

SOME ASPECTS OF THE SEISMIC INELASTIC RESPONSE OF A SIMPLE GENERAL PURPOSE TORSIONALLY COUPLED MODEL

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SUMMARY

The seismic non-linear inelastic response of a simple general purpose torsionally coupled model defined in a companion paper is further investigated. A classification into various simple types of coupled structures is presented, and the characteristic behaviour of structures in some of these categories is studied. The responses of stiffness eccentric and mass eccentric structures are compared. Strength eccentricities and plastic centroid considerations are addressed. The effect of increasing earthquake excitation intensities and variations in strain hardening values are examined. Each of those limited parametric studies was conducted to improve the understanding of the fundamental behaviour of simple torsionally structures, all while specializing the proposed model into an elementary structural model useful for future more comprehensive research on this topic.

INTRODUCTION

A simple general purpose model for the study of the non-linear inelastic seismic response of torsionally coupled structures has been proposed in a companion paper [1]. Nonetheless, as revealed by the requirements for the geometric equivalence of non-linear structures developed in that companion paper, small changes in the characteristics of even simple structures are sufficient to ensure that identical non-linear response of lateral-load-resisting structural elements (LLRSEs) is not possible. Consequently, even the minimal structural system proposed in that aforementioned paper, and deemed sufficient to satisfactorily capture the non-linear inelastic characteristics of torsionally coupled structures, can be configured in a variety of different ways for which dissimilar response is unavoidable.

Thus, simple torsionally coupled structures must be further categorized. A classification is proposed herein. It is the scope of this paper to present results from

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a number of simple analyses conducted to improve the understanding of the behavioral characteristic of structures in some of the proposed categories. In this respect, the responses of stiffness eccentric and mass eccentric structures are compared, strength eccentricities and plastic centroid considerations are addressed, the effect of increasing earthquake excitation intensities and variations in strain hardening values are examined. The significance of these results in light of the proposed simple general purpose model is examined.

Restrictions developed in the preceding paper regarding the monosymmetric model configurations, unidirectional excitations, and other assumptions and simplifications, are carried through for continuity in this paper. Parameters, variables and other terms previously described remain unchanged as well.

TYPES OF NON-LINEAR INELASTIC TORSIONALLY COUPLED STRUCTURES

For the structures with only two LLRSEs, a few general categories of plan eccentricities are possible, as shown on Figure 1. First, considering only elastic response characteristics, a structure can be considered symmetric or eccentric, i.e. the centres of stiffness and mass coincide or not. Realizing that elastic symmetry is not necessarily preserved in the inelastic domain, these systems are actually no more than "initially symmetric" or, by opposition, "initially eccentric".

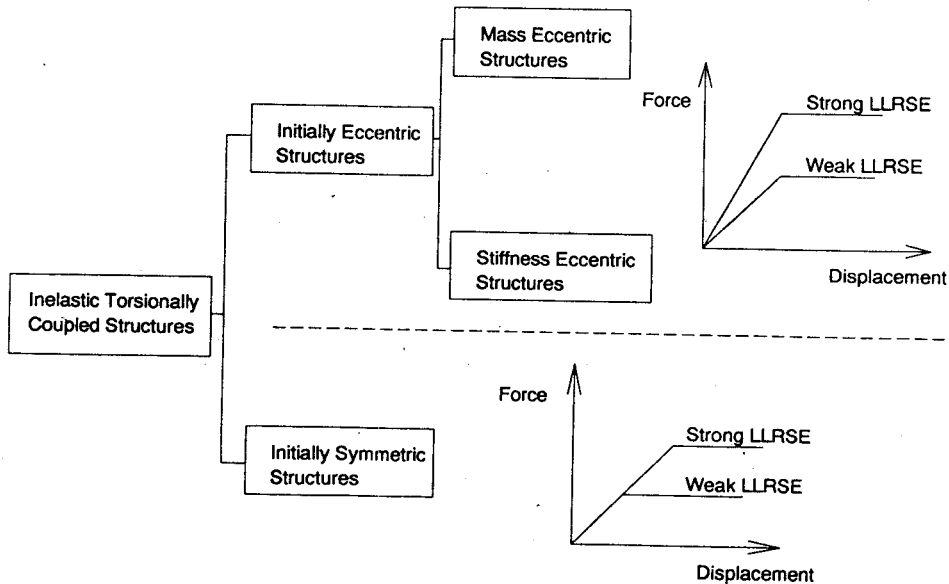


Figure 1 General classification of plan eccentricity types, and corresponding element models.

Theoretically, the symmetric nature of the structures of interest can only be extended in the non-linear domain if both LLRSEs are of the same hysteretic model and yield displacement; strength is related to stiffness, which itself is constrained by the requirement of coincidence of the centres of stiffness and mass. Even under these circumstances, some researchers [2,3] have demonstrated that torsional response can potentially be excited, but this level of refinement is neglected herein.

Strength eccentricity can occur for an initially symmetric structural system with LLRSEs not yielding simultaneously. In that case, a transient state of torsional coupling is excited by the inelastic response; i.e. torsion is introduced when one LLRSE is yielded while the other one remains elastic, creating an instantaneous eccentricity. When the structural response returns to the elastic range, torsional movements are eventually damped out, provided the LLRSEs are of bilinear hysteretic models. Thus, eccentricity here exists primarily in the non-linear phase of the response, and only coupled torsional/translational non-linear inelastic analyses can provide an estimate of the maximum deviation from the otherwise predicted purely translational movement. This has been illustrated in the companion paper [1] when the effect of rotational mass of inertia was examined.

It is sometimes perceived that an initially eccentric structure with only two LLRSEs is not possible since it constitutes a statically determinate structure for which the distribution of the lateral load in plan is fixed by the given geometry. Following this reasoning, an initially symmetric structure will automatically be generated by the design process. This would be true if the designer could exactly control all aspects influencing the structural design, thus ensuring perfect superposition of both centres of mass and stiffness, but experience shows that, in practice, a multitude of factors can prevent this ideal case to occur.

For the case at hand, mass eccentricity occurs if the centre of mass is not at mid-distance between two LLRSEs of equal stiffness. This is possible for irregularly shaped floor plans or nonuniform mass distributions. It also occurs when the centre of mass is not contained between the two LLRSEs, but this case is beyond the scope of this study. Stiffness eccentricity occurs when the centre of mass is equidistant from the two LLRSEs of different stiffnesses. This is often the case for structures with regular floor plan for which architectural requirements, for example, force the use of dissimilar LLRSEs at opposed ends of the structure.

No actual structure perfectly falls in any of these categories; each is rather a mix of all of those conditions. Nonetheless, these idealized categories are useful in the determination of the principal behavioral characteristics of inelastic torsional coupling. The important consideration of accidental eccentricity, omitted from the above

classification but properly recognized by building codes, is lumped with the other initially eccentric cases for the purpose of this study.

