Orthogonal-Like Space–Time-Coded CPM Systems With Fast Decoding for Three and Four Transmit Antennas

Genyuan Wang, *Member, IEEE*, Weifeng Su, *Member, IEEE*, and Xiang-Gen Xia, *Fellow, IEEE*

***Abstract*—The Alamouti orthogonal space–time block code for two transmit antennas was designed primarily for QAM and PSK modulations, and we have previously generalized it for the continuous phase modulation (CPM), denoted as OST-CPM, by maintaining the orthogonality (for the fast ML decoding/demodulation) and the phase continuity of two signals from two transmit antennas. In this paper, we design orthogonal-like space–time coded CPM systems for three and four transmit antennas based on orthogonal and quasi-orthogonal space–time codes. Although the signals from transmit antennas in the proposed orthogonal-like space–time coded CPM systems are not orthogonal, the fast decoding/demodulation is maintained like the two transmit antenna case. Simulation results show that the performance of the proposed orthogonal-like space–time coded CPM systems for four transmit antennas is much better than that of the OST-CPM systems for two transmit antennas.**

***Index Terms*—Continuous phase modulation (CPM), orthogonal space–time block codes, quasi-orthogonal space–time block codes, space–time coding.**

I. INTRODUCTION

CONTINUOUS phase modulation (CPM) systems with single transmit antenna have been widely used in wireless systems due to its spectral efficiency and resistance to wireless channel fading [1]. In recent years, space–time coding for multiple transmit antennas has attracted much attention due to its capability of combating severe channel fading and increasing system capacity in wireless communications, see, for example, [2]–[29], and the references therein. A natural and interesting idea is to consider space–time coded CPM systems to take advantages of both spectral efficiency and system performance improvement. In [14], Zhang and Fitz proposed trellis space–time coding for CPM systems. A similar scheme was also proposed in [15]. Due to the computational complexity issue, in this paper, we consider block space–time coded CPM systems that have fast decoding/demodulation algorithms.

Based on the Alamouti's scheme [5], we have previously proposed a CPM system with orthogonal space–time (OST) coding for two transmit antennas [18], [28], [29] where the orthogonality and the continuity of the two signal phases from two transmit antennas at any time t are maintained. The orthogonality provides us a fast maximum-likelihood (ML) decoding which is similar to the Alamouti's scheme with QAM modulations. The difficulty of the design comes from the maintaining of both the phase continuity and the orthogonality of the signals from two transmit antennas.

As it is already a challenge task to design high rate orthogonal space–time codes for more than two transmit antennas for QAM modulations [6], [10]–[13], it is even more challenging to keep the continuity of the signal phases if we apply the codes for CPM systems. Although there exist orthogonal space–time codes of rate 3/4 for three and four transmit antennas, unfortunately, they cannot be directly used in the OST-CPM systems. For example, for four transmit antennas, the following well-known orthogonal space–time code [7]–[10]

$$\begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 \\ x_2 & x_1^* & 0 & -x_3^* \\ x_3 & 0 & -x_1^* & x_2^* \\ 0 & x_3 & x_2 & x_1 \end{bmatrix} \tag{1}$$

does not suit for CPM systems, since there are some zero values in the code matrix which affects the continuity of the signal phases in each antenna transmissions. Notice that for 4 transmit antennas, there are other orthogonal space–time codes with linear processing of symbols, for example in [6], but it is also hard to use them in the OST-CPM systems because it is hard to guarantee the phase continuity of the transmission signals if each signal is a linear combination of several symbols.

In this paper, for 4 transmit antennas, we modify the orthogonal space–time code (1) to have the following format

$$\begin{bmatrix} x_1 & -x_2^* & x_3^* & e^{j\phi_1} \\ x_2 & x_1^* & e^{j\phi_2} & -x_3^* \\ x_3 & e^{j\phi_3} & -x_1^* & x_2^* \\ e^{j\phi_4} & x_3 & x_2 & x_1 \end{bmatrix} \tag{2}$$

where ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are some real constants which will be specified later. Clearly, it has the same full diversity as the code in (1) with symbols x_1, x_2, and x_3. Notice that, the modified code (2) does not satisfy the orthogonality condition as the code

Manuscript received April 27, 2004; revised October 03, 2009. Current version published March 10, 2010. This work was supported in part by the Air Force Office of Scientific Research (AFOSR) by Grant No. F9550-08-1-0219. The material in this paper was presented in part at IEEE Globecom 2003, San Francisco, December 2003.

G. Wang was with the Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716 USA. He is now with Cisco Systems, Richardson, TX 75082 USA (e-mail: genyuanwangzz@gmail.com).

W. Su was with the Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716 USA. He is now with the Department of Electrical Engineering, State University of New York at Buffalo, Buffalo, NY 14260 USA (e-mail: weifeng@eng.buffalo.edu).

X.-G. Xia is with the Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716 USA (e-mail: xxia@ee.udel.edu).

Communicated by B. S. Rajan, Associate Editor for Coding Theory.

Color versions of Figures 2 and 3 in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TIT.2009.2039058

(1), but its behavior in ML decoding is similar to that of the code (1) and the fast ML decoding is maintained as we shall see in Section III-B. In this paper, we design a CPM system based on the modified code in (2), which guarantees a fast decoding algorithm. Specifically, if we let $s_m(t)$ be the signal transmitted at the mth antenna, they are designed such that their phases are continuous and the signal matrix S in (3) at the bottom of this page follows the form of the code in (2) for any $t \in [0, T]$ and any integer l. Similar to the OST-CPM for two transmit antennas in [18], [28], and [29], the main difficulty is to maintain the phase continuity for the signals $s_m(t)$ at each transmit antenna while preserving certain orthogonality for fast ML decoding.

One of the most important advantages of orthogonal space–time block codes (from orthogonal designs) is that they have the fast ML decoding and all information symbols can be decoded individually. However, the shortcoming of complex orthogonal space–time codes is its rate limitation. In [20], it was shown that the rates are upper bounded by 3/4 for three or more transmit antennas with or without linear processing in the code design, and this bound was first shown in [10] for codes without linear processing. In other words, the rate of the code in (1) is already optimal no mater how large a block size or time delay is. To increase the code rate, quasi-orthogonal space–time codes have been proposed by Jafarkhani [16], and Tirkkonen, Boariu and Hottinen [17] by relaxing the orthogonality. They constructed quasi-orthogonal space–time block codes for four transmit antennas with rate 1 from quasi-orthogonal designs. With the relaxed orthogonality, the ML decoding of 4 information symbols becomes the decoding of two independent information symbol pairs. The decoding complexity is higher than that of the orthogonal spca-time block code for four transmit antennas.

The quasi-orthogonal space–time codes for 4 transmit antennas and 4 information symbols proposed by Jafarkhani, and Tirkkonen, Boariu and Hottinen have rank 2, i.e., they do not have full diversity. In [21], rate-1 quasi-orthogonal space–time codes with full diversity were designed and optimized for any QAM constellation and constellations on square lattice or equilateral triangular lattice.

In this paper, we also design CPM systems based on the quasi-orthogonal space–time coding for three and four transmit antennas. The resulting CPM systems have better performance than the CPM system using the modified orthogonal space–time code of rate 3/4 in (2). The CPM systems with quasi-orthogonal space–time coding still have a fast decoding algorithm, but the decoding complexity is higher than that of the CPM system based on the code in (2) and the difference is similar to that between the orthogonal and the quasi-orthogonal space–time codes as mentioned above.

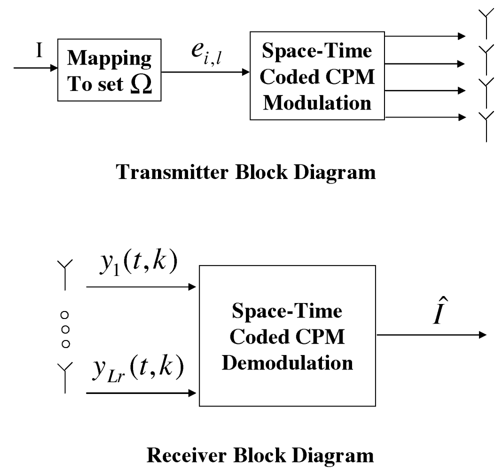

Fig. 1. Space-Time CPM Diagram.

In the following, we discuss the design of space–time coded CPM systems primarily for four transmit antennas and the design for three transmit antennas can be obtained by simply deleting one of the four columns in each code. The paper is organized as follows. In Section II, we describe the system model with a general block space–time coding. In Section III, we design a full response CPM system with the modified orthogonal space–time code for four transmit antennas, and also present a fast decoding algorithm. In Section IV, we design a quasi-orthogonal space–time coded CPM system and present a fast decoding algorithm accordingly. We present simulation and comparison results in Section V, and finally, conclude in Section VI.

Notations: We denote $q(t)$ and $q_0(t)$ as phase smoothing response functions in the CPM systems; denote h_m as the modulation index of the CPM system; and T as the symbol time duration.

