Koopman Based Control of a Boost Converter Feeding a Constant Power Load

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Introduction

- DC based power networks are composed of an interconnection of converters feeding Constant Power Loads (CPLs).
- Due to the non-linear nature of these systems, stability cannot be guaranteed for all operating conditions, e.g. large load powers can make the system unstable.
- To reduce computational complexity for non-linear controllers, a linear Koopman model predictive control strategy is proposed for a highly nonlinear system: boost converter feeding a CPL.

DC Systems with CPLs

- To improve power quality at the load side, loads are tightly regulated (high bandwidth), behaving as Constant Power Loads (CPLs).
- The CPL appears as a dynamic negative resistance, reducing the damping of the system and causing instability in the network.

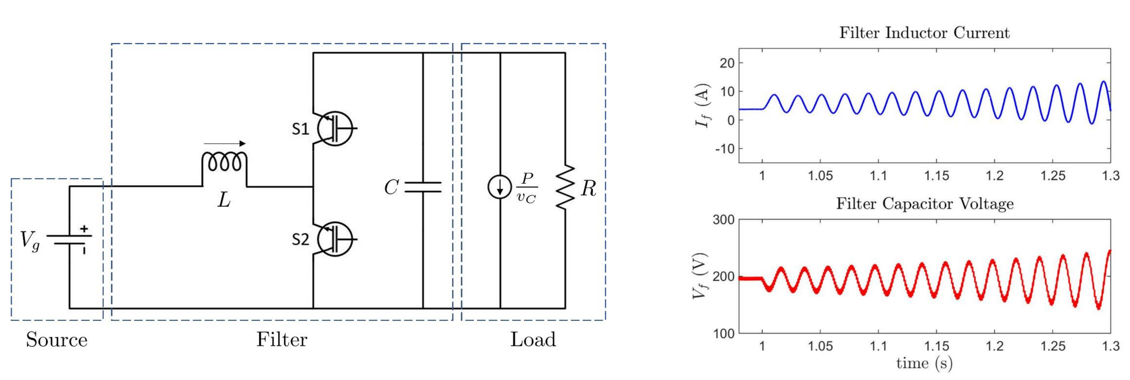

Fig. 1. Boost converter with CPL (left) and example of unstable load change (right).

- Controlling a dc boost converter with CPL can be challenging due to the nonlinearities in both the converter and the load.
- This system can be described as follows:

$$\frac{di_L}{dt} = \frac{-v_C(1-u)}{L} + \frac{V_g}{L}$$
$$\frac{dv_C}{dt} = \frac{i_L(1-u)}{C} - \frac{v_C}{RC} - \frac{P}{C \cdot v_C}$$

- Traditional nonlinear control strategies (e.g. passivity based control, feedback linearization, etc.) are highly dependent on model parameters.

Koopman Operator Theory

- Consider a nonlinear dynamic system characterized by the following equation in discrete time:

$$x_{k+1} = f\left(x_k\right)$$

where $x \in \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}^n$.

- Define the set of observable functions $h \in \mathscr{F}$, as a function $h : \mathbb{R}^n \to \mathbb{R}$.
- The Koopman operator $\mathcal{K} : \mathscr{F} \to \mathscr{F}$ advances the observations w.r.t. the flow of the system as follows:

$$\left(\mathcal{K}h\right)\left(x_k\right) = h\left(f\left(x_k\right)\right) = h\left(x_{k+1}\right)$$

- Using a truncated basis for $\mathscr{F}$ as $z_k \triangleq \begin{pmatrix}\psi_1(x_k) & \cdots & \psi_{N_b}(x_k)\end{pmatrix}^T$, we can obtain a linear approximation of the original nonlinear system: $z_{k+1} = \mathcal{K}z_k \approx Az_k$

Koopman Based Learning

- Consider a set of data obtained from experiments or simulations as follows:

$$X = \begin{pmatrix}x_0 & \cdots & x_T\end{pmatrix}, \quad Y = \begin{pmatrix}x_1 & \cdots & x_{T+1}\end{pmatrix}$$
$$U = \begin{pmatrix}u_0 & \cdots & u_T\end{pmatrix}, \quad W = \begin{pmatrix}w_0 & \cdots & w_T\end{pmatrix}$$

- Let $X_{\text{lift}} = \begin{pmatrix}z(x_0) & \cdots & z(x_T)\end{pmatrix}$ and $Y_{\text{lift}} = \begin{pmatrix}z(x_1) & \cdots & z(x_{T+1})\end{pmatrix}$, where $z_k \triangleq \begin{pmatrix}\psi_1(x_k) & \cdots & \psi_{N_b}(x_k)\end{pmatrix}^T$ (truncated basis)
- We can then obtain a linear approximation of the nonlinear system in this truncated basis of the form:

$$\left\{ x_{k+1} = f(x_k,\ u_k,\ w_k) \right. \quad \Rightarrow$$
$$\left\{ \begin{aligned} z_{k+1} &= Az_k + Bu_k + Dw_k \\ \hat{x}_k &= Cz_k \end{aligned} \right.$$

where

$$\min_{A,B,D} ||Y_{\text{lift}} - AX_{\text{lift}} - BU - DW||$$
$$\min_{C} ||X - CX_{\text{lift}}||$$

Koopman Model Predictive Control

- Having a linear approximation of the nonlinear system, we can now design linear based controllers, e.g. state feedback, LQR, Model Predictive Control (MPC), etc.