COMPARATIVE RESPONSE OF SIMPLE MASS AND STIFFNESS ECCENTRIC STRUCTURES

In the elastic domain, there need not be a distinction made between mass eccentricity and stiffness eccentricity. As described in a companion paper [1] all structures sharing the same characteristic parameters ω_x , Ω and (e/r) will have similar response $v_x(t)$ and $rv_\theta(t)$ at their centre of mass, and the response of an individual LLRSE is only affected by its distance from the centre of mass.

In order to assess differences in non-linear inelastic responses, two sets of mass and stiffness eccentric structures, equivalent in the elastic domain, have been selected. Both sets share an uncoupled translational period of 0.1 seconds, bilinear hysteretic element model with 5% strain hardening, a Rayleigh-type damping of 2% at the two true periods of each structure, and were subjected to the same N-S component 1940 El Centro earthquake record arbitrarily scaled to largely exceed the yield strength of all LLRSEs.

The first set has a ratio of uncoupled frequencies Ω of 1.5 and a normalized eccentricity (e/r) of 0.3. For LLRSEs 100 units apart in plan, this translates into static eccentricities of 10.2 units and 10 units for the mass and stiffness eccentric structures respectively. The true periods, as obtained from the eigensolution of the equations of motion, are 0.1036 and 0.0657 seconds.

For the second set, the ratio of uncoupled frequencies Ω equals 0.5 and the normalized eccentricity (e/r) remains 0.3, which implies that for LLRSEs 100 units apart, the resulting static eccentricities for the mass and stiffness eccentric structures are 37.5 units and 30 units, respectively. The true periods are 0.2628 and 0.0951 seconds.

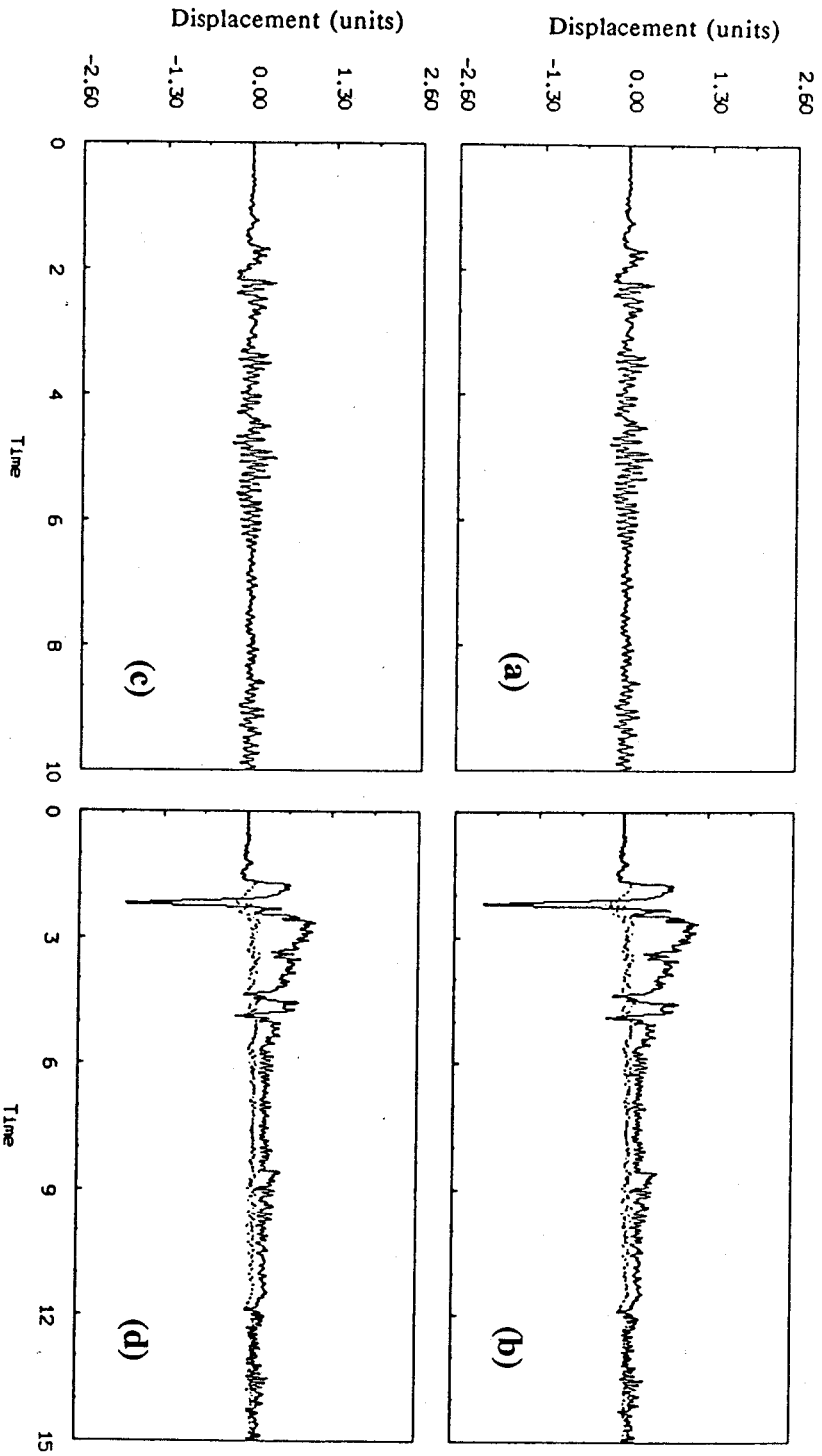
Results from elastic and inelastic step-by-step dynamic analyses are numerically presented in Table 1, and plotted all to the same scale in Figures 2(a)-(d) and 3(a)-(d). For each set, the linear elastic response at the centre of mass is numerically identical; maximum values presented in Table 1 compare well. As expected, the inelastic responses differ at their centres of mass, the most significant departures with equivalent elastic structures occurring when static eccentricities are the largest. Also, comparing the various non-linear inelastic time histories of LLRSE, the more severe element ductility demand occurs in the weak LLRSE of the stiffness eccentric structure.

Table 1. Mass and Stiffness Eccentric Structures Results *

Response Item (1)	Linear Elastic Reponse		Non-Linear Inelastic Response	
	Mass Eccentric (2)	Stiffness Eccentric (3)	Mass Eccentric (4)	Stiffness Eccentric (5)
Case 1: $T_x = 0.1$ s., $\Omega = 1.5$, $(e/r) = 0.3$, 2% Damping, 5% Strain Hardening				
Strong Element	0.205	0.220	0.239	0.212
Displacement	-0.149	-0.156	-0.197	-0.294
Center of Mass	0.307	0.307	0.701	0.655
Displacement	-0.215	-0.215	-1.19	-1.21
Weak Element	0.374	0.394	1.05	1.15
Displacement	-0.273	-0.289	-1.88	-2.15
Rotation rv_θ	0.058	0.058	0.303	0.340
	-0.051	-0.051	-0.602	-0.640
Case 2: $T_x = 0.1$ s., $\Omega = 0.5$, $(e/r) = 0.3$, 2% Damping, 5% Strain Hardening				
Strong Element	0.424	0.282	1.46	0.978
Displacement	-0.458	-0.234	-0.366	-0.512
Center of Mass	0.481	0.481	1.08	1.29
Displacement	-0.397	-0.397	-1.71	-1.24
Weak Element	0.593	1.07	1.24	2.54
Displacement	-0.501	-0.916	-1.96	-2.61
Rotation rv_θ	1.22	1.22	1.58	2.51
	-1.04	-1.04	-3.55	-3.41

* Results of Analysis in generic units for comparison purposes only.

