3,d_3} = 4$. In this case, the optimal values of the parameter that minimize the total power consumption are $x^* = 2.34$, $x^* = 2.77$ and $x^* = 3.17$ for the three sets of SINR requirements, respectively. Based on the optimal values, the optimum power assignments P_1, P_2, P_3 and P_r can

TABLE I
OPTIMUM POWER ASSIGNMENT THAT MINIMIZES THE TOTAL POWER UNDER GIVEN SINR CONSTRAINTS USING THE PROPOSED METHOD AND THE EXHAUSTIVE SEARCH METHOD FOR AN *ASYMMETRIC* SETTING ($K = 2$)

	(γ_1, γ_2)(dB)	P_1(dB)	P_2(dB)	P_r(dB)
Proposed method	(10,10)	13.14	19.89	20.54
Exhaustive search	(10,10)	13.22	19.91	20.41
Proposed method	(10,20)	11.59	27.34	29.74
Exhaustive search	(10,20)	11.76	27.28	29.70

TABLE II
OPTIMUM POWER ASSIGNMENT THAT MINIMIZES THE TOTAL POWER UNDER GIVEN SINR CONSTRAINTS USING THE PROPOSED METHOD AND THE EXHAUSTIVE SEARCH METHOD FOR A *SYMMETRIC* SETTING ($K = 2$)

	(γ_1, γ_2)(dB)	P_1(dB)	P_2(dB)	P_r(dB)
Proposed method	(10,10)	14.56	14.56	15.86
Exhaustive search	(10,10)	14.91	14.91	15.31
Proposed method	(10,20)	13.25	23.25	22.44
Exhaustive search	(10,20)	13.32	23.32	22.34

TABLE III
COMPARISON OF THE TOTAL POWER CONSUMPTION RESULTING FROM THE OPTIMUM POWER ASSIGNMENT SCHEME AND THE EQUAL POWER ASSIGNMENT SCHEME ($K = 3$)

	$\gamma_1, \gamma_2, \gamma_3$ (dB)	Total Power Consumption (dB)	
		Optimum	Equal
Asymmetric System	[20,20,20]	37.15	41.58
	[10,20,20]	36.82	41.58
	[10,10,20]	35.63	41.58
Symmetric System	[20,20,20]	29.79	30.30
	[10,20,20]	28.25	30.30
	[10,10,20]	25.85	30.30

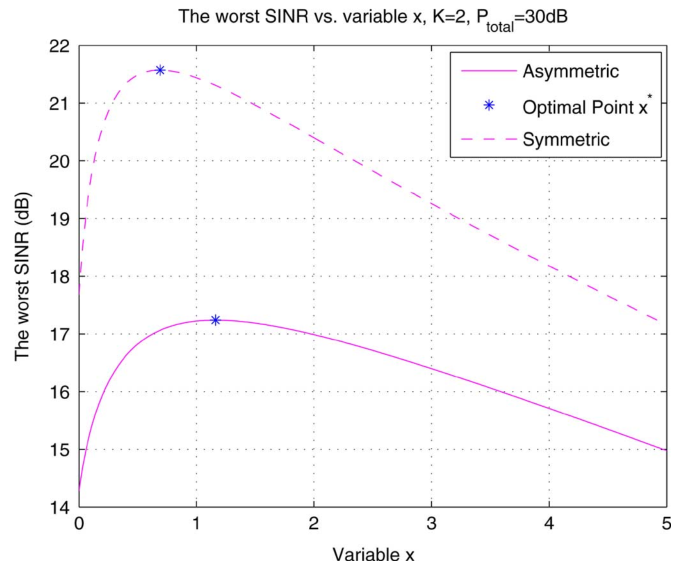

Fig. 6. Maximization of the minimum SINR among all source-destination pairs under given total power budget $P_{\text{total}} = 30$ dB ($K = 2$).

TABLE IV
OPTIMUM POWER ASSIGNMENT THAT MAXIMIZES THE MINIMUM SINR AMONG TWO SOURCE-DESTINATION PAIRS UNDER GIVEN TOTAL POWER BUDGET $P_{\text{total}} = 30$ DB, BY USING THE PROPOSED METHOD AND THE EXHAUSTIVE SEARCH METHOD ($K = 2$)

		P_1(dB)	P_2(dB)	P_r(dB)
Asymmetric System	Proposed method	19.22	25.34	27.59
	Exhaustive search	19.37	25.45	27.66
Symmetric System	Proposed method	24.49	24.49	26.41
	Exhaustive search	24.64	24.64	26.50

be obtained accordingly based on (31) and (29). In Fig. 5, we consider a symmetric system with $\delta_{s_k,r} = \delta_{r,d_k} = 1, (k = 1,2,3)$ and $\delta_{s_k,d_k} = 2, (k = 1,2,3)$. The optimal values of the parameter are $x^* = 0.77$, $x^* = 0.76$ and $x^* = 0.75$ for the three sets of SINR requirements, respectively. We can see that the optimal values x^* in the symmetric system are almost the same for the three sets of SINR requirements. This result is consistent with the theoretical discussion that the optimal value x^* is independent of the SINR requirement γ_k but it depends on the channel condition.

