Problem 1 A two-wire copper transmission line is embedded in a dielectric material with $\varepsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}$ S/m. Its wires are separated by 3 cm and their radii are 1 mm each.

(a) Calculate the line parameters R', L', G', and C' at 2 GHz.

(b) Compare your results with those based on CD Module 2.1. Include a printout of the screen display.

Solution:

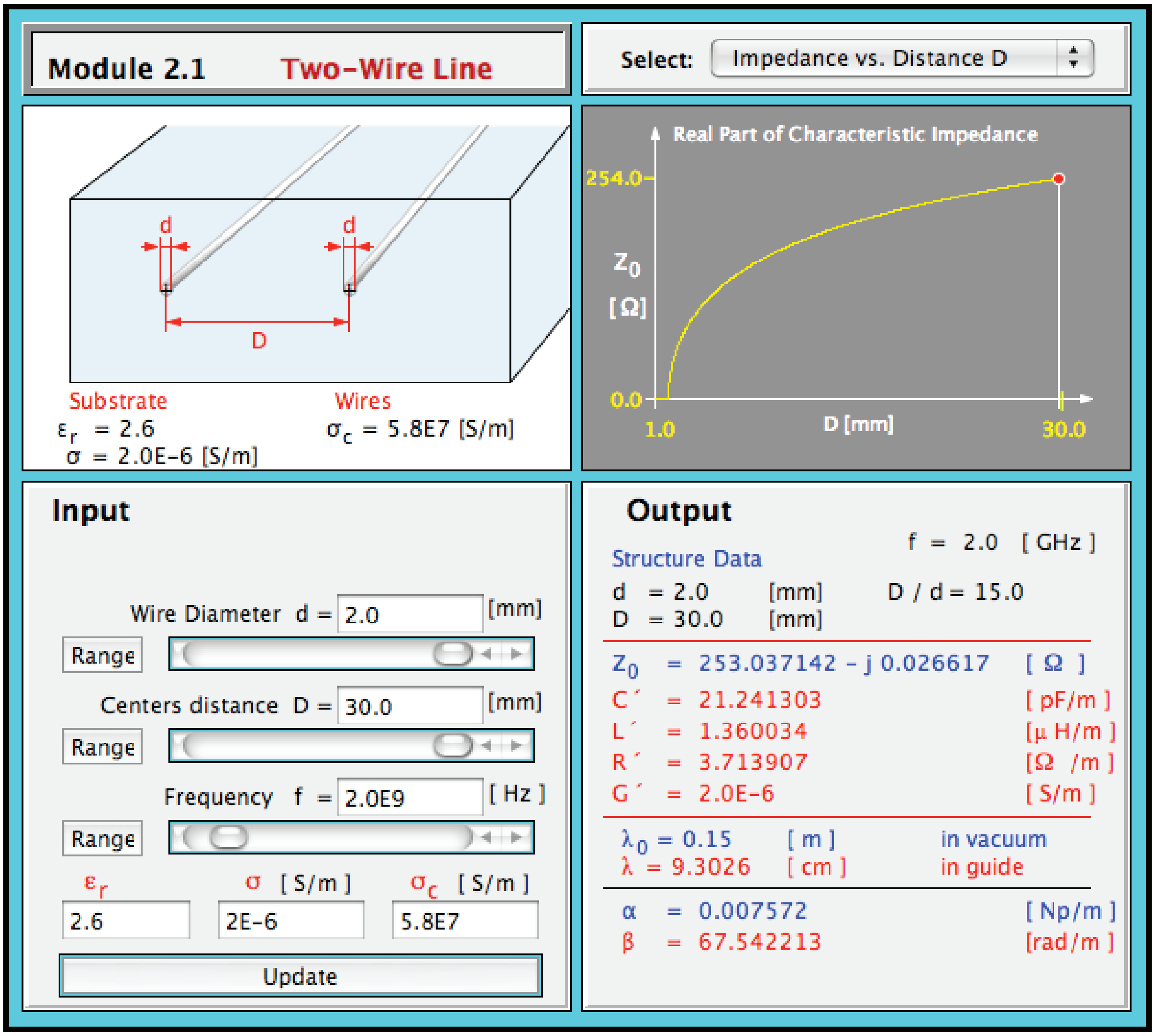
(a) Given:

$$
\begin{aligned}
f &= 2 \times 10^9 \text{ Hz},\\
d &= 2 \times 10^{-3} \text{ m},\\
D &= 3 \times 10^{-2} \text{ m},\\
\sigma_c &= 5.8 \times 10^7 \text{ S/m (copper)},\\
\varepsilon_r &= 2.6,\\
\sigma &= 2 \times 10^{-6} \text{ S/m},\\
\mu &= \mu_c = \mu_0.
\end{aligned}
$$

