EE 459/559: Control and Applications of Power Electronics

3ϕ

Topic 4: DC/AC (AC/DC) Converters Analysis, Control, and Applications

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Reference reading:
[1] H. Akagi, E. Watanabe, and M. Aredes, "Instantaneous Power Theory and Applications to Power Conditioning", IEEE Press, 2007. Chapter 3.

[2] Y. Yang, W. Chen, and F. Blaabjerg, "Advanced Control of Photovoltaic and Wind Turbines Power Systems", Springer, 2014. Chapter 2.

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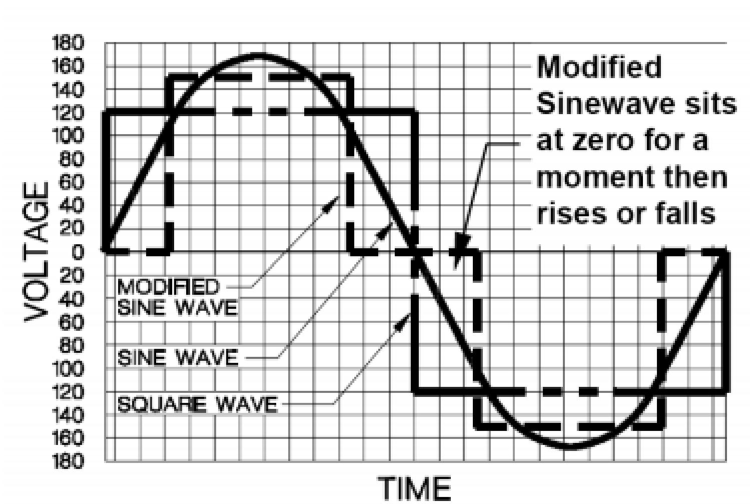
Outline

- **Review of dc/ac (ac/dc) converters**
- **abc to dq transformation – Clarke and Park Transformations**
  $\alpha\beta 0$ $dq0$
- **Space vector PWM** (SVPWM)
- **Controller Design Overview** 3ϕ
- **Applications** PV, HVDC, Motor Drives.

Small Power Inverters

Small power inverters

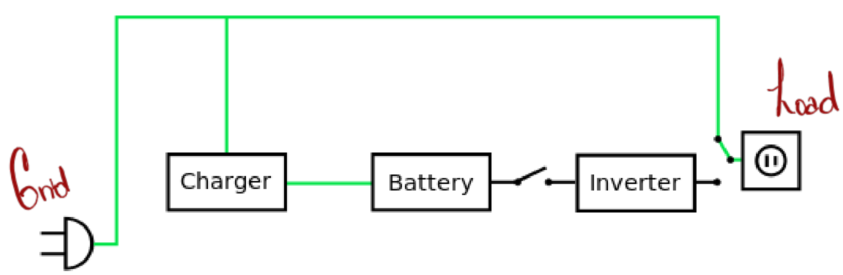


Square, Modified, and Pure Sine Wave

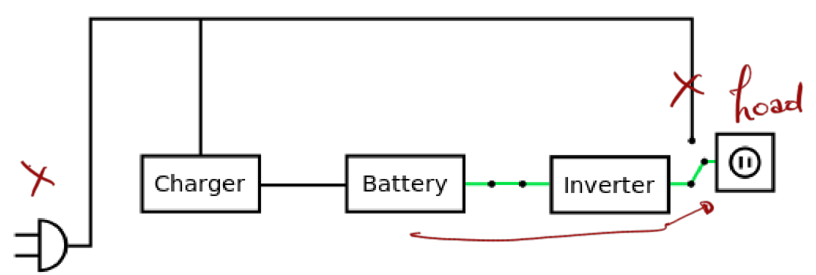
- Small power inverters
  - Take dc power supplied by a battery, such as a 12 V car battery
  - Transform it to a 120 V ac power source t 60 Hz
  - Emulate the power available at an ordinary household electrical outlet
- Applications of small power inverters
  - Camping vehicles, boats
  - Power appliances in a car: cell phones, radios and televisions
- Pure sine wave inverter
  - More expensive due to added circuitry
  - Can provide power to all ac electronic devices
  - Reduce audible and electric noise

Uninterruptible Power Supply (UPS)

- An electrical apparatus that provides emergency power to a load when the input power source or mains power fails
  - Instantaneous protection from input power interruptions
  - The on-battery runtime of most UPS is relatively short: a few minutes
  - Used to protect computers, data centers, telecommunication equipment, etc



Normal AC Power



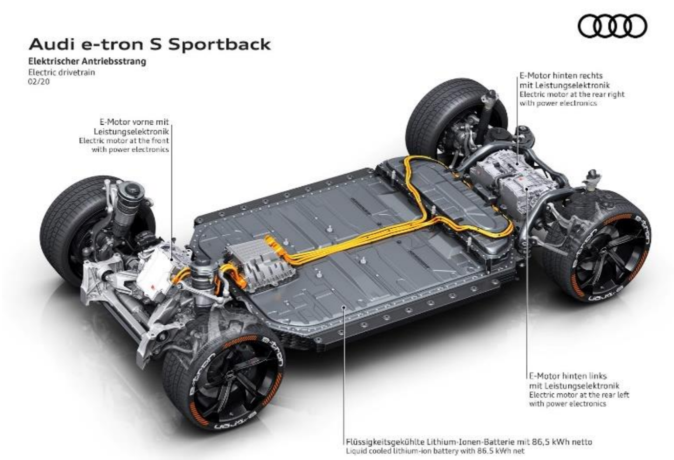
Over/Undervoltage; Loss of Power

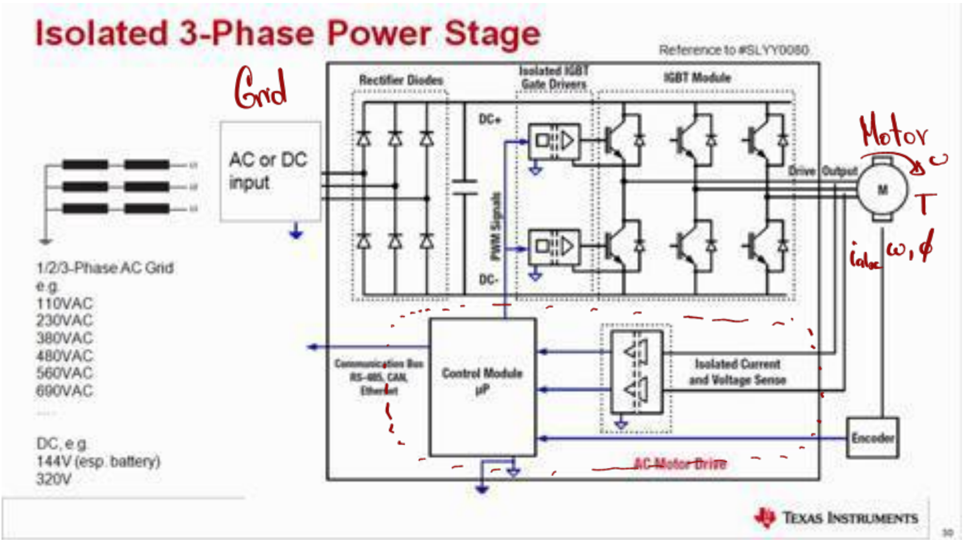
Uninterruptible Power Supply (UPS)

- Example: the largest UPS
  - Fairbanks, Alaska
  - Powers the entire city and nearby rural communities during outages
  - Built by ABB and commissioned in 2003
  - Battery is made up of almost 14,000 nickel-cadmium batteries that can provide 26 MW of power for 15 mins, or up to 40 MW for 7 mins.
  - Facility covers an area bigger than a soccer field.

Motor Drive

- Electric motor speed control
  - Control and feedback circuitry to adjust the final output of the inverter
  - The inverter output determine the speed of the motor
- Applications
  - Industrial motor driven equipment
  - Electric vehicles
  - Rail transport system
  - Power tools
  - Inverter compressors



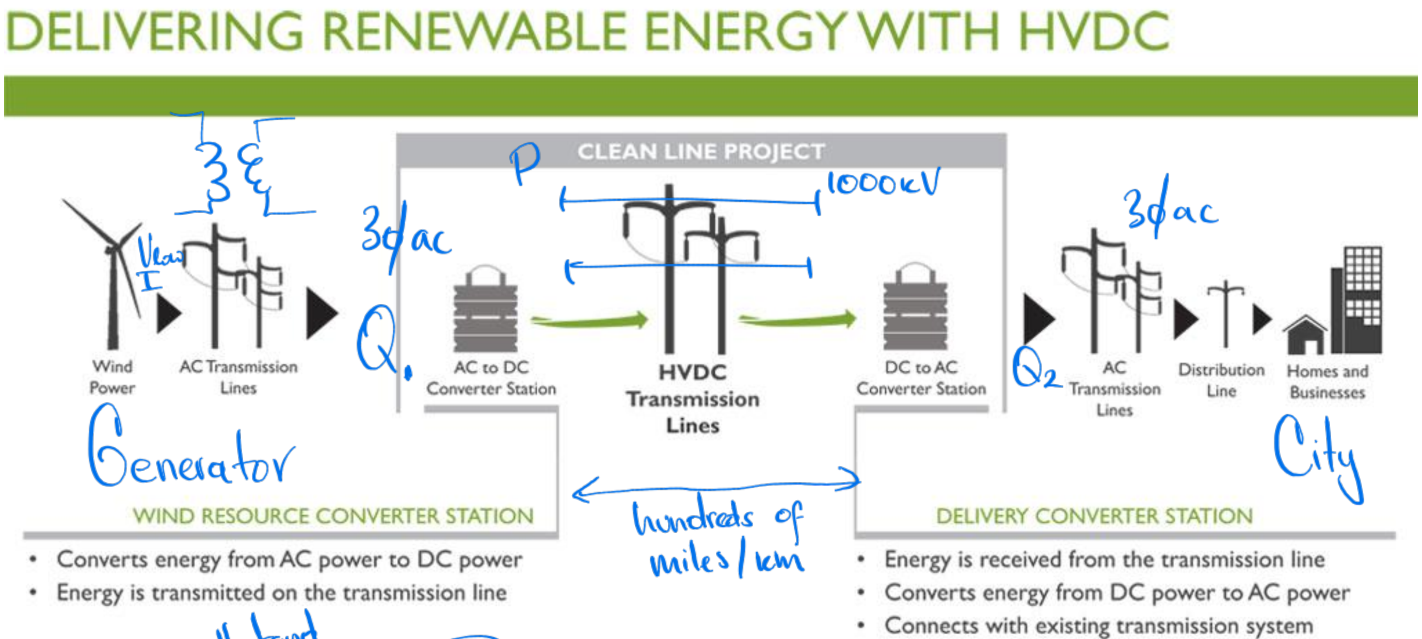


Motor Drive : control ω, φ, torque

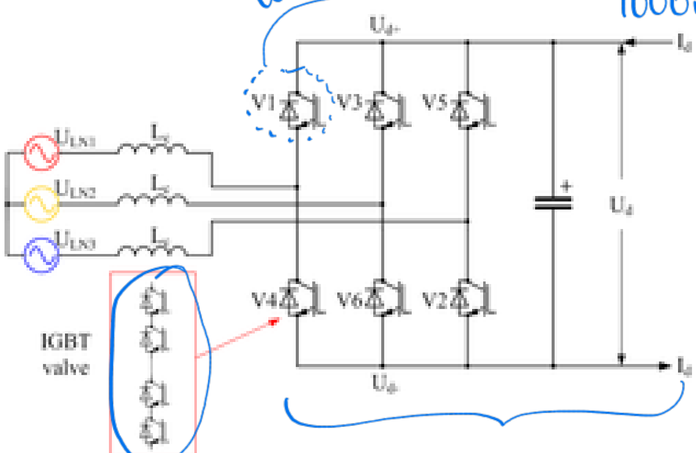
Servo motor

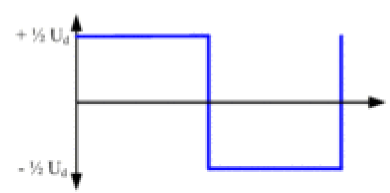
HVDC

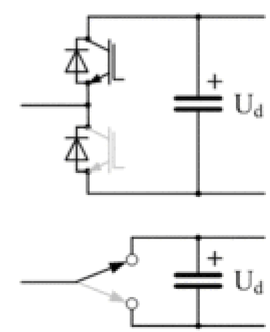
(fixed) $P = \downarrow I \, V \uparrow$ $\downarrow P_{loss} = \downarrow I^2 R$



withstand 1000kV



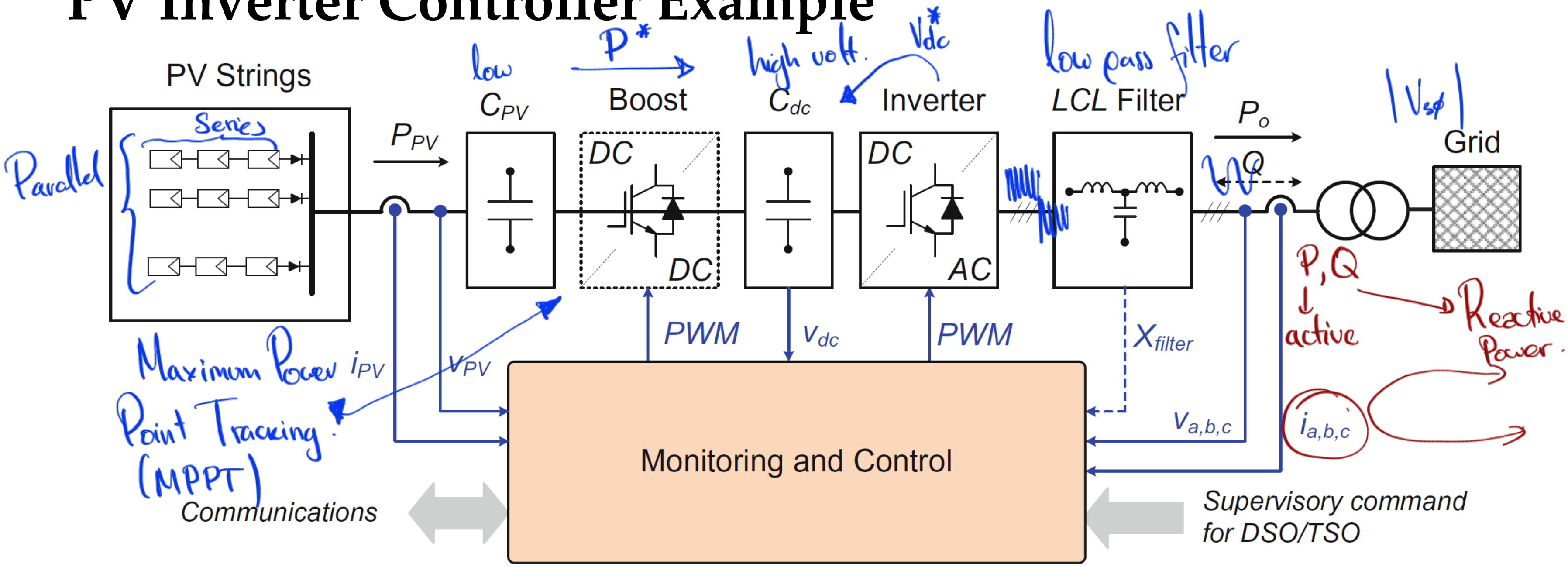


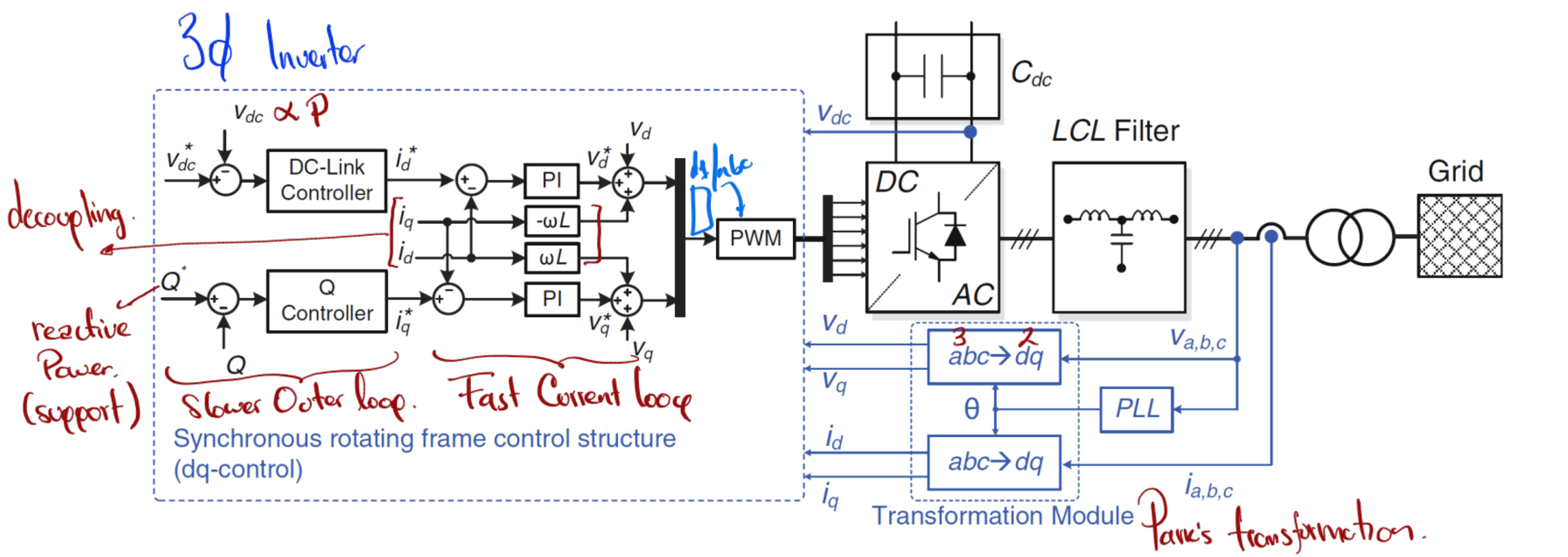


or use Multilevel Converters.

