

SOME ASPECTS OF DEFINING A SIMPLE GENERAL PURPOSE MODEL FOR THE STUDY OF SEISMIC INELASTIC TORSIONAL COUPLING

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SUMMARY

A wide variety of structural models and configurations have been used in past studies of the seismic inelastic response of torsionally coupled structures. The minimal properties of a simple general purpose model capable of capturing the essential features of the inelastic torsional response of structures are sought. Factors responsible for the added complexity in structural response due to the introduction of non-linear hysteretic models are reviewed. The indirect promotion of least plan redundancy by building codes is demonstrated. The monosymmetric two lateral-load-resisting-structural-element structure is proposed as an acceptable system for the fundamental study of inelastic torsional coupling, and a simple comparison of static, dynamic elastic and dynamic inelastic response is performed to support this proposition. The usefulness of perfect geometric equivalence is evaluated. The importance of the rotational inertia on the response of torsionally coupled structures is also presented. Finally, limitations of the proposed model are discussed.

INTRODUCTION

The considerable difference in the seismic dynamic response between symmetric structures and structures with stiffness and/or mass eccentricities in plan has long been recognized. More than 50 years ago, based on dynamic response studies of irregular building forms in small scale experiments, the fundamental concept of structural torsional coupling was enunciated [1], i.e. whenever the centres of rigidity of a structure do not coincide with its centres of mass, translation in horizontal directions will be accompanied by torsional movement in plan, whether or not there is rotation in the ground motion. Fifteen years later, it was demonstrated [2] that, for structures where plan eccentricity is created by a stiffness mismatch of shear walls in the direction parallel to the earthquake excitation, dynamic analysis calculated forces would exceed those otherwise obtained from static analysis. This, as well as observation and analysis of buildings damaged by torsional oscillations during severe earthquakes such as the 1957 Mexico City earthquake [3] led to the adoption of building codes provisions for magnifying the calculated static eccentricity, and

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ensuring that some minimum eccentricity is considered during design. The introduction of modern formulations of matrix theory and increased computer power further fuelled research on torsionally coupled structures.

While many of aspects of the behaviour, analysis and design of such structures have been studied since, as reflected by the long list of valuable contributions presented elsewhere [4], considerable research efforts are still currently invested to improve the understanding of both the linear elastic [5] and non-linear inelastic [6,7] response of such structures. This paper is concerned with the non-linear inelastic aspects of this response; an understanding of this behaviour is essential as these structures will be excited near their ultimate strength/ductility capabilities during large and infrequent earthquakes.

It is well established that equations of motion of torsionally coupled structures are generally amenable to a format for which all structures sharing the same values for a few key parameters encapsulating their structural characteristics, irrespectively of their geometry, will share the same linear elastic response at a given reference point. Unfortunately, the same cannot be said of non-linear inelastic structures; a complex interdependence of the number, location, and hysteretic characteristics of structural elements directly impact on the behaviour of these structures.

Consequently, a wide variety of structural models and configurations have been recently used to investigate the inelastic response of torsionally coupled structures. *Kan and Chopra* [8] replaced the physical structure by a single-element model having an interaction surface mapping the shear-torsion space. *Tso and Sadek* [9] studied the behaviour of a unisymmetric structure having four identical columns of circular cross-section under bi-directional earthquake excitations. *Bozorgnia and Tso* [10] investigated the behaviour of monosymmetric structures having three lateral load resisting structural elements (LLRSE) sharing identical yield displacements and bilinear hysteretic model under unidirectional earthquake excitations. Others [11,12,4,6] among many concerned with various forms of inelastic response, including also that of structures symmetric in the elastic sense, have used monosymmetric structures with only two LLRSEs, considering various element models and types of dynamic excitations. In most of these studies, LLRSEs perpendicular to the direction of earthquake excitation were located such as not to contribute to the torsional resistance of the structures.

It is noteworthy that resulting observations on the effect of various parameters on the inelastic response of torsionally coupled structures have generally not been in agreement. This can be partly attributed to the diverse analytical assumptions and approaches that were adopted in each study.