From Table 2-1:

$$
\begin{aligned}
R_s &= \sqrt{\pi f \mu_c / \sigma_c}\\
&= [\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7} / 5.8 \times 10^7]^{1/2}\\
&= 1.17 \times 10^{-2}\ \Omega,\\
R' &= \frac{2R_s}{\pi d} = \frac{2 \times 1.17 \times 10^{-2}}{2\pi \times 10^{-3}} = 3.71\ \Omega/\text{m},\\
L' &= \frac{\mu}{\pi} \ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right]\\
&= 1.36 \times 10^{-6} \text{ H/m},\\
G' &= \frac{\pi\sigma}{\ln[(D/d) + \sqrt{(D/d)^2 - 1}]}\\
&= 1.85 \times 10^{-6} \text{ S/m},\\
C' &= \frac{G'\varepsilon}{\sigma}\\
&= \frac{1.85 \times 10^{-6} \times 8.85 \times 10^{-12} \times 2.6}{2 \times 10^{-6}}\\
&= 2.13 \times 10^{-11} \text{ F/m}.
\end{aligned}
$$

(b) Solution via Module 2.1:


Module 2.1 Two-Wire Line
Select: Impedance vs. Distance D
d
d
D
Substrate
εr = 2.6
σ = 2.0E-6 [S/m]
Wires
σc = 5.8E7 [S/m]
Real Part of Characteristic Impedance
254.0
Z0 [Ω]
0.0
1.0
D [mm]
30.0
Input
Wire Diameter d = 2.0 [mm]
Range
Centers distance D = 30.0 [mm]
Range
Frequency f = 2.0E9 [Hz]
Range
εr σ [S/m] σc [S/m]
2.6 2E-6 5.8E7
Update
Output
f = 2.0 [GHz]
Structure Data
d = 2.0 [mm] D / d = 15.0
D = 30.0 [mm]
Z0 = 253.037142 - j 0.026617 [Ω]
C´ = 21.241303 [pF/m]
L´ = 1.360034 [μ H/m]
R´ = 3.713907 [Ω /m]
G´ = 2.0E-6 [S/m]
λ0 = 0.15 [m] in vacuum
λ = 9.3026 [cm] in guide
α = 0.007572 [Np/m]
β = 67.542213 [rad/m]

Problem 2 Find α, β, u_p, and Z_0 for the coaxial line of Problem 2.6. Verify your results by applying CD Module 2.2. Include a printout of the screen display.

Solution: From Eq. (2.22),

$$\begin{aligned}\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\ &= \sqrt{(0.788\ \Omega/\text{m}) + j(2\pi \times 10^9\ \text{s}^{-1})(139 \times 10^{-9}\ \text{H/m})} \\ &\quad \times \sqrt{(9.1 \times 10^{-3}\ \text{S/m}) + j(2\pi \times 10^9\ \text{s}^{-1})(362 \times 10^{-12}\ \text{F/m})} \\ &= (109 \times 10^{-3} + j44.5)\ \text{m}^{-1}.\end{aligned}$$

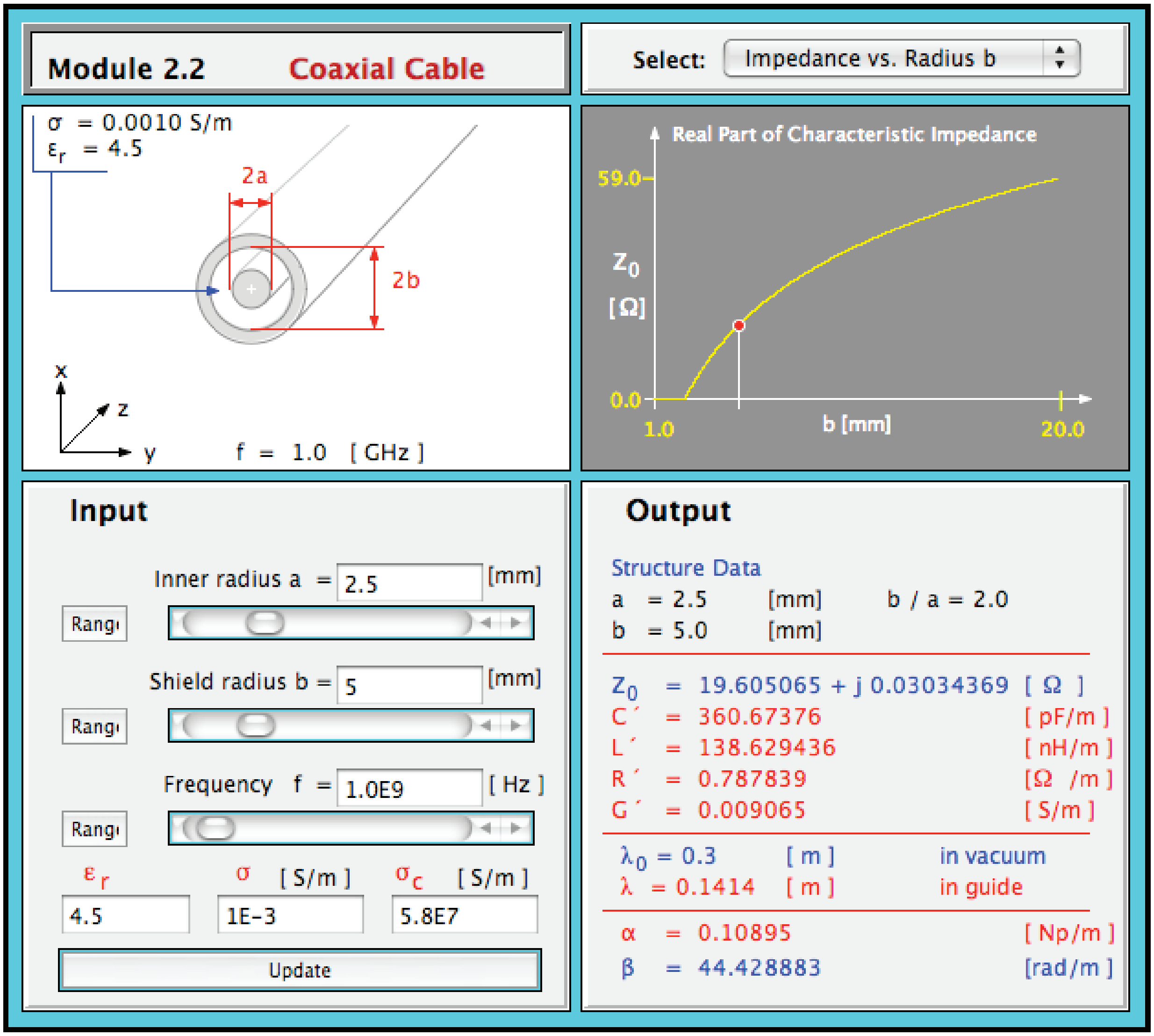
Thus, from Eqs. (2.25a) and (2.25b), $\alpha = 0.109$ Np/m and $\beta = 44.5$ rad/m.