PV Inverters

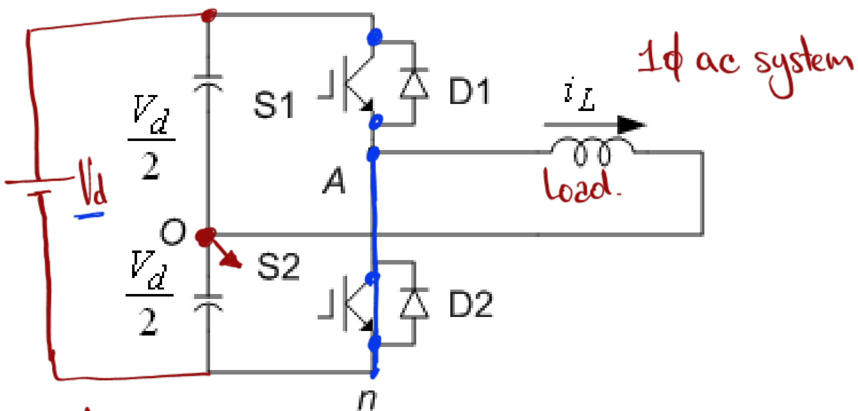
PV Inverter Controller Example

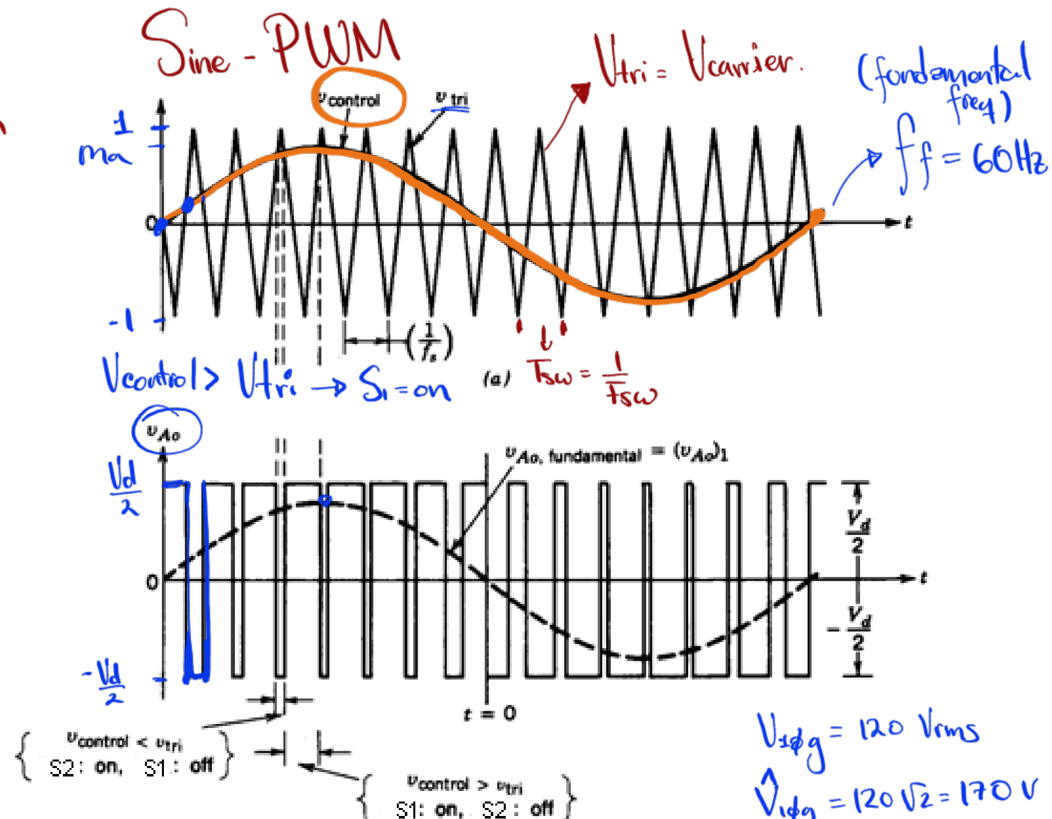




Single Phase: Half Bridge Converter and Sine PWM

- Let's first review the different types of dc/ac (ac/dc) converters and their modulation strategies





$\hat{V}_{Ao}$ (peak)

- What is $\hat{V}_{Ao}$ when S_1=on (S_2=off)? $\frac{V_d}{2}$

- " " $\hat{V}_{Ao}$ " S_1=off (S_2=on)? $\frac{-V_d}{2}$

$V_{1\phi g}$ = 120 Vrms

$\hat{V}_{1\phi g} = 120\sqrt{2} = 170$ V

min V_d = 170×2

- When $\hat{V}_{control} \leq \hat{V}_{tri}$ the modulation index can be defined as follows:

if $m_a = 1 \rightarrow \hat{V}_{Ao} = \frac{V_d}{2}$

max peak voltage

$$m_a = \cancel{\frac{\hat{V}_{control}}{\hat{V}_{tri}}} = \underbrace{\frac{\hat{V}_{A0}}{\hat{V}_d/2}}, \quad 0 \leq m_a \leq 1$$

(Peak) $\hat{V}_{AO} = m_a \cdot \frac{V_d}{2}$

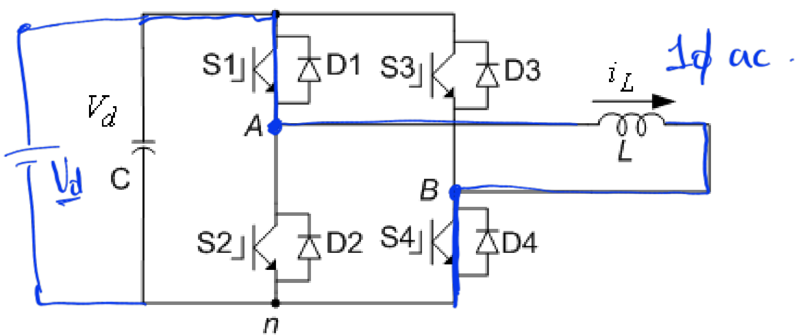
$m_a \in [0,1]$

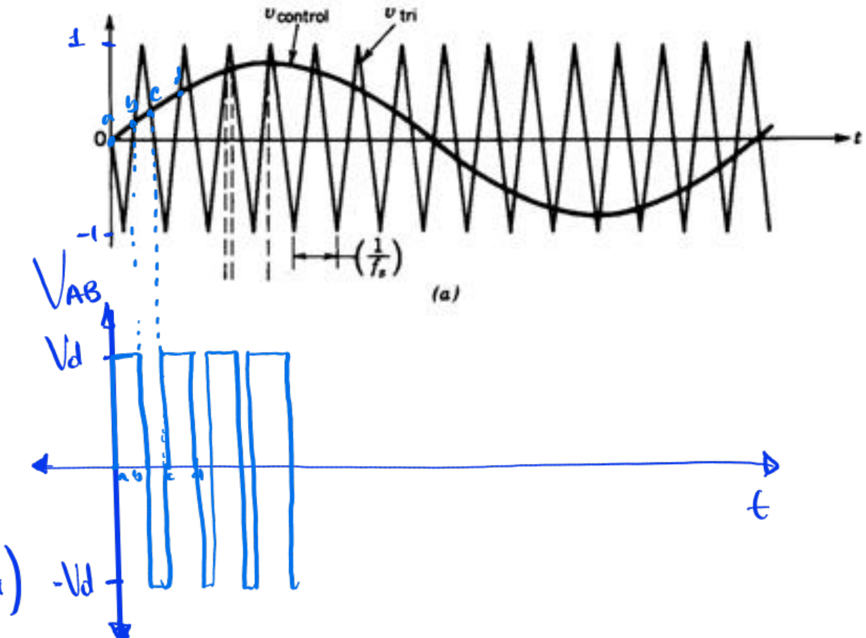
Single Phase: Full Bridge Converter and Sine PWM

H bridge

- Review the operation of the full bridge converter

$V_{control} > V_{tri} \rightarrow S_1$ & S_4 on





- Operation: • S_1 and S_4 operate at same time
  - S_2 and S_3 operate at same time (complementary to S_1 & S_4)

- What is $\hat{V}_{AB}$ when S_1 & S_4 on? V_d
- " " " " S_2 & S_3 on? $-V_d$

$\hat{V}_{g\text{-}1\phi} = 170 \longrightarrow$ min $V_d = 170V$

- When $\hat{V}_{\text{control}} \leq \hat{V}_{\text{tri}}$ the modulation index can be defined as follows:

$$m_a = \cancel{\frac{\hat{V}_{\text{control}}}{\hat{V}_{\text{tri}}}} = \frac{\hat{V}_{AB}}{\hat{V}_d}, \quad 0 \leq m_a \leq 1$$

$\hat{V}_{AB} = m_a \cdot V_d$

if $m_a = 1$ $\boxed{\hat{V}_{AB} = V_d}$ max peak ac voltage

$m_a \in [0, 1]$

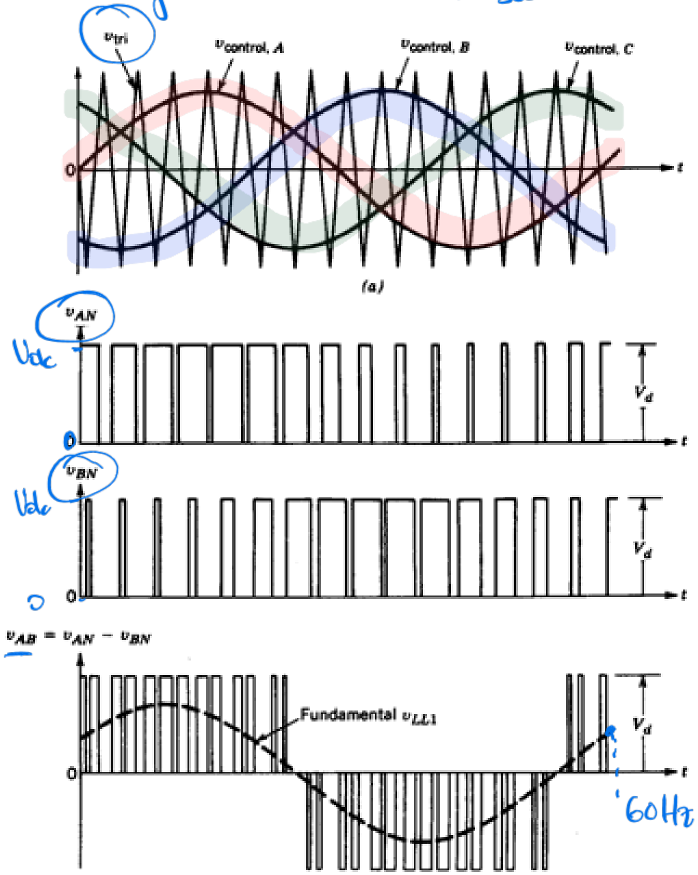
$m_a = \frac{120(\sqrt{2})V}{300V} = .56$

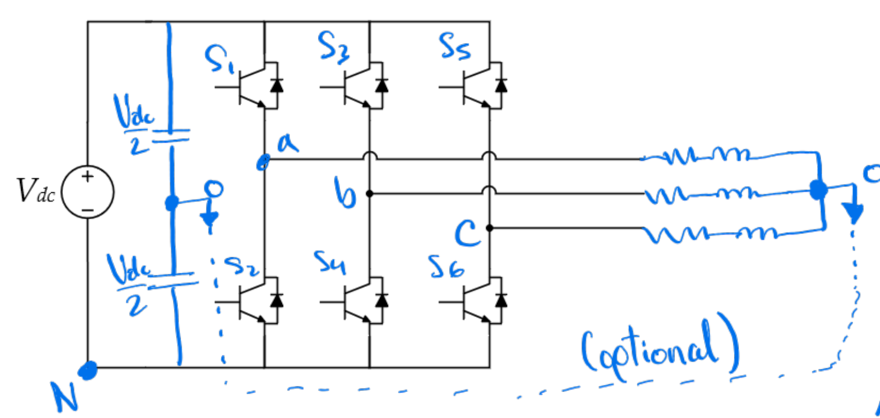
- Assume $V_d = 300V$, if we want $V_{AB} = 120V$ (rms), what is m_a? $m_a \approx .56$

Three Phase Inverter

- A three phase inverter is a combination of three (half - bridge)

1 triangular = carrier = decides F_{sw}





$\Rightarrow$ maximum line to neutral peak voltage $\hat{V}_{ao} = \frac{V_{dc}}{2}$ $= \hat{V}_{bo}$ " $= \hat{V}_{co}$ "

- Whenever $V_{control\,A} > V_{tri} \Rightarrow S_1$ on (S_2 off)

  $V_{control\,B} > V_{tri} \Rightarrow S_3$ on (S_4 off)

  $V_{control\,C} > V_{tri} \Rightarrow S_5$ on (S_6 off)

Line to line $\Rightarrow V_{AB} = V_{AN} - V_{BN} = V_{AO} - V_{BO}$

$V_{BC} = V_{BN} - V_{CN}$

$V_{CA} = V_{CN} - V_{AN}$

$\hat{V}_{ao} = \sqrt{2}\, V_{ao}$ $\quad V_{ao}$ (rms)

(rms) $|V_{ab}| = \sqrt{3}\, |V_{ao}|$

$|V_{LL}| = \sqrt{3}\, |V_{LN}|$

$\hat{V}_{LN} = \sqrt{2}\, V_{LN}$

$\hat{V}_{ao} = m_a \frac{V_{dc}}{2}$

- 3ϕ power supply, w/ rated voltage of $V_{LL} = 408V$
- Suppose we wanted $m_{a,sf} = 0.5$, what is V_{dc} needed?

Outline

- Review of dc/ac (ac/dc) converters

  (Parc) (Intermediate)

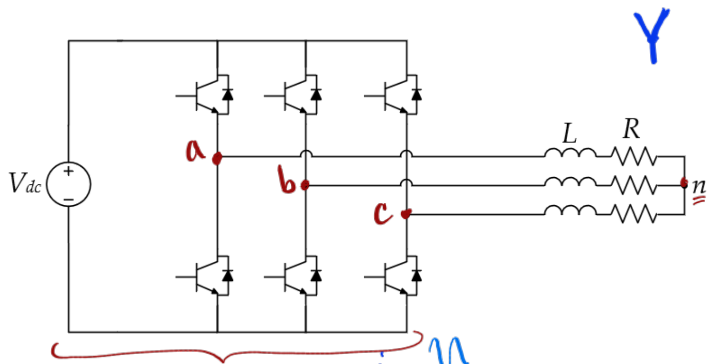
- **abc to dq transformation – Clarke and Park Transformations**

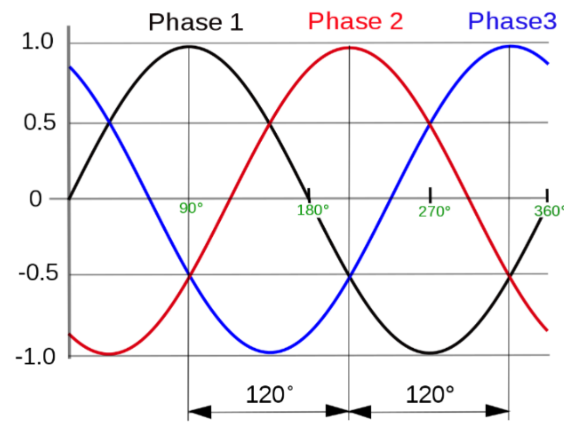
  $\alpha\beta 0$ $dq0$

- Space vector PWM
- Controller Design Overview
- Applications

Balanced Three Phase System

- Let's look at a **balanced** three phase system:
  - Three voltage sources with equal magnitude but with phase shift of 120°
  - Equal loads on each phase (a, b, c)
  - Equal impedance on the lines





$V_{an}(t) = \hat{V}_{LN}\cos(\omega t)$ ~ $\tilde{V}_{an} = \frac{\hat{V}_{LN}}{\sqrt{2}}\angle 0^\circ$

$V_{bn}(t) = \hat{V}_{LN}\cos(\omega t - \frac{2\pi}{3})$ ~ $\tilde{V}_{bn} = \frac{\hat{V}_{LN}}{\sqrt{2}}\angle -120^\circ$

$V_{cn}(t) = \hat{V}_{LN}\cos(\omega t + \frac{2\pi}{3})$ ~ $\tilde{V}_{cn} = \frac{\hat{V}_{LN}}{\sqrt{2}}\angle +120^\circ$

$p_a(t) = V_{an}(t)\cdot i_a(t)$

Advantages of Balanced 3ϕ systems

1) $i_a + i_b + i_c = 0 = i_N$
   Zero neutral current
2) Rotating mag. field ⇒ easier motor start
3) Const. Inst. Power: $P_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t) = P_{dc}$
   → Const. Torque (motor)

Integral Control

- As we have seen before, integrator (e.g. in PI or integral + state feedback) can be used for tracking constant or step references $V_{out}^{*} = r(t)$
- However, in ac systems the signals are **not** constant

$i_a^*(t) = r_a(t)$ are sinusoidal signals. $r_a(t) = \hat{i}_a \cos(\omega t + \phi)$

- An integrator **cannot** be directly use to track a sine/ac reference

Internal Model Principle: → for whatever $r(t)$ you want to track, the controller must internally generate the same type of signal.

$r(t)$	$R(s)$	
step	$\frac{\hat{r}}{s}$	(Integrator)
sinusoid	$\frac{\omega^2}{s^2+\omega^2}$	(resonant)

- What can we do?

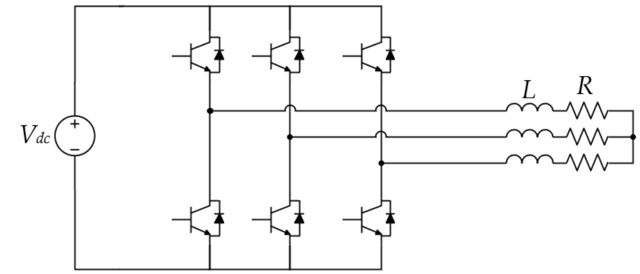
One approach

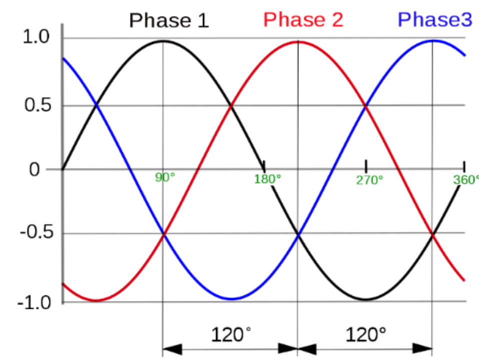
1) Resonant Controller (R)

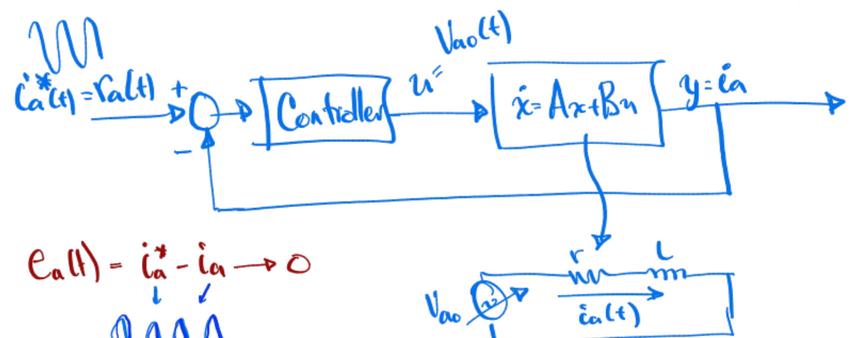
Prop. + Resonant (PR)

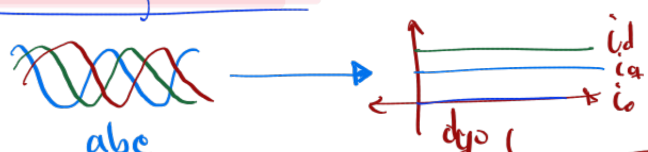
2) Coordinate Transformation — ac → dc (abc — dq0)

$e_a(t) = i_a^* - i_a \to 0$









Use PI, or State + Integral control

Transformations

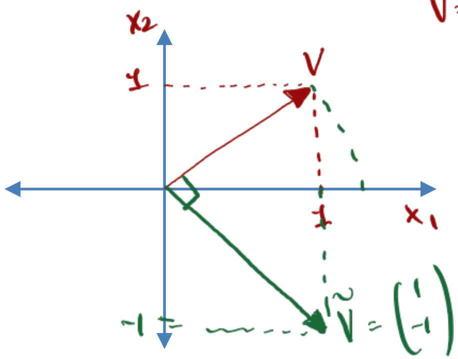
2 Dimension $\cong \overset{\text{real numbers}}{\mathbb{R}^2}$, $x \in \mathbb{R}^2 \Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $T(x): \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ $\quad Tx = y \quad y \in \mathbb{R}^2$

- One particular type of transformation that we will use later is known as angle rotation

$v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad \underbrace{T_\theta = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}}$

θ: specified

Clockwise rotation by θ



$\tilde{V} = \underline{T_\theta V}$ $\qquad \tilde{V}$ = vector v rotated clockwise by θ (radians)

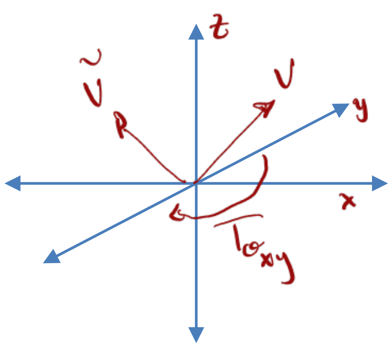
- Example Rotate V by 90° ($\frac{\pi}{2}$) clockwise

$$\tilde{V} = T_{\frac{\pi}{2}} V = \begin{bmatrix} \overset{0}{\cancel{\cos(\pi/2)}} & \overset{1}{\sin(\pi/2)} \\ \underset{-1}{-\sin(\frac{\pi}{2})} & \underset{0}{\cancel{\cos(\pi/2)}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

3 Dimensions $\cong \mathbb{R}^3$

- In three dimensions, we can also define similar transformations

$v \in \mathbb{R}^3 \rightarrow v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $\qquad T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \Rightarrow Tv = \tilde{v} \in \mathbb{R}^3$



- Rotate along the x-y plane (clockwise)

$$T_{\theta_{xy}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

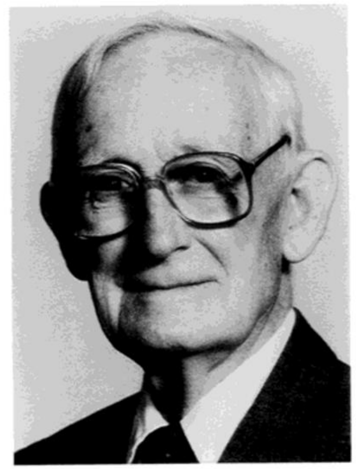
Clarke and Park Transformation

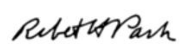
Edith Clarke

Born	February 10, 1883 Howard County, Maryland
Died	October 29, 1959 (aged 76)
Residence	Massachusetts, United States
Nationality	American
Fields	Electrical Engineering
Institutions	General Electric University of Texas at Austin
Alma mater	Vassar College Massachusetts Institute of Technology
Notable awards	National Inventors Hall of Fame

First female professor in Electrical Engineering in the country

$abc - \alpha\beta 0$





Robert H. Park (March 15, 1902 – February 18, 1994) was an American electrical engineer and inventor, best known for the Park's transformation, used to simplify the analysis of three-phase electric circuits. His related 1929 concept paper ranked second, when looking at the impact of all twentieth century power engineering papers.[1] [2] Park was an IEEE Fellow and a member of the National Academy of Engineering.[3][4]

Park was born on March 15, 1902, in Strasbourg, when his father urban sociologist Robert E. Park was studying in Germany. Back in the United States Park lived in Wollaston, Massachusetts and earned in 1923 a degree in electrical engineering at the Massachusetts Institute of Technology. After this he went to the Royal Institute of Technology in Stockholm, Sweden to improve his knowledge on operational calculus.[3][4][5]

$abc - dq0$

$* \; abc \rightarrow \alpha\beta 0 \rightarrow dq0$

Park's 1929 paper is voted the second most important paper in Power Engineering (1900-1999)

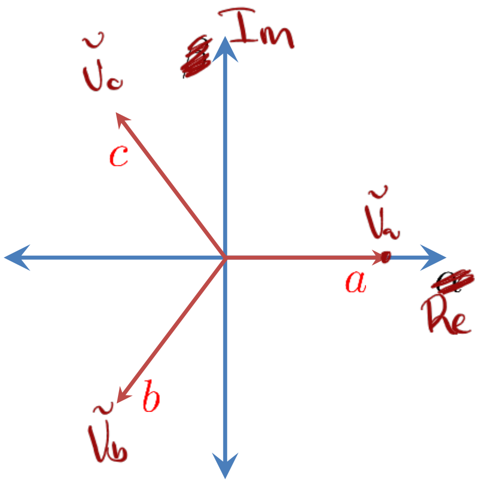
Milestone Achievement Papers

The final four papers are:

- **Charles L. Fortescue, "Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks,"** *Transactions of the AIEE*, vol. 37, pp. 1027-1140, 1918
- **Robert Park, "Two Reaction Theory of Synchronous Machines,"** *Transactions of the AIEE*, vol. 48, pp. 716-730, 1929
- **James Ward and Harry Hale, "Digital Computer Solution of Power Flow Problems,"** *Transactions of the AIEE*, vol. 75, Pt. iii, pp. 398-404, January 1956
- **John R. Carson, "Wave Propagation in Overhead Wires with Ground Return,"** *Bell System Technical Journal*, vol. 5, pp. 539-554, October 1926.