It is noteworthy that two contradicting effects physically interact. On one hand, the distance from the centre of mass to the LLRSE estimated to yield first, as assessed by static analysis and referred to as the weak element hereafter, is greater in the case of stiffness eccentric structure; Considering additive torsional and rotational dynamic response for that LLRSE and adopting the identical yield displacements model of Figure 1, yielding of the edge weak LLRSE would therefore occur sooner for that structure, and ductility demand of this LLRSE could be anticipated to be larger than that for the equivalent mass eccentric structure. On the other hand, once the edge weak LLRSE has yielded, the instantaneous eccentricity (e') would become the largest in the mass eccentric structure, with potential for a larger concentration of inelastic demand in this element. As shown in Figures 2 and 3, the limited analysis conducted here indicate that the first effect more than overcomes the second.



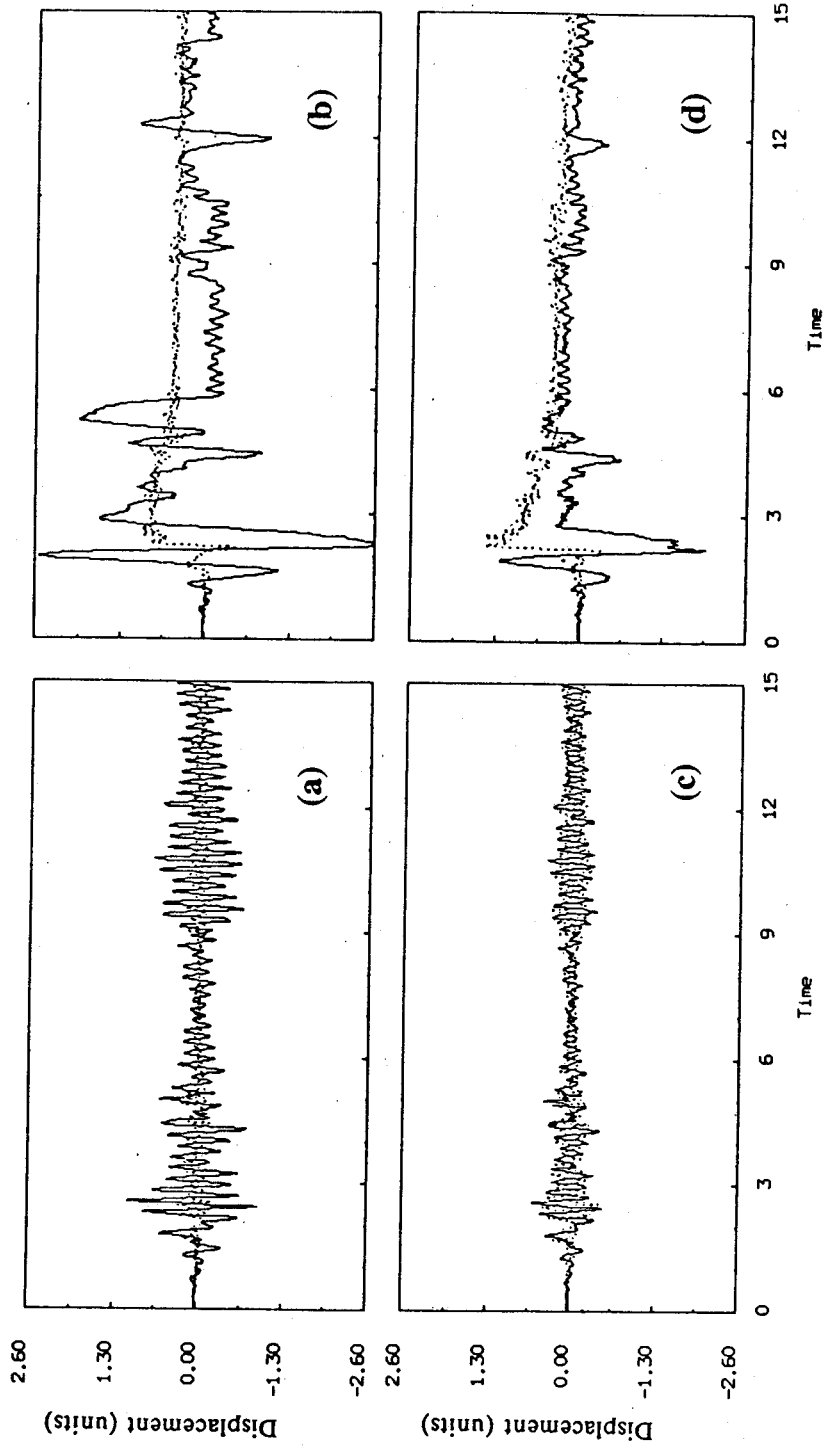


Figure 3 Time history response of (a) Elastic and (b) Inelastic stiffness eccentric structures, and (c) Elastic and (d) Inelastic mass eccentric structures, for $T_x = 0.1$ seconds, $\Omega = 0.5$, and $(e/r) = 0.3$; Yield displacement is 0.12 units; Solid line is weak LLRSE; Dotted line is strong LLRSE.

Conclusions obtained from the results of studies performed on stiffness eccentric structures could conservatively be extended to mass eccentric structures, assuming the comparison remains within the aforementioned limitations. These limitations are important. If instead earthquake excitations were scaled with intent to match the maximum edge weak LLRSE elastic response of the mass and stiffness eccentric structures, the mass eccentric structure would be thus subjected to a much stronger earthquake, and comparison made on this different basis could show the inelastic response of the mass eccentric structure exceeding that of the stiffness eccentric structure. Such a case was not studied.

PLASTIC CENTROID — INTER-ELEMENT MODEL RELATIONS

Admittedly, the type of hysteretic element model will have a considerable effect on the seismic inelastic response of torsionally coupled structures. Nonetheless, since current research still focuses on improving the basic understanding of the behaviour of torsionally coupled structures in the inelastic domain, the consideration of very complex hysteretic models remains premature. Some studies have briefly examined the influence of more complex models in relation to comprehensive parametric studies on simpler model [4] but at this time, bilinear hysteretic models have been the basis of most research on torsionally coupled structures [5,6 among others].

The concept of strength eccentricity [5], equivalent to the corollary concept of plastic centroid, provides a representation of the relationship between LLRSEs' strength within a same structure. It quantitatively expresses the observation that the relative yielding levels between different LLRSEs will directly affect the global inelastic behaviour. For a bilinear hysteretic model with two LLRSE and a given set of Ω , (e/r) and T_x , the respective yield displacements between the two LLRSEs will completely define this inter-element model relation, and simultaneously locate the plastic centroid.