From Eq. (2.29),

$$\begin{aligned}Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} &= \sqrt{\frac{(0.788\ \Omega/\text{m}) + j(2\pi \times 10^9\ \text{s}^{-1})(139 \times 10^{-9}\ \text{H/m})}{(9.1 \times 10^{-3}\ \text{S/m}) + j(2\pi \times 10^9\ \text{s}^{-1})(362 \times 10^{-12}\ \text{F/m})}} \\ &= (19.6 + j0.030)\ \Omega.\end{aligned}$$

From Eq. (2.33),

$$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^9}{44.5} = 1.41 \times 10^8\ \text{m/s}.$$


Module 2.2 Coaxial Cable
Select: Impedance vs. Radius b
σ = 0.0010 S/m
εr = 4.5
2a
2b
x
z
y
f = 1.0 [GHz]
Real Part of Characteristic Impedance
59.0
Z0
[Ω]
0.0
1.0
b [mm]
20.0
Input
Inner radius a = 2.5 [mm]
Shield radius b = 5 [mm]
Frequency f = 1.0E9 [Hz]
εr
σ [S/m]
σc [S/m]
4.5
1E-3
5.8E7
Update
Output
Structure Data
a = 2.5 [mm] b / a = 2.0
b = 5.0 [mm]
Z0 = 19.605065 + j 0.03034369 [Ω]
C´ = 360.67376 [pF/m]
L´ = 138.629436 [nH/m]
R´ = 0.787839 [Ω /m]
G´ = 0.009065 [S/m]
λ0 = 0.3 [m] in vacuum
λ = 0.1414 [m] in guide
α = 0.10895 [Np/m]
β = 44.428883 [rad/m]

Problem 3 Polyethylene with $\varepsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with characteristic impedance of 50 Ω. The radius of the inner conductor is 1.2 mm.

(a) What is the radius of the outer conductor?

(b) What is the phase velocity of the line?

Solution: Given a lossless coaxial line, $Z_0 = 50\ \Omega$, $\varepsilon_r = 2.25$, $a = 1.2$ mm:

(a) From Table 2-2, $Z_0 = (60/\sqrt{\varepsilon_r}) \ln(b/a)$ which can be rearranged to give

$$b = a e^{Z_0 \sqrt{\varepsilon_r}/60} = (1.2 \text{ mm}) e^{50\sqrt{2.25}/60} = 4.2 \text{ mm}.$$

(b) Also from Table 2-2,

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25}} = 2.0 \times 10^8 \text{ m/s}.$$

Problem 4 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)$ Ω. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,
(b) the standing-wave ratio on the line,
(c) the position of the voltage maximum nearest the load,
(d) the position of the current maximum nearest the load.
(e) Verify quantities in parts (a)–(d) using CD Module 2.4. Include a printout of the screen display.