II. System Model

In this paper, we consider a CPM communication system with four transmit antennas and one receive antenna as shown in Fig. 1. It can be straightforwardly extended to a system with more than one receive antennas. We adopt some notations from [14]. For an information sequence $\mathbf{I} = (I_{1,0}, \ldots, I_{L,0}, \ldots, I_{1,l}, \ldots, I_{L,l}, \ldots)$, each information block $I_{1,l}, \ldots, I_{L,l}$ of length L is mapped to an information symbol matrix such as

$$\mathbf{d}_l = \begin{bmatrix} d_{1,1}(l) & d_{1,2}(l) & d_{1,3}(l) & d_{1,4}(l) \\ d_{2,1}(l) & d_{2,2}(l) & d_{2,3}(l) & d_{2,4}(l) \\ d_{3,1}(l) & d_{3,2}(l) & d_{3,3}(l) & d_{3,4}(l) \\ d_{4,1}(l) & d_{4,2}(l) & d_{4,3}(l) & d_{4,4}(l) \end{bmatrix} \qquad (4)$$

$$S = \begin{bmatrix} s_1(t+4lT) & s_1(t+(4l+1)T) & s_1(t+(4l+2)T) & s_1(t+(4l+3)T) \\ s_2(t+4lT) & s_2(t+(4l+1)T) & s_2(t+(4l+2)T) & s_2(t+(4l+3)T) \\ s_3(t+4lT) & s_3(t+(4l+1)T) & s_3(t+(4l+2)T) & s_3(t+(4l+3)T) \\ s_4(t+4lT) & s_4(t+(4l+1)T) & s_4(t+(4l+2)T) & s_4(t+(4l+3)T) \end{bmatrix}. \qquad (3)$$

where all entries $d_{m,n}(l)$ are modulation symbols coming from a signal constellation, for example from the following pulse-amplitude-modulated (PAM) signal constellation with a constellation size $2M$:

$$\Omega \triangleq \{-2M+1, -2M+3, \ldots, -1, 1, \ldots, 2M-1\}. \quad (5)$$

During the lth time period $[4lT, 4(l+1)T]$ with symbol time duration T, the information symbol matrix $\mathbf{d}_l$ is used to generate the following signal matrix (6) at the bottom of the page. The mth row of the signal matrix S is transmitted by the mth transmit antenna. In time period $(4l+n-1)T \le t \le (4l+n)T$, all signals in the nth column of the matrix S are transmitted simultaneously, and we denote this time period as the $(4l+n)$th time slot for $n = 1, 2, 3, 4$.

For any $n = 1, 2, 3, 4$, the received signal $y(t,k)$ at time slot $k \triangleq 4l + n$ can be written as [1], [14]:

$$y(t,k) = \sum_{m=1}^{4} \alpha_m(t) s_m(t,k) + w(t) \quad (7)$$

where $w(t)$ is the additive noise, $\alpha_m(t)$ is the channel gain from the mth transmit antenna to the receive antenna, and $s_m(t,k)$ is the transmitted signal from the mth transmit antenna at time slot k which is given by

$$s_m(t,k) = \sqrt{\frac{1}{T}} \exp\{j2\pi \Phi_m(t,k)\}. \quad (8)$$

The phase term $\Phi_m(t,k)$ in (8) contains the modulation symbols $d_{m,n}(l)$ and is specified as follows:

$$\Phi_m(t,k) = \sum_{i=0}^{k} \{h_m d_{m,i} q(t-(i-1)T) + c_{m,i} q_0(t-(i-1)T)\}, \quad (9)$$

where $(k-1)T \le t \le kT$, $d_{m,i} \triangleq d_{m,n}(l)$ for any $i = 4l+n$, and h_m is the modulation index of the CPM system. For simplicity, the phase smoothing response functions $q(t)$ and $q_0(t)$ in (9) are selected as the follows

$$q(t) = q_0(t) = \begin{cases} 0 & \text{if } t \le 0, \\ 1/2 & \text{if } t \ge T. \end{cases} \quad (10)$$

In (9), $c_{m,i} \triangleq c_{m,n}(l)$ for any $i = 4l + n$ is generated by the following matrix

$$\mathbf{c}_l = \begin{bmatrix} c_{1,1}(l) & c_{1,2}(l) & c_{1,3}(l) & c_{1,4}(l) \\ c_{2,1}(l) & c_{2,2}(l) & c_{2,3}(l) & c_{2,4}(l) \\ c_{3,1}(l) & c_{3,2}(l) & c_{3,3}(l) & c_{3,4}(l) \\ c_{4,1}(l) & c_{4,2}(l) & c_{4,3}(l) & c_{4,4}(l) \end{bmatrix} \quad (11)$$

which depends on the information symbol matrix $\mathbf{d}_l$ and will be specified later. The choice of matrix $\mathbf{c}_l$ plays a critical role and it is used to ensure that the rows of the transmitted signal matrix S in (6) have some orthogonality, and therefore a fast decoding algorithm can be developed. Notice that, the transmitted signal here can be viewed as a nontrivial extension[1] of that in [18], [28], [29] from two transmit antennas to four transmit antennas.

If the modulation index h_m is chosen as $h_m = h \triangleq m_0/p$ for two relatively prime integers m_0 and p, then the phase $\Phi_m(t,k)$ at time period $(k-1)T \le t \le kT$ can be expressed as [1]:

$$\Phi_m(t,k) = \theta_m(k-\gamma) + \sum_{i=k-\gamma+1}^{k} \{h d_{m,i} q(t-(i-1)T) + c_{m,i} q_0(t-(i-1)T)\}, \quad (12)$$

where γ is the modulation memory size and

$$\theta_m(k-\gamma) = \frac{h}{2} \sum_{i \le k-\gamma} d_{m,i} + \frac{1}{2} \sum_{i \le k-\gamma} c_{m,i} \quad (13)$$

belongs (after modulo 1) to the set Ω_θ defined as:

$$\Omega_\theta \triangleq \left\{0, \frac{1}{p}, \frac{2}{p}, \ldots, \frac{p-1}{p}\right\}. \quad (14)$$

When $\gamma = 1$, the system is called a *full response* CPM system. When $\gamma > 1$, the system is called a *partial response* CPM system. In this paper, we focus on the full response CPM systems. The discuss of a partial response STC-CPM design is similar but much more complicated, see for example [19] for two transmit antennas.

In a full response CPM system, the phase $\Phi_m(t,k)$ at time period $(k-1)T \le t \le kT$ is given by

$$\Phi_m(t,k) = \theta_m(k-1) + h d_{m,k} q(t-(k-1)T) + c_{m,k} q_0(t-(k-1)T). \quad (15)$$

Thus, $\Phi_m(t,k)$ has a trellis structure with states in the set Ω_θ, and for the above space–time coded CPM system, $(\Phi_1(t,k), \Phi_2(t,k), \Phi_3(t,k), \Phi_4(t,k))$ has a trellis structure with states in the product set $\Omega_\theta \times \Omega_\theta \times \Omega_\theta \times \Omega_\theta$. One can see that, in general, the number of states increases exponentially with the number of transmit antennas which is 4 in this case. The current symbol tuple $(d_{1,k}, d_{2,k}, d_{3,k}, d_{4,k})$ drives a state transfer and generates a branch from current state to next state.

[1]Note that the orthogonal space–time code (2) for 4 transmit antennas is not a trivial extension of the one for two transmit antennas.

$$S = \begin{bmatrix} s_1(t,4l+1) & s_1(t,4l+2) & s_1(t,4l+3) & s_1(t,4l+4) \\ s_2(t,4l+1) & s_2(t,4l+2) & s_2(t,4l+3) & s_2(t,4l+4) \\ s_3(t,4l+1) & s_3(t,4l+2) & s_3(t,4l+3) & s_3(t,4l+4) \\ s_4(t,4l+1) & s_4(t,4l+2) & s_4(t,4l+3) & s_4(t,4l+4) \end{bmatrix}. \quad (6)$$

The ML demodulation of the information sequence $\mathbf{I}$ over time period $0 \leq t \leq 4lT$ is [1], [14]

$$\hat{\mathbf{I}} = \arg\min_{\mathbf{I}} \left\{ \sum_{k=1}^{4l} \int_0^T \left| y(t,k) - \sum_{m=1}^{4} \alpha_m(t) s_m(t,k) \right|^2 dt \right\}. \quad (16)$$

When a Viterbi algorithm is considered to solve the above ML demodulation, each state in the trellis structure has $(2M)^{L_0}$ coming branches and $(2M)^{L_0}$ leaving branches, where L_0 is the number of independent symbols $d_{i,k}$ in the symbol tuple $(d_{1,k}, d_{2,k}, d_{3,k}, d_{4,k})$. The decoding complexity is thus prohibitive if there is no fast searching algorithm for the trellis branches in the ML decoding. In the following sections, we propose two different designs for the information symbol matrix $\mathbf{d}_l$ for two space–time coded CPM schemes, respectively. In our designs, the branches at each state can be decomposed into several independent sets, and thus the branch searching (therefore the ML demodulation complexity) can be greatly reduced as we shall see in more details in next sections.

III. Full Response CPM System With Modified Orthogonal Space–Time Coding

In this section, we design a CPM system based on the modified orthogonal space–time code (2) for four transmit antennas and propose a fast decoding/demodulation algorithm.

A. Design CPM Signals

A binary information sequence $\{\ldots, I_{1,l}, \ldots, I_{L,l}, \cdots\}$ is mapped to a symbol sequence $\{\ldots, e_{1,l}, e_{2,l}, e_{3,l}, \cdots\}$, where $L = 3\log_2(2M)$ and symbols are chosen from the signal constellation Ω specified in (5). The information symbol matrix $\mathbf{d}_l$ in (4) is constructed as follows:

$$\mathbf{d}_l = \begin{bmatrix} e_{1,l} & -e_{2,l} & -e_{3,l} & 0 \\ e_{2,l} & -e_{1,l} & 0 & -e_{3,l} \\ e_{3,l} & 0 & -e_{1,l} & -e_{2,l} \\ 0 & e_{3,l} & e_{2,l} & e_{1,l} \end{bmatrix}. \quad (17)$$

In this case, the information symbol tuple $(d_{1,k}, d_{2,k}, d_{3,k}, d_{4,k}) = (e_{1,k}, e_{2,k}, e_{3,k}, 0)$ and $e_{i,k}, k = 1,2,3$, are three independent symbols from the constellation Ω.