Clarke Transformation

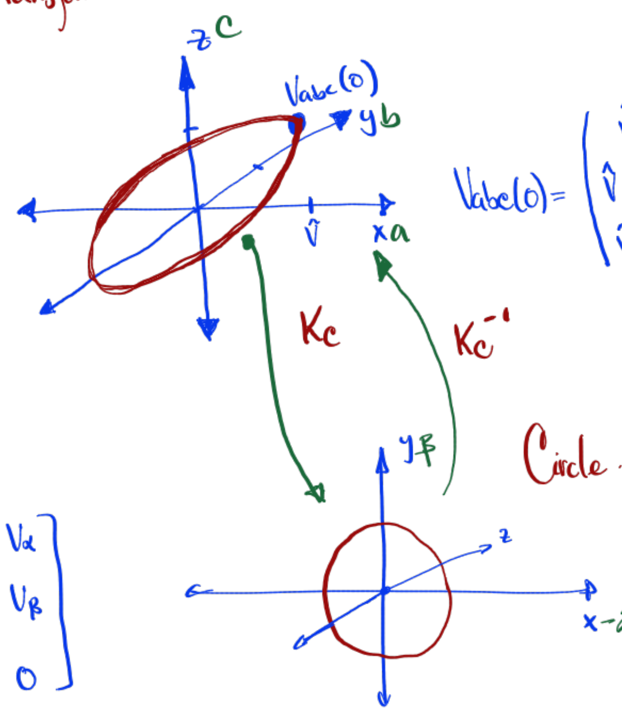
- Clarke transformation (alpha-beta transformation)
  - A transformation matrix to change three phase signals onto the $\alpha\beta$ axes



$$V_{\alpha\beta 0} = K_C V_{abc}$$

$$\underbrace{K_C}_{\text{Clarke's Transform.}} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

- $V_{abc} = \begin{pmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{pmatrix} \in \mathbb{R}^3$
- $V_{abc} = \begin{pmatrix} \hat{V}\cos(\omega t) \\ \hat{V}\cos(\omega t - 2\pi/3) \\ \hat{V}\cos(\omega t + \frac{2\pi}{3}) \end{pmatrix}$



$$V_{abc}(0) = \begin{pmatrix} \hat{V} \\ \hat{V}\cdot\cos(-\frac{2\pi}{3}) \\ \hat{V}\cos(\frac{2\pi}{3}) \end{pmatrix}$$

Circle.

$$V_a(t) = V\cos(\theta(t)) \longrightarrow \tilde{V}_a = \frac{V}{\sqrt{2}}\angle 0^\circ$$
$$V_b(t) = V\cos(\theta(t) - 2\pi/3) \quad \tilde{V}_b = \frac{V}{\sqrt{2}}\angle -120^\circ$$
$$V_c(t) = V\cos(\theta(t) + 2\pi/3) \quad \tilde{V}_c = \frac{V}{\sqrt{2}}\angle +120^\circ$$

$$V_{abc} = \begin{pmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{pmatrix} \quad V_{\alpha\beta 0} = \begin{pmatrix} V_\alpha(t) \\ V_\beta(t) \\ V_0(t) \end{pmatrix}$$

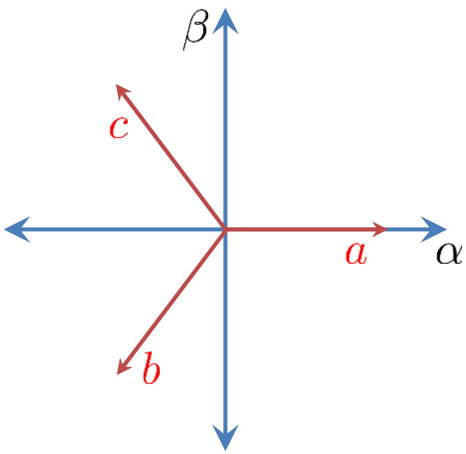
$$\begin{bmatrix} V_\alpha(t) \\ V_\beta(t) \\ V_0(t) \end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{bmatrix} = \begin{bmatrix} V_\alpha \\ V_\beta \\ 0 \end{bmatrix}$$

- $V_0(t) = \frac{1}{\sqrt{2}}\left(V_a(t) + V_b(t) + V_c(t)\right) = 0$ (Balanced), $V_\alpha = \sqrt{\frac{2}{3}}\cdot\frac{3}{2}\hat{V}\cos(\omega t)$, $V_\beta = \sqrt{\frac{2}{3}}\,\frac{3}{2}\hat{V}\sin(\omega t)$

Inverse Clarke Transformation

- The inverse Clarke transformation can then be used to obtain the abc values from the alpha/beta components

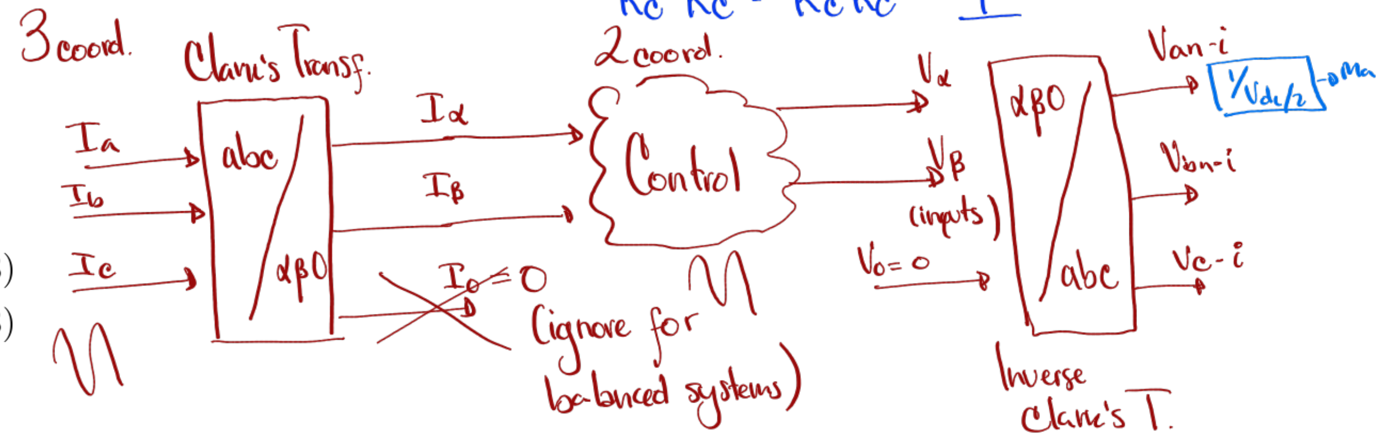
Inverse Clarke Transformation



$$V_{abc} = K_C^{-1} V_{\alpha\beta 0} \qquad \underbrace{K_C^{-1}}_{} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\alpha\beta 0 \longrightarrow abc$

$K_C^{-1} K_C = K_C K_C^{-1} = I$



$$V_a(t) = V\cos(\theta(t))$$
$$V_b(t) = V\cos(\theta(t) - 2\pi/3)$$
$$V_c(t) = V\cos(\theta(t) + 2\pi/3)$$

$$V_{abc} = \begin{pmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{pmatrix} \qquad V_{\alpha\beta 0} = \begin{pmatrix} V_\alpha(t) \\ V_\beta(t) \\ V_0(t) \end{pmatrix}$$

- Advantages: Design controller for 2 coordinates only (α, β)
- Disadvantage: $\alpha\beta$ components are still ac (Sinusoidal)

Summary of Clarke Transformation

- Clarke transformation can be used to reduce a three phase system into orthogonal components (alpha, beta, 0)

  $\perp$ (separated by 90°)

- **If the system is balanced, the V_0 component is always 0**

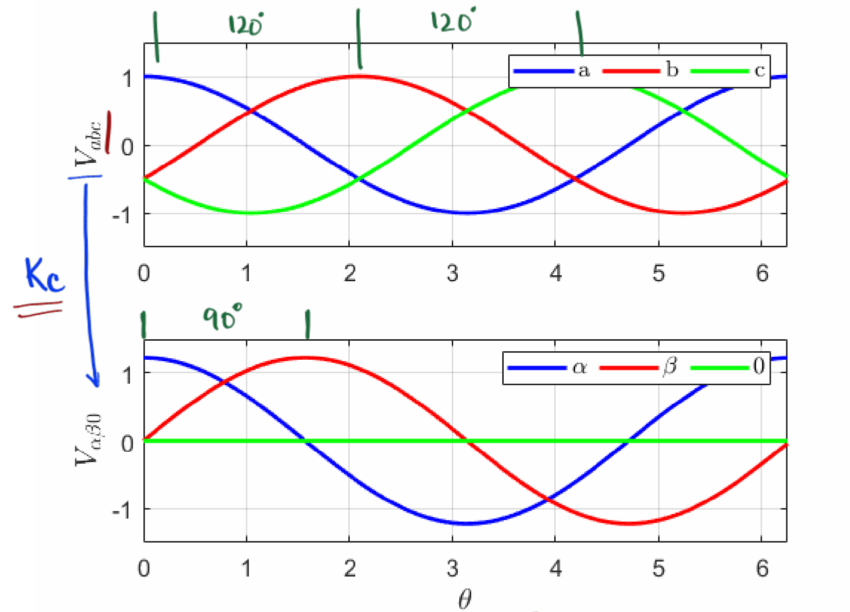
$$V_a(t) = V\cos(\theta(t))$$
$$V_b(t) = V\cos(\theta(t) - 2\pi/3)$$
$$V_c(t) = V\cos(\theta(t) + 2\pi/3)$$

$$V_{abc} = \begin{pmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{pmatrix} \qquad V_{\alpha\beta 0} = \begin{pmatrix} V_\alpha(t) \\ V_\beta(t) \\ V_0(t) \end{pmatrix}$$

$$V_{\alpha\beta 0} = K_C V_{abc} \qquad V_{abc} = K_C^{-1} V_{\alpha\beta 0}$$

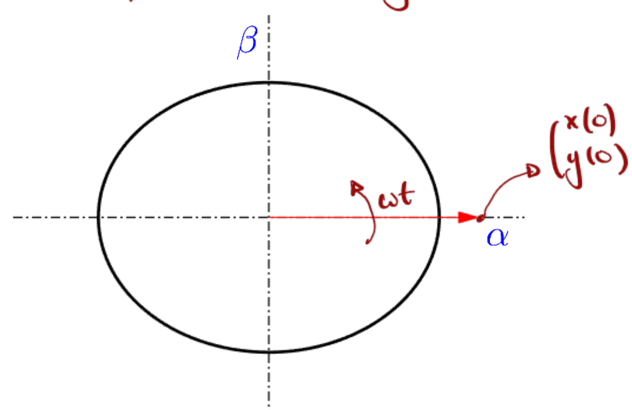
$$K_C = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Power Invariant Clarke's Transformation



$V_\alpha = \sqrt{\frac{2}{3}} \cdot \frac{3}{2}\cos(\omega t)$ $\quad V_\beta = \sqrt{\frac{2}{3}} \cdot \frac{3}{2}\sin(\omega t)$

$x(t) \qquad y(t)$

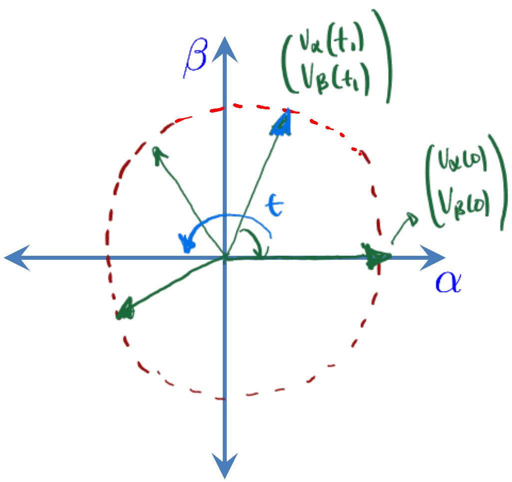


Park Transformation

- Park transformation (direct-quadrature transformation (dq))
  - The dq transformation changes a three phase system into **dc values**/ Signals. (PI, Integrators)
  - This can be done by first converting to alpha/beta/0 components, and then do an angle rotation matrix!

$\theta = \omega t = 2\pi f t \quad f = 60\text{Hz}$

$\alpha, \beta, 0$ first



$$V_{dq0} = K_{PC} V_{\alpha\beta 0}$$

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix}$$

$t = 0$ $\quad V_\alpha(0) = \sqrt{\frac{2}{3}}\frac{3}{2}\hat{V}$

$V_\beta(0) = 0$

$V_\alpha = \sqrt{\frac{2}{3}} \cdot \frac{3}{2}\hat{V}\cos(\omega t)$

$V_\beta = \sqrt{\frac{2}{3}}\frac{3}{2}\hat{V}\sin(\omega t)$

- Ignore V_0 component (balanced systems)

$\tilde{V} = \sqrt{\frac{2}{3}} \cdot \frac{3}{2}\hat{V}$

$+\phi$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} V_\alpha = \tilde{V}\cos(\omega t) \\ V_\beta = \tilde{V}\sin(\omega t) \end{bmatrix} = \begin{bmatrix} \tilde{V} \\ 0 \end{bmatrix}$$

$+\phi$

- for this case ($\phi = 0$), $\quad V_d = \tilde{V}$, $V_q = 0$

$V_d = \tilde{V}\cos^2(\omega t) + \tilde{V}\sin^2(\omega t) = \tilde{V}$

$V_q = \underline{-\tilde{V}\cos(\omega t)\sin(\omega t)} + \underline{\tilde{V}\cos(\omega t)\sin(\omega t)} = 0$

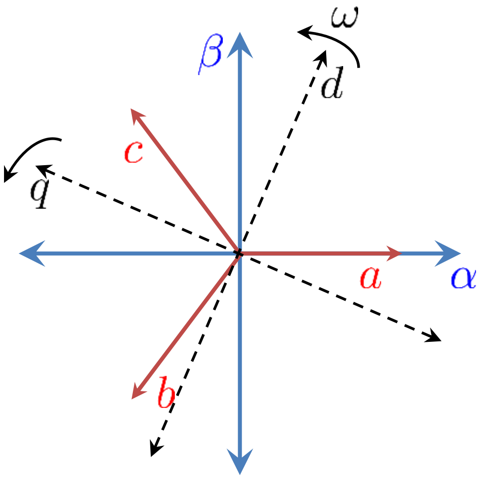
- If $\phi \neq 0$, $V_d \neq 0$ $V_q \neq 0$ (constant values)

$$V_a(t) = V\cos(\theta(t))$$
$$V_b(t) = V\cos(\theta(t) - 2\pi/3)$$
$$V_c(t) = V\cos(\theta(t) + 2\pi/3)$$

$$V_{abc} = \begin{pmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{pmatrix} \quad V_{dq0} = \begin{pmatrix} V_d(t) \\ V_q(t) \\ V_0(t) \end{pmatrix}$$

Park Transformation (2)

- Park transformation (direct-quadrature transformation (dq))
  - The dq transformation changes a three phase system into **dc values**



$$V_{dq0} = K_P V_{abc}$$ (Skips Clarke's transformation)

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin(\theta) & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_0 = \sqrt{\frac{2}{3}} \cdot \frac{\sqrt{2}}{2} (V_a + V_b + V_c) = 0$$

$$V_a(t) = \hat{V} \cos(\theta(t) + \phi)$$
$$V_b(t) = \hat{V} \cos(\theta(t) - 2\pi/3 + \phi)$$
$$V_c(t) = V \cos(\theta(t) + 2\pi/3 + \phi)$$

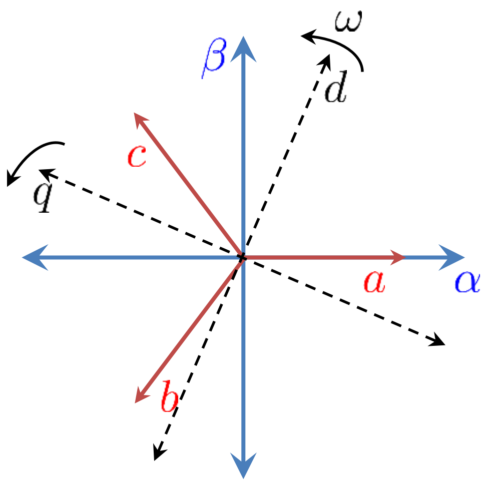
when $\phi = 0$ $\quad V_d = \sqrt{\frac{2}{3}} \cdot \frac{3}{2} \hat{V}$, $V_q = 0$

when $\phi \neq 0$ $\quad V_d \neq 0$ $\quad V_q \neq 0$ Constant elements. (DC components)

$$V_{abc} = \begin{pmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{pmatrix} \quad V_{dq0} = \begin{pmatrix} V_d(t) \\ V_q(t) \\ V_0(t) \end{pmatrix}$$

Inverse Park Transformation

- The inverse park transformation converts the *dq0* components back to *abc*

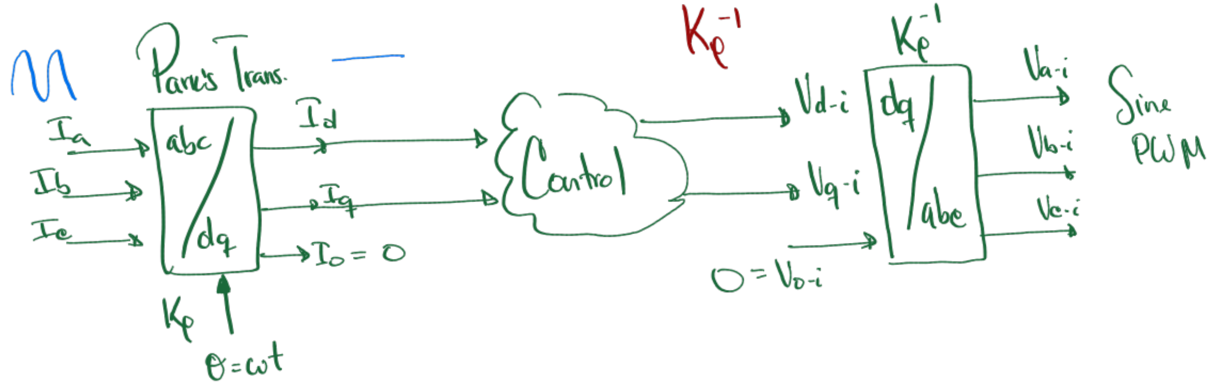


$$V_{abc} = K_P^{-1} V_{dq0}$$

Power invariant Transf.