Solution:

(a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57e^{-j79.8^\circ}.$$

(b) From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

(c) From Eq. (2.70)

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8 \text{ cm}}{4\pi} \frac{\pi \text{ rad}}{180^\circ} + \frac{n \times 8 \text{ cm}}{2}$$
$$= -0.89 \text{ cm} + 4.0 \text{ cm} = 3.11 \text{ cm}.$$

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.72),

$$d_{min} = d_{max} - \lambda/4 = 3.11 \text{ cm} - 8 \text{ cm}/4 = 1.11 \text{ cm}.$$

(e) The problem statement does not specify the frequency, so in Module 2.4 we need to select the combination of f and ε_r such that $\lambda = 5$ cm. With ε_r chosen as 1,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-2}} = 3.75 \text{ GHz}.$$

The generator parameters are irrelevant to the problem.

The results listed in the output screens are very close to those given in parts (a) through (d).

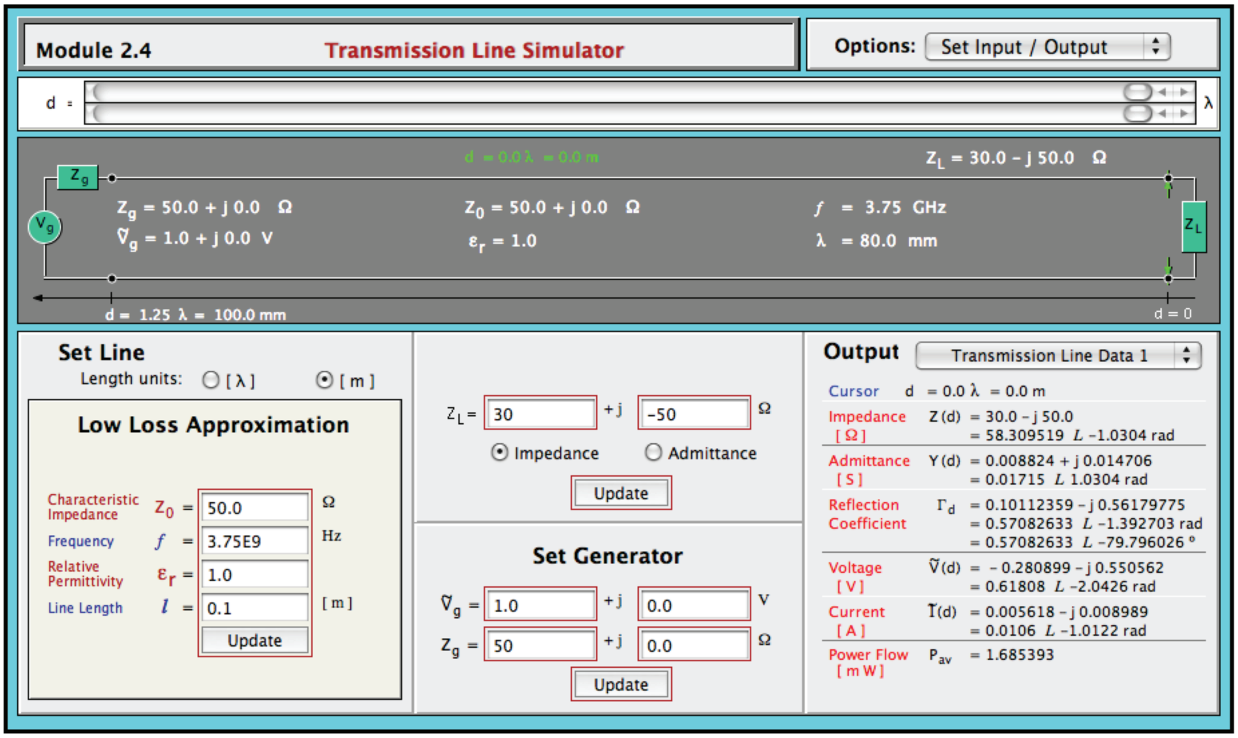


Figure P2.19(a)