To generate the CPM signal waveforms $s_m(t,k)$ in (8), we also need the matrix $\mathbf{c}_l$ in (11), which is related to the information symbol matrix $\mathbf{d}_l$ and is specified as follows: [see (18) at the bottom of the page], where

$$a_{i,l} = \mathrm{mod}\left(\frac{e_{i,l} m_0}{p}, 2\right), \quad i = 1,2,3 \quad (19)$$

where $\mathrm{mod}(x,y)$ is the modulo operation of x with base y and $m_0/p = h$ is the modulation index of the CPM system. The reason of taking modulo 2 rather modulo 1 in the phase component is due to the fact that the smoothing response function is $q(T) = q_0(T) = 1/2$ in (10) and $1/2$ appears in the phase modulation in (13). We can see that the matrix $\mathbf{c}_l$ depends only on $a_{1,l}, a_{2,l}$ and $a_{3,l}$, and all of $a_{i,l}$ have at most $2p_0$ possible values for all possible values of $e_{i,l}$ in Ω, where

$$p_0 = \begin{cases} p, & \text{if } p \text{ is odd} \\ p/2, & \text{if } p \text{ is even} \end{cases} \quad (20)$$

since all of $e_{1,l}, e_{2,l}$ and $e_{3,l}$ are odd numbers, and m_0 and p are relatively prime integers.

We now specify the transmission signals. At the time period between $4lT$ and $4(l+1)T$, the following signals are sent through the mth transmit antenna

$$s_m(t, 4l+n) = \exp\{j2\pi\Phi_m(t, 4l+n)\} \quad n = 1,2,3,4 \quad (21)$$

in which $(4l+n-1)T \leq t \leq (4l+n)T$

$$\begin{aligned} \Phi_m(t, 4l+n) &= \theta_m(4l+n-1)) \\ &\quad + h d_{m,n}(l) q(t - (4l+n-1))T) \\ &\quad + c_{m,n}(l) q_0(t - (4l+n-1)T) \end{aligned} \quad (22)$$

and

$$\theta_m(4l+n-1) = \frac{h}{2} \sum_{i \leq 4l+n-1} d_{m,i} + \frac{1}{2} \sum_{i \leq 4l+n-1} c_{m,i} \quad (23)$$

where for any $i' = 4l + n', d_{m,i'} = d_{m,n'}(l)$ and $c_{m,i'} = c_{m,n'}(l)$ come from the matrices $\mathbf{d}_l$ and $\mathbf{c}_l$ in (17) and (18), respectively.

One can check that the transmitted signals have continuous phases at each transmit antenna. In the following, we want to check that during time period $4lT \leq t \leq 4(l+1)T$, the transmitted signal matrix S in (6) has a special structure like (2), and, therefore, a fast decoding algorithm can be developed as we shall see later. In fact, the 4×4 transmitted signal matrix

$$\mathbf{c}_l = \begin{bmatrix} 1 - a_{1,l} & 1 + a_{2,l} + a_{3,l} & 1 + a_{2,l} + a_{3,l} & 1 + a_{1,l} \\ -a_{2,l} & a_{1,l} + a_{3,l} & 1 + a_{2,l} & a_{1,l} + a_{3,l} \\ 1 - a_{3,l} & a_{3,l} & 1 + a_{1,l} + a_{2,l} & 1 + a_{1,l} + a_{2,l} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (18)$$

can be further specified in (24) at the bottom of this page, where $0 \leq \tau < T$. For simplicity, let

$$x_i = e^{j2\pi[e_{i,l}hq(\tau) - a_{i,l}q_0(\tau)]}, \quad i = 1, 2, 3 \tag{25}$$

then the above signal matrix S can be written as the form in (26) at the bottom of the page, where

$$E = \mathrm{diag}\left\{e^{j2\pi[\theta_1(4l)+q_0(\tau)]}, e^{j2\pi\theta_2(4l)} \right.$$
$$\left. e^{j2\pi[\theta_3(4l)+q_0(\tau)]}, e^{j2\pi\theta_4(4l)}\right\} \tag{27}$$

and

$$F = \mathrm{diag}\left\{1, e^{j2\pi a_{3,l}q_0(\tau)}, e^{j2\pi a_{2,l}q_0(\tau)} \right.$$
$$\left. e^{j2\pi a_{1,l}q_0(\tau)}\right\}. \tag{28}$$

Let

$$C(x_1, x_2, x_3) \triangleq \begin{bmatrix} x_1 & -x_2^* & x_3^* & -1 \\ x_2 & x_1^* & e^{j2\pi q_0(\tau)} & -x_3^* \\ x_3 & -e^{-j2\pi q_0(\tau)} & -x_1^* & x_2^* \\ 1 & x_3 & x_2 & x_1 \end{bmatrix}. \tag{29}$$

Then, according to (23), it is easy to check that

$$S = EC(x_1, x_2, x_3)F_1 \tag{30}$$

where

$$F_1 = \mathrm{diag}\left\{1, 1, e^{j\pi a_{3,l}}, e^{j\pi(a_{3,l}+a_{2,l})}\right\} F. \tag{31}$$

Notice that $C(x_1, x_2, x_3)$ in (29) has the same structure as the code in (2) and the matrices E and F_1 in S are diagonal and unitary.

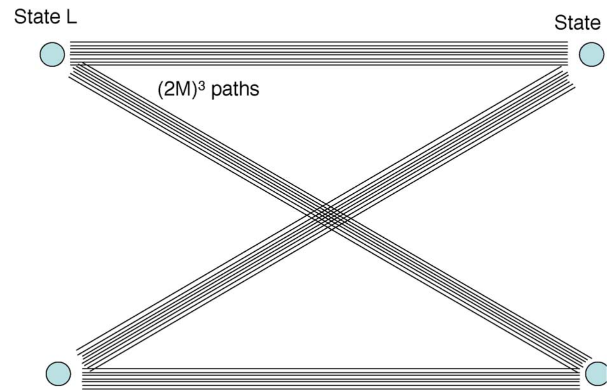

Fig. 2. Trellis structure of STC-CPM without fast-demodulation algorithm.

B. Fast Demodulation Algorithm

By the trellis structure of the CPM transmission signals, the sequence detection in (16) can be implemented using the Viterbi algorithm. The trellis structure of the STC-CPM demodulation is illustrated in Fig. 2. For each state of the trellis, there are $(2M)^3$ coming branches and $(2M)^3$ leaving branches since $L_0 = 3$ in this case. In order to search the survivor paths, the input symbol block $(e_{1,l}, e_{2,l}, e_{3,l})$ and the branch metric from one state $\theta_m(4l)$ to the next state $\theta_m(4(l+1))$ needs to be calculated and compared, where the input symbol block $(e_{1,l}, e_{2,l}, e_{3,l})$ drives the state transfer from $\theta_m(4l)$ to $\theta_m(4(l+1))$. Thus, we need to search all the branch metrics at the stage l as follows:

$$(\hat{e}_{1,l}, \hat{e}_{2,l}, \hat{e}_{3,l}) = \arg\min_{(e_{1,l}, e_{2,l}, e_{3,l}) \in \Omega\times\Omega\times\Omega} \left\{ \sum_{k=4l+1}^{4(l+1)} \int_{(k-1)T}^{kT} \left| y(t,k) - \sum_{m=1}^{4} \alpha_m(t) s_m(t,k) \right|^2 dt \right\}. \tag{32}$$

$$S = \begin{bmatrix} e^{j2\pi[\theta_1(4l)+e_{1,l}hq(\tau)+(1-a_{1,l})q_0(\tau)]} & e^{j2\pi[\theta_1(4l+1)-e_{2,l}hq(\tau)+(1+a_{2,l}+a_{3,l})q_0(\tau)]} & e^{j2\pi[\theta_1(4l+2)-e_{3,l}hq(\tau)+(1+a_{2,l}+a_{3,l})q_0(\tau)]} & e^{j2\pi[\theta_1(4l+3)+(1+a_{1,l})q_0(\tau)]} \\ e^{j2\pi[\theta_2(4l)+e_{2,l}hq(\tau)-a_{2,l}q_0(\tau)]} & e^{j2\pi[\theta_2(4l+1)-e_{1,l}hq(\tau)+(a_{1,l}+a_{3,l})q_0(\tau)]} & e^{j2\pi[\theta_2(4l+2)+(1+a_{2,l})q_0(\tau)]} & e^{j2\pi[\theta_2(4l+3)-e_{3,l}hq(\tau)+(a_{1,l}+a_{3,l})q_0(\tau)]} \\ e^{j2\pi[\theta_3(4l)+e_{3,l}hq(\tau)+(1-a_{3,l})q_0(\tau)]} & e^{j2\pi[\theta_3(4l+1)+a_{3,l}q_0(\tau)]} & e^{j2\pi[\theta_3(4l+2)-e_{1,l}hq(\tau)+(1+a_{1,l}+a_{2,l})q_0(\tau)]} & e^{j2\pi[\theta_3(4l+3)-e_{2,l}hq(\tau)+(1+a_{1,l}+a_{2,l})q_0(\tau)]} \\ e^{j2\pi\theta_4(4l)} & e^{j2\pi[\theta_4(4l+1)+e_{3,l}hq(\tau)]} & e^{j2\pi[\theta_4(4l+2)+e_{2,l}hq(\tau)]} & e^{j2\pi[\theta_4(4l+3)+e_{1,l}hq(\tau)]} \end{bmatrix} \tag{24}$$

$$S = E \begin{bmatrix} x_1 & e^{j2\pi[\theta_1(4l+1)-\theta_1(4l)]}x_2^* & e^{j2\pi[\theta_1(4l+2)-\theta_1(4l)]}x_3^* & e^{j2\pi[\theta_1(4l+3)-\theta_1(4l)]} \\ x_2 & e^{j2\pi[\theta_2(4l+1)-\theta_2(4l)]}x_1^* & e^{j2\pi[\theta_2(4l+2)-\theta_2(4l)+q_0(\tau)]} & e^{j2\pi[\theta_2(4l+3)-\theta_2(4l)]}x_3^* \\ x_3 & e^{j2\pi[\theta_3(4l+1)-\theta_3(4l)-q_0(\tau)]} & e^{j2\pi[\theta_3(4l+2)-\theta_3(4l)]}x_1^* & e^{j2\pi[\theta_3(4l+3)-\theta_3(4l)]}x_2^* \\ 1 & e^{j2\pi[\theta_4(4l+1)-\theta_4(4l)]}x_3 & e^{j2\pi[\theta_4(4l+2)-\theta_4(4l)]}x_2 & e^{j2\pi[\theta_4(4l+3)-\theta_4(4l)]}x_1 \end{bmatrix} F \tag{26}$$