By analogy with reinforced concrete theory, the plastic centroid is defined as the point where a static lateral load must be applied in order to produce a purely translational displacement when all elasto-perfectly plastic elements are yielded. The plastic centroid distance from the centre of mass can be used as another indicator of the severity of a structure's inelastic torsional behaviour. A plastic centroid distance of zero would produce simultaneous yielding of both LLRSEs under a monotonically increasing static loading, although under dynamic excitation it is not necessarily the case.

To assess the significance of the plastic centroid distance on the global structural response, the two stiffness eccentric structures used previously, with $\Omega = 0.5$ & 1.5 ,

$(e/r) = 0.3$, $T_x = 0.1$, and the same arbitrarily scaled magnitude of the NS component of the 1940 El Centro earthquake, were reanalysed using various yield strength levels for the stronger LLRSE. The strength of the weaker LLRSE remained unchanged throughout all cases. The various strength eccentricities considered are schematically illustrated in Figure 4. It is noteworthy that for a given relative strength configuration in that Figure, a larger spread of the absolute strength results from the larger static eccentricity attached to lower values of Ω . For example, the ratio R_3/R_1 equals 1.5 and 4 for Ω values of 1.5 and 0.5 respectively.

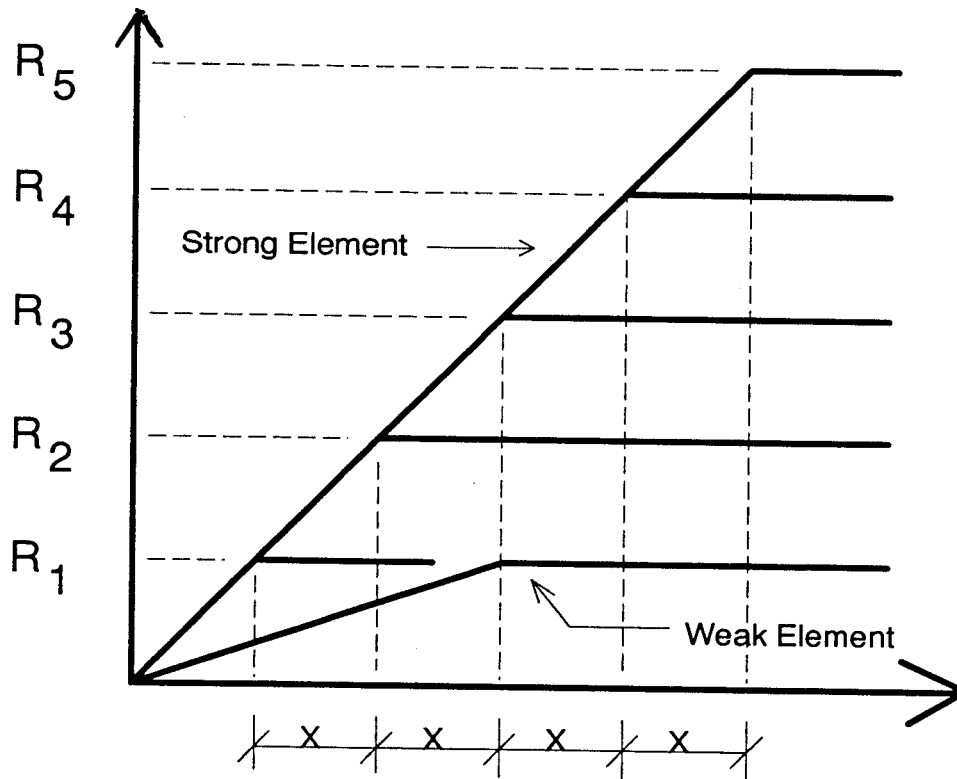


Figure 4 Various strength eccentricity cases studied for initially eccentric structures, constant weak element strength and yield displacement step increment (X).

The complete displacement time histories for the cases defined as R_1 , R_2 , R_4 and R_5 in Figure 4 are presented in Figures 5 and 6. Rotation time histories are presented elsewhere [7]. For the R_3 case, corresponding to the proposed model with equal yield displacement LLRSEs, results have already been presented in the previous section.