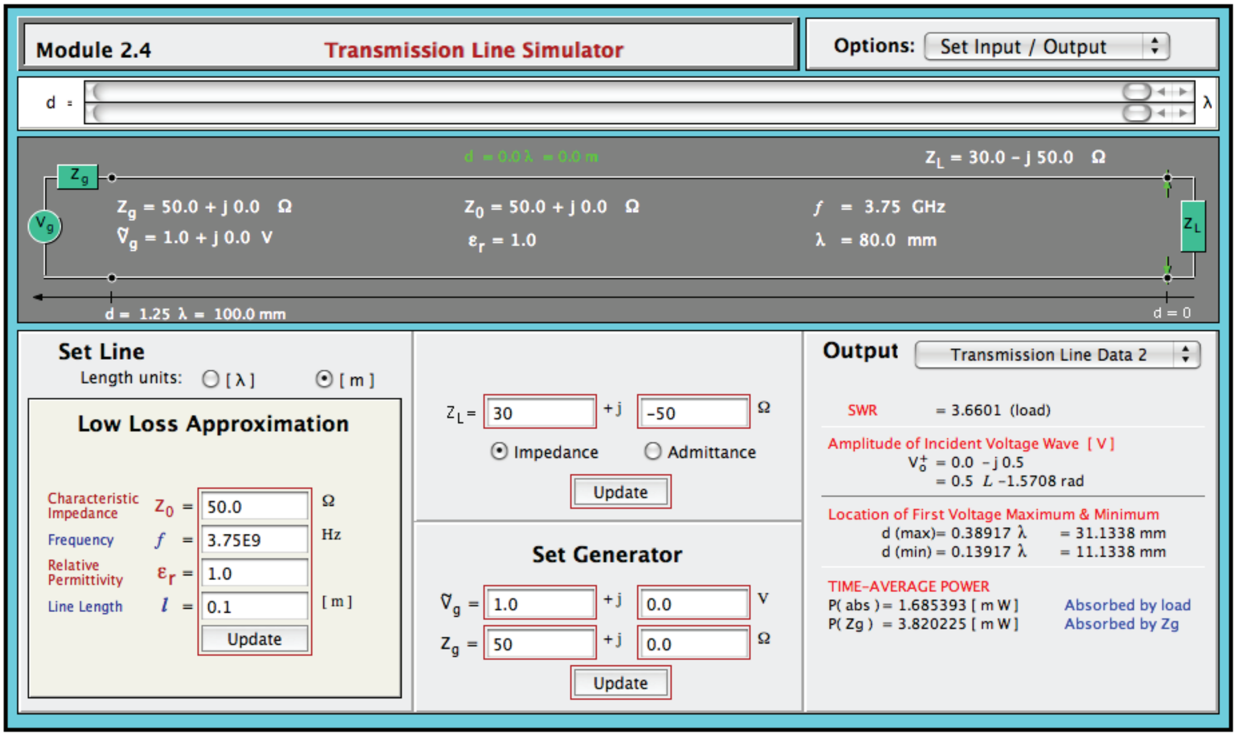


Figure P2.19(b)

Problem 5 A 50-Ω lossless line terminated in a purely resistive load has a voltage standing-wave ratio of 3. Find all possible values of Z_L.

Solution:

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5.$$

For a purely resistive load, $\theta_r = 0$ or π. For $\theta_r = 0$,

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 50 \left[\frac{1+0.5}{1-0.5}\right] = 150\ \Omega.$$

For $\theta_r = \pi$, $\Gamma = -0.5$ and

$$Z_L = 50 \left[\frac{1-0.5}{1+0.5}\right] = 15\ \Omega.$$

Problem 6 At an operating frequency of 300 MHz, a lossless 50-Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)\ \Omega$. Find the input impedance.

Solution: Given a lossless transmission line, $Z_0 = 50\ \Omega$, $f = 300$ MHz, $l = 2.5$ m, and $Z_L = (40 + j20)\ \Omega$. Since the line is air filled, $u_p = c$ and therefore, from Eq. (2.48),

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \text{ rad/m}.$$

Since the line is lossless, Eq. (2.79) is valid:

$$\begin{aligned} Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) &= 50 \left[\frac{(40 + j20) + j50 \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})}{50 + j(40 + j20) \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})} \right] \\ &= 50 \left[(40 + j20) + j50 \times 0 \right] 50 + j(40 + j20) \times 0 \\ &= (40 + j20)\ \Omega. \end{aligned}$$

Problem 7 A lossless transmission line of electrical length $l = 0.35\lambda$ is terminated in a load impedance as shown in Fig. P2.28. Find Γ, S, and Z_{in}. Verify your results using CD Modules 2.4 or 2.5. Include a printout of the screen's output display.

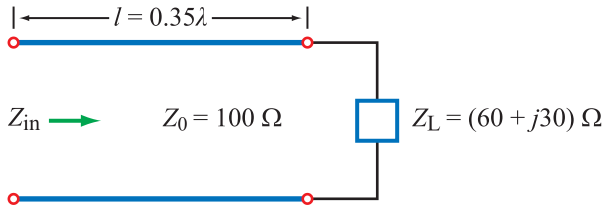


Figure P2.28: Circuit for Problem 2.28.