$K_P K_P^{-1} = K_P^{-1} K_P = I$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \underbrace{\sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \frac{\sqrt{2}}{2} \\ \cos(\theta - 2\pi/3) & -\sin(\theta - 2\pi/3) & \frac{\sqrt{2}}{2} \\ \cos(\theta + 2\pi/3) & -\sin(\theta + 2\pi/3) & \frac{\sqrt{2}}{2} \end{bmatrix}}_{K_P^{-1}} \begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix}$$



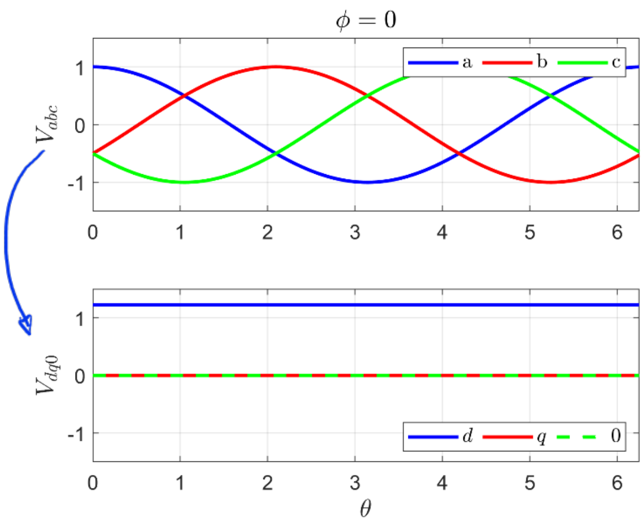
$$V_a(t) = V\cos(\theta(t))$$
$$V_b(t) = V\cos(\theta(t) - 2\pi/3)$$
$$V_c(t) = V\cos(\theta(t) + 2\pi/3)$$

$$V_{abc} = \begin{pmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{pmatrix} \quad V_{dq0} = \begin{pmatrix} V_d(t) \\ V_q(t) \\ V_0(t) \end{pmatrix}$$

Summary of Park Transformation

- Park transformation can be used to transform a three phase system into **dc** components!
- **If the system is balanced, the V_0 component is always 0!**

$$V_a(t) = V\cos(\theta(t) + \phi)$$
$$V_b(t) = V\cos(\theta(t) - 2\pi/3 + \phi)$$
$$V_c(t) = V\cos(\theta(t) + 2\pi/3 + \phi)$$

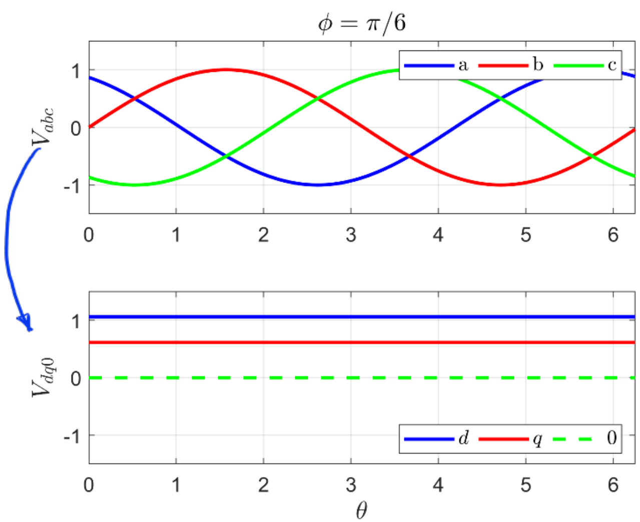


Kp

$$V_{abc} = \begin{pmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{pmatrix} \quad V_{dq0} = \begin{pmatrix} V_d(t) \\ V_q(t) \\ V_0(t) \end{pmatrix}$$

$X_{dq0} = K_p X_{abc}$

$X = V, I$



Kp

$$V_{dq0} = K_P V_{abc} \qquad V_{abc} = K_P^{-1} V_{dq0}$$

$$K_P = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin(\theta) & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$\theta = \omega t = 2\pi f t \qquad f = 60\text{Hz}$

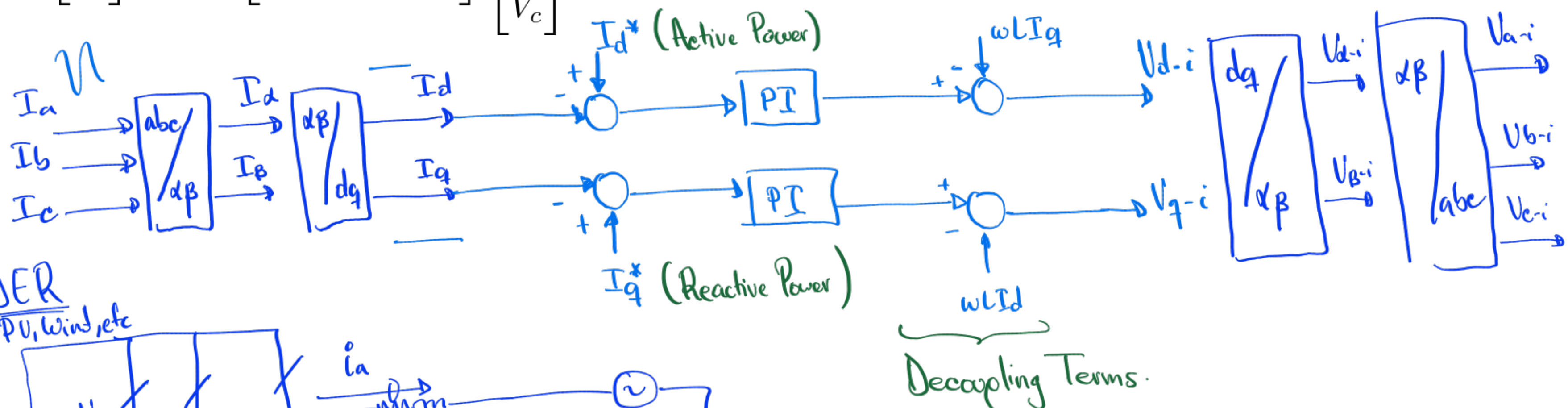
Clarke and Park Transformation for Balanced Systems

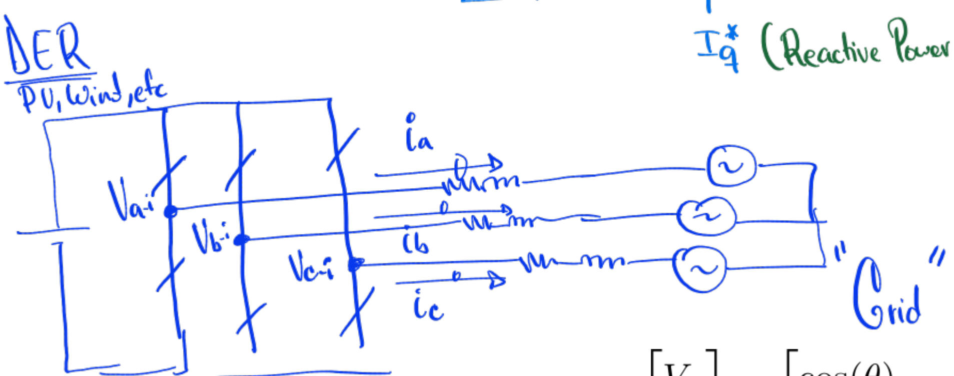
- If the three phase system is known to be balanced, we can ignore the 0 component
- This simplifies the equations significantly

Current Control in dq frame.

$$\begin{bmatrix}V_\alpha\\ V_\beta\end{bmatrix} = \overbrace{\sqrt{\frac{2}{3}}\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}}^{\text{Clarke's}} \begin{bmatrix}V_a\\ V_b\\ V_c\end{bmatrix}$$

$$\begin{bmatrix}V_d\\ V_q\end{bmatrix} = \overbrace{\begin{bmatrix}\overset{*}{\cos}(\theta) & \overset{*}{\sin}(\theta)\\ -\sin(\theta) & \cos(\theta)\end{bmatrix}}^{\text{Park's}} \begin{bmatrix}V_\alpha\\ V_\beta\end{bmatrix}$$





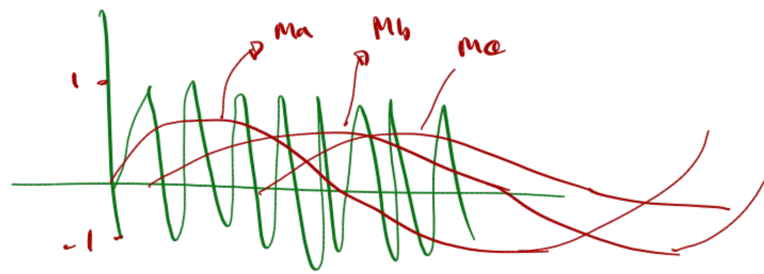
$$\begin{bmatrix}V_\alpha\\ V_\beta\end{bmatrix} = \begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix} \begin{bmatrix}V_d\\ V_q\end{bmatrix}$$

$$\begin{bmatrix}V_a\\ V_b\\ V_c\end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix}1 & 0\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{1}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix} \begin{bmatrix}V_\alpha\\ V_\beta\end{bmatrix}$$

Outline

- Review of dc/ac (ac/dc) converters
- abc to dq transformation – Clarke and Park Transformations
- **Space vector PWM**
- Controller Design Overview *
- Applications

Sine-PWM



$V_{an}(t) = m_a(t) \cdot \frac{V_{dc}}{2}, \quad m_a(t) \in [-1, 1]$

Max, peak Line to neutral voltage using Sine PWM is $\frac{V_{dc}}{2}$

Space Vector PWM: Switching Signals to Vabc-o

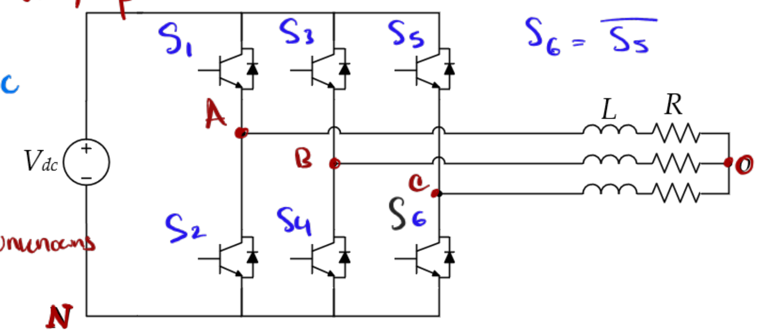
- The alpha/beta transformation can be used to modulate a three phase inverter
- Let's analyze all of the switching signals for a three phase inverter!

$S_2 = \overline{S_1}$

$S_4 = \overline{S_3}$

$S_6 = \overline{S_5}$

- What are V_{AO}, V_{BO}, V_{CO} as a function of S_1, S_3, S_5? $\Rightarrow V_{\alpha i}, V_{\beta i}$



- $V_{AO} = f(S_1, S_3, S_5; V_{dc})$ (Not trivial) if $S_1 = 1 \Rightarrow V_{AN} = V_{dc}$; $S_1 = 0 \Rightarrow V_{AN} = 0$
- $V_{AN} = S_1 V_{dc}$, $V_{BN} = S_3 V_{dc}$, $V_{CN} = S_5 V_{dc}$

3 eqns, 3 unknowns

- $V_{AB} = V_{AO} - V_{BO} = V_{AN} - V_{BN} = (S_1 - S_3)V_{dc}$
- $V_{BC} = V_{BO} - V_{CO} = V_{BN} - V_{CN} = (S_3 - S_5)V_{dc}$
- $V_{CA} = V_{CO} - V_{AO} = V_{CN} - V_{AN} = (S_5 - S_1)V_{dc}$

$\rightarrow$ 3ϕ balanced system: $V_{AO} + V_{BO} + V_{CO} = 0$

$$V_{BC} - V_{AB} = V_{BO} - V_{CO} - (V_{AO} - V_{BO}) = 2V_{BO} - V_{CO} - V_{AO}$$
$$= 2V_{BO} - (V_{AO} + V_{CO}) = 3V_{BO}$$

$$V_{BC} - V_{AB} = 3V_{BO} = [(S_3 - S_5) - (S_1 - S_3)]V_{dc} = [2S_3 - S_1 - S_5]V_{dc}$$

$\rightarrow$ $V_{BO} = \frac{V_{dc}}{3}(-S_1 + 2S_3 - S_5)$, Similarly:

$V_{AO} = \frac{V_{dc}}{3}(2S_1 - S_3 - S_5)$

$V_{CO} = \frac{V_{dc}}{3}(-S_1 - S_3 + 2S_5)$

$$\boxed{\begin{bmatrix} V_{AO} \\ V_{BO} \\ V_{CO} \end{bmatrix} = \frac{V_{dc}}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} S_1 \\ S_3 \\ S_5 \end{bmatrix}}$$ *

- 3 independent switches: S_1, S_3, S_5

$S_{1,3,5} \in \{0, 1\}$.

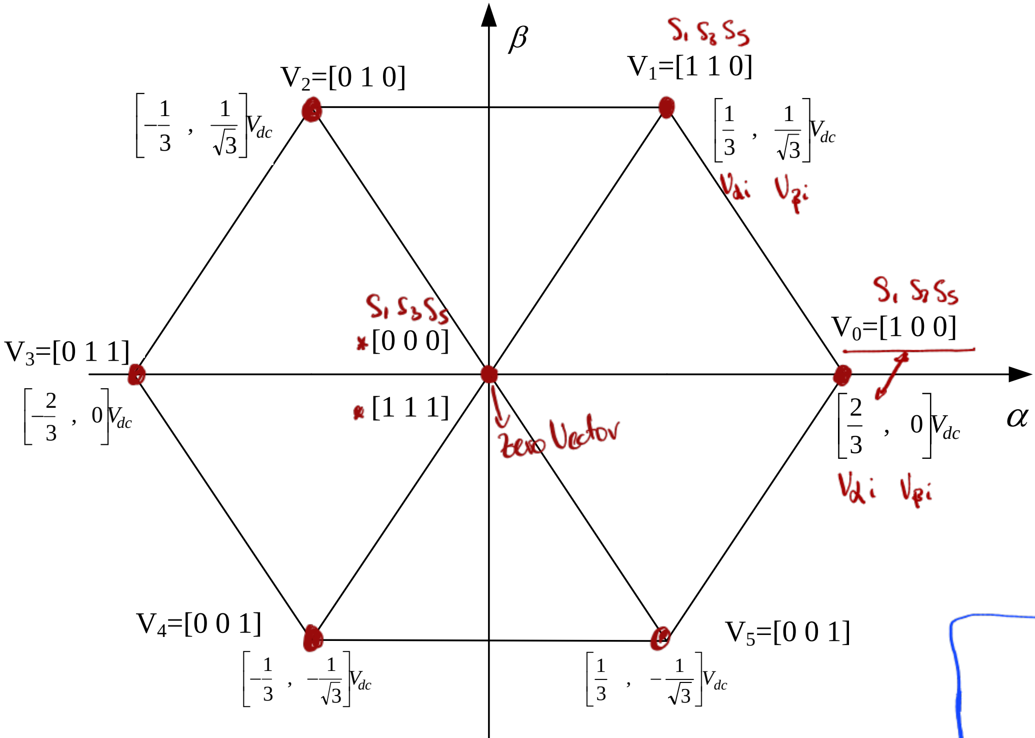
- How many switching combinations do we have? $2^3 = 8$

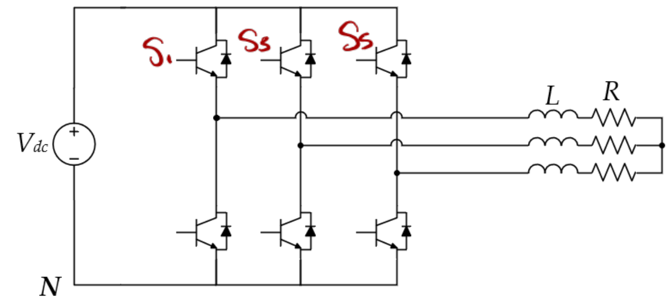
S_5	S_3	S_1
0	0	0
0	0	1
0	1	0
⋮		

What about $\underbrace{\begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix} = \begin{bmatrix} ? \end{bmatrix}\begin{bmatrix} S_1 \\ S_3 \\ S_5 \end{bmatrix}}_{*}$?

Space Vector PWM: Switching Signals to Alpha/Beta

- We can then convert each possible switching combination (8) to alpha/beta component





Goal: Obtain $\begin{pmatrix} V_\alpha \\ V_\beta \end{pmatrix}$ as a function of S_1, S_3, S_5.

- We can use Clarke's transformation (Not Power Inv.)

$$\begin{bmatrix} V_{\alpha i} \\ V_{\beta i} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\underbrace{\begin{bmatrix} V_{AO} \\ V_{BO} \\ V_{CO} \end{bmatrix}}_{\text{prev slide*}}$$

$S_{1,3,5} \in \{0, 1\}$. only 8 possible combinations!

$$\begin{bmatrix} V_{\alpha i} \\ V_{\beta i} \end{bmatrix} = \frac{2}{3}\cdot\frac{V_{dc}}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} S_1 \\ S_3 \\ S_5 \end{bmatrix}$$

$=$

Try a few cases..

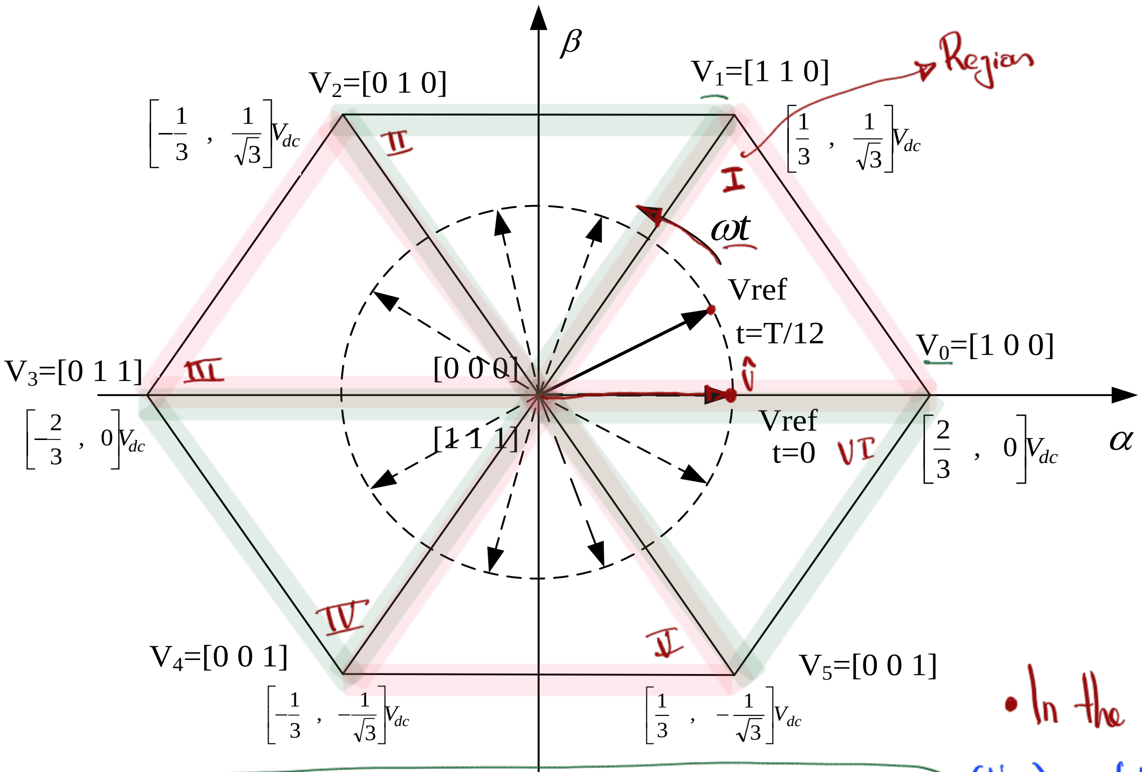
for $S_1S_3S_5 = 001$

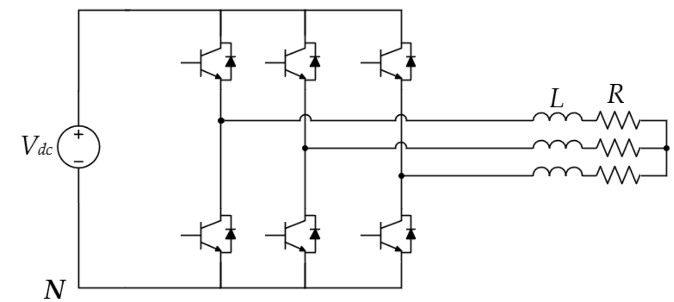
$$\begin{bmatrix} V_{\alpha i} \\ V_{\beta i} \end{bmatrix} = V_{dc}\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \end{bmatrix}\begin{bmatrix} S_1 \\ S_3 \\ S_5 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ 0 \end{bmatrix}\cdot V_{dc}$$

	S_1	S_3	S_5	$V_{\alpha i}$	$V_{\beta i}$	
1	0	0	0	0	0	→ Zero Vector
2	0	0	1	-1/3	$-1/\sqrt{3}$	
3	0	1	0	-1/3	$1/\sqrt{3}$	
	0	1	1	-2/3	0	
	1	0	0	2/3	0	
	1	0	1	1/3	$-1/\sqrt{3}$	
	1	1	0	1/3	$1/\sqrt{3}$	
8	1	1	1	0	0	→ Zero Vector

Space Vector PWM: Rotating Reference Signal

- **Rotating Reference:** in the αβ plane formed by the Clark transformation of balanced three phase voltages (currents).