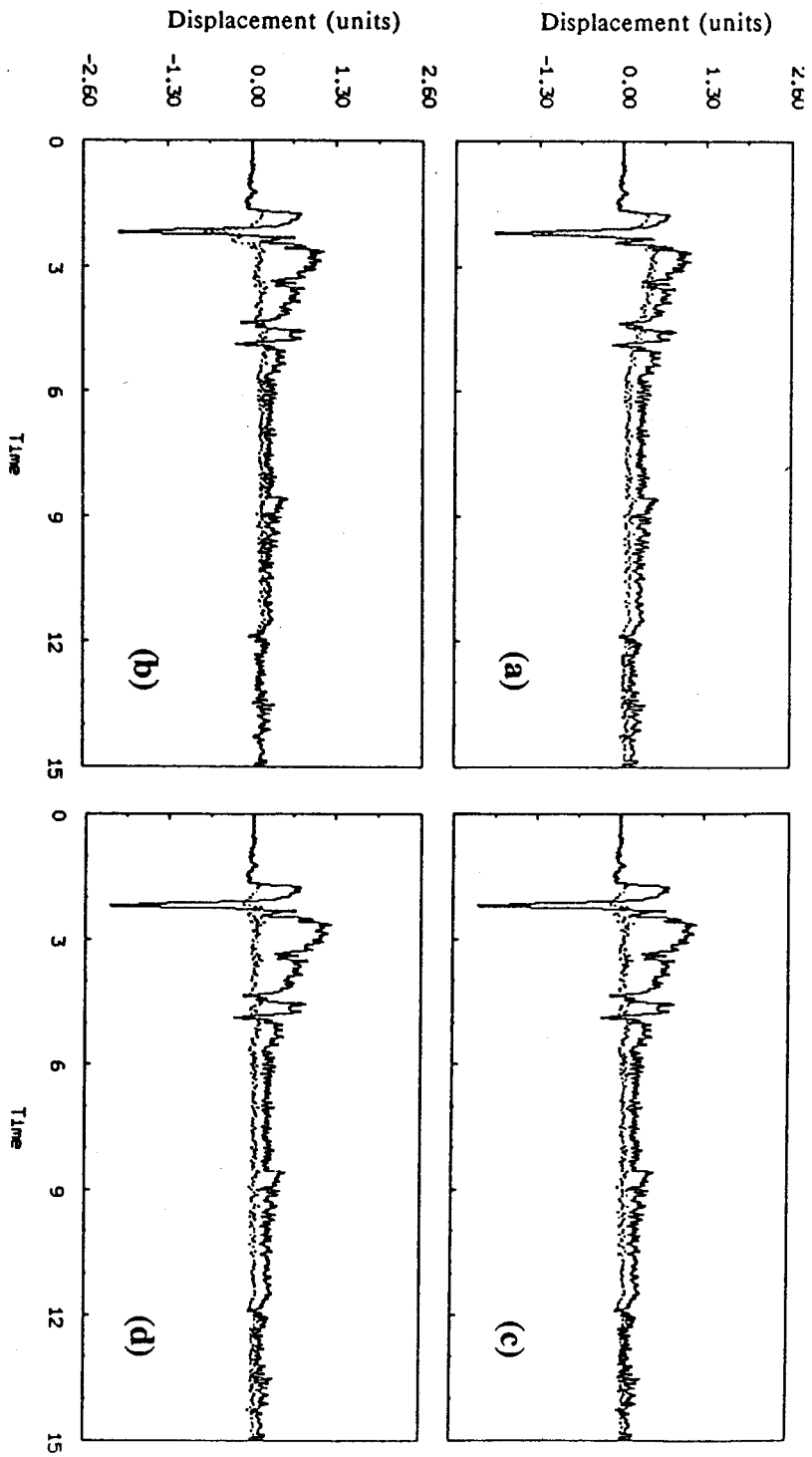


Figure 5 Inelastic displacement time histories for strength eccentric case (a) $R_p = 1.5$, (b) $R_p = 1.5$, (c) $R_p = 1.5$, (d) $R_p = 1.5$ of stiffness eccentric structures for $T_x = 0.1$ seconds, $\Omega = 1.5$, $(e/r) = 0.3$; Yield Displacement is 0.12 units; Solid line is weak LLRSE; Dotted line is strong LLRSE.

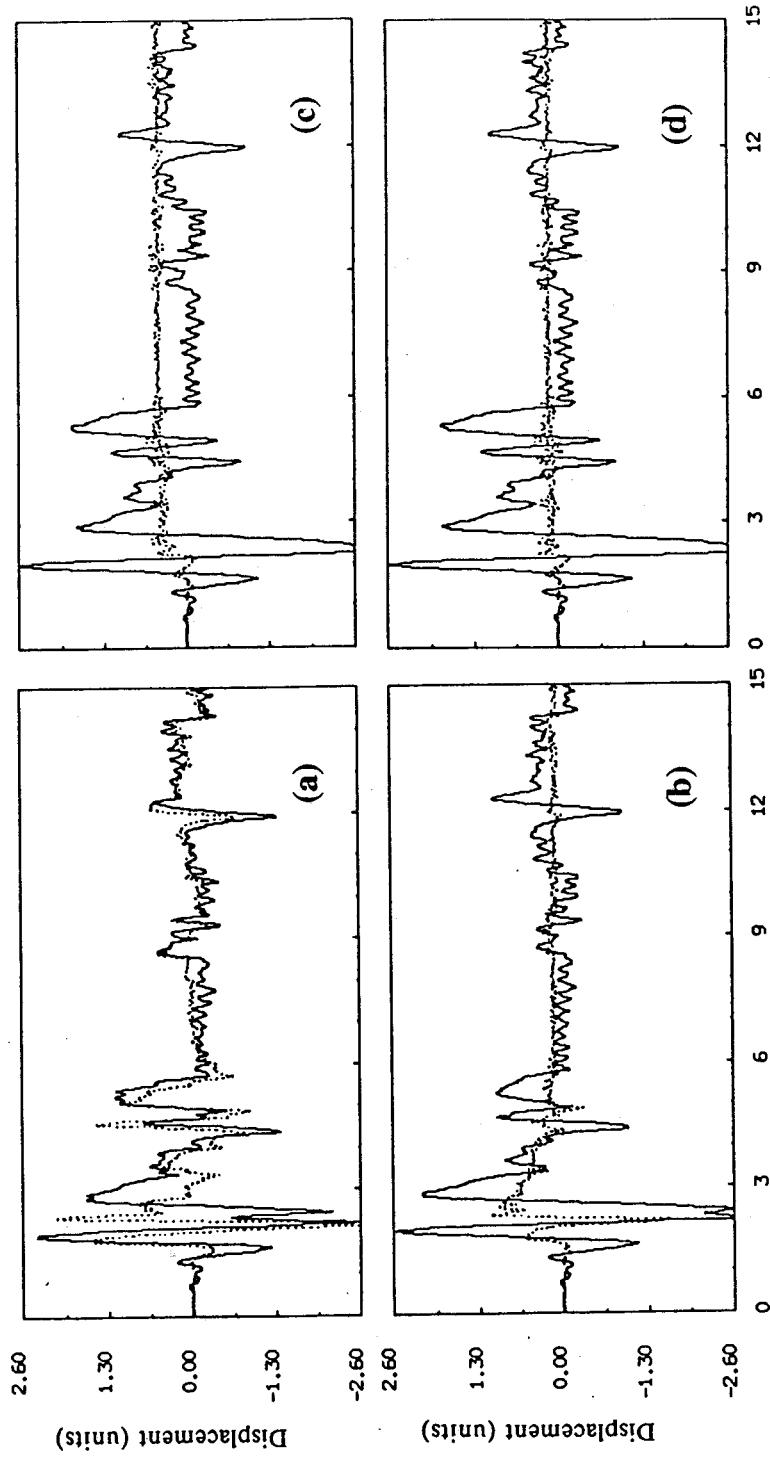


Figure 6 Inelastic displacement time histories for strength eccentric case (a) R_1 , (b) R_2 , (c) R_4 , (d) R_5 , of stiffness eccentric structures for $T_x = 0.1$ seconds, $\Omega = 0.5$, $(e/r) = 0.3$; Yield Displacement is 0.12 units; Solid line is weak LLRSE; Dotted line is strong LLRSE.

Interestingly, changes in plastic centroid distance seriously affect the LLRSE whose yielding strength is varied, but have relatively little effect on the weak LLRSE whose yield strength is kept constant. The maximum variation of the weak LLRSE response is less than 16% whereas the strong LLRSE response variation exceeds 500% when comparing the various results obtained.

Therefore, if the maximum response of the weak LLRSE is of concern, the proposed simple model with LLRSEs sharing equal yield displacements is generally adequate. If the strong LLRSE's response is also of interest, its high sensitivity to the plastic centroid distance makes the inter-element model relationship a more important issue. These observations come from inelastic analyses conducted at large ductility demand levels; likewise, others have reported [5] that strength eccentricity concepts are most meaningful at higher ductility levels.

EFFECT OF DIFFERENT EARTHQUAKE INTENSITIES

Initially Symmetric Structures

A two LLRSE system with an uncoupled period T_x of 0.4 seconds, 2% viscous damping, a geometric ratio (d/r) of 1.6, a ratio of uncoupled frequency Ω of 1.6, bilinear hysteretic element model with 0.5% strain hardening, and subjected to the N-S component of the 1940 El Centro earthquake record, is arbitrarily selected in this case. Inelastic torsional coupling is produced by the unequal yield strength of two LLRSEs of identical stiffness and equidistant from the centre of mass: one LLRSE yield level is 80% of the other one. An earthquake level of unity here is defined as that producing a maximum displacement equal to the yield displacement of the stronger LLRSE. Other earthquake levels are obtained directly by proportional scaling of the earthquake excitation by that value.