In Table III, we show the power efficiency of the proposed optimum power assignment scheme by listing the resulting total power consumption, compared to the total power consumption that results from the equal power assignment scheme. We consider both an asymmetric case (the system setup is the same as that in Fig. 4) and a symmetric case (the system setup is the same as that in Fig. 5). We can see that in the asymmetric system, the power savings of the optimum power assignment scheme is 4–6 dB for the three sets of SINR requirements. The more unbalanced the SINR requirements of the three source-destination pairs are, the more power savings of the optimum power assignment scheme compared to the equal power assignment scheme can be achieved. In the symmetric system, the power savings of the optimum power assignment scheme is 0.5–4.5dB for the three sets of SINR requirements. Comparing the results between the asymmetric system and the symmetric system, we observe that the optimum power assignment scheme gains more power savings in the asymmetric system than in the symmetric system. Also, we observe that the total power consumption of the equal power assignment scheme is the same in each of the asymmetric and symmetric systems, which is consistent with our previous discussion that the equal power assignment depends only on the most challenging/weakest source-destination pair.

B. Max-Min SINR Optimization

In the second set of numerical studies, we illustrate the optimum power assignment algorithm that maximizes the minimum SINR among all source-destination pairs under a given total power budget. We consider a system with two source-destination pairs and one relay, i.e., $K = 2$, with a total power budget $P_{\text{total}} = 30$ dB. We assume that the cross-correlation of the two source codes is $\rho = 0.25$. We consider both an asymmetric case (the system setup is the same as that in Fig. 2) and a symmetric case (the system setup is the same as that in Fig. 3).

Fig. 6 shows the maximization of the minimum SINR with varying parameter $x \geq 0$ for both the asymmetric and symmetric systems, respectively. The optimal values of the parameter that maximize the minimum SINR of all source-destination pairs are $x^* = 1.16$ for the asymmetric system and $x^* = 0.69$ for the symmetric system. For both the asymmetric and symmetric cases, the optimum power assignments P_1, P_2 and P_r are determined based on (58) and (59) with the corresponding optimal value x^*, listed in Table IV. In the table, we also compare the optimal power values obtained by the proposed approximation method and those obtained by exhaustive search based on the optimization in (51). We observe that the optimal power

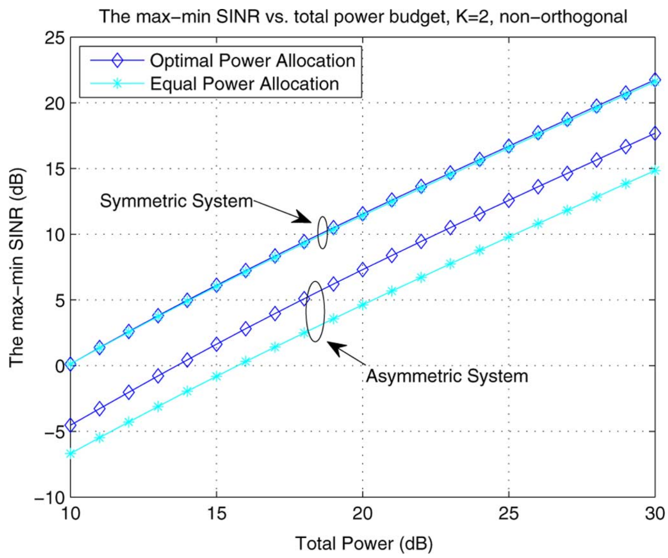

Fig. 7. Comparison of the minimum SINR resulting from the proposed optimum power assignment scheme and the equal power assignment scheme with any given total power budget ($K = 2$).

values of the power assignment obtained by the two methods are indistinguishable. We note that for the symmetric case, the sources are allocated equal power under the max-min SINR optimization scheme, while the relay utilizes transmission power which is some what different from that allocated to the sources.

Finally, in Fig. 7, we compare the minimum SINR resulting from the proposed optimum power assignment scheme and the minimum SINR resulting from the equal power assignment scheme. We consider both an asymmetric case and a symmetric case, and the system setup is the same as that in Fig. 6. We can see that in the asymmetric case, the minimum SINR of the two source-destination pairs is improved significantly (by at least 2 dB) when we use the proposed optimum power assignment scheme instead of the equal power assignment scheme. In the symmetric case, we can see that the performance of the equal power assignment scheme is very close to that of the optimum power assignment scheme (actually, in this case all the source-destination pairs have the same performance).

VII. Conclusion

In this paper, we analyzed and optimized a multi-source multi-destination relay network where a relay amplifies and forwards simultaneously the signals received from all sources. We developed optimum power assignment schemes for two scenarios. The first scheme minimizes the total power consumption of all sources and the relay under the constraint that the SINR requirement of each source-destination pair is satisfied, while the second scheme maximizes the minimum SINR of all source-destination pairs with any given total power budget. Clearly, both optimization problems as stated above involve K power variables, where K is the number of source-destination pairs in the network, which implies that an exhaustive search approach is prohibitive for large K. In this paper, we derived an asymptotically tight approximation of the SINR that allows us to reformulate the original optimization problems, and eventually reduce them to single-parameter optimization problems, which can be easily solved by numerical search of the single parameter. Then, the corresponding optimal transmission power at each source and at the relay can be calculated directly. Each of the proposed optimization scheme is scalable and the power assignment algorithm has the same optimization complexity for any number of source-destination pairs in the network. For the special case of transmission over orthogonal channels under quasi-static channel conditions, we were able to further simplify the proposed single-variable optimization scheme, and obtain an analytical solution for a symmetric system. Numerical studies were provided to illustrate and validate our theoretical developments.

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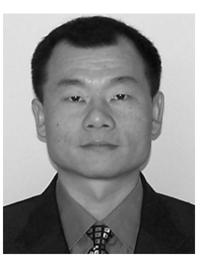

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