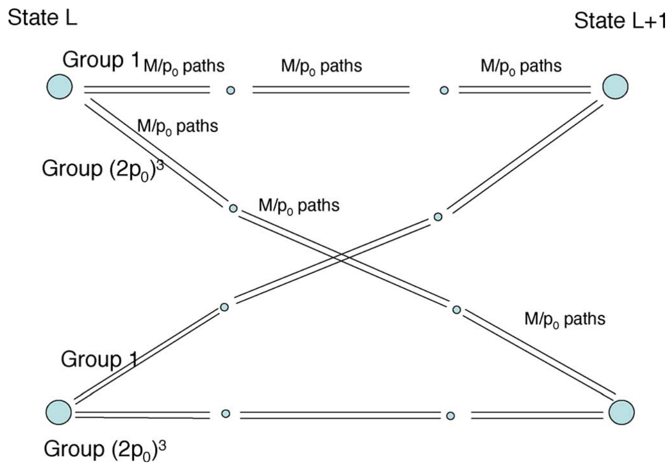

Fig. 3. Trellis structure of STC-CPM with fast demodulation algorithm.

We can see that the complexity of the branch searching in this case is $(2M)^3$.

In the following, we simplify the above branch searching by taking advantage of the special trellis structure of the proposed STC-CPM system, as illustrated in Fig. 3. The basic idea is to divide the total $(2M)^3$ paths into several groups and fast searching can be implemented in each group. The idea is further elaborated as follows.

Assume that the channel state information $\alpha_m(t)$ does not change in each space–time block duration $[4lT, 4(l+1)T]$. Let $A=[\alpha_1(t)\alpha_2(t)\alpha_3(t)\alpha_4(t)]$, and $Y(l)=[y(t,4l+1)y(t,4l+2)y(t,4l+3)y(t,4l+4)]$, then the branch metric (32) can be rewritten as

$$\begin{aligned}(\hat{e}_{1,l},\hat{e}_{2,l},\hat{e}_{3,l}) &= \arg\min_{(e_{1,l},e_{2,l},e_{3,l})\in\Omega\times\Omega\times\Omega}\int_0^T\sum_{n=1}^{4}\left|y(\tau,4l+n)\right.\\ &\quad\left.-\sum_{m=1}^{4}\alpha_m(\tau)s_m(\tau,4l+n)\right|^2 d\tau\\ &= \arg\min_{(e_{1,l},e_{2,l},e_{3,l})\in\Omega\times\Omega\times\Omega}\int_0^T\|Y(l)-AS\|_F^2 d\tau \end{aligned}\tag{33}$$

where $\|V\|_F$ is the Frobenius norm[2] of matrix V. Notice that

$$\begin{aligned}\|Y(l)-AS\|_F^2 &= \mathrm{tr}[(Y(l)-AS)^{\mathcal{H}}(Y(l)-AS)]\\ &= \mathrm{tr}[Y(l)^{\mathcal{H}}Y(l)]-2Re\{\mathrm{tr}[Y(l)^{\mathcal{H}}AS]\}\\ &\quad+\mathrm{tr}[A^{\mathcal{H}}ASS^{\mathcal{H}}]\end{aligned}\tag{34}$$

where $^{\mathcal{H}}$ stands for the complex conjugate and transpose of a matrix. From (30), we know that $S=EC(x_1,x_2,x_3)F_1$, where E and F_1 depend only on $a_{1,l}, a_{2,l}$ and $a_{3,l}$ as we can see from (27)(29)–(30). We observe that for any fixed $a_{1,l}, a_{2,l}$ and $a_{3,l}$, $\mathrm{tr}[Y(l)^{\mathcal{H}}AS+S^{\mathcal{H}}A^{\mathcal{H}}Y(l)]$ is a linear combination of the first order of x_1,x_2,x_3 or their conjugates x_1^*,x_2^*,x_3^*, and $\mathrm{tr}[A^{\mathcal{H}}ASS^{\mathcal{H}}]$ is a linear combination of the second order of them. Furthermore, there are no terms of $x_ix_j, x_ix_j^*$ and $x_i^*x_j^*$ with $i\neq j$ in $\mathrm{tr}[A^{\mathcal{H}}ASS^{\mathcal{H}}]$ (see Appendix for the proof). Thus, the branch metric in (33) can be written as the following sum of three functions that depend only on each of the three variables x_1, x_2 and x_3, respectively,

$$\begin{aligned}\int_0^T\|Y(l)-AS\|_F^2 d\tau &= f_1(x_1,a_{1,l},a_{2,l},a_{3,l})\\ &\quad+f_2(x_2,a_{1,l},a_{2,l},a_{3,l})+f_3(x_3,a_{1,l},a_{2,l},a_{3,l}).\end{aligned}\tag{35}$$

From (25), we know that $x_i=e^{j2\pi[e_{i,l}hq(\tau)-a_{i,l}q_0(\tau)]}, i=1,2,3$, are independent each other if the information symbols $e_{1,l}, e_{2,l}$ and $e_{3,l}$ are independent each other. Therefore, for any fixed $a_{1,l},a_{2,l},a_{3,l}$, the three functions $f_i(x_i,a_{1,l},a_{2,l},a_{3,l}), i=1,2,3$, are independent each other.

Recall that all of $a_{i,l}, i=1,2,3$, have only $2p_0$ possible values, where p_0 is specified in (20). More precisely, since $a_{i,l}=\mathrm{mod}(e_{i,l}m_0/p,2), i=1,2,3$, every $a_{i,l}$ belongs to the following set G:

$$G\triangleq\begin{cases}\{0,\frac{1}{p_0},\frac{2}{p_0},\ldots,\frac{2p_0-1}{p_0}\} & \text{if } p \text{ is odd}, p_0=p;\\ \left\{\frac{1}{p},\frac{1}{p}+\frac{1}{p_0},\frac{1}{p}+\frac{2}{p_0},\ldots,\frac{1}{p}+\frac{2p_0-1}{p_0}\right\} & \text{if } p \text{ is even}, p_0=\frac{p}{2}.\end{cases}\tag{36}$$

Again, since $a_{i,l}=\mathrm{mod}(e_{i,l}m_0/p,2)$, for a fixed $a_{i,l}$, symbol $e_{i,l}$ has to be in the following set $\Omega(a_{i,l})$:

$$\Omega(a_{i,l})\triangleq\{n\in\Omega:\mathrm{mod}(nm_0/p,2)=a_{i,l}\}\tag{37}$$

where Ω is specified in (5). The number of elements in $\Omega(a_{i,l})$ is at most $2M/(2p_0)$. Thus, the branch metric minimization in (33) can be simplified as

$$\begin{aligned}&\min_{(e_{1,l},e_{2,l},e_{3,l})\in\Omega\times\Omega\times\Omega}\int_0^T\|Y(l)-AS\|_F^2 d\tau\\ &=\min_{(a_{1,l},a_{2,l},a_{3,l})\in G\times G\times G}\\ &\quad\left\{\min_{e_{1,l}\in\Omega(a_{1,l}),e_{2,l}\in\Omega(a_{2,l}),e_{3,l}\in\Omega(a_{3,l})}\sum_{i=1}^{3}f_i(x_i,a_{1,l},a_{2,l},a_{3,l})\right\}\\ &=\min_{(a_{1,l},a_{2,l},a_{3,l})\in G\times G\times G}\\ &\quad\times\left\{\min_{e_{1,l}\in\Omega(a_{1,l})}f_1(x_1,a_{1,l},a_{2,l},a_{3,l})\right.\\ &\quad+\min_{e_{2,l}\in\Omega(a_{2,l})}f_2(x_2,a_{1,l},a_{2,l},a_{3,l})\\ &\quad\left.+\min_{e_{3,l}\in\Omega(a_{3,l})}f_3(x_3,a_{1,l},a_{2,l},a_{3,l})\right\}.\end{aligned}\tag{38}$$

The first equation is due to the definition of $a_{i.l}=\mathbf{mod}(e_{i,l}mo/p,2), i=1,2,3$, and the definition of $\Omega(a_{i,l})$ in (3.19). The last equation hold because $f_1(x_1,a_{1,l},a_{2,l},a_{3,l})$ depends only on $e_{1,l}$, $f_2(x_2,a_{1,l},a_{2,l},a_{3,l})$ depends only on $e_{2,l}$, and $f_3(x_3,a_{1,l},a_{2,l},a_{3,l})$ depends only on $e_{3,l}$.