Nonlinear MPC

$$\min_{y_p,u_p} \sum_{k=0}^{N-1} \left[||C_y y_k - y_{ref}||_{Q_y}^2 + ||u_k||_R^2\right]$$
$$\text{s.t. } x_{k+1} = f_d(x_k, u_k, w_k), k = 0, ..., N-1 \quad \text{(Nonlinear equality constraint)}$$
$$y_k = C_d x_k, k = 0, ..., N-1$$
$$x_0 = x_c$$
$$y_k \in \mathcal{Y}, k = 0, ..., N-1$$
$$u_k \in \mathcal{U}, k = 0, ..., N-1$$
$$\text{where, } \mathcal{Y} = \{y_k | y_{min} \le y_k \le y_{max}\}, \quad \mathcal{U} = \{u_k | u_{min} \le u_k \le u_{max}\}$$

↓

Koopman MPC

$$\min_{z_p,u_p} \sum_{i=0}^{N-1} [(Cz_i - r)^T Q_i (Cz_i - r) + u_i^T R u_i] + (Cz_N - r)^T Q_N (Cz_N - r)$$
$$\text{s.t. } z_{i+1} = Az_i + Bu_i + Gw_i,\ i = 0, \cdots, N-1 \quad \text{(Linearization using Koopman operator)}$$
$$Ez_i + Fu_i + Mw_i \le b_i,\ i = 0, \cdots, N-1$$
$$Kz_N \le b_N$$
$$z_0 = \psi(x_k)$$

- We can then apply Koopman MPC for the controller design of boost converters with CPLs.

Simulation Results

- A Koopman MPC was designed to regulate the capacitor voltage in the presence of CPLs.
- At time $t = 0.1$ s, a reference change occurs and a load change at time $t = 0.15$ s.