- Consider a certain "desired" inverter voltage:

$$V_{ao\text{-}i} = \hat{V}\cos(\omega t+\phi)$$
$$V_{bo\text{-}i} = \hat{V}\cos(\omega t+\phi-2\pi/3)$$
$$V_{co\text{-}i} = \hat{V}\cos(\omega t+\phi+2\pi/3)$$

- In the α-β frame (not power invariant transformation):

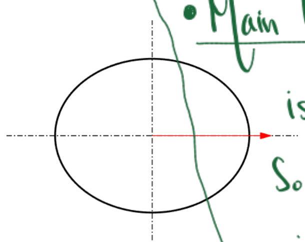
$$\begin{pmatrix} V_{\alpha i} \\ V_{\beta i} \end{pmatrix} = \underbrace{\frac{2}{3}}_{\text{Standard Clarke}} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \rightarrow \begin{matrix} V_{\alpha i} = \hat{V}\cos(\omega t+\phi) \\ V_{\beta i} = \hat{V}\sin(\omega t+\phi) \end{matrix}$$

Same magnitude.

$Mag = \sqrt{V_{\alpha i}^2 + V_{\beta i}^2} = \hat{V}$

$ang = \tan^{-1}\left(\frac{V_{\beta i}}{V_{\alpha i}}\right) = \omega t + \phi$

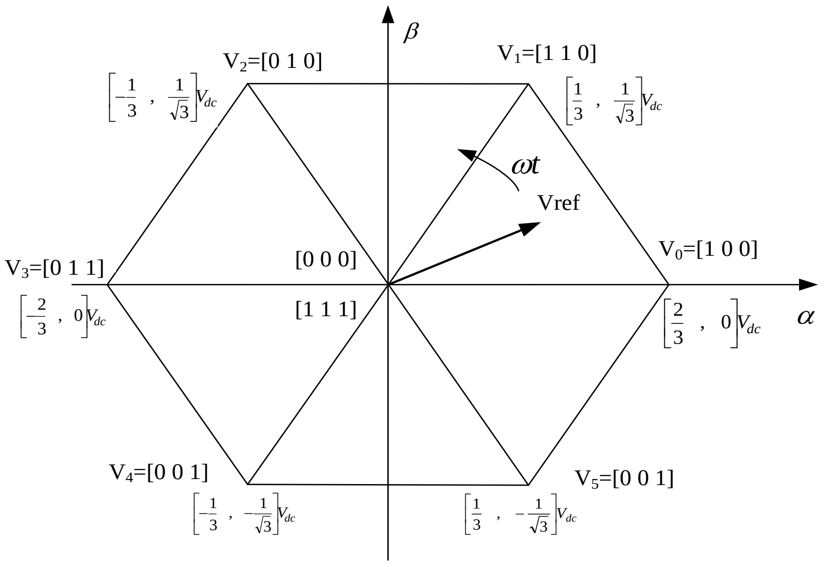
if $\phi = 0 \Rightarrow Mag = \hat{V}$, angle $= \omega t$. (rotating vector)

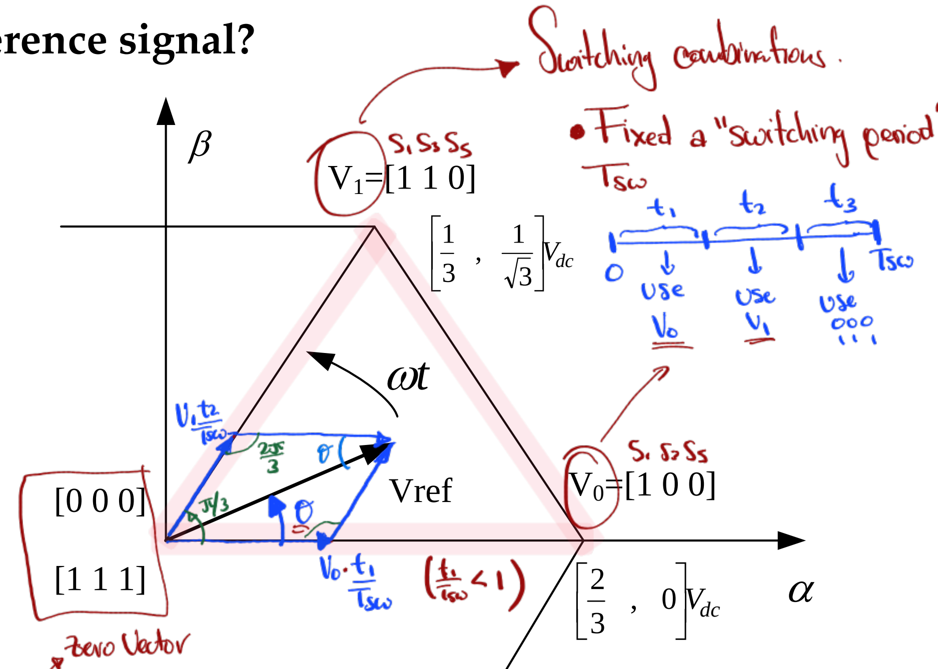


- Main Idea: if the vector $\begin{pmatrix} V_{\alpha i} \\ V_{\beta i} \end{pmatrix} = \begin{pmatrix} \hat{V}\cos(\omega t+\phi) \\ \hat{V}\sin(\omega t+\phi) \end{pmatrix}$ is in region I, use switching combinations. $S_0 \triangleq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $S_1 \triangleq \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ to "approximate" (average) it! Similar for other regions.

SVPWM: Reference Synthesis with Switching Vectors

- **How can we approximate a rotating reference signal?**





Zero Vector

- Fix a switching period T_{sw} and define 3 times t_1, t_2, t_3 s.t. $t_1+t_2+t_3 = T_{sw}$
- If $T_{sw} << \frac{1}{f} = T_f$ then V_{ref} is approximately constant during T_{sw}.
- Goal: Approximate V_{ref} on "average" during one T_{sw}, i.e.:

$V_{dc}\begin{pmatrix} \frac{2}{3} \\ 0 \end{pmatrix} = \frac{2V_{dc}}{3} + j0$ $\quad \frac{1}{3}V_{dc} + j\frac{V_{dc}}{\sqrt{3}}$ $\quad 0+0j$

$$\vec{V}_{ref} \approx \vec{V}_0 \cdot \frac{t_1}{T_{sw}} + \vec{V}_1 \cdot \frac{t_2}{T_{sw}} + \vec{V}_z \cdot \frac{t_3}{T_{sw}}$$

$\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\quad \begin{pmatrix}1\\1\\0\end{pmatrix}$ $\quad \begin{pmatrix}1\\1\\1\end{pmatrix}.\begin{pmatrix}0\\0\\0\end{pmatrix}$

- Unknowns: t_1, t_2, t_3, We can use the "law of sines"

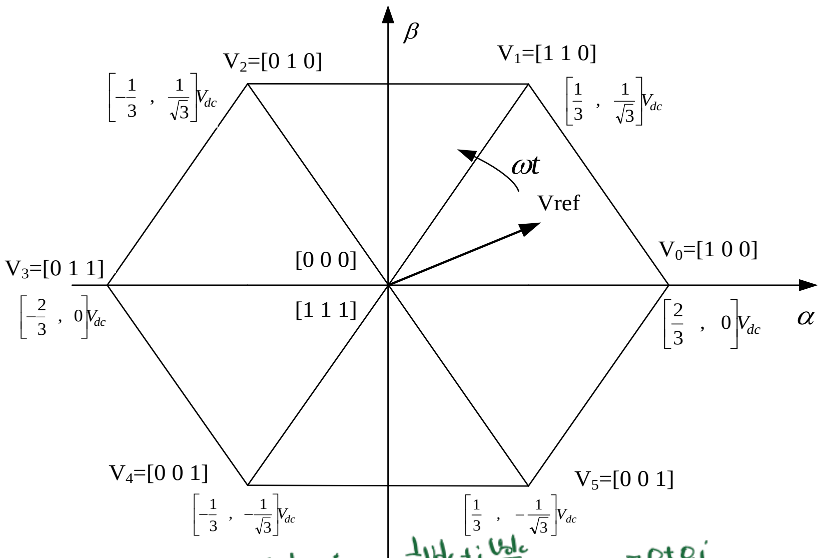
$$\frac{|V_{ref}|}{\sin\left(\frac{2\pi}{3}\right)} = \frac{|V_0| \frac{t_1}{T_{sw}}}{\sin\left(\frac{\pi}{3} - \theta\right)} = \frac{|V_1| \frac{t_2}{T_{sw}}}{\sin(\theta)}$$

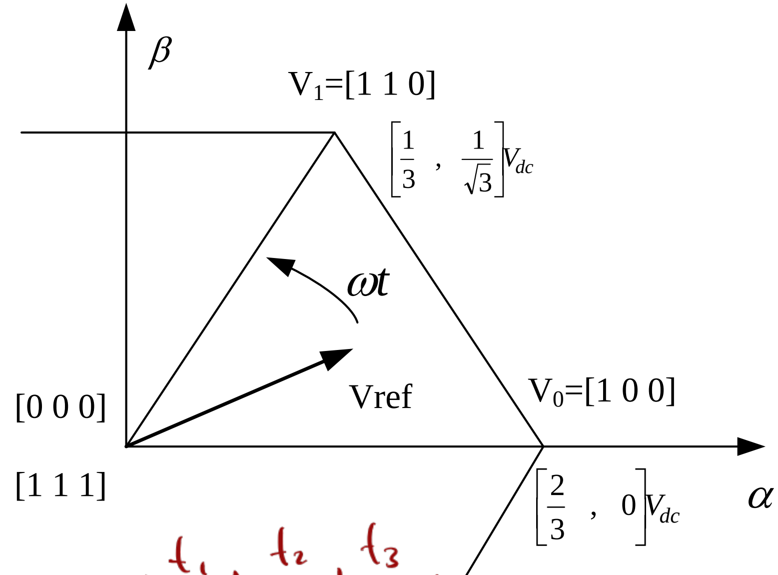
$$|V_{ref}| = \sqrt{V_{\alpha i}^2 + V_{\beta i}^2}$$

$$\theta = \tan^{-1}\left(\frac{V_{\beta i}}{V_{\alpha i}}\right)$$

SVPWM: Reference Synthesis with Switching Vectors

- **How can we approximate a rotating reference signal?**





Goal: $\vec{V}_{ref} = \underbrace{\vec{V_0}}_{\frac{2}{3}V_{dc}+0j}\cdot\frac{t_1}{T_{sw}} + \underbrace{\vec{V_1}}_{\frac{1}{3}V_{dc}+j\frac{V_{dc}}{\sqrt{3}}}\cdot\frac{t_2}{t_{sw}} + \underbrace{\vec{V}_{zr}}_{0+0j}\cdot\frac{t_3}{T_{sw}}$

$S_1 = 1$
$S_3 = 0$
$S_5 = 0$

$t_1 \quad t_2 \quad t_3$

$S_0 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad \begin{pmatrix}1\\1\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix} \quad T_{sw}$

- for t_1: $\dfrac{|V_{ref}|}{\sin\left(\frac{2\pi}{3}\right)} = \dfrac{|V_0|\cdot\frac{t_1}{T_{sw}}}{\sin\left(\frac{\pi}{3}-\theta\right)} = \dfrac{\frac{2}{3}\cdot V_{dc}\cdot\frac{t_1}{T_{sw}}}{\sin\left(\frac{\pi}{3}-\theta\right)} \Rightarrow \boxed{t_1 = \frac{3}{2}\cdot\frac{T_{sw}}{V_{dc}}\cdot\frac{\sin\left(\pi/3-\theta\right)}{\sin\left(2\pi/3\right)}\cdot|V_{ref}|}$?

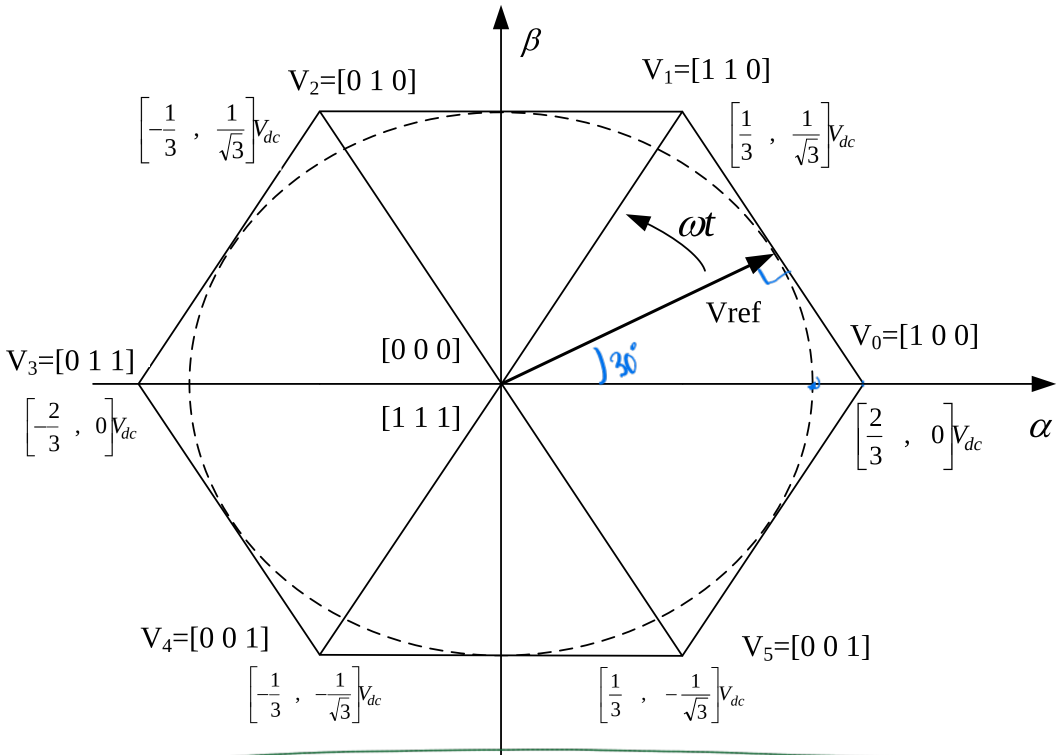
- for t_2: $\dfrac{|V_{ref}|}{\sin\left(2\pi/3\right)} = \dfrac{|V_1|\cdot\frac{t_2}{T_{sw}}}{\sin(\theta)} = \dfrac{\frac{2}{3}V_{dc}\cdot\frac{t_2}{T_{sw}}}{\sin(\theta)} \Rightarrow \boxed{t_2 = \frac{3}{2}\cdot\frac{T_{sw}}{V_{dc}}\cdot\frac{\sin(\theta_{ref})}{\sin\left(\frac{2\pi}{3}\right)}\cdot|V_{ref}|}$

- What about t_3?

$T_{sw} = t_1 + t_2 + t_3 \Rightarrow \boxed{t_3 = T_{sw} - t_1 - t_2}$

SVPWM: Voltage Capability

- **How does SVPWM compare to Sine PWM?**



• SVPWM has a better dc bus
Voltage utilization factor

.577 > .5

.577Vdc > .5 Vdc
SPVWM   SPWM

Sine-PWM

$\hat{V}_i = \hat{m}_i \cdot \frac{V_{dc}}{2}$ i=a,b,c

$\hat{m}_i \in [0,1] \rightarrow$ max peak Volt. is $\boxed{\frac{V_{dc}}{2}}$

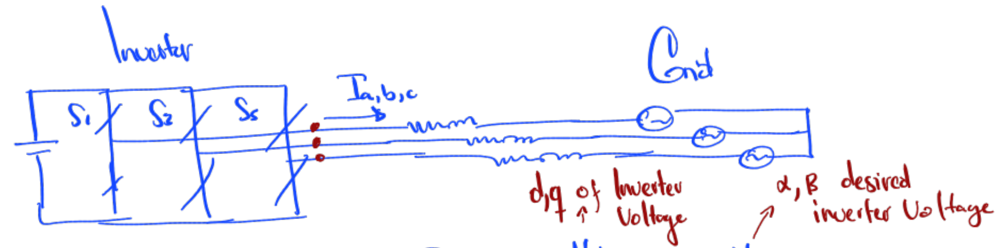
SVPWM

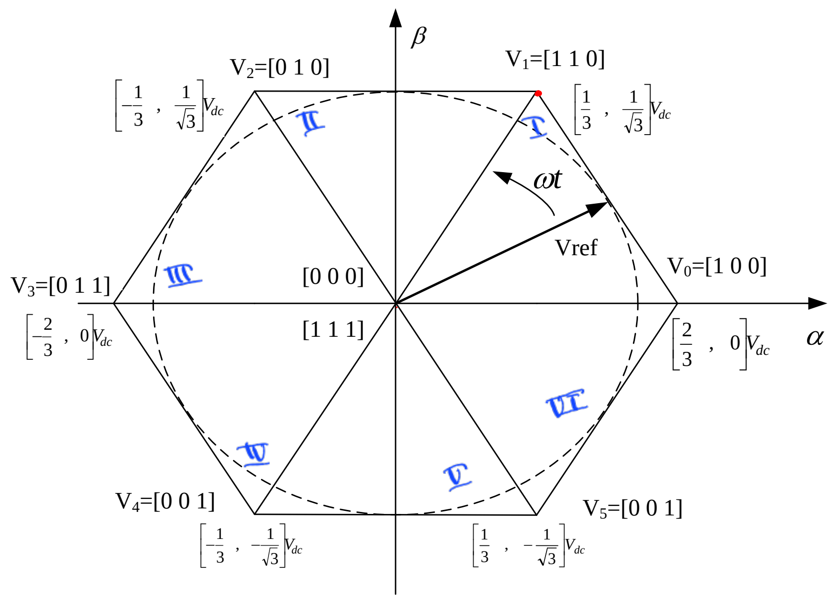
• max peak voltage is $\hat{V}_i = \frac{2}{3} \cdot V_{dc} \cdot \cos(30^\circ)$

$\boxed{\hat{V}_i = .577\, V_{dc}}$

SVPWM: Summary

- **Steps to implement SVPWM**





1) Obtain desired inverter voltage in α/β frame, $V_{\alpha i}, V_{\beta i}$

2) Compute $|V_{ref}| = \sqrt{V_{\alpha i}^2 + V_{\beta i}^2}$ $\quad \theta = \tan^{-1}\left(\frac{V_{\beta i}}{V_{\alpha i}}\right)$

3) Decide which sector you are on (I, II, ---, VI) and switching combinations to use:

I: $V_0 = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $V_1 = \begin{pmatrix}1\\1\\0\end{pmatrix}$, $\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}0\\0\\0\end{pmatrix}$

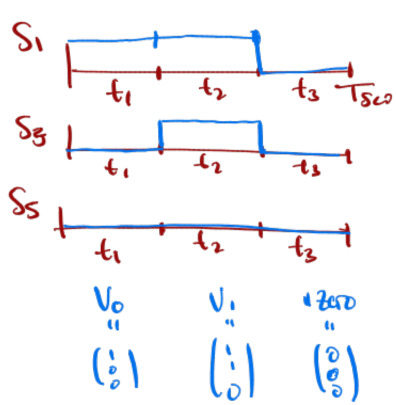
II: V_1, V_2

III: V_2, V_3

4) Compute the times t_1, t_2, t_3 given fixed T_{sw}.