Solution: From Eq. (2.59),

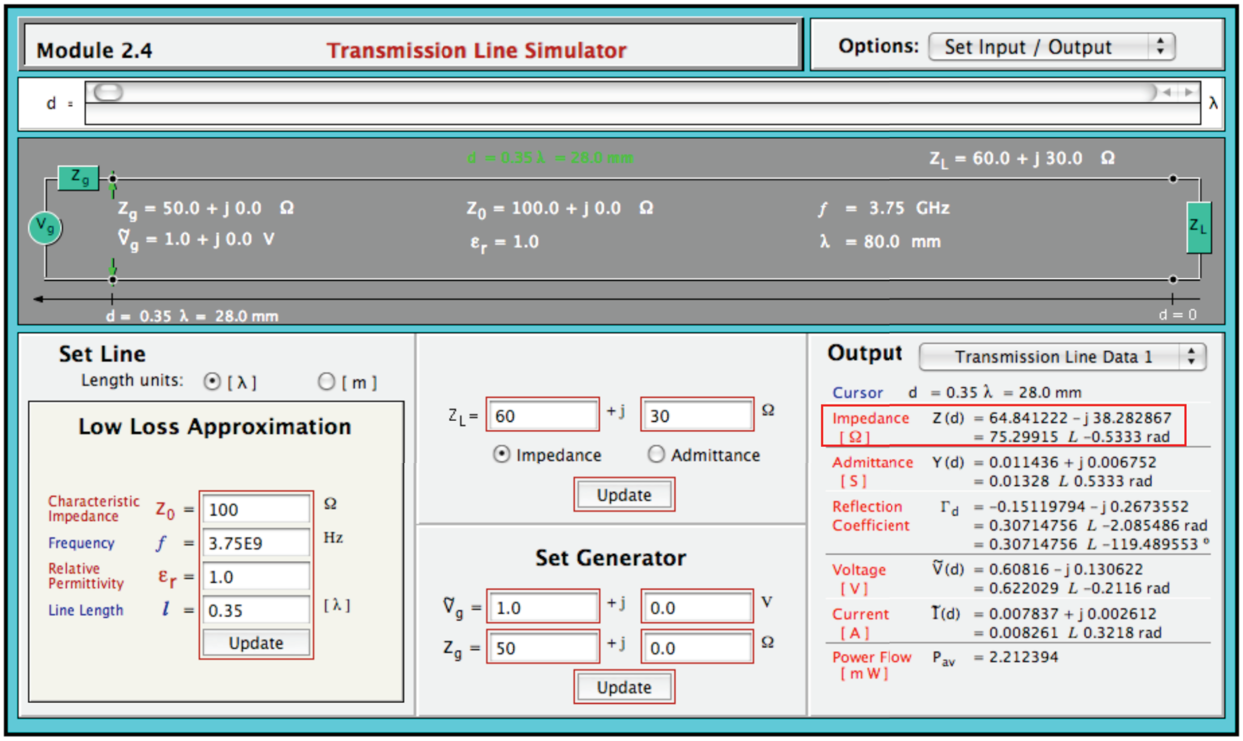
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(60 + j30) - 100}{(60 + j30) + 100} = 0.307e^{j132.5^\circ}.$$

From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.307}{1 - 0.307} = 1.89.$$

From Eq. (2.79)

$$\begin{aligned} Z_{in} &= Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \\ &= 100 \left[\frac{(60 + j30) + j100 \tan\left(\frac{2\pi \text{ rad}}{\lambda} 0.35\lambda\right)}{100 + j(60 + j30) \tan\left(\frac{2\pi \text{ rad}}{\lambda} 0.35\lambda\right)} \right] = (64.8 - j38.3)\ \Omega. \end{aligned}$$


Module 2.4
Transmission Line Simulator
Options: Set Input / Output
Z_L = 60.0 + j 30.0 Ω
Z_g = 50.0 + j 0.0 Ω
Ṽ_g = 1.0 + j 0.0 V
Z_0 = 100.0 + j 0.0 Ω
ε_r = 1.0
f = 3.75 GHz
λ = 80.0 mm
d = 0.35 λ = 28.0 mm
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z_0 = 100 Ω
Frequency f = 3.75E9 Hz
Relative Permittivity ε_r = 1.0
Line Length l = 0.35 [λ]
Update
Z_L = 60 + j 30 Ω
Impedance
Admittance
Update
Set Generator
Ṽ_g = 1.0 + j 0.0 V
Z_g = 50 + j 0.0 Ω
Update
Output
Transmission Line Data 1
Cursor d = 0.35 λ = 28.0 mm
Impedance [Ω] Z(d) = 64.841222 – j 38.282867
= 75.29915 ∠ –0.5333 rad
Admittance [S] Y(d) = 0.011436 + j 0.006752
= 0.01328 ∠ 0.5333 rad
Reflection Coefficient Γ_d = –0.15119794 – j 0.2673552
= 0.30714756 ∠ –2.085486 rad
= 0.30714756 ∠ –119.489553 °
Voltage [V] Ṽ(d) = 0.60816 – j 0.130622
= 0.622029 ∠ –0.2116 rad
Current [A] Ĩ(d) = 0.007837 + j 0.002612
= 0.008261 ∠ 0.3218 rad
Power Flow [m W] P_av = 2.212394

Problem 8 A 6-m section of 150-Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \quad \text{(V)}$$

and $Z_g = 150\ \Omega$. If the line, which has a relative permittivity $\varepsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)\ \Omega$, determine:

(a) λ on the line.

(b) The reflection coefficient at the load.

(c) The input impedance.

(d) The input voltage $\widetilde{V}_i$.

(e) The time-domain input voltage $v_i(t)$.

(f) Quantities in (a) to (d) using CD Modules 2.4 or 2.5.