[2]The Frobenius norm of V is given by

$$\|V\|_F^2=\mathrm{tr}(V^{\mathcal{H}}V)=\mathrm{tr}(VV^{\mathcal{H}})=\sum_{i,j}|v_{i,j}|^2.$$

Therefore, the branch searching in (32), or equivalently in (33), can be simplified as

$$\begin{aligned}(\hat{e}_{1,l}, \hat{e}_{2,l}, \hat{e}_{3,l}) &= \arg \min_{(a_{1,l},a_{2,l},a_{3,l})\in G\times G\times G} \\ &\left\{ \min_{e_{1,l}\in\Omega(a_{1,l})} f_1(x_1, a_{1,l}, a_{2,l}, a_{3,l}) \right. \\ &+ \min_{e_{2,l}\in\Omega(a_{2,l})} f_2(x_2, a_{1,l}, a_{2,l}, a_{3,l}) \\ &\left. + \min_{e_{3,l}\in\Omega(a_{3,l})} f_3(x_3, a_{1,l}, a_{2,l}, a_{3,l}) \right\}. \end{aligned} \tag{39}$$

We can see that the complexity of the above searching algorithm is at most $(2p_0)^3 3(2M/(2p_0)) = 24p_0^2 M$, while the complexity of the original branch searching in (32) is $(2M)^3$. We note that, p_0 depends only on the CPM modulation index h, not on the signal constellation size $2M$, and p_0 is usually much smaller than $2M$. Therefore, the complexity of the new search algorithm is, in general, much less than that of the original algorithm. For example, when $h = 1/2$ is considered in a CPM system, $p_0 = 1$. In this case, the complexity of the new branch searching is at most $24M$ while the original one is $8M^3$.

IV. Full Response CPM System With Quasi-Orthogonal Space–Time Coding

Since the rate of the space–time block codes from orthogonal designs cannot be greater than 3/4 for more than two transmit antennas [10], [20], the following quasi-orthogonal space–time codes were proposed by relaxing the orthogonality constraint [16], [17]

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2^* & -x_1^* & x_4^* & -x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ x_4^* & -x_3^* & x_2^* & -x_1^* \end{bmatrix} \tag{40}$$

with rate 1 for four transmit antennas. The code (40) also has a fast decoding, but does not have full diversity and the diversity is only 2 if all 4 information symbols $x_i, i = 1, 2, 3, 4$, are independently from the same constellation. Later, a quasi-orthogonal space–time code with full diversity based on (40) was proposed in [21], where the basic idea is that the information symbols x_1 and x_2 are chosen independently from a signal constellation $\mathcal{A}$ while the information symbols x_3 and x_4 are chosen independently from a rotated version of the constellation $\mathcal{A}$. The optimal rotation angles, in the sense of achieving the maximal diversity product or coding gain, of QAM and equilateral triangular constellations were also obtained in [21]. Similar to the idea used in [21], by using the quasi-orthogonal design (40) we try to design a quasi-orthogonal space–time coded CPM system with full diversity for 4 transmit antennas.

A. Design CPM Signals

A binary information sequence $\{\ldots, I_{1,l}, \ldots, I_{L,l}, \cdots\}$ is mapped to a symbol sequence $\{\ldots, e_{1,l}, e_{2,l}, e_{3,l}, e_{4,l}, \cdots\}$, where $e_{1,l}$ and $e_{2,l}$ are chosen from the following signal constellation

$$\Omega = \{-2M+1, \ldots, -1, 1, \ldots, 2M-1\} \tag{41}$$

while $e_{3,l}$ and $e_{4,l}$ are chosen from another signal constellation as follows:

$$\tilde{\Omega} = \{-2(\tilde{M}-1), \ldots, -2, 0, 2, \ldots, 2\tilde{M}\} \tag{42}$$

and $L = 2\log_2(2M) + 2\log_2(2\tilde{M})$, where M and $\tilde{M}$ may be the same. From (41) and (42), one can see that, if $M = \tilde{M}$, then, the constellation $\tilde{\Omega}$ is a shift of Ω in the phase domain, i.e., $\tilde{\Omega} = \Omega + 1$, which is corresponding to a rotation in the signal domain. This part is different from that for the modified orthogonal space–time block coding proposed in Section III, where all information symbols $e_{i,l}$ are taken from the same constellation Ω. The reason of choosing the above two different constellations Ω and $\tilde{\Omega}$ is that we want to produce a quasi-orthogonal block code for the transmitted signal matrix S such that symbols x_1 and x_2 are chosen from a constellation while symbols x_3 and x_4 are chosen from a rotated version of the constellation for the purpose of achieving the full diversity [21]. With the information symbols $e_{1,l}, \ldots, e_{4,l}$, the matrix $\mathbf{d}_l$ in (4) can be constructed as follows:

$$\mathbf{d}_l = \begin{bmatrix} e_{1,l} & e_{2,l} & e_{3,l} & e_{4,l} \\ -e_{2,l} & -e_{1,l} & -e_{4,l} & -e_{3,l} \\ e_{3,l} & e_{4,l} & e_{1,l} & e_{2,l} \\ -e_{4,l} & -e_{3,l} & -e_{2,l} & -e_{1,l} \end{bmatrix}. \tag{43}$$

Similar to Section III, to generate the CPM signal waveforms $s_m(t, k)$ in (8), we also need matrix $\mathbf{c}_l$ in (11), which is related to the symbol matrix $\mathbf{d}_l$ and can be specified as in (44) at the bottom of the page, where

$$a_{i,l} = \mathrm{mod}\left(\frac{e_{i,l} m_0}{p}, 2\right), \quad i = 1, 2, 3, 4 \tag{45}$$

where $m_0/p = h$ is the modulation index. Similar to (18)–(19), matrix $\mathbf{c}_l$ depends only on $a_{1,l}, a_{2,l}, a_{3,l}$ and $a_{4,l}$, and all of $a_{i,l}$ have at most $2p_0$ possible values, where

$$p_0 = \begin{cases} p, & \text{if } p \text{ is odd} \\ p/2, & \text{if } p \text{ is even.} \end{cases} \tag{46}$$

$$\mathbf{c}_l = \begin{bmatrix} a_{3,l} - a_{1,l} & a_{4,l} - a_{2,l} & a_{1,l} - a_{3,l} & a_{2,l} - a_{4,l} \\ 1 + a_{2,l} + a_{3,l} & 1 + a_{1,l} + a_{4,l} & 1 + a_{1,l} + a_{4,l} & 1 + a_{2,l} + a_{3,l} \\ 0 & 0 & 0 & 0 \\ 1 + a_{3,l} + a_{4,l} & 1 + a_{3,l} + a_{4,l} & 1 + a_{1,l} + a_{2,l} & 1 + a_{1,l} + a_{2,l} \end{bmatrix} \tag{44}$$

For simplicity, let

$$x_i = e^{j2\pi[e_{i,l}hq(\tau)-a_{i,l}q_0(\tau)]}, \quad i=1,2,3,4 \tag{47}$$

and

$$C(x_1,x_2,x_3,x_4) = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2^* & -x_1^* & x_4^* & -x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ x_4^* & -x_3^* & x_2^* & -x_1^* \end{bmatrix} \tag{48}$$

then, the transmitted signal matrix S at time period between $4lT$ and $4(l+1)T$ can be written as

$$S = EC(x_1,x_2,x_3,x_4)F_1 \tag{49}$$

where

$$E = \mathrm{diag}\left\{e^{j2\pi\theta_1(4l)}, e^{j2\pi[\theta_2(4l)+q_0(\tau)]}, e^{j2\pi\theta_3(4l)}, e^{j2\pi[\theta_4(4l)+q_0(\tau)]}\right\} \tag{50}$$

and

$$F_1 = \mathrm{diag}\left\{e^{j2\pi a_{3,l}q_0(\tau)}, e^{j\pi[2a_{4,l}q_0(\tau)+a_{3,l}]}, e^{j\pi[2a_{1,l}q_0(\tau)+a_{3,l}+a_{4,l}]}, e^{j\pi[2a_{2,l}q_0(\tau)+a_{1,l}+a_{3,l}+a_{4,l}]}\right\}. \tag{51}$$

One can see that the space–time code $C(x_1,x_2,x_3,x_4)$ in (48) has the same form as the quasi-orthogonal design in (40). Notice that $e_{1,l}$ and $e_{2,l}$ are chosen from Ω while $e_{3,l}$ and $e_{4,l}$ are chosen from $\tilde{\Omega}$, the resulting signal constellation for x_3 and x_4 is a rotated version of the constellation for x_1 and x_2. It is not difficult to check that the quasi-orthogonal space–time code $C(x_1,x_2,x_3,x_4)$ in (48) achieves the full diversity [21]. If all information symbols $e_{i,l}, i=1,2,3,4$, are from the same constellation Ω, then it is easy to see that the 4×4 space–time code C or S has only rank 2 at any time τ, which would result in degraded performance as we will see in the simulations in Section V. A remark here is that although the minimum rank of S for a nonzero information symbol vector and the diversity order of code C, i.e., the minimum rank of the difference matrix of two distinct matrices C, are both 2 at any time τ, the diversity order of S may not be 2 at any time τ, since the CPM is a nonlinear modulation and different from linear modulations. In other words, the diversity order of the quasi-orthogonal ST-CPM (nonrotated) may be higher than 2.