I: $\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\begin{pmatrix}1\\1\\0\end{pmatrix}$ zero $\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}$

- for sector 1:

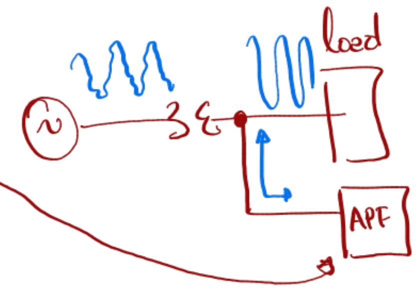


Outline

- Review of dc/ac (ac/dc) converters
- abc to dq transformation – Clarke and Park Transformations
- Space vector PWM
- **Controller Design Overview**
- Applications

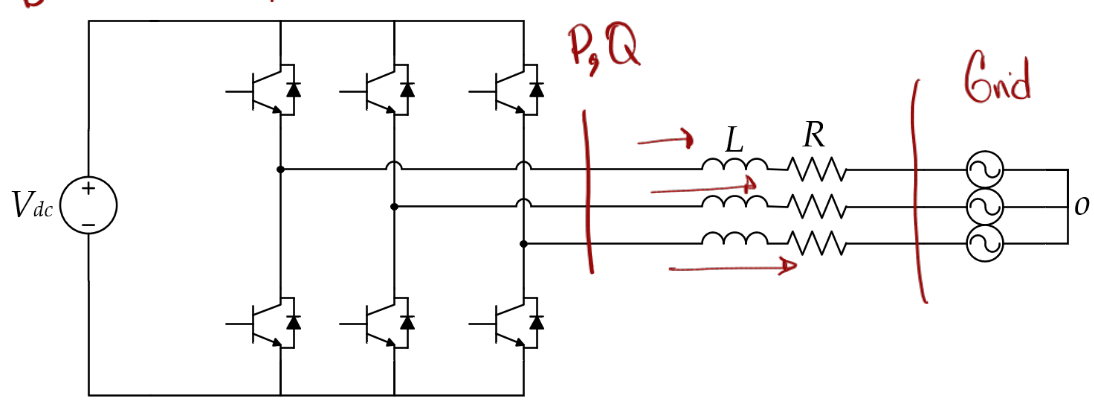
Motivation for Inverter Control

- The transformations we have learned can help us in controlling a three phase inverter to the grid
- Motivations for grid connections:
  - Send power to the grid (renewable sources)
  - Receive power from the grid (loads, batteries, etc.)
  - Improve grid power quality Active Power filter



  - Help with reactive power (power factor) support.
  - ….

DER (PV, Wind)



Active Power in Alpha/Beta and DQ Coordinates

- The instantaneous power of a three phase system can be computed as follows:

$$P_{3\phi}(t) = P_{3\phi} = V_{an} I_a + V_{bn} I_b + V_{cn} I_c = \underline{\text{Constant}}$$

$V_{an}(t) I_a(t) + \dots + \dots$

Power Invariant

$$K_C = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

- Derive this power in terms of α/β and dq coordinates

- Instantaneous Power: $P_{3\phi}(t) = V_{an}(t) I_a(t) + V_{bn}(t) I_b(t) + V_{cn}(t) I_c(t)$

$$V_{abc} \triangleq \begin{pmatrix} V_{an}(t) \\ V_{bn}(t) \\ V_{cn}(t) \end{pmatrix}, \quad I_{abc} = \begin{pmatrix} I_a(t) \\ I_b(t) \\ I_c(t) \end{pmatrix}$$

$$\Rightarrow P_{3\phi}(t) = \langle V_{abc}, I_{abc} \rangle = V_{abc}^T I_{abc} = (V_{an} \; V_{bn} \; V_{cn}) \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \; /\!/$$

$V_{abc} \in \mathbb{R}^3 \quad I_{abc} \in \mathbb{R}^3$

- In $\alpha\beta 0$ frame: $P_{3\phi}(t) = (K_C^{-1} V_{\alpha\beta 0})^T (K_C^{-1} I_{\alpha\beta 0})$

$$= V_{\alpha\beta 0}^T \underbrace{(K_C^{-1})^T K_C^{-1}}_{I_{3\times 3}} I_{\alpha\beta 0}$$

$$V_{\alpha\beta 0} = \begin{pmatrix} V_\alpha \\ V_\beta \\ V_0 \end{pmatrix} = K_C V_{abc} \Leftrightarrow V_{abc} = K_C^{-1} V_{\alpha\beta 0}$$

$I_{abc} = K_C^{-1} I_{\alpha\beta 0}$

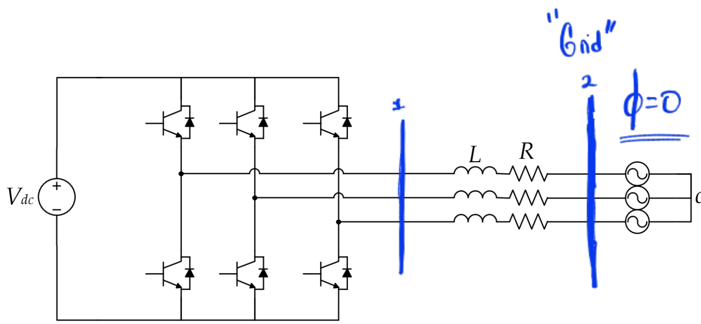
$(AB)^T = B^T A^T$

$K_C^{-1} = K_C^T$ (Power Invariant)

$$\Rightarrow P_{3\phi}(t) = V_{\alpha\beta 0}^T I_{\alpha\beta 0} = V_\alpha I_\alpha + V_\beta I_\beta + \cancel{V_0 I_0}$$

$$P_{3\phi} = V_\alpha I_\alpha + V_\beta I_\beta \quad \text{(Balanced)}$$

- In $dq0$ frame: $\boxed{P_{3\phi}(t) = V_d I_d + V_q I_q \quad \text{(Balanced)}}$



Complex Power: Active and Reactive Power

H. Akagi

- For ac systems, we can also define the complex/apparent power as follows:

(Apparent Power)
$$S_{3\phi} = 3\tilde{V}\tilde{I}^* \quad \text{(phasors)} = (\tilde{V}_a\tilde{I}_a + \tilde{V}_b\tilde{I}_b + \tilde{V}_c\tilde{I}_c)$$
$$= \underbrace{P_{3\phi}}_{\text{Real}} + \underbrace{jQ_{3\phi}}_{\text{Imag.}}$$

$\tilde{V}_a = V_{rms}\angle\theta_V$

$\tilde{I}_a = I_{rms}\angle\theta_I$

- When we transform a set of three phase signals into α/β or dq, we can also think of complex numbers (why? α,β and d,q are separated by $90°$ or orthogonal components)

$\rightarrow V_{\alpha\beta} = V_\alpha + jV_\beta$

$V_{dq} = V_d + jV_q$

$I_{\alpha\beta} = I_\alpha + jI_\beta$

- Apparent Power in $\alpha\beta$: $S_{3\phi} = (V_{\alpha\beta})(I_{\alpha\beta})^*$

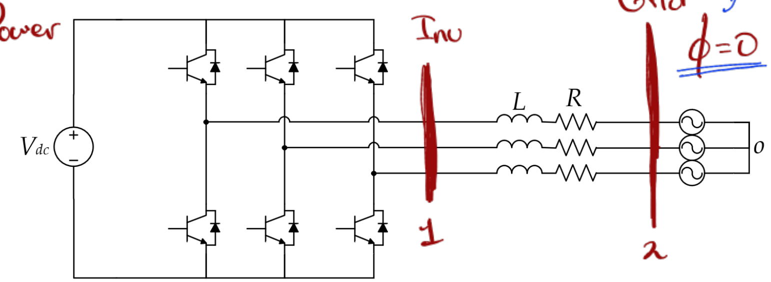
$$S_{3\phi} = (V_\alpha + jV_\beta)(I_\alpha - jI_\beta)$$
$$= (V_\alpha I_\alpha - jV_\alpha I_\beta + jV_\beta I_\alpha - j^2 V_\beta I_\beta)$$
$$S_{3\phi} = \underbrace{(V_\alpha I_\alpha + V_\beta I_\beta)}_{\text{Active Power}} + j\underbrace{(V_\beta I_\alpha - V_\alpha I_\beta)}_{\text{Reactive Power}}$$

$$\boxed{\begin{aligned} P_{3\phi} &= V_\alpha I_\alpha + V_\beta I_\beta = V_d I_d + V_q I_q \\ Q_{3\phi} &= V_\beta I_\alpha - V_\alpha I_\beta = V_q I_d - V_d I_q \end{aligned}}$$

Grid $\phi = 0$

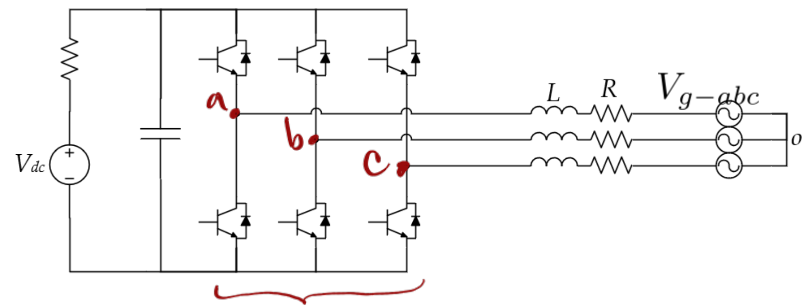
$V_d = \bar{V}$

$V_q = 0$



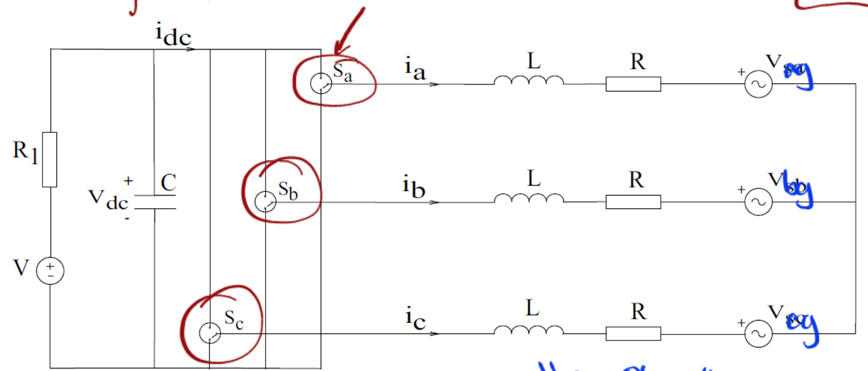
Three Phase Inverter State Space Modeling

- Analyze a grid connected three phase inverter
- Simplify the model by adding voltage and current sources
- → Derive a "dynamic" model of 3φ-inverter

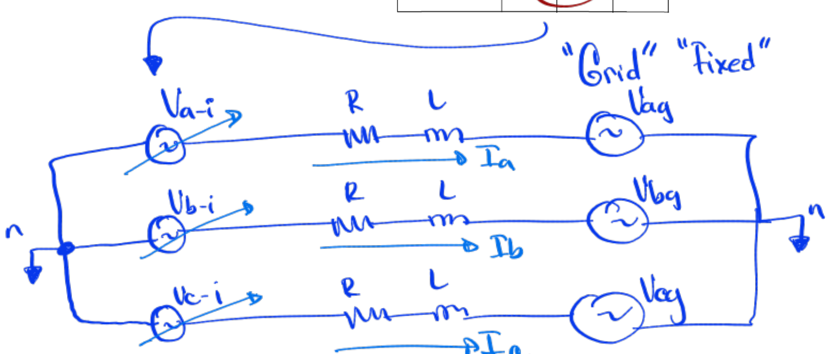


Linear: $\dot{x} = Ax + Bu$

nonlinear: $\dot{x} = f(x, u)$



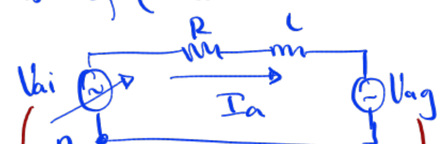
"Grid" "fixed"



$\underbrace{\quad}_{\text{3φ-inverter input}}$

→ 3rd order System $x = \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix}$

"Per Phase" of phase "a"



input

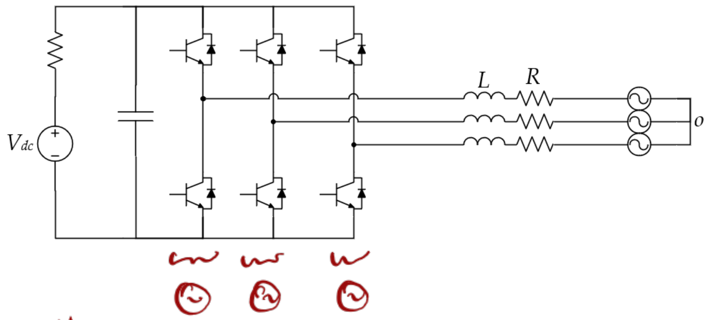
grid/disturbance

1st order system (1 inductor)

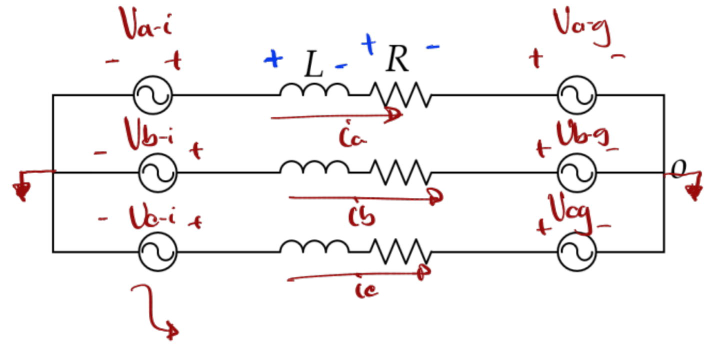
$x = I_a$

AC Side State Space Equations (abc)

- Derive the state space equations in "$\underline{abc}$"



$\approx$



- Derive the current equation for phase a, b, c.

$\underline{a}$: $-V_{a\text{-}i} + L\dfrac{di_a}{dt} + Ri_a + V_{a\text{-}g} = 0$

$\underline{b}$: $-V_{b\text{-}i} + L\dfrac{di_b}{dt} + Ri_b + V_{b\text{-}g} = 0$

$\underline{c}$: $-V_{c\text{-}i} + L\dfrac{di_c}{dt} + Ri_c + V_{c\text{-}g} = 0$

$$\left\{ \frac{di_j}{dt} = -\frac{R}{L} i_j + \frac{1}{L} V_{j\text{-}i} - \frac{1}{L} V_{j\text{-}g} \qquad j = a, b, c \right.$$

$$\Rightarrow \frac{d}{dt} \underbrace{\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}}_{i_{abc}} = \underbrace{\begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & -\frac{R}{L} & 0 \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}}_{A} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} & 0 & 0 \\ 0 & \frac{1}{L} & 0 \\ 0 & 0 & \frac{1}{L} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} V_{a\text{-}i} \\ V_{b\text{-}i} \\ V_{c\text{-}i} \end{bmatrix}}_{V_{abc\text{-}i}} + \begin{bmatrix} -\frac{1}{L} & 0 & 0 \\ 0 & -\frac{1}{L} & 0 \\ 0 & 0 & -\frac{1}{L} \end{bmatrix} \underbrace{\begin{bmatrix} V_{a\text{-}g} \\ V_{b\text{-}g} \\ V_{c\text{-}g} \end{bmatrix}}_{V_{abc\text{-}g}}$$

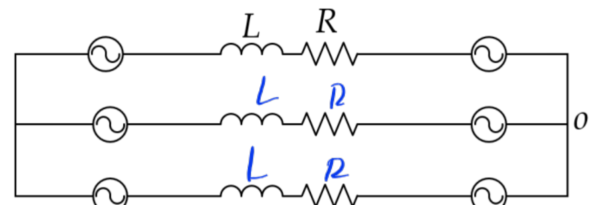
3^{rd} order Linear Time Invariant system

$$\dot{x} = Ax + Bu + PV_g$$

- The system of equations are decoupled $\Rightarrow$ "a" doesn't affect "b", "c"
- $\underline{\text{Disadvantages}}$ i) 3 states, ii) $V_{a\text{-}g}, V_{b\text{-}g}, V_{c\text{-}g}$ are $\underline{\text{sinusoidal}}$ ac

AC Side State Space Equations (Alpha/Beta)

- Derive the state space equations in α/β frame.



- Assumption: The 3 phase system is "balanced" $\longrightarrow$ V_0 (zero component) $= 0$
- The State Space model in abc can be written as:

$X = I, V$ $\left[X_{\alpha\beta o} = K_c X_{abc} \right]$

$X_{abc} = K_c^{-1} X_{\alpha\beta o}$ $\quad V_{\alpha\beta 0} = K_C V_{abc}$

$$L \frac{d i_{abc}}{dt} = -R i_{abc} + V_{abc\text{-}i} - V_{abc\text{-}g}$$

Constant Transform $\left[K_C = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \right.$

- In $\alpha\beta 0$ frame, the system becomes as follows:

$$i_{abc} = \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} \quad V_{abc\text{-}i} = \begin{pmatrix} V_{a\text{-}i} \\ V_{b\text{-}i} \\ V_{c\text{-}i} \end{pmatrix} \quad V_{abc\text{-}g} = \begin{pmatrix} V_{a\text{-}g} \\ V_{b\text{-}g} \\ V_{c\text{-}g} \end{pmatrix}$$

$$L \frac{d(K_c^{-1} i_{\alpha\beta 0})}{dt} = -R K_c^{-1} i_{\alpha\beta 0} + K_c^{-1} V_{\alpha\beta 0\text{-}i} - K_c^{-1} V_{\alpha\beta 0\text{-}g}$$

$$\Rightarrow K_c \left[L K_c^{-1} \frac{d i_{\alpha\beta 0}}{dt} = -R K_c^{-1} i_{\alpha\beta 0} + K_c^{-1} V_{\alpha\beta 0\text{-}i} - K_c^{-1} V_{\alpha\beta 0\text{-}g} \right]$$

$$\rightarrow \boxed{L \frac{d i_{\alpha\beta 0}}{dt} = -R i_{\alpha\beta 0} + V_{\alpha\beta 0\text{-}i} - V_{\alpha\beta 0\text{-}g}}$$

$$\left.\begin{aligned} &\rightarrow L \frac{d i_\alpha}{dt} = -R i_\alpha + V_{\alpha\text{-}i} - V_{\alpha\text{-}g} \\ &\rightarrow L \frac{d i_\beta}{dt} = -R i_\beta + V_{\beta\text{-}i} - V_{\beta\text{-}g} \end{aligned}\right\} \rightarrow$$

Second Order system

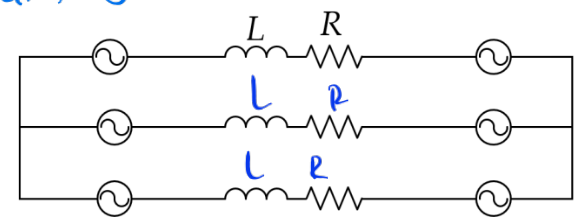
Drawback: ac, sinusoidal signals.

$$\rightarrow \cancel{L \frac{d i_0}{dt} = -R i_0 + V_{0\text{-}i} - V_{0\text{-}g}} = 0$$ (ignore for balanced systems)

AC Side State Space Equations (dq)

"Product Rule"

$$\frac{d}{dt} f(t)g(t) = \frac{df(t)}{dt} \cdot g(t) + f(t) \cdot \frac{dg(t)}{dt}$$



- Transform the equations from α/β to dq
  - The state space equation for the inverter in $\alpha\beta$ frame can be written as:

$$i_{\alpha\beta} = \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}$$

$$L \frac{di_{\alpha\beta}}{dt} = -R\, i_{\alpha\beta} + V_{\alpha\beta\text{-}i} - V_{\alpha\beta\text{-}g}$$

$$V_{\alpha\beta\text{-}i} = \begin{pmatrix} V_{\alpha\text{-}i} \\ V_{\beta\text{-}i} \end{pmatrix} \qquad V_{\alpha\beta\text{-}g} = \begin{pmatrix} V_{\alpha\text{-}g} \\ V_{\beta\text{-}g} \end{pmatrix}$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix}}_{K_{pc}(t)} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{bmatrix}}_{K_{pc}^{-1}(t)} \begin{bmatrix} V_d \\ V_q \end{bmatrix}$$

- Transform to dq frame:

$$X_{dq} = K_{pc}(t) X_{\alpha\beta}$$

$$X_{\alpha\beta} = K_{pc}^{-1}(t) X_{dq}$$

$$X_{dq} = \begin{pmatrix} X_d \\ X_q \end{pmatrix}, \quad X_{\alpha\beta} = \begin{pmatrix} X_\alpha \\ X_\beta \end{pmatrix}$$

$$L \frac{d\left(\overset{*}{\overbrace{K_{pc}^{-1}(t)\, i_{dq}(t)}}\right)}{dt} = -R\, K_{pc}^{-1}(t)\, i_{dq} + K_{pc}^{-1}(t) V_{dq\text{-}i} - K_{pc}^{-1}(t) V_{dq\text{-}g}$$

$$K_{pc}\left[\underbrace{L \frac{d \overset{?}{\overbrace{K_{pc}^{-1}(t)}}}{dt} i_{dq}}_{\text{new/extra}} + L K_{pc}^{-1}(t) \cdot \frac{di_{dq}}{dt} = \text{SAME} \right]$$

$$\frac{d}{dt} K_{pc}^{-1}(t) = \frac{d}{dt} \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{pmatrix} = \omega \begin{pmatrix} -\sin(\omega t) & -\cos(\omega t) \\ \cos(\omega t) & -\sin(\omega t) \end{pmatrix}$$

$$\Rightarrow \left\{ L \frac{di_{dq}}{dt} = -R\, i_{dq} + V_{dq\text{-}i} - V_{dq\text{-}g} - \overset{\text{* Coupling term*}}{\overbrace{L \underbrace{K_{pc} \cdot \frac{dK_{pc}^{-1}}{dt}}} \cdot i_{dq}} \right.$$

$$\omega \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \cdot \begin{bmatrix} -\sin(\omega t) & -\cos(\omega t) \\ \cos(\omega t) & -\sin(\omega t) \end{bmatrix}$$

$$= \omega \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

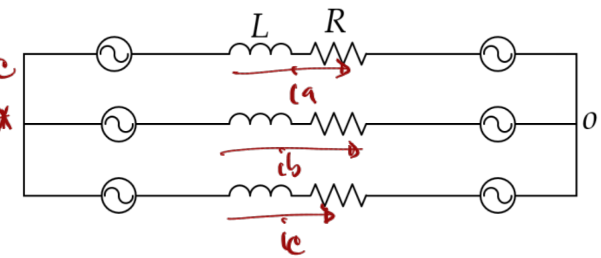
AC Side State Space Equations (dq) (0 component is ignored (balanced))

- Transform the equations from α/β to dq

$K_{pc}\frac{dK_{pc}^{-1}}{dt}$

abc

dq



$$\begin{cases} \dfrac{di_d}{dt} = \dfrac{-R}{L} i_d + \dfrac{1}{L} V_{d-i} - \dfrac{1}{L} V_{d-g} + \omega i_q \\ \\ \dfrac{di_q}{dt} = \dfrac{-R}{L} i_q + \dfrac{1}{L} V_{q-i} - \dfrac{1}{L} V_{q-g} - \omega i_d \end{cases}$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix}$$

- Advantages: 2nd order system, in steady state, we have dc signals.
- Disadvantage: The two equations are now coupled.