Solution:

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°)\ \text{V},$$
$$\widetilde{V}_g = 5e^{-j30°}\ \text{V}.$$

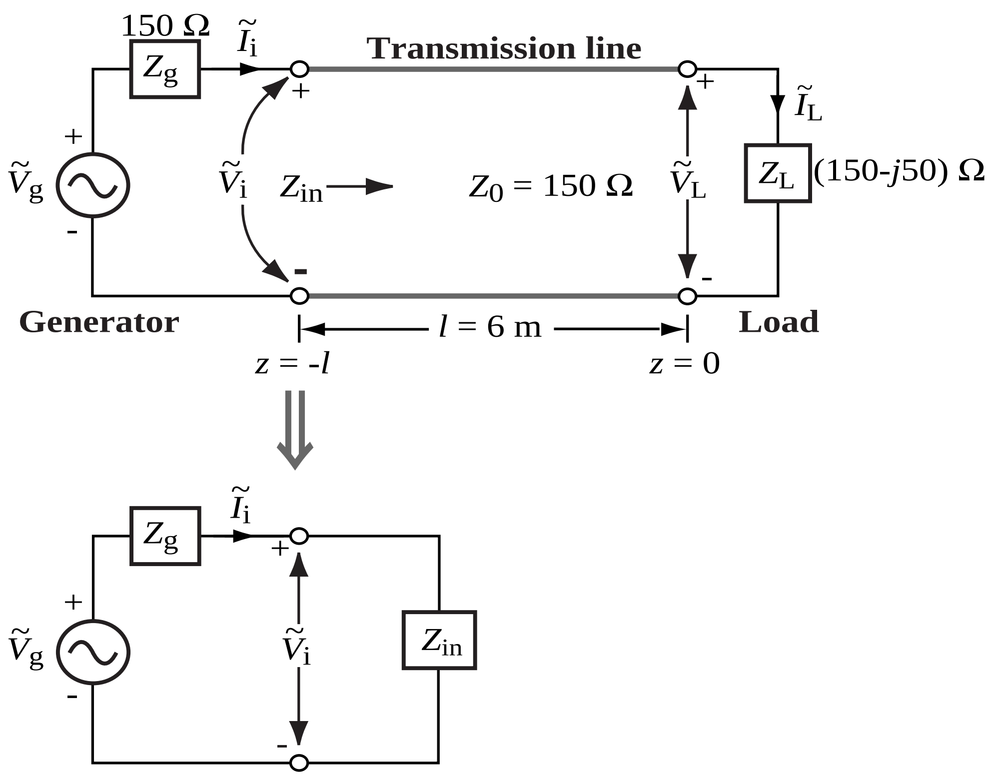


Figure P2.32: Circuit for Problem 2.32.

(a)

$$u_{\mathrm{p}} = \frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}} = \frac{3\times 10^8}{\sqrt{2.25}} = 2\times 10^8 \quad \text{(m/s)},$$
$$\lambda = \frac{u_{\mathrm{p}}}{f} = \frac{2\pi u_{\mathrm{p}}}{\omega} = \frac{2\pi \times 2\times 10^8}{8\pi\times 10^7} = 5 \ \text{ m},$$
$$\beta = \frac{\omega}{u_{\mathrm{p}}} = \frac{8\pi\times 10^7}{2\times 10^8} = 0.4\pi \quad \text{(rad/m)},$$
$$\beta l = 0.4\pi \times 6 = 2.4\pi \quad \text{(rad)}.$$

Since this exceeds 2π (rad), we can subtract 2π, which leaves a remainder $\beta l = 0.4\pi$ (rad).

(b) $\Gamma = \frac{Z_{\mathrm{L}} - Z_0}{Z_{\mathrm{L}} + Z_0} = \frac{150 - j50 - 150}{150 - j50 + 150} = \frac{-j50}{300 - j50} = 0.16\,e^{-j80.54^\circ}$.

(c)