B. Fast Demodulation Algorithm

Similar to the fast demodulation algorithm developed in Section III, we assume that the channel state information $\alpha_m(t)$ is constant during a space–time coding block $[4lT, 4(l+1)T]$. Let $A = [\alpha_1(t)\alpha_2(t)\alpha_3(t)\alpha_4(t)]$, and $Y(l) = [y(t,4l+1)y(t,4l+2)y(t,4l+3)y(t,4l+4)]$, then the branch metric at stage l can be calculated as

$$\begin{aligned}(\hat{e}_{1,l},\hat{e}_{2,l},\hat{e}_{3,l},\hat{e}_{4,l}) &= \arg\min_{(e_{1,l},e_{2,l},e_{3,l},e_{4,l})\in\Omega\times\Omega\times\tilde{\Omega}\times\tilde{\Omega}} \int_0^T \sum_{n=1}^{4}\left|y(\tau,4l+n) - \sum_{m=1}^{4}\alpha_m(\tau)s_m(\tau,4l+n)\right|^2 d\tau \\ &= \arg\min_{(e_{1,l},e_{2,l},e_{3,l},e_{4,l})\in\Omega\times\Omega\times\tilde{\Omega}\times\tilde{\Omega}} \int_0^T \|Y(l)-AS\|_F^2 d\tau.\end{aligned} \tag{52}$$

We can see that the decoding complexity of the above branch searching is $(2M)^2(2\tilde{M})^2 = 16(M\tilde{M})^2$. Next, we would like to simplify the branch searching. Notice that

$$\|Y(l)-AS\|_F^2 = \mathrm{tr}[Y(l)^{\mathcal{H}}Y(l)] - \mathrm{Re}\{\mathrm{tr}[Y(l)^{\mathcal{H}}AS]\} + \mathrm{tr}[A^{\mathcal{H}}ASS^{\mathcal{H}}]. \tag{53}$$

Because of the quasi-orthogonal structure of the signal matrix S in (49), for any fixed $a_{i,l}, i=1,2,3,4$, the branch metric in (52) can be written as a sum of two functions whose variables depend on pairs (x_1,x_3) and (x_2,x_4), respectively, i.e.,

$$\begin{aligned}\int_0^T \|Y(l)-AS\|_F^2 d\tau &= f_{13}(x_1,x_3,a_{1,l},a_{2,l},a_{3,l},a_{4,l}) \\ &\quad + f_{24}(x_2,x_4,a_{1,l},a_{2,l},a_{3,l},a_{4,l}).\end{aligned} \tag{54}$$

For more details about the decomposition of a quasi-orthogonal code, we refer the reader to [16], [17], [21]. From (47), we have $x_i = e^{j2\pi[e_{i,l}hq(\tau)-a_{i,l}q_0(\tau)]}, \quad i=1,2,3,4$. Clearly, for any fixed $a_{i,l}, \quad i=1,2,3,4$, the above two functions f_{13} and f_{24} are independent since information symbol pairs $(e_{1,l},e_{3,l})\in\Omega\times\tilde{\Omega}$ and $(e_{2,l},e_{4,l})\in\Omega\times\tilde{\Omega}$ are independent.

Recall that all of $a_{i,l}, i=1,2,3,4$, have at most $2p_0$ possible values, where p_0 is specified in (46). More precisely, $a_{1,l}$ and $a_{2,l}$ belong to the following set G

$$G = \begin{cases} \{0, \frac{1}{p_0}, \frac{2}{p_0}, \ldots, \frac{2p_0-1}{p_0}\}, & \text{if } p \text{ is odd } p_0 = p \\ \left\{\frac{1}{p}, \frac{1}{p}+\frac{1}{p_0}, \frac{1}{p}+\frac{2}{p_0}, \ldots, \frac{1}{p}+\frac{2p_0-1}{p_0}\right\}, & \text{if } p \text{ is even } p_0 = \frac{p}{2} \end{cases} \tag{55}$$

and $a_{3,l}$ and $a_{4,l}$ belong to the following set $\tilde{G}$

$$\tilde{G} = \left\{0, \frac{1}{p_0}, \frac{2}{p_0}, \ldots, \frac{2p_0-1}{p_0}\right\} \tag{56}$$

which is different from the set G in (55) because constellation $\tilde{\Omega}$ in (42) is different from constellation Ω in (41). Since $a_{i,l} = \mathrm{mod}(e_{i,l}m_0/p, 2), i=1,2,3,4$, if $a_{1,l}$ and $a_{2,l}$ are fixed, then $e_{1,l}$ and $e_{2,l}$ belong to the following sets $\Omega(a_{i,l}), i=1,2$, respectively

$$\Omega(a_{i,l}) = \{n \in \Omega : \mathrm{mod}(nm_0/p, 2) = a_{i,l}\} \qquad i = 1,2 \quad (57)$$

where Ω is specified in (41). The number of elements in $\Omega(a_{i,l})$ is at most $2M/(2p_0)$. If $a_{3,l}$ and $a_{4,l}$ are fixed, then $e_{3,l}$ and $e_{4,l}$ belong to the following sets $\tilde{\Omega}(a_{i,l}), i=3,4$, respectively

$$\tilde{\Omega}(a_{i,l}) = \{n \in \tilde{\Omega} : \mathrm{mod}(nm_0/p, 2) = a_{i,l}\} \qquad i = 3,4 \quad (58)$$

where $\tilde{\Omega}$ is specified in (42). The number of elements in $\tilde{\Omega}(a_{i,l})$ is at most $2\tilde{M}/(2p_0)$.

From (54), the minimization of the branch metric in (52) can be rewritten as

$$\begin{aligned} &\min_{(e_{1,l},e_{2,l},e_{3,l},e_{4,l})\in\Omega\times\Omega\times\tilde{\Omega}\times\tilde{\Omega}} \int_0^T \|Y(l)-AS\|_F^2 d\tau \\ &= \min_{(a_{1,l},a_{2,l},a_{3,l},a_{4,l})\in G\times G\times\tilde{G}\times\tilde{G}} \Bigg\{ \min_{(e_{1,l},e_{3,l})\in\Omega(a_{1,l})\times\tilde{\Omega}(a_{3,l})} f_{13}(x_1,x_3,a_{1,l},a_{2,l},a_{3,l},a_{4,l}) \\ &\quad + \min_{(e_{2,l},e_{4,l})\in\Omega(a_{2,l})\times\tilde{\Omega}(a_{4,l})} f_{24}(x_2,x_4,a_{1,l},a_{2,l},a_{3,l},a_{4,l}) \Bigg\}. \end{aligned} \quad (59)$$

Therefore, the branch searching (52) can be simplified as

$$\begin{aligned} &(\hat{e}_{1,l},\hat{e}_{2,l},\hat{e}_{3,l},\hat{e}_{4,l}) \\ &= \arg \min_{(a_{1,l},a_{2,l},a_{3,l},a_{4,l})\in G\times G\times\tilde{G}\times\tilde{G}} \Bigg\{ \min_{(e_{1,l},e_{3,l})\in\Omega(a_{1,l})\times\tilde{\Omega}(a_{3,l})} f_{13}(x_1,x_3,a_{1,l},a_{2,l},a_{3,l},a_{4,l}) \\ &\quad + \min_{(e_{2,l},e_{4,l})\in\Omega(a_{2,l})\times\tilde{\Omega}(a_{4,l})} f_{24}(x_2,x_4,a_{1,l},a_{2,l},a_{3,l},a_{4,l}) \Bigg\}. \end{aligned} \quad (60)$$

The decoding complexity of the above branch searching is $(2p_0)^4(2M+2\tilde{M})/(2p_0) = 16p_0^3(M+\tilde{M})$, while the original one is $16(M\tilde{M})^2$. Notice that, p_0 depends only on the CPM modulation index h, not on the signal constellation size $2M$ or $2\tilde{M}$, and p_0 is usually much smaller than $2M$ and $2\tilde{M}$. For example, when $h = 1/2$ is considered in a CPM system, $p_0 = 1$. In this case, the complexity of the new branch searching is at most $16(M+\tilde{M})$ while the original one is always $16(M\tilde{M})^2$.

V. Simulation Results

In this section, we compare the performances of the modified orthogonal ST-CPM system for four transmit antennas, the quasi-orthogonal ST-CPM system also for four transmit antennas, and the OST-CPM system [18], [28], [29] for two transmit antennas. One receive antenna is used in all the simulations. The channel coefficients are zero mean complex Gaussian random variables with variance 1. We assume the channel is *quasi-static*, i.e., the channel coefficients are constant during one block transmission, and change independently from one block to another. In all simulations, we set the full response CPM systems with the modulation index $h = 1/2$ and the smoothing phase functions $q(t) = \frac{1}{2T}t, q_0(t) = \frac{t}{2T} - \frac{T}{2\pi}\sin(2\pi\frac{t}{T})$ if $t \in [0,T]$; $q_0(t) = q(t) = 0$ if $t \le 0$; and $q_0(t) = q(t) = 1/2$ if $t > T$. The initial phases for all 4 transmit antennas are set to 0.

The signal constellation $\Omega = \{-2M+1,\ldots,-1,1,2M-1\}$ is used in the conventional one transmitter CPM system, the OST-CPM system for two transmit antennas in [18], [28], and [29] the modified orthogonal ST-CPM system for four transmit antennas, and the quasi-orthogonal ST-CPM system without full diversity for four transmit antennas. For the quasi-orthogonal ST-CPM system with full diversity for four transmit antennas, signal constellation $\Omega = \{-2M+1,\ldots,-1,1,\ldots,2M-1\}$ is used for $e_{1,l}$ and $e_{2,l}$, and signal constellation $\tilde{\Omega} = \{-2(\tilde{M}-1),\ldots,-2,0,2,\ldots,2\tilde{M}\}$ with $\tilde{M} = M$ is used for $e_{3,l}$ and $e_{4,l}$.