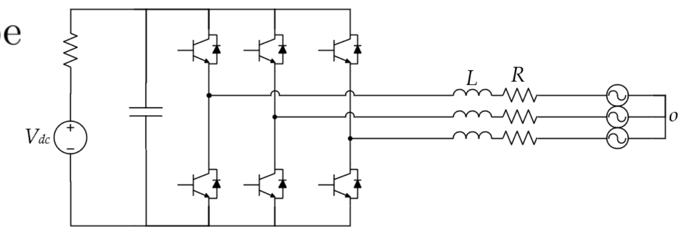
- Can we decouple the two equations? yes, by changing our inputs. $V_{dq} = \begin{pmatrix} V_{d-i} \\ V_{q-i} \end{pmatrix}$

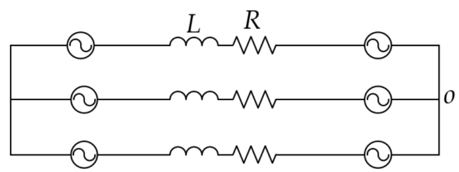
- Want to use V_{d-i}, V_{q-i} to cancel our coupling terms. For example:

$$V_{d-i} = \underbrace{-L \cdot \omega i_q}_{\text{cancel coup.}} + \underbrace{u_{d-i}}_{\text{controller design}}$$

$$V_{q-i} = \underbrace{+L\omega i_d}_{\text{cancel coup.}} + \underbrace{u_{q-i}}_{\text{controller design}}$$

Summary of the Dynamic Equations in DQ Frame





- The state space equations for a three phase inverter can be summarized as follows:

$$\left\{ \frac{dI_{abc}}{dt} = \frac{-R}{L} I_{abc} + \frac{1}{L} V_{abc} - \frac{1}{L} V_{g-abc} \right.$$

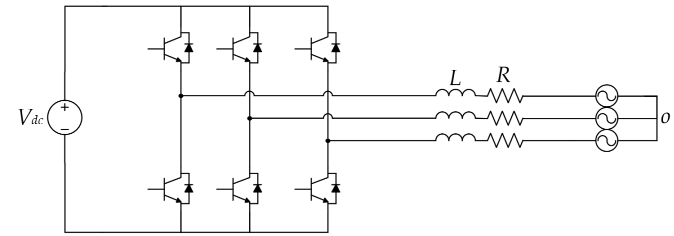
- In dq reference frame, the equations become:

$$\left\{ \begin{aligned} \frac{dI_d}{dt} &= -\frac{R}{L} I_d + \frac{1}{L} V_d - \frac{1}{L} V_{g-d} + \omega I_q \\ \frac{dI_q}{dt} &= -\frac{R}{L} I_q + \frac{1}{L} V_q - \frac{1}{L} V_{g-q} - \omega I_d \end{aligned} \right.$$

Coupling terms.

- The cross-coupling terms will become important in the controller design!

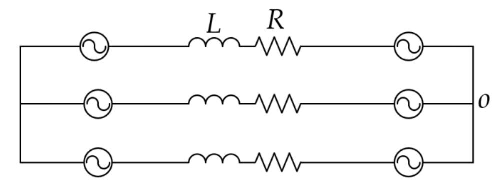
Controller Design – DQ Decoupling



- In dq reference frame, the equations become:

$$\frac{dI_d}{dt} = -\frac{R}{L}I_d + \frac{1}{L}V_{d\text{-}i} - \frac{1}{L}V_{d\text{-}g} + \omega I_q$$

$$\frac{dI_q}{dt} = -\frac{R}{L}I_q + \frac{1}{L}V_{q\text{-}i} - \frac{1}{L}V_{q\text{-}g} - \omega I_d$$



- Is it possible to separate the equations?

- Want to use $V_{d\text{-}i}, V_{q\text{-}i}$ to cancel our coupling terms. For example:

$$V_{d\text{-}i} = \underbrace{-L\omega i_q}_{\text{cancel coup.}} + \underbrace{u_{d\text{-}i}}_{\text{controller design}}$$

$$V_{q\text{-}i} = \underbrace{+L\omega i_d}_{} + \underbrace{u_{q\text{-}i}}_{}$$

- Simplify the system by adding the new inputs.

$$\frac{di_d}{dt} = -\frac{R}{L}i_d + \frac{1}{L}\left(\cancel{-L\omega i_q} + u_{d\text{-}i}\right) - \frac{1}{L}V_{d\text{-}g} + \cancel{\omega i_q} = -\frac{R}{L}i_d + \frac{1}{L}u_{d\text{-}i} - \frac{1}{L}V_{d\text{-}g}$$

$$\frac{di_q}{dt} = -\frac{R}{L}i_q + \frac{1}{L}\left(\cancel{L\omega i_d} + u_{q\text{-}i}\right) - \frac{1}{L}V_{q\text{-}g} - \cancel{\omega i_d} = -\frac{R}{L}i_q + \frac{1}{L}u_{q\text{-}i} - \frac{1}{L}V_{q\text{-}g}$$

Decoupled!

Controller Design – DQ Decoupling (cont'd)

- In dq reference frame, the equations become:

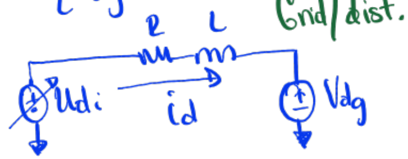
$$\frac{dI_d}{dt} = -\frac{R}{L}I_d + \frac{1}{L}U_{d\text{-}i} - \frac{1}{L}V_{d\text{-}g}, \text{ where } \underline{U_{di} = V_d + L\omega I_q}$$ (New input)

$$\frac{dI_q}{dt} = -\frac{R}{L}I_q + \frac{1}{L}U_{q\text{-}i} - \frac{1}{L}V_{q\text{-}g}, \text{ where } \underline{U_{qi} = V_q - L\omega I_d}$$ (New Input)

D-Axis

$$\frac{di_d}{dt} = -\frac{R}{L} i_d + \frac{1}{L} U_{d\text{-}i} - \frac{1}{L} V_{dg}$$

Grid/dist.



Goal: Design a controller to regulate the "i_d"

$i_d \rightarrow i_d^r$

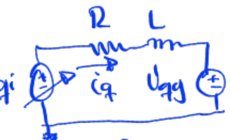
- The reference i_d^r is a constant or step function

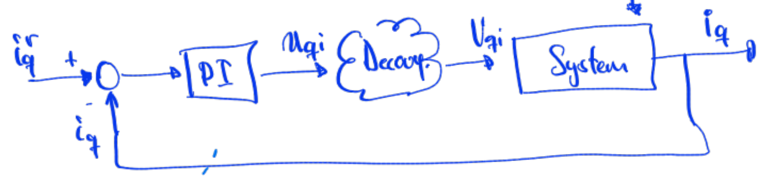
$\Rightarrow$ An integrator term is sufficient to ensure $i_d \rightarrow i_d^r$

$e = i_d^r - i_d \rightarrow 0$

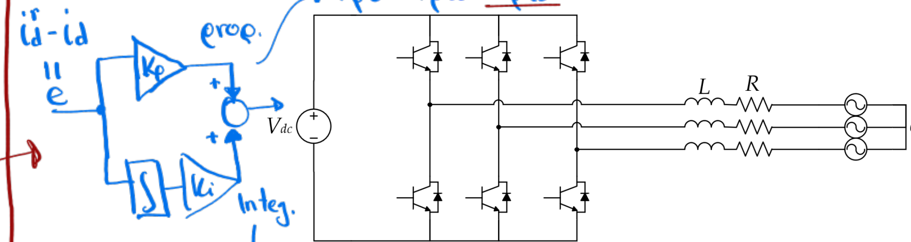
Q-axis

$$\frac{di_q}{dt} = -\frac{R}{L} i_q + \frac{1}{L} U_{qi} - \frac{1}{L} V_{qg}$$



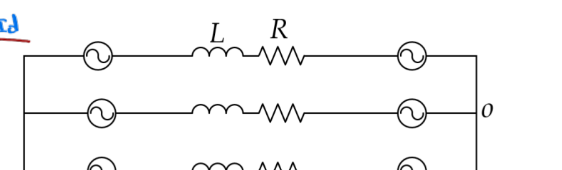


$K_p e = K_p i_d^r - K_p i_d$



$K_I e_{Id}$

$\int i_d^r - i_d$

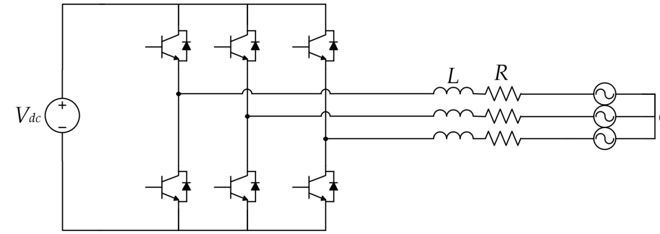


Controller Design – Current Controller Design

- In dq reference frame, the equations become:

$$\frac{dI_d}{dt} = -\frac{R}{L}I_d + \frac{1}{L}U_d - \frac{1}{L}V_{d-g}, \text{ where } U_d = V_d + L\omega I_q$$

$$\frac{dI_q}{dt} = -\frac{R}{L}I_q + \frac{1}{L}U_q - \frac{1}{L}V_{q-g}, \text{ where } U_q = V_q - L\omega I_d$$



- We can regulate each current independently! PI Controller for each current (i_d, i_q)
- How to obtain our Proportional (K_P) + Integral (K_I) gains?
- Derive the model for d-axis current (q-axis analysis is similar)

1) Manual tuning → try different K_P, K_I
(Small K_P, larger K_I) → good settling time

Model Based

* 2) Pole placement for a state feedback + integral control

3) LQR + integral

4) ⋮

2nd order system
$$\begin{cases} \dfrac{di_d}{dt} = -\dfrac{R}{L} i_d + \dfrac{1}{L} U_{di} - \dfrac{1}{L} V_{dg} \\ \dot{e}_{Id} = i_d^r - i_d \end{cases} \longrightarrow e_I = \int (i_d^r - i_d)\,dt$$

Goal $U_{di} = (K_1 \;\; K_2)\begin{pmatrix} i_d \\ e_{Id} \end{pmatrix} = K\tilde{x}_d \qquad \tilde{x}_d = \begin{pmatrix} i_d \\ e_{Id} \end{pmatrix}$

$$\boxed{\begin{aligned} K_P &= -K_1 \\ K_I &= K_2 \end{aligned}}$$

$$\begin{pmatrix} \dot{i}_d \\ \dot{e}_{Id} \end{pmatrix} = \underbrace{\begin{pmatrix} -\frac{R}{L} & 0 \\ -1 & 0 \end{pmatrix}}_{A} \begin{pmatrix} i_d \\ e_{Id} \end{pmatrix} + \underbrace{\begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}}_{B} U_{di} + \begin{pmatrix} -\frac{1}{L} \\ 0 \end{pmatrix} V_{dg} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} i_d^r$$

$$\boxed{K = -\text{place}\left(A, B, [\lambda_1^{des} \;\; \lambda_2^{des}]\right)}$$

$$\dot{\tilde{x}}_d = A\tilde{x}_d + BU_{di} \overset{U_{di} = K\tilde{x}_d}{=} (A + BK)\tilde{x}_d \quad \text{s.t.} \quad \text{eig}(A+BK) \subset \mathbb{C}^- \text{ (Neg. real part)}$$

Controller Design – Overall Controller Diagram

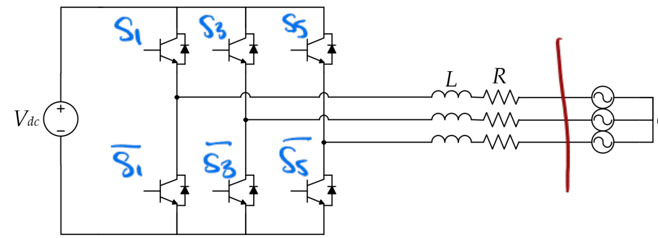
- In dq reference frame, the equations become:

$$\frac{dI_d}{dt} = -\frac{R}{L}I_d + \frac{1}{L}U_d - \frac{1}{L}V_{g-d}, \text{ where } U_d = V_d + L\omega I_q^*$$

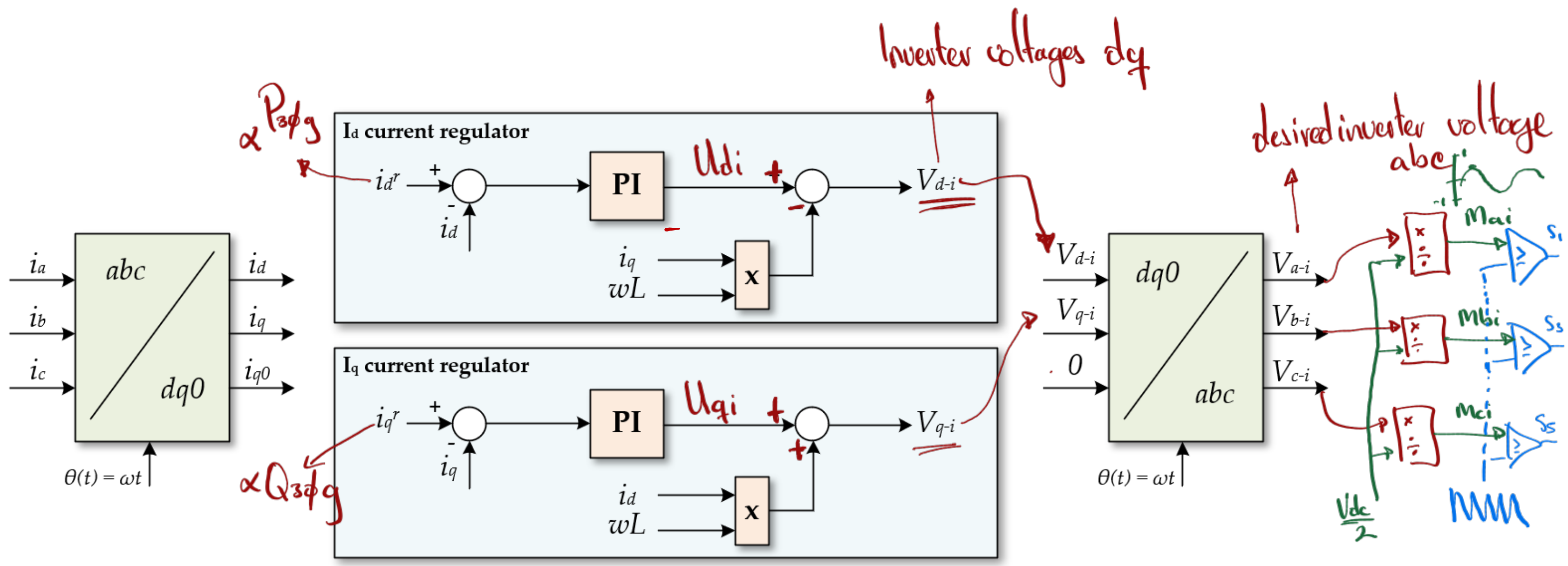
$$\frac{dI_q}{dt} = -\frac{R}{L}I_q + \frac{1}{L}U_q - \frac{1}{L}V_{g-q}, \text{ where } U_q = V_q - L\omega I_d$$

$V_{di} = U_{di} - L\omega i_q$

$V_{qi} = U_{qi} + \omega L i_d$



- We can regulate each current independently!



Inner Current Loops - Fast

- How to compute i_d^r, i_q^r? How are they related to $P_{3\phi}$, $Q_{3\phi}$?

Outline

- Review of dc/ac (ac/dc) converters
- abc to dq transformation – Clarke and Park Transformations
- Space vector PWM
- Controller Design Overview
- **Applications**

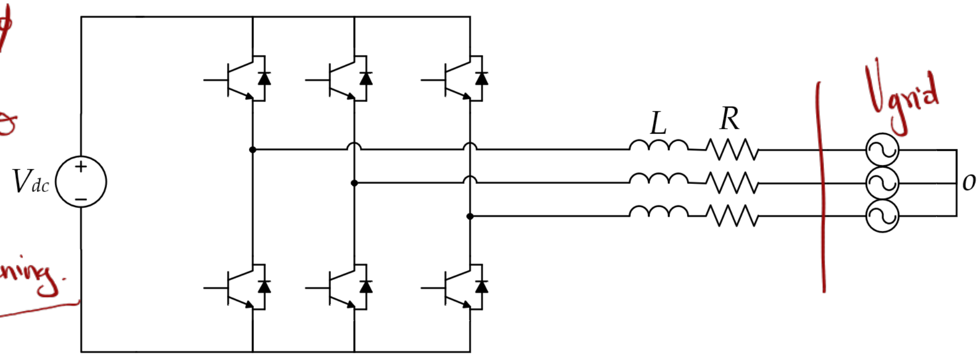
Overview

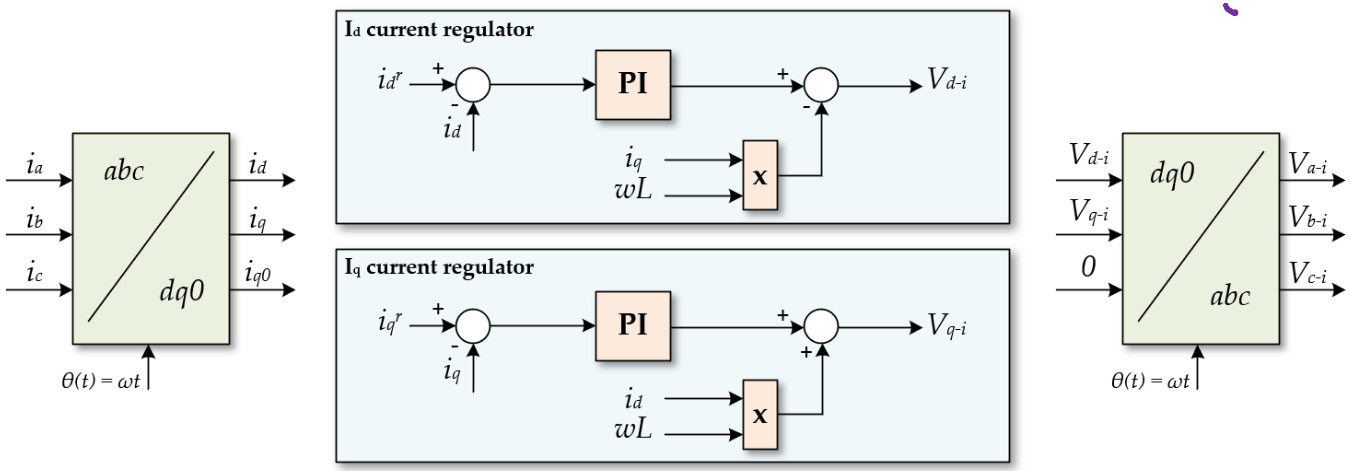
- A **fast** current regulator is crucial for many applications (Utility, Motor drives → EV, Actuators, Appliances ⋮ ,)
- Once it has been designed, many **slower** outer controllers can be developed to suit the application

Utility: $P_{3\phi}$, $Q_{3\phi}$

Motor Drives: T_e, ω, θ, λ_m (flux), field weakening.