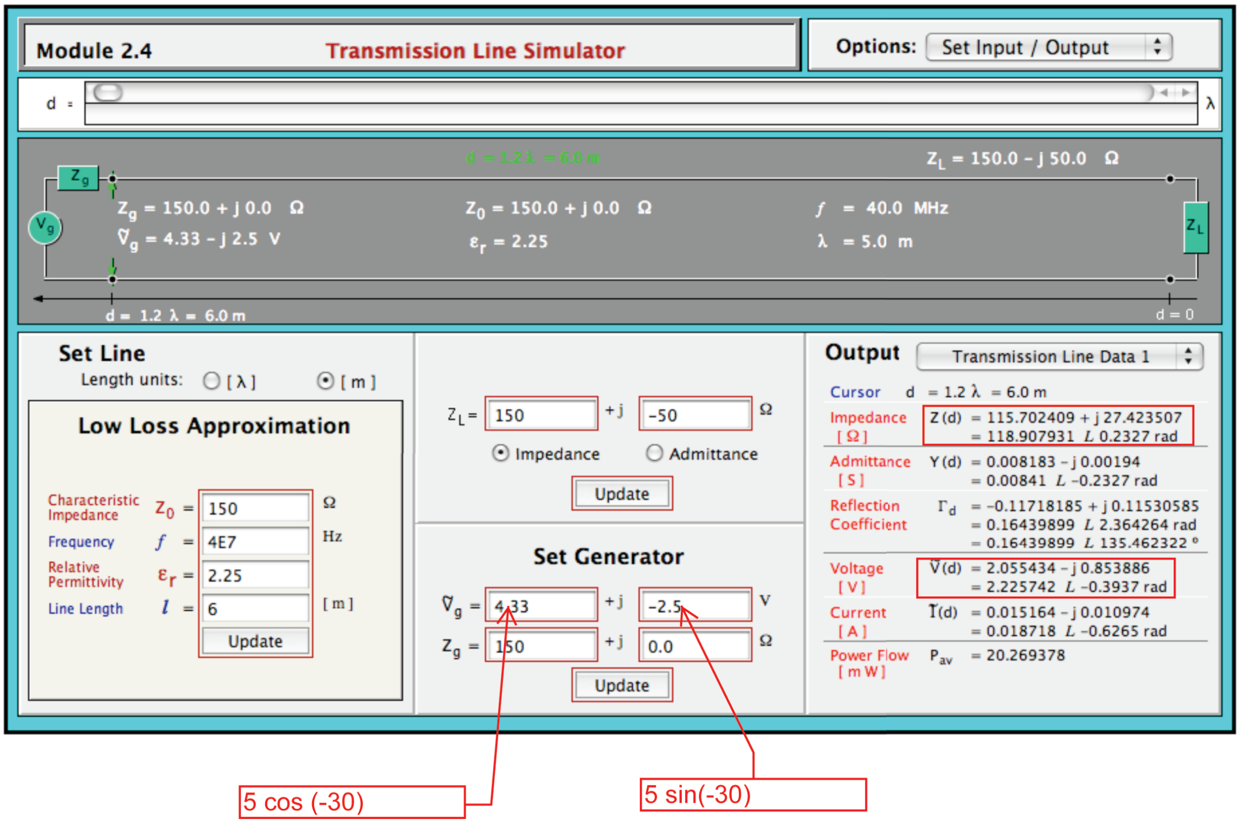
$$Z_{\mathrm{in}} = Z_0 \left[\frac{Z_{\mathrm{L}} + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$
$$= 150 \left[\frac{(150 - j50) + j150\tan(0.4\pi)}{150 + j(150 - j50)\tan(0.4\pi)}\right] = (115.70 + j27.42)\ \Omega.$$

(d)

$$\widetilde{V}_{\mathrm{i}} = \frac{\widetilde{V}_{\mathrm{g}} Z_{\mathrm{in}}}{Z_{\mathrm{g}} + Z_{\mathrm{in}}} = \frac{5e^{-j30^\circ}(115.7 + j27.42)}{150 + 115.7 + j27.42}$$
$$= 5e^{-j30^\circ}\left(\frac{115.7 + j27.42}{265.7 + j27.42}\right)$$
$$= 5e^{-j30^\circ} \times 0.44\,e^{j7.44^\circ} = 2.2\,e^{-j22.56^\circ} \quad \text{(V)}.$$

(e)

$$v_{\mathrm{i}}(t) = \mathfrak{Re}[\widetilde{V}_{\mathrm{i}} e^{j\omega t}] = \mathfrak{Re}[2.2\,e^{-j22.56^\circ} e^{j\omega t}] = 2.2\cos(8\pi\times 10^7 t - 22.56^\circ) \ \text{ V}.$$


Module 2.4
Transmission Line Simulator
Options: Set Input / Output
Z_L = 150.0 – j 50.0 Ω
Z_g = 150.0 + j 0.0 Ω
V_g = 4.33 – j 2.5 V
Z_0 = 150.0 + j 0.0 Ω
ε_r = 2.25
f = 40.0 MHz
λ = 5.0 m
d = 1.2 λ = 6.0 m
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z_0 = 150 Ω
Frequency f = 4E7 Hz
Relative Permittivity ε_r = 2.25
Line Length l = 6 [m]
Update
Z_L = 150 +j -50 Ω
Impedance Admittance
Update
Set Generator
V_g = 4.33 +j -2.5 V
Z_g = 150 +j 0.0 Ω
Update
Output Transmission Line Data 1
Cursor d = 1.2 λ = 6.0 m
Impedance [Ω] Z(d) = 115.702409 + j 27.423507 = 118.907931 ∠ 0.2327 rad
Admittance [S] Y(d) = 0.008183 – j 0.00194 = 0.00841 ∠ -0.2327 rad
Reflection Coefficient Γ_d = -0.11718185 + j 0.11530585 = 0.16439899 ∠ 2.364264 rad = 0.16439899 ∠ 135.462322 °
Voltage [V] V(d) = 2.055434 – j 0.853886 = 2.225742 ∠ -0.3937 rad
Current [A] I(d) = 0.015164 – j 0.010974 = 0.018718 ∠ -0.6265 rad
Power Flow [m W] P_av = 20.269378
5 cos (-30)
5 sin(-30)