Results for this initially symmetric structure for earthquake levels ranging from 1 to 12 are presented in Figures 7 and 8. Response of elastic and inelastic single-degree-of-freedom (SDOF) systems are represented, in Figure 7, by solid lines, whereas the weak and strong LLRSEs response are shown by dotted lines. As seen in that Figure, the mean value of the LLRSE's individual ductility demands is approximately equal or slightly greater than that of the inelastic SDOF systems. At all levels, this inelastic SDOF ductility demand curve is bounded by the ones for the LLRSEs of the uncoupled structure, the element with smaller yield strength having consistently the largest ductility demand.

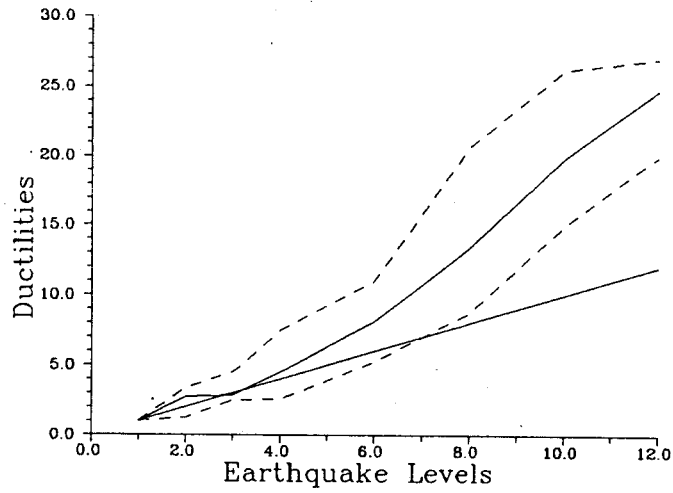


Figure 7 Displacement ductility demands as affected by earthquake intensity; Linear and non-linear solid lines for SDOF elastic and inelastic response respectively; Dotted lines for weak (top) and strong (bottom) LLRSE of an example initially symmetric structure.

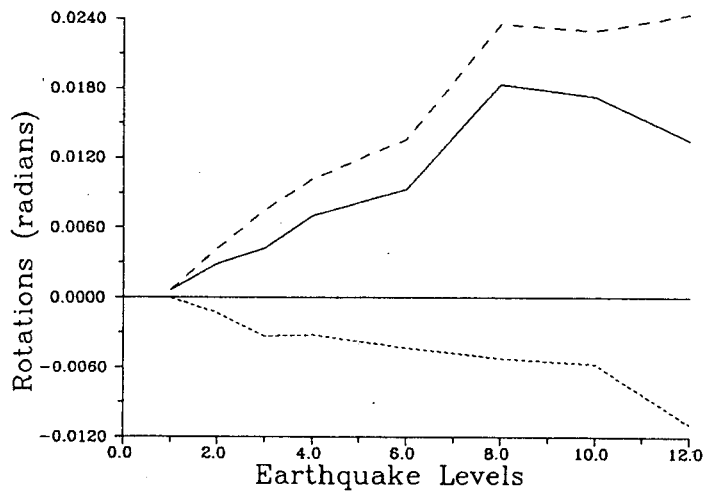


Figure 8 Maximum plan rotations of an example initially symmetric structure as affected by earthquake intensity; Solid line is maximum positive, dotted line is maximum negative, and dashed line is maximum absolute sum.

For completeness, the effect of earthquake excitation level on the maximum rotations recorded during the response are presented as well (Figure 8). As expected,

the maximum attained rotation angles increase somewhat proportionally with the earthquake level in a well behaved manner.

Initially Eccentric Structures

For this study, a two LLRSE stiffness eccentric structure with uncoupled period T_x of 0.1 seconds, 2% damping, (e/r) of 0.3, Ω of 1.5, and bilinear hysteretic element model with 0.5% strain hardening, subjected to the N-S component of the 1940 El Centro earthquake record, was selected. Here, an earthquake level of one brings the structure to first yield of a LLRSE.

The results under earthquake levels ranging from 1 to 12 are presented in Figures 9 and 10. In Figure 9, the weak and strong LLRSE responses are plotted with dotted lines, the weaker element always having the larger ductility demand. The response of a SDOF having a 0.1 second period, along with the coupled elastic response of the weak element (in which case elastic overstress is plotted instead of ductility) are the solid lines on the same figure. Although, the true coupled structure periods are

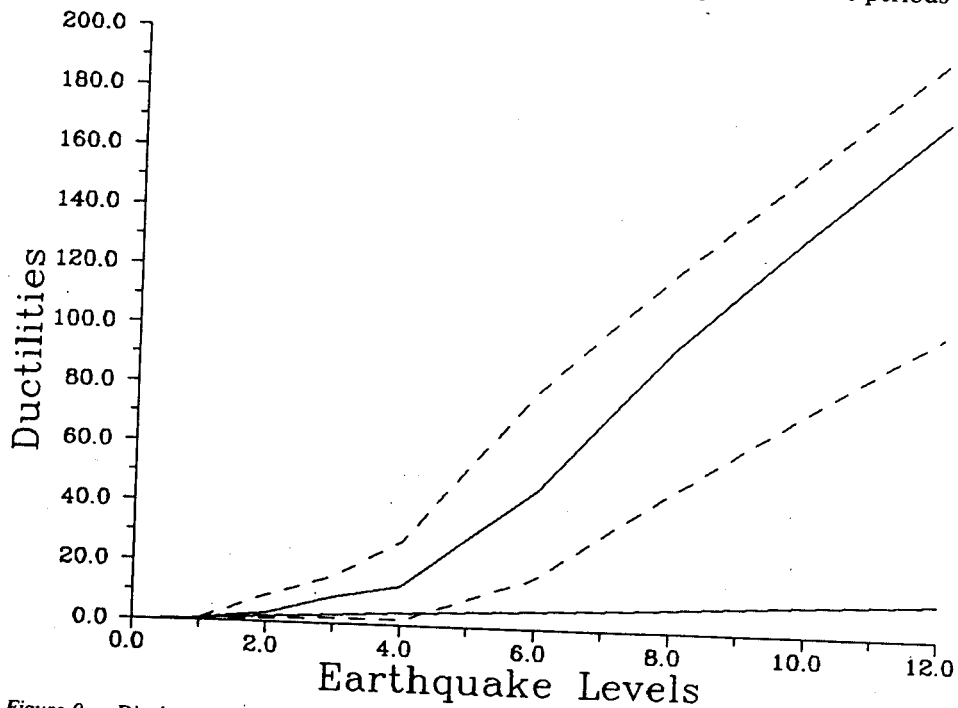


Figure 9 Displacement ductility demands as affected by earthquake intensity; Dotted lines for weak (top) and strong (bottom) LLRSE of an example initially eccentric structure; Solid linear line for elastic response of the weak LLRSE; Solid non-linear line for inelastic response of SDOF of 0.1 second period.

0.1036 and 0.0657 seconds, the ductility demand of a SDOF of 0.1 second period was found to be within 15% of that of a 0.1036 second SDOF.