Outer loop(s)





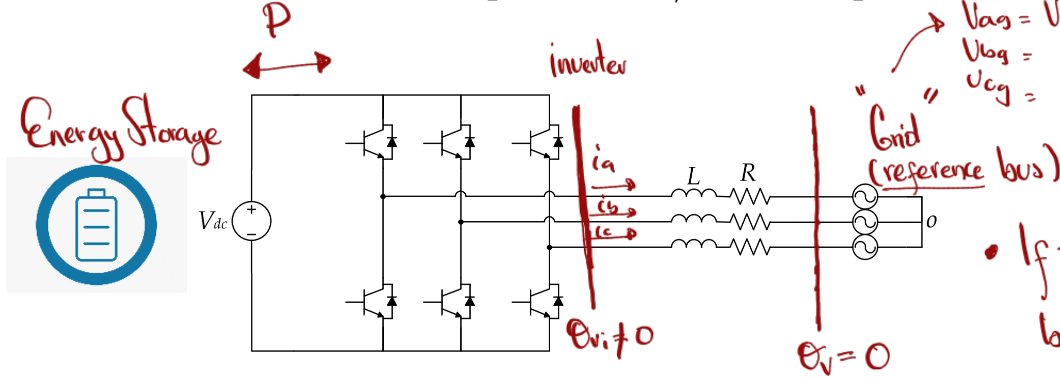
Energy Storage Application – Output Active/Reactive Power

- A **fast** current regulator is crucial for many applications
- Once it has been designed, many **slower** outer controllers can be developed to suit the application
- Let's consider having a source with a well defined dc voltage
- Design a controller to track the output active/reactive power!

$V_{ag} = V_{pk\text{-}ln}\cos(\omega t) \rightarrow \bar{V}_{ag} = \frac{V_{pk}}{\sqrt{2}}\angle 0^\circ$

$V_{bg} = $ " " $\cos(\omega t - \frac{2\pi}{3})$

$V_{cg} = $ " " $\cos(\omega t + \frac{2\pi}{3})$



- If the Grid voltage is our reference bus $\rightarrow$ $V_{dg} = V_{dg} \neq 0 = ?$ (refer to abc/dq slides)
  $V_{qg} = 0$

- What is $P_{3\phi g}$ and $Q_{3\phi g}$ at grid side?

$P_{3\phi g} = V_{dg} i_d + \cancel{V_{qg} i_q}^{\,0} = V_{dg} i_d \Rightarrow i_d \propto P_{3\phi g}$ $\quad$ V_{dg} is typically fixed

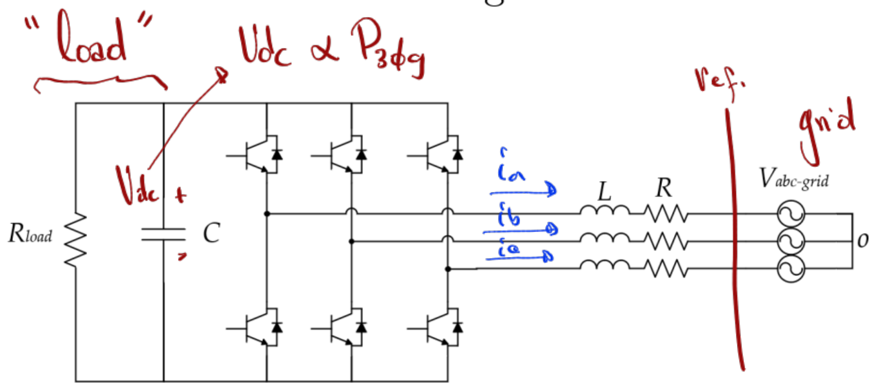
$Q_{3\phi g} = \cancel{V_{qg} i_d}_{\,0} - V_{dg} i_q = -V_{dg} i_q \Rightarrow i_q \propto Q_{3\phi g}$

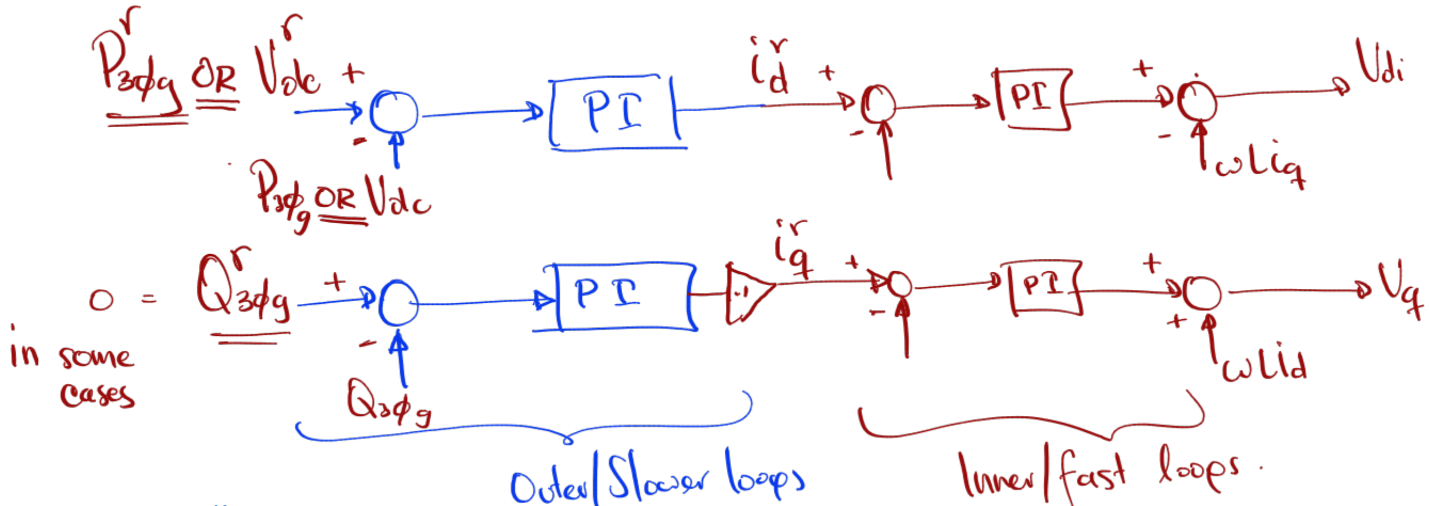
$$\Rightarrow \boxed{\begin{aligned} P_{3\phi g} &= V_{dg} \cdot I_d \\ Q_{3\phi g} &= -V_{dg} I_q \end{aligned}}$$

- We will use i_d to control Active power sent/received to/from grid
- " " " i_q " " Reactive power " " " " " .

Active Rectification

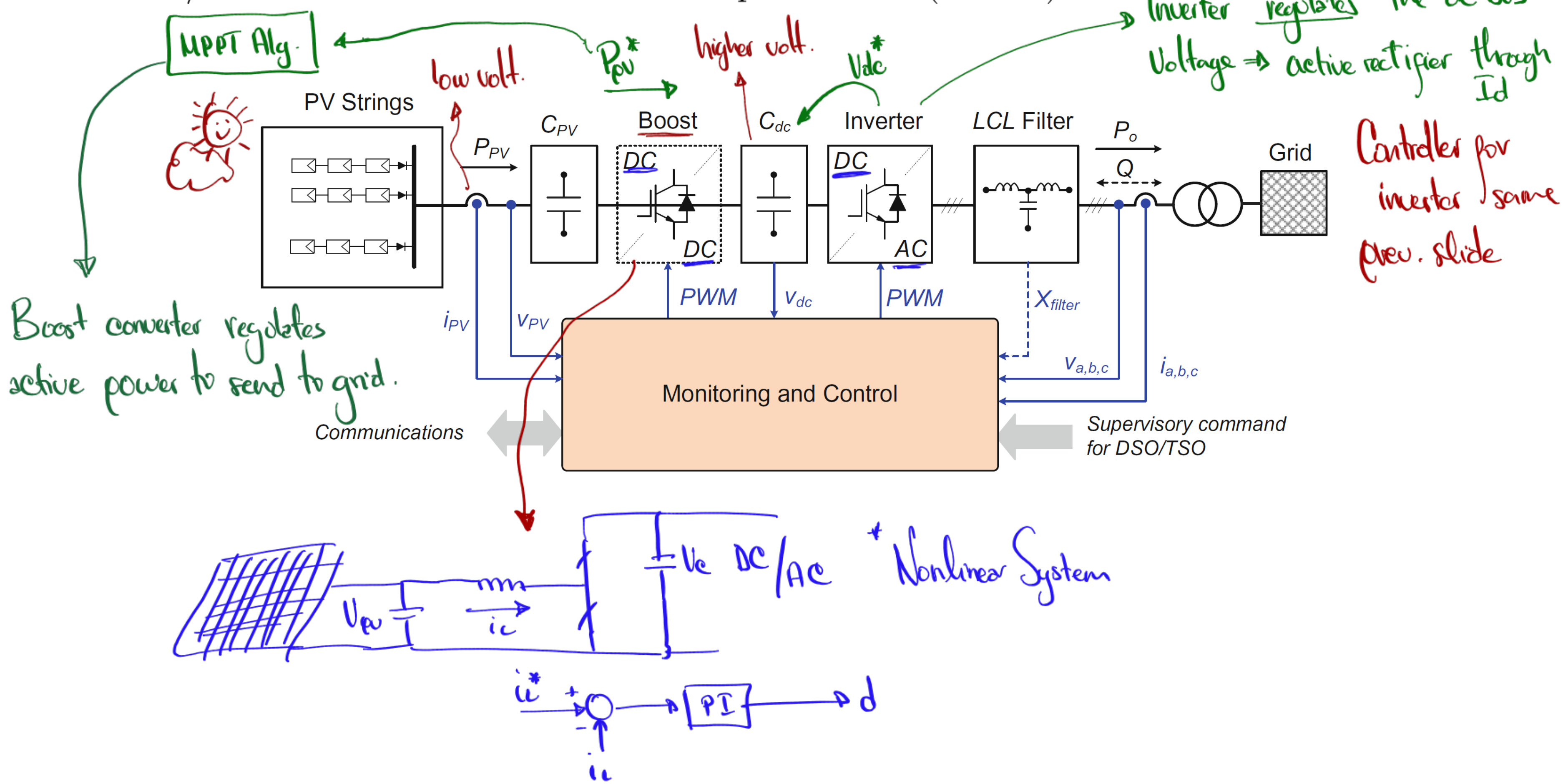
- Let's consider now an **active rectifier**
- At the dc side, only active power is consumed
- The dc bus voltage can therefore be controlled by the d axis current
- The reactive power at the ac side can still be regulated





PV/Renewable Energy Example

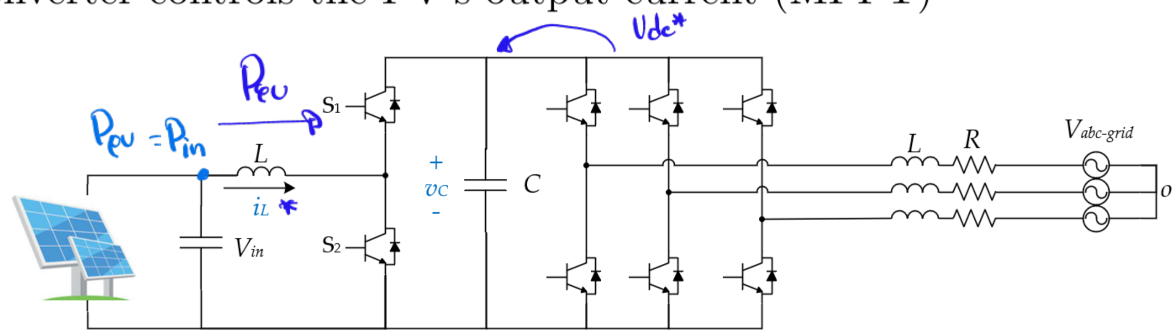
- For PV applications, the power electronics are generally composed of a dc/dc + dc/ac converter
- The dc/ac converter controls the dc bus voltage, V_{dc}
- The dc/dc converter controls the PV's output current (MPPT)



PV/Renewable Energy Example (cont'd)

- For PV applications, the power electronics are generally composed of a dc/dc + dc/ac converter
- The dc/ac converter controls the dc bus voltage, V_{dc}
- The dc/dc converter controls the PV's output current (MPPT)

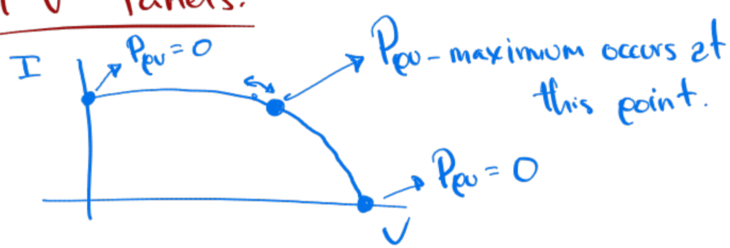
MPPT



- for a Boost Converter:

Input Power: $P_{pv} = P_{in} = V_{in} \cdot I_L$ $\qquad P_{in} \propto I_L \qquad$ by regulating I_L we can decide how much Power from PV panels to send to grid.

- PV Panels:



MPPT: Max. Power Point tracking

- Perturb and Observe

High Voltage DC Transmission

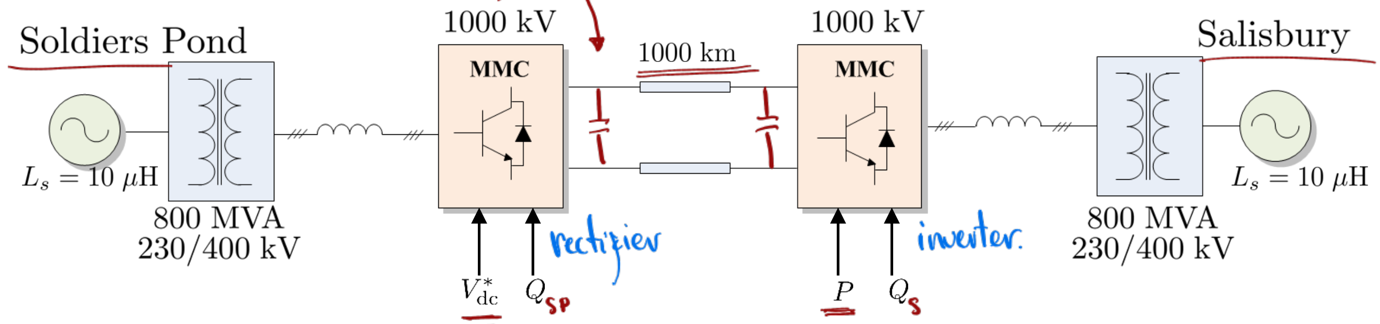
$P = VI = I^2R$ if $V\uparrow$ $I\downarrow$ $\Rightarrow$ line losses are reduced.

Point-to-Point.

- In High Voltage DC Transmission (HVDC), we typically need back-to-back converters!

(CANADA)

Master V_{dc}^* P Slave



- This control strategy where one station regulates Vdc and another the P is refer to as Master/Slave control.

  Master: Regulates Vdc (active rectifier)

  Slave: Regulates P (inverter)

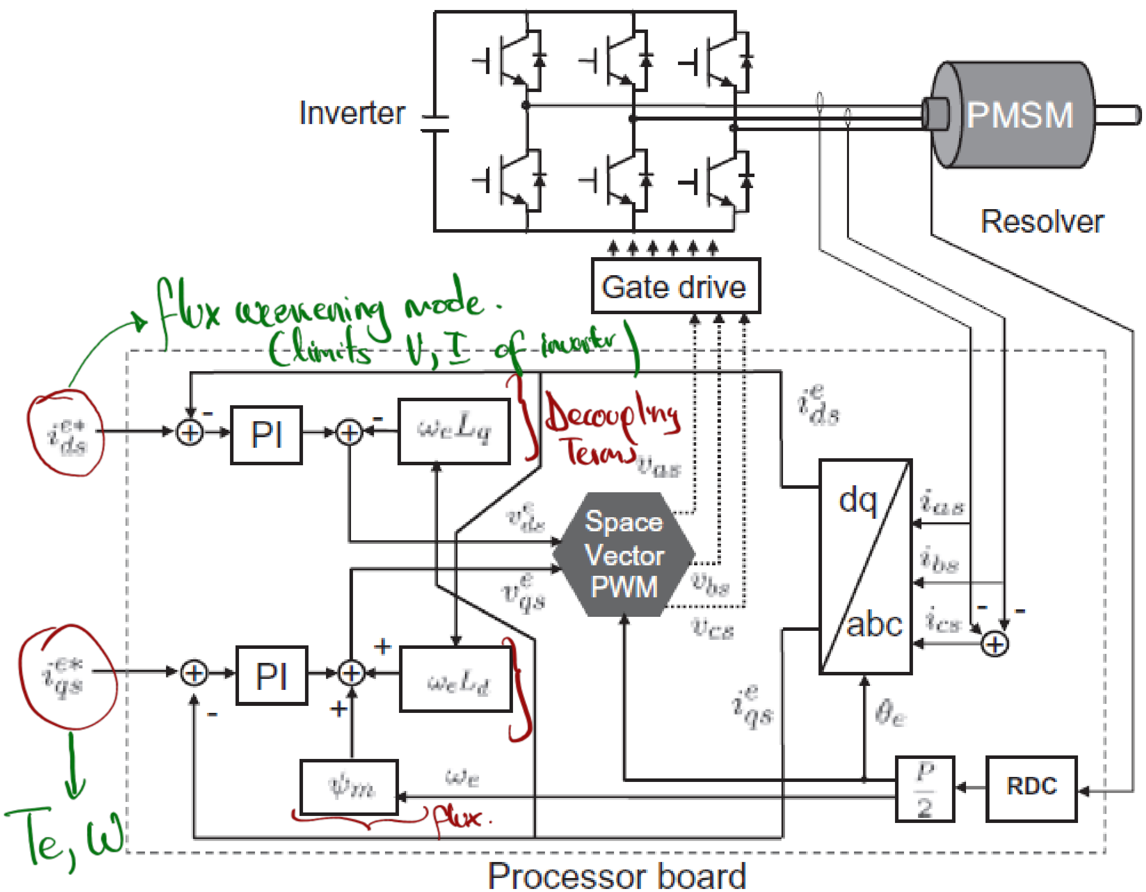
- One advantage of HVDC transmission is that reactive power is contained within each AC system.

- Other types of strategies include Droop control P-Vdc droop

  Dynamic Consensus (requires communication)

PMSM Motor Drives

- Suppose that we would like to control the speed of a Permanent Magnet Synchronous Machine (PMSM)
- For speed tracking, torque control, etc. An inner current regulator is very important!



State Space model for PMSM

Nonlinear System. (ac)

$$\begin{cases} \dot{i}_d = \frac{-R_s}{L_d} i_d + \omega_r \frac{L_q}{L_d} i_q + \frac{1}{L_d} v_d \\ \dot{i}_q = \frac{-R_s}{L_q} i_q - \omega_r \frac{L_d}{L_q} i_d - \frac{\omega_r}{L_q} \lambda_m + \frac{1}{L_q} v_q \end{cases}$$

$$\dot{\omega}_r = \frac{1}{J}(T_e - T_L)$$

$$T_e = \frac{3}{2}\frac{P}{2}(\lambda_m i_q + \overset{0}{(L_d - L_q)} i_q i_d) \quad \text{(Nm)}$$

ω_r: rotor speed (not constant!)

P: # of Poles.

λ_m: motor flux due to magnets (fixed)

L_d, L_q : inductance on d, q frame.

In some cases (Surface Mounted PMSM) $L_d \approx L_q$

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