We plot symbol error rate verses the SNR at the receiver in Fig. 4(a) and (b) for signal constellations with size 4 (i.e., $M = 2$) and size 8 (i.e., $M = 8$), respectively. From the simulation results, we can see that the performance of the modified orthogonal ST-CPM system for four transmit antennas is much better than that of the OST-CPM system for two transmit antennas, and it shows a higher diversity order in the performance curves. Moreover, the quasi-orthogonal ST-CPM system further outperforms the modified orthogonal ST-CPM system. This may be due to the fact that in the modified orthogonal space–time code in (2), there is 1/4 of power being used for the noninformation symbol transmission along the skew-diagonal of the code matrix. Another reason is that the diversity order of the quasi-orthogonal SP-CPM (nonrotated) may not be as lower as 2 as we explained at the end of Section IV-A. Also, one can also see that the quasi-orthogonal ST-CPM system with full diversity outperforms the quasi-orthogonal ST-CPM system without full diversity. Finally, we would like to point out that both with their own fast decoding algorithms, the decoding complexity of the quasi-orthogonal ST-CPM system $(16p_0^3(M+\tilde{M}))$ is higher than that of the modified orthogonal ST-CPM system $(24p_0^2M)$. In the simulated examples ($h = 1/2$ and $\tilde{M} = M$), the decoding complexity of the quasi-orthogonal ST-CPM system is $32M$ while the decoding complexity of the modified orthogonal ST-CPM system is $24M$.

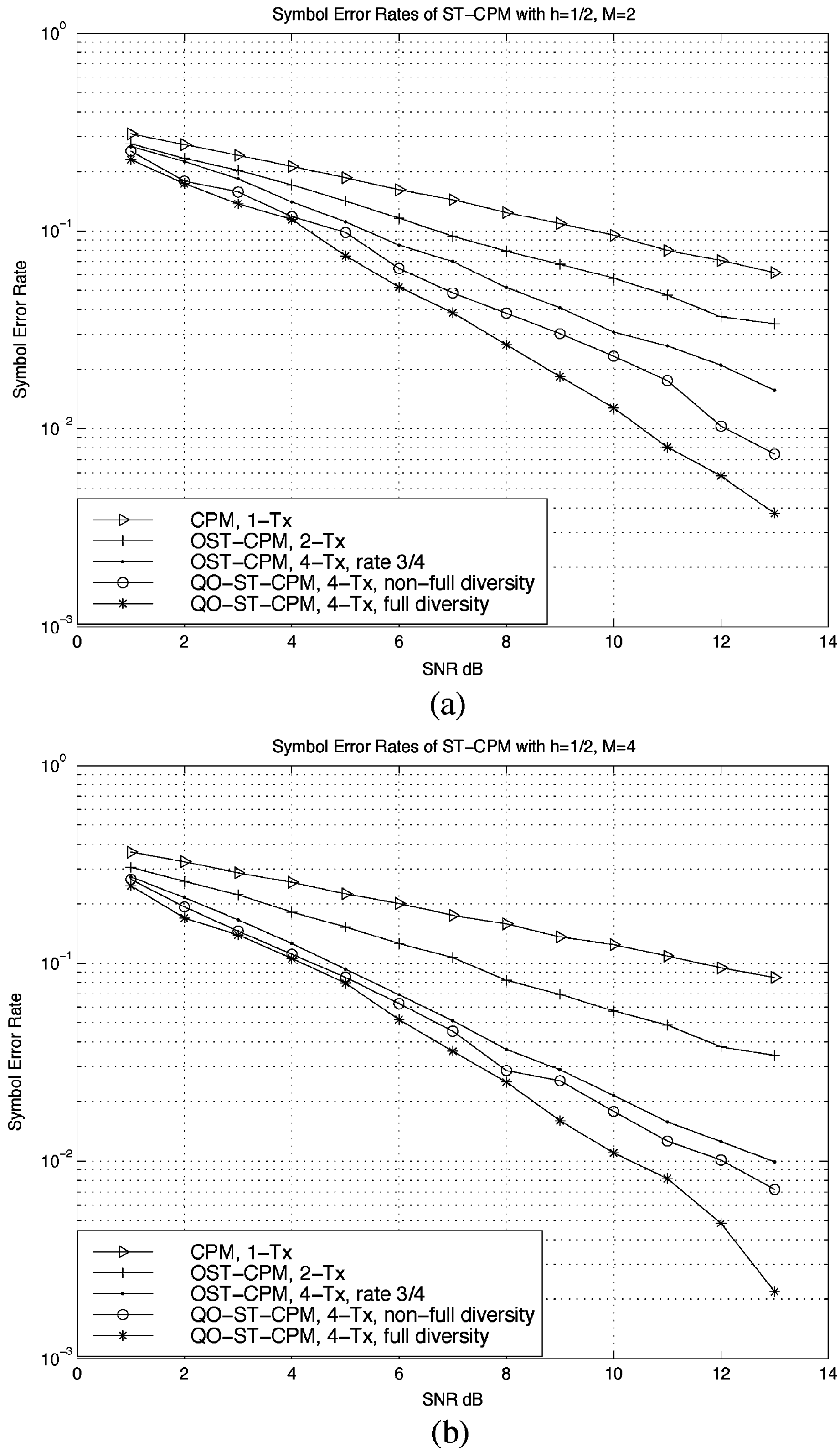

Fig. 4. Performances of the conventional CPM with 1 Tx antenna (line with ▹), the OST-CPM with 2 Tx antennas (line with $+$), the modified OST-CPM with 4 Tx antennas (line with $\cdot$), and the quasi-orthogonal ST-CPM with 4 Tx antennas (line with $\circ$ for that *without* full diversity, and line with $*$ for that with full diversity). (a) Constellation size 4 (i.e., $M = 2$). (b) Constellation size 8 (i.e., $M = 4$).

VI. Conclusion

In this paper, we proposed a modified orthogonal ST-CPM system and a quasi-orthogonal ST-CPM system for three and four transmit antennas, and derived fast ML demodulation algorithms for the proposed two systems accordingly. Simulation results showed that the performances of the proposed ST-CPM schemes for four transmit antennas are much better than that of the OST-CPM system for two transmit antennas. We also observed that the quasi-orthogonal ST-CPM system outperforms the modified orthogonal ST-CPM system, which is due to the noninformation symbol transmission in the modified orthogonal space–time code. However, both with their own fast decoding algorithms, the decoding complexity of the quasi-orthogonal ST-CPM system is higher than that of the modified orthogonal ST-CPM system. The proposed two ST-CPM systems provide a good tradeoff between decoding complexity and performance improvement in practical system implementation.

We would like to comment that there are some other quasi-orthogonal type space–time codes proposed recently in for example [22]–[26] with some good properties, but most of these codes cannot be applied directly to the ST-CPM systems since they may have some zero entries in the code matrix. However, it may be possible to modify these codes like the one in (2) for applying them to the ST-CPM systems, which would be interesting to consider. Regarding to the rotations and linear transforms for QOSTBC with minimum decoding complexity (MDC) proposed in [22]–[26], it would be interesting to consider their corresponding CPM schemes as well.

Appendix

Claim: There are no terms of $x_ix_j, x_ix_j^*$ and $x_i^*x_j^*$ with $i \neq j$ in the term $\mathrm{tr}(A^{\mathcal{H}}ASS^{\mathcal{H}})$ in (34).

Proof: From (30) we have

$$S = EC(x_1, x_2, x_3)F_1.$$

Notice that $F_1 = \mathrm{diag}(1, 1, e^{j\pi a_{3,l}}, e^{j\pi(a_{3,l}+a_{2,l})}) \cdot \mathrm{diag}\{1, e^{j2\pi a_{3,l}q_0(\tau)}, e^{j2\pi a_{2,l}q_0(\tau)}, e^{j2\pi a_{1,l}q_0(\tau)}\}$. Clearly, $F_1F_1^{\mathcal{H}} = I_4$. Thus, we have

$$\begin{aligned}\mathrm{tr}(A^{\mathcal{H}}ASS^{\mathcal{H}}) &= \mathrm{tr}(A^{\mathcal{H}}AECFF^{\mathcal{H}}C^{\mathcal{H}}E^{\mathcal{H}}) \\ &= \mathrm{tr}(E^{\mathcal{H}}A^{\mathcal{H}}AECC^{\mathcal{H}}).\end{aligned}$$

Therefore, to prove the claim, it is sufficient to prove that there are no terms of $x_ix_j, x_ix_j^*$ and $x_i^*x_j^*$ with $i \neq j$ in the entries of $CC^{\mathcal{H}}$.

We denote C as $C = C_0 + C_1$, where

$$C_0 = \begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 \\ x_2 & x_1^* & 0 & -x_3^* \\ x_3 & 0 & -x_1^* & x_2^* \\ 0 & x_3 & x_2 & x_1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & e^{j2\pi q_0(\tau)} & 0 \\ 0 & -e^{-j2\pi q_0(\tau)} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

It is easy to check that $C_0C_0^{\mathcal{H}} = (|x_1|^2 + |x_2|^2 + |x_3|^2)I_4$ and $C_1C_1^{\mathcal{H}} = I_4$. Therefore, we have

$$\begin{aligned}CC^{\mathcal{H}} &= (C_0 + C_1)(C_0 + C_1)^{\mathcal{H}} \\ &= C_0C_0^{\mathcal{H}} + C_0C_1^{\mathcal{H}} + C_1C_0^{\mathcal{H}} + C_1C_1^{\mathcal{H}} \\ &= (1 + |x_1|^2 + |x_2|^2 + |x_3|^2)I_4 + C_0C_1^{\mathcal{H}} + C_1C_0^{\mathcal{H}}.\end{aligned}$$

We can see that the entries of $C_0C_1^{\mathcal{H}} + C_1C_0^{\mathcal{H}}$ are some linear combinations of the first order of x_1, x_2, x_3 or their conjugates x_1^*, x_2^*, x_3^*. So there are no terms of $x_ix_j, x_ix_j^*$ and $x_i^*x_j^*$ with $i \neq j$ in the entries of $CC^{\mathcal{H}}$. This concludes the proof.