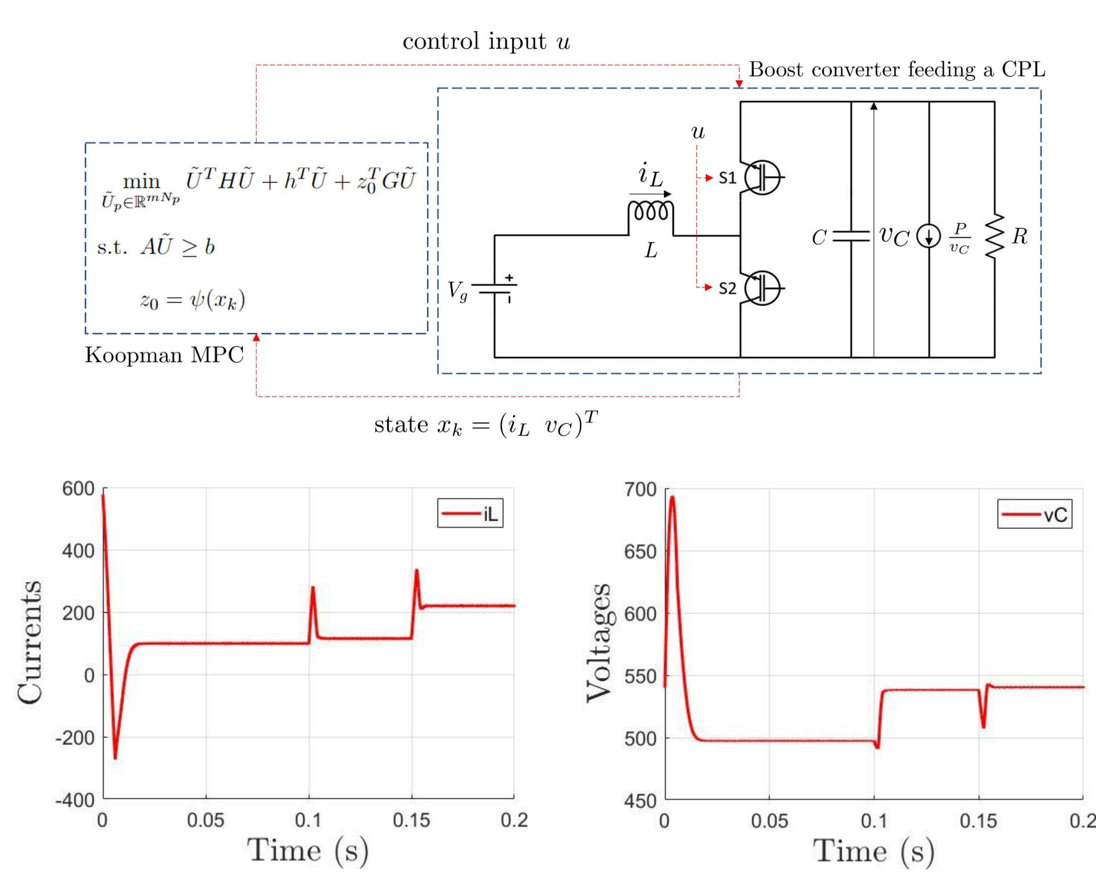

Fig. 2. Koopman based MPC for boost converter with CPL.

Experimental Results

- The Koopman MPC was tested experimentally as shown in Fig. 3.
- The boost converter is based on SiC Semikron stack and the electronic load is a Magna Power dc load.
- The controller was implemented in an Opal RT OP4510.

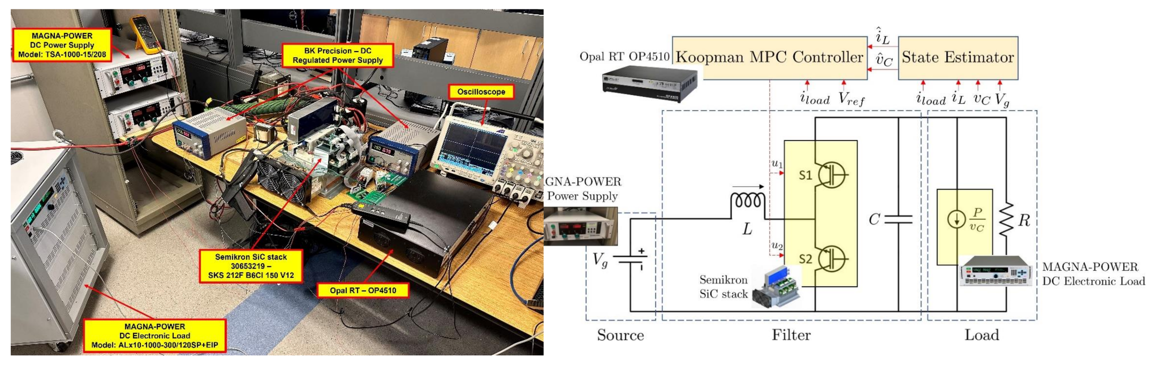

Fig. 3. Experimental testbed for controller verification.

- The experimental results are shown in Fig. 4.
- The results show similar performance as the simulation results.

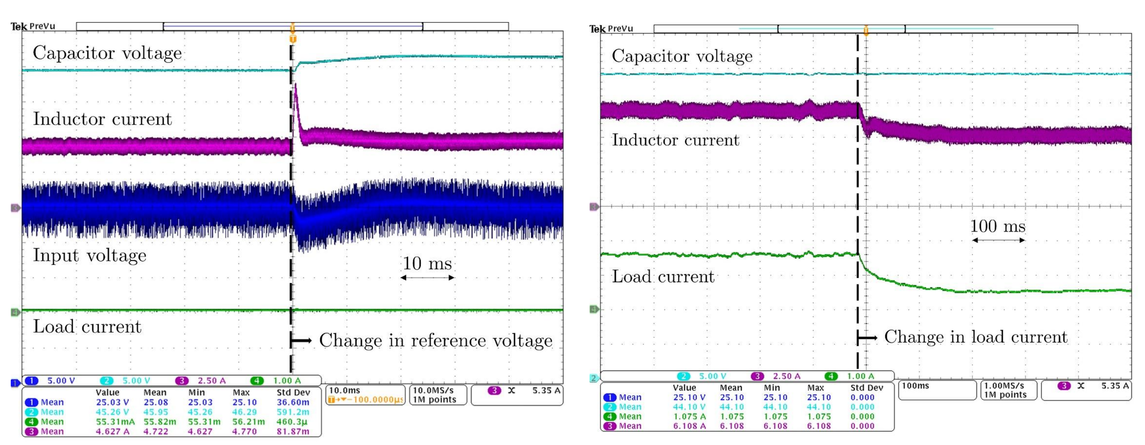

Fig. 4. Experimental testbed during reference change (left) and CPL load change (right).

Conclusion

- A linear Koopman model predictive control strategy is proposed for the stabilization of the system with constant power loads.
- The linear Koopman approximation can be obtained from data (simulation or experimental), hence decreasing the dependency on model parameters.
- The closed-loop performance of the control strategy is demonstrated by tracking a desired current and voltage, for changes in the load.
- Experimental results verified the feasibility of implementing the proposed strategy in real operating systems.