Here, the average between the strong and weak element maximum response of the inelastic torsionally coupled structure is approximately equal to the uncoupled inelastic system response, except for large earthquake levels above 6 where the SDOF system somewhat exceeds that average. At all times, the SDOF ductility demand is less than the weak LLRSE ductility demand, and more than the strong LLRSE's one. Also, the maximum rotations again increase proportionally with the earthquake level (Figure 10). Note that, in this case, the maximum ductility demand and rotations are very severe for both the uncoupled and coupled structures past moderate excitation levels, and actual structures are not expected to be excited to the extreme ductility values indicated on these plots.

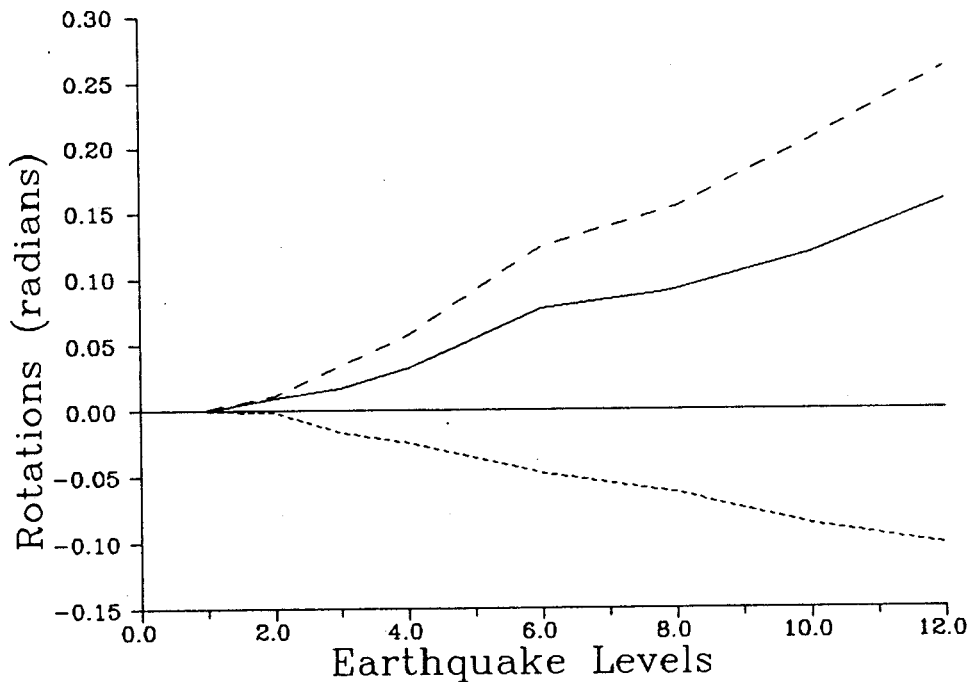


Figure 10 Maximum plan rotations of an example initially eccentric structure as affected by earthquake intensity; Solid line is maximum positive, dotted line is maximum negative, and dashed line is maximum absolute sum.

These above results for both initially symmetric and eccentric structures confirm that, for the simple model adopted, ductility demand of the weaker LLRSE is more of concern whereas the stronger LLRSE ductility demand can conservatively be estimated from that of a corresponding SDOF.

EFFECT OF STRAIN HARDENING

One additional aspect of the simple bilinear hysteretic element model deserving consideration is the effect of strain hardening. Again, predictions in behaviour from study under monotonically increasing loading are not corroborated by the observed actual dynamic inelastic behaviour. For a two LLRSE structure under monotonically increasing loading at the centre of mass, should the bilinear hysteretic element model be elasto-perfectly plastic (i.e. no strain hardening), the structure becomes unstable as soon as only one of the elements yields; the maximum angle of rotation becomes infinite. Increases in strain hardening progressively reduce resulting displacements for a given static loading beyond the point of first yield. Also, larger strain hardening will reduce the magnitude of the instantaneous eccentricity when only one of the LLRSE is yielded, with corresponding further reduction in response.

All this suggests that increased strain hardening will necessarily be beneficiary in reducing ductility demand. Unfortunately, and much like what was observed of torsional coupling behaviour in general in a companion paper [1], static analysis based observations do not properly predict the behaviour of inelastic torsionally coupled structures.

Two sets of initially eccentric structures have been analyzed under the arbitrarily scaled NS component of the 1940 El Centro earthquake. Within each set, all characteristic response parameters are made identical except for strain hardening values selected as 5% and 0.5% of the initial stiffness respectively. As shown in Figures 11 and 12, an increase in strain hardening can either reduce or increase the maximum response. There is no predictable result. Very minor differences in the early response of comparable structures with different strain hardening values have major effects on the overall behaviour, especially under large ductility demands. After an initial yield excursion, the bilinear hysteretic model will make the element with the largest strain hardening yield at a lower strength during load reversal; this may account for the striking differences in behaviour. Oddly enough, maximum values of response in this particular case, in spite of much difference in time history signatures, are very similar in magnitude.

CONCLUSIONS

Inelastic torsional coupling of structures can be categorized into initially symmetric and initially eccentric structures, this last group itself subdivided into mass eccentric and stiffness eccentric cases. For the simple general purpose model used herein, a number of useful observations are made from the few selective analyses conducted.