In this paper, it is demonstrated that a simple two LLRSE system can adequately capture the essential non-linear behavioral characteristics of torsionally coupled structures and be used in fundamental research on this seismic inelastic structural response. Valuable information obtained from parametric studies conducted on such simple two LLRSE systems has actually led to the development of a method, presented elsewhere [7] to predict the inelastic response of torsionally coupled structures based on the availability of elastic and inelastic response spectra for equivalent single-degree-of-freedom (SDOF) systems.

Based on extensions of the equations of motion into the inelastic domain, the additional geometric dependence necessary to ensure identical response for non-linear inelastic structures are described in this paper. Factors found to steer designs towards lesser redundancy, the influence of the rotational inertia on the inelastic response of torsionally coupled structures, and limitations of the proposed model, are also presented. In a companion paper [13] the effect of the type of eccentricity (mass eccentric versus stiffness eccentric), strain-hardening for bilinear hysteretic model, earthquake intensities, and plastic centroid, are examined to establish which modelling considerations are most significantly affecting the non-linear inelastic response of torsionally coupled structures.

EQUATIONS OF MOTION

The general equations of motion around the centre of mass for single-story torsionally coupled structures are well known and have been derived by others. [14] For monosymmetric structures (i.e. structures having one axis of symmetry) and neglecting torsional seismic excitation, the equations along the y-axis (axis of symmetry) are decoupled, and the resulting coupled translational-torsional equations of motion are simplified to:

$$\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{bmatrix} \ddot{v}_x \\ \ddot{v}_\theta \end{bmatrix} + \begin{bmatrix} K_x & -K_x e \\ -K_x e & K_\theta \end{bmatrix} \begin{bmatrix} v_x \\ v_\theta \end{bmatrix} = \begin{bmatrix} -m\ddot{v}_{gx} \\ 0 \end{bmatrix} \quad (1)$$

and, equivalently,

$$\begin{bmatrix} \ddot{v}_x \\ r\ddot{v}_\theta \end{bmatrix} + \omega_x^2 \begin{bmatrix} 1 & -e/r \\ -e/r & \Omega^2 \end{bmatrix} \begin{bmatrix} v_x \\ r v_\theta \end{bmatrix} = \begin{bmatrix} -\ddot{v}_g \\ 0 \end{bmatrix} \quad (2)$$

4

with

$$\Omega = \omega_{\theta} / \omega_x = T_x / T_{\theta} \quad (3)$$

$$\omega_x^2 = K_x / m \quad (4)$$

$$\omega_{\theta}^2 = K_{\theta} / mr^2 \quad (5)$$

Where K_x and K_{θ} are the system's translational (along X) and rotational (around θ) stiffness for the resulting two degrees-of freedom system, and e is the static eccentricity of this system, expressed by:

$$K_x = \sum_i K_{ix} \quad (6)$$

$$K_{\theta} = \sum_i K_{ix} y_i^2 + \sum_j K_{jy} x_j^2 \quad (7)$$

$$e = \frac{1}{K_x} \sum_i y_i K_{ix} \quad (8)$$

The mass of the floor is m , its radius of gyration r , v_x and \ddot{v}_x are the translation displacement and acceleration of the centre of mass in direction x , v_{θ} and \ddot{v}_{θ} are the rotational displacement and acceleration of the floor around a vertical axis, \ddot{v}_{gx} is the ground acceleration in direction x , y_i and x_j are the distances from elements i and j to the centre of mass, and K_{ix} and K_{jy} are the translational stiffness of elements i and j in the x and y directions respectively. The translational and torsional uncoupled frequencies, ω_x and ω_{θ} , the corresponding uncoupled periods, T_x and T_{θ} and the ratio of those uncoupled frequencies Ω , are defined in Eqs. (3) to (5). The actual periods of vibration of these structures would be obtained from the eigensolution of Eq. (2). The torsional stiffness of individual lateral load resisting elements is neglected.

Obviously, from Eq. (2), in the linear elastic domain, all structures sharing the same (e/r) , ω_x and Ω will have the same response v_x and rv_{θ} at their centre of mass. As will be demonstrated later, this does not hold in the non-linear inelastic domain.

Also, similar equations of motion could have been derived around any other reference point, the centre of rigidity being another logical choice of coordinates origin is often selected. Nonetheless, transformation functions can easily be developed, if necessary, to relate the structure's descriptive parameters and response around those various centres of reference.

BUILDING CODES INDIRECT INFLUENCE ON TORSIONAL REDUNDANCY

It is noteworthy that many current building codes indirectly promote the reduction of plan redundancy. This is illustrated in the short example following.

In an apparently symmetric building, the accidental eccentricity provision mandated by the equivalent static seismic lateral force design method of most building codes provides a minimum design eccentricity which is thought to account for uncertainties in mechanical properties, mass distribution, and ground motion. This accidental eccentricity is usually set by different codes to a small percentage of the maximum plan dimension.

For the monosymmetric structures studied herein, should only two lateral load resisting elements be present in the principal direction (Figure 1a), an accidental eccentricity of 5% of the maximum plan dimensions will increase the design forces in each element by 10%. Consequently, the design translational and torsional strength will also increase by 10%. If, instead, four equally spaced elements with equal stiffness are now considered (Figure 1b), the same accidental eccentricity requirements will

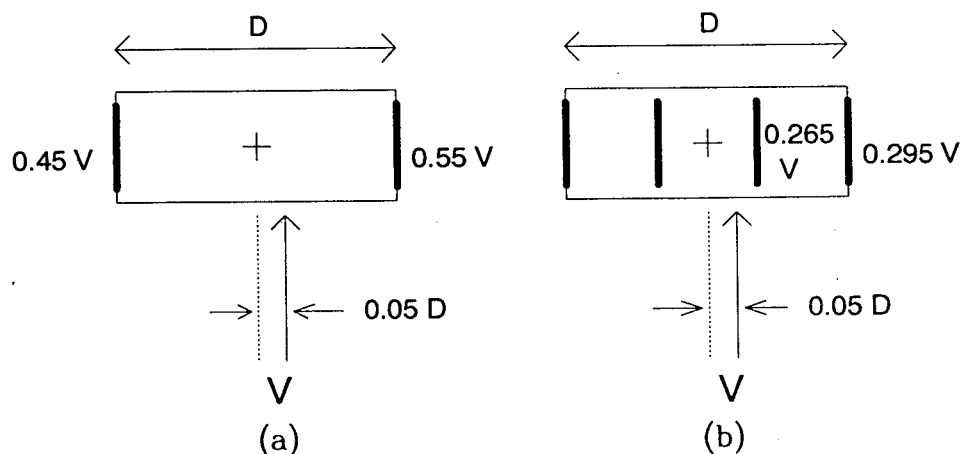


Figure 1. Monosymmetric structures with (a) two LLRSEs and (b) four LLRSEs, for the study of building codes' indirect influence on torsional redundancy - Plan views.

increase the design forces by 18% for the edge elements and by 6% for the inside elements. The resulting design translational strength is thereby increased 12% and the net torsional strength is increased 17%, and, therefore, the more redundant structure is only achieved at a premium in material and labour. Consequently, strict adherence to building codes' seismic provisions could make the two element system a more economical design alternative, which is apparently discordant with earthquake engineering's traditional wisdom that redundancy improves the ultimate seismic resistance of structures.

In light of this least cost observation, the proposed two LLRSE monosymmetric model shown in Figure 2 appears appropriate. In this model, all floor diaphragms

INITIALLY ECCENTRIC, TWO - ELEMENT MODEL

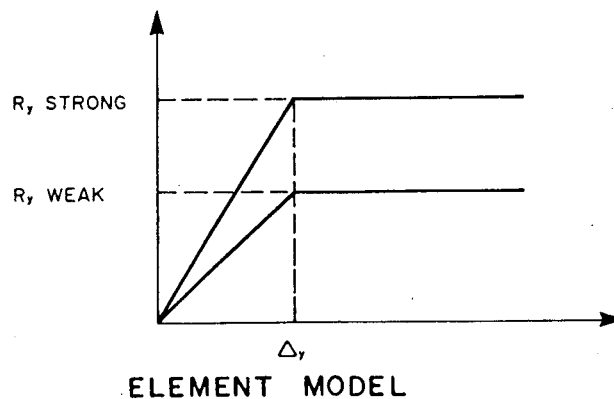