Acknowledgment

The authors would like to thank the anonymous reviewers for their useful comments and suggestions that have helped the clarity of the presentation of this paper.

References

[1] J. B. Anderson, T. Aulin, and C. Sunberg, *Digital Phase Modulation*. New York: Plenum, 1986.
[2] E. Teletar, "Capacity of multi-antenna Gaussian channels," AT&T Bell Labs, Tech. Rep. Jun. 1995.
[3] J.-C. Guey, M. P. Fitz, M. R. Bell, and W.-Y. Kuo, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," in *Proc. IEEE VTC'96*, pp. 136–140.
[4] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, 1998.
[5] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, 1998.
[6] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, 1999.
[7] B. M. Hochwald, T. L. Marzetta, and C. B. Papadias, "A transmitter diversity scheme for wideband CDMA systems based on space-time spreading," *IEEE J. Sel. Areas Commun.*, vol. 19, pp. 48–60, Jan. 2001.
[8] G. Ganesan and P. Stoica, "Space-time block codes: A maximum SNR approach," *IEEE Trans. Inf. Theory*, vol. 47, pp. 1650–1656, May 2001.
[9] O. Tirkkonen and A. Hottinen, "Square-matrix embeddable space-time block codes for complex signal constellations," *IEEE Trans. Inf. Theory*, vol. 48, pp. 384–395, Jan. 2002.
[10] W. Su and X.-G. Xia, "On space-time block codes from complex orthogonal designs," *Wireless Pers. Commun. (Springer)*, vol. 25, no. 1, pp. 1–26, Apr. 2003.
[11] X.-B. Liang, "Orthogonal designs with maximal rates," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2468–2503, Oct. 2003.
[12] W. Su, X.-G. Xia, and K. J. R. Liu, "A systematic design of high-rate complex orthogonal space-time block codes," *IEEE Commun. Lett.*, vol. 8, no. 6, pp. 380–382, Jun. 2004.
[13] K. Lu, S. Fu, and X.-G. Xia, "Closed-form designs of complex orthogonal space-time block codes of rate $(2k+1)/2k$ for $2k-1$ or $2k$ transmit antennas," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 4340–4347, Oct. 2005.
[14] X. Zhang and M. P. Fitz, "Space-time coding for Rayleigh fading channels in CPM system," in *Proc. 38th Annual Allerton Conf. Commun., Control, and Comput.*, Monticello, IL, Oct. 2000.
[15] J. Tan and G. L. Stüber, "Space-time coded CPM," *IEEE Trans. Wireless Comm.* Dec. 2001 [Online]. Available: http://users.ece.gatech.edu/juntan/publications.html
[16] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 1–4, 2001.

[17] O. Tirkkonen, A. Boariu, and A. Hottinen, "Minimal nonorthogonality rate 1 space-time block code for 3+ Tx antennas," in *Proc. IEEE 6th Int. Symp. on Spread-Spectrum Technol. Appl. (ISSSTA 2000)*, Sep. 2000, pp. 429–432.

[18] G. Wang and X.-G. Xia, "Orthogonal space-time coding for CPM system with fast decoding," in *Proc. ICC'2002*, New York, May 2002.

[19] D. Wang, G. Wang, and X.-G. Xia, "An orthogonal space-time coded partial response CPM system with fast decoding for two transmit antennas," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2410–2422, Sep. 2005.

[20] H. Wang and X.-G. Xia, "Upper bounds of rates of complex orthogonal space-time block codes," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2788–2796, Oct. 2003.

[21] W. Su and X.-G. Xia, "Signal constellations for quasi-orthogonal space-time block codes with full diversity," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2331–2347, Oct. 2004.

[22] C. Yuen, Y. L. Guan, and T. T. Tjhung, "Optimizing quasi-orthogonal STBC through group-constrained linear transformation," *IET Commun.*, vol. 1, pp. 373–381, Jun. 2007.

[23] C. Yuen, Y. L. Guan, and T. T. Tjhung, "Quasi-orthogonal STBC with minimum decoding complexity," *IEEE Trans. Wireless Comms.*, vol. 4, pp. 2089–2094, 2005.

[24] Z. A. Khan and B. S. Rajan, "Space-time block codes from coordinate interleaved orthogonal designs," *Proc. IEEE ISIT*, p. 275, 2002.

[25] Z. A. Khan and B. S. Rajan, "Single-symbol maximum-likelihood decodable linear STBCs," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 2062–2091, May 2006.

[26] H. Wang, D. Wang, and X.-G. Xia, "On optimal quasi-orthogonal space-time block codes with minimum decoding complexity," *IEEE Trans. Inf. Theory*, vol. 55, no. 3, pp. 1104–1130, Mar. 2009.

[27] J.-C. Guey, M. P. Fitz, M. R. Bell, and W.-Y. Kuo, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 47, pp. 527–537, Apr. 1999.

[28] G. Wang and X.-G. Xia, "An orthogonal space-time coding for CPM systems," in *Proc. Int. Symp. Inf. Theory*, Lausanne, Switzerland, Jun. 30–Jul. 5, 2002.

[29] G. Wang and X.-G. Xia, "An orthogonal space-time coded CPM system with fast decoding for two transmit antennas," *IEEE Trans. Inf. Theory*, vol. 50, pp. 486–493, Mar. 2004.

Genyuan Wang (M'04) received the B.Sc. and M.S. degrees in mathematics from Shaanxi Normal University, Xi'an, China, in 1985 and 1988, respectively, and the Ph.D. degree in electrical engineering from Xidian University, Xi'an China, in 1998.

From June 1988 to December 2003, he was a Postdoctoral Fellow with the Department of Electrical and Computer Engineering, University of Delaware, Newark. From January 2004 to April 2006, he was a Research Associate with the Center for Advanced Communications, Villanova University, Villanova, PA. Since May, 2006, he has been with Cisco Systems as a Senior System Engineer working on physical and MAC layer system designs. His research interests are radar imaging and radar signal processing, and MIMO-wireless systems.

Weifeng Su (M'03) received the Ph.D. degree in electrical engineering from the University of Delaware, Newark, in 2002. He received the B.S. and Ph.D. degrees in mathematics from Nankai University, Tianjin, China, in 1994 and 1999, respectively.

He is an Assistant Professor with the Department of Electrical Engineering, the State University of New York (SUNY) at Buffalo. From June 2002 to March 2005, he was a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering and the Institute for Systems Research (ISR), University of Maryland, College Park. His research interests span a broad range of areas from signal processing to wireless communications and networking, including space–time coding and modulation for MIMO wireless communications, MIMO-OFDM systems, ultrawideband (UWB) communications, and cooperative communications for wireless networks.

Dr. Su received the Signal Processing and Communications Faculty Award from the University of Delaware in 2002 as an outstanding graduate student in the field of signal processing and communications. In 2005, he received the Invention of the Year Award from the University of Maryland. He serves as an Associate Editor for the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY and IEEE SIGNAL PROCESSING LETTERS. He also organized two Special Issues for IEEE journals on cooperative communications and networking.

Xiang-Gen Xia (M'97–SM'00–F'09) received the B.S. degree in mathematics from Nanjing Normal University, Nanjing, China, and the M.S. degree in mathematics from Nankai University, Tianjin, China, and the Ph.D. degree in electrical engineering from the University of Southern California, Los Angeles, in 1983, 1986, and 1992, respectively.

He was a Senior/Research Staff Member with Hughes Research Laboratories, Malibu, CA, during 1995-1996. In September 1996, he joined the Department of Electrical and Computer Engineering, University of Delaware, Newark, where he is the Charles Black Evans Professor. He was a Visiting Professor with the Chinese University of Hong Kong during 2002-2003, where he is an Adjunct Professor. Before 1995, he held visiting positions in a few institutions. His current research interests include space–time coding, MIMO and OFDM systems, and SAR and ISAR imaging. He has over 200 refereed journal articles published and accepted, and seven U.S. patents awarded and is the author of the book *Modulated Coding for Intersymbol Interference Channels* (New York: Marcel Dekker, 2000).

Dr. Xia received the National Science Foundation (NSF) Faculty Early Career Development (CAREER) Program Award in 1997, the Office of Naval Research (ONR) Young Investigator Award in 1998, and the Outstanding Overseas Young Investigator Award from the National Nature Science Foundation of China in 2001. He also received the Outstanding Junior Faculty Award of the Engineering School of the University of Delaware in 2001. He is currently an Associate Editor of the IEEE TRANSACTIONS ON SIGNAL PROCESSING, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, *Signal Processing (EURASIP)*, and the *Journal of Communications and Networks (JCN)*. He was a guest editor of Space-Time Coding and Its Applications in the EURASIP *Journal of Applied Signal Processing* in 2002. He served as an Associate Editor of the IEEE TRANSACTIONS ON SIGNAL PROCESSING during 1996 to 2003, the IEEE TRANSACTIONS ON MOBILE COMPUTING during 2001 to 2004, the IEEE SIGNAL PROCESSING LETTERS during 2003 to 2007, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY during 2005 to 2008, and the EURASIP *Journal of Applied Signal Processing* during 2001–2004. He is also a Member of the Sensor Array and Multichannel (SAM) Technical Committee in the IEEE Signal Processing Society. He is the General Co-Chair of ICASSP 2005 in Philadelphia, PA.