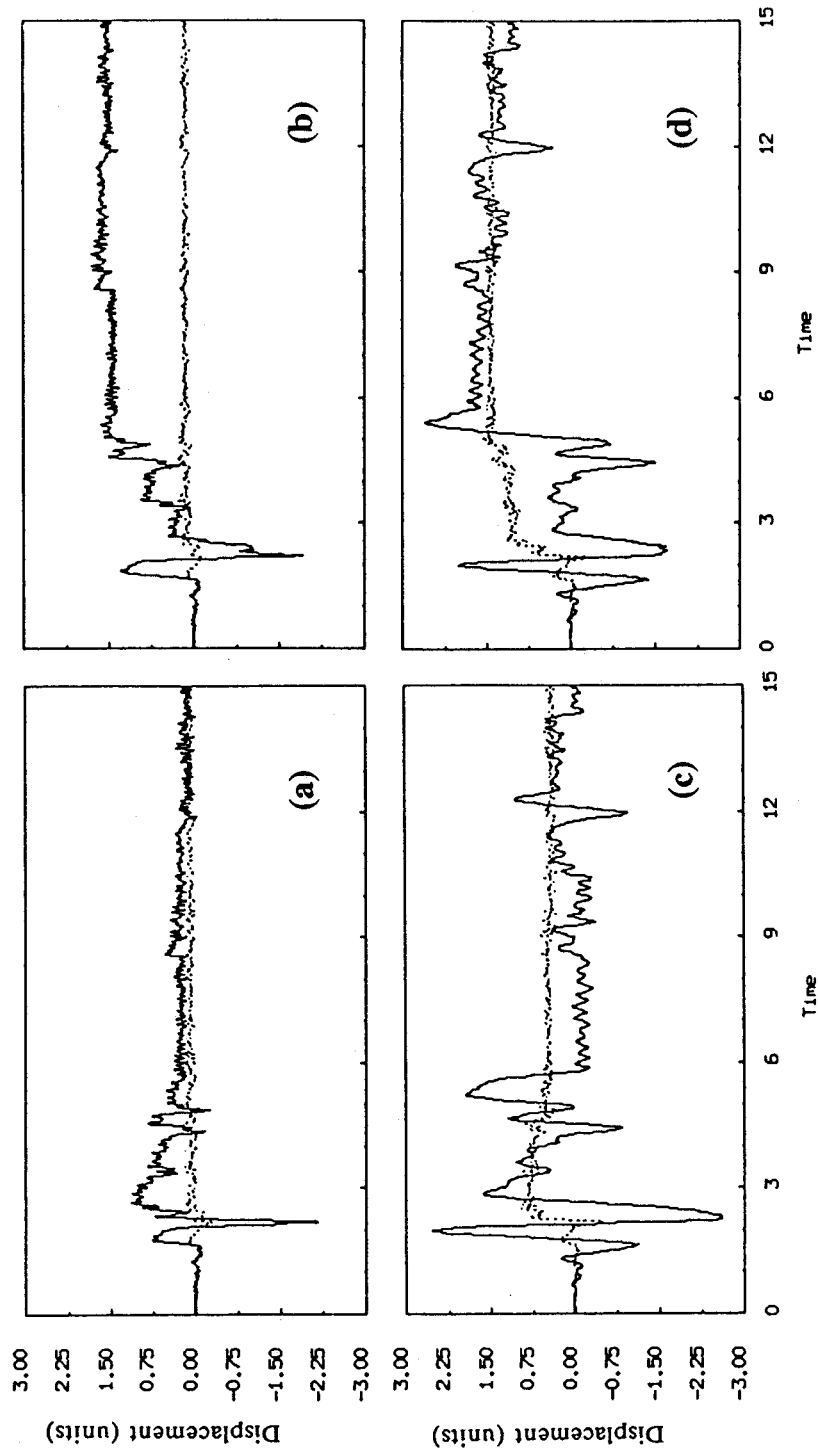


Figure 11 Displacement time history response of arbitrarily selected initially eccentric structures; First sample structure with (a) 5% and (b) 0.5% strain hardening; Second sample structure with (c) 5% and (d) 0.5% strain hardening; Yield displacement is 0.12 units; Solid line is weak LLRSE and dotted line is strong LLRSE.

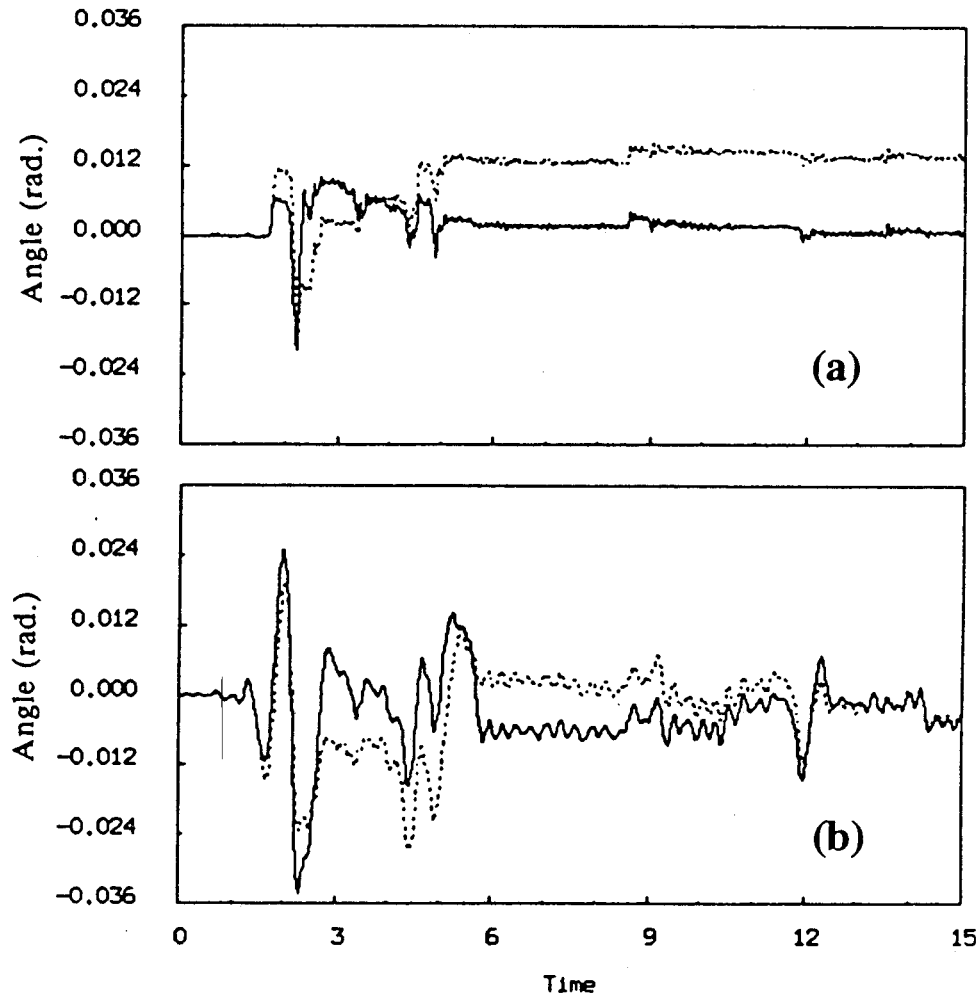


Figure 12 Rotation time history response of arbitrarily selected initially eccentric structures, with 5% (solid line) and 0.5% (dotted line) for (a) first sample structure and (b) second sample structure.

At identical level of earthquake excitation, stiffness eccentric structures have more severe LLRSE ductility demand than equivalent mass eccentric structures. Results from the study of stiffness eccentric structures can therefore be conservatively extended to other initially eccentric structures.

Variation in strength of the stronger LLRSE is found to insignificantly influence the behaviour of the weaker LLRSE; plastic centroid considerations are thus irrelevant if concerned with inelastic response of weaker LLRSE.

Irrespectively of earthquake intensity level, the weaker and stronger LLRSE ductility demands always bound that of the equivalent SDOF system. The strong LLRSE ductility demand can therefore be conservatively estimated by that of the equivalent SDOF, leaving only the weaker LLRSE ductility demand magnitude to assess by other means.

For bilinear hysteretic element model, modifications of the strain hardening value have unpredictable effects on the LLRSEs time history signatures; non-linear inelastic static analysis is found to be deficient in predicting the effect of strain hardening on the global behaviour. The high sensitivity of response to hysteretic model characteristics requires further consideration in future studies on inelastic torsional coupling.

The current findings based on a simple general purpose torsionally coupled model, along with those of the companion paper [1], provide useful data on the general behaviour of torsionally coupled structures seismically excited in the non-linear inelastic domain. This information has already been used in more conventional parametric studies [4,7] assessing the influence of the characteristic response parameters Ω , (e/r) and T_x on the inelastic response of these structures.

Additional research on all aspects of the non-linear inelastic response of torsionally coupled structures is undoubtedly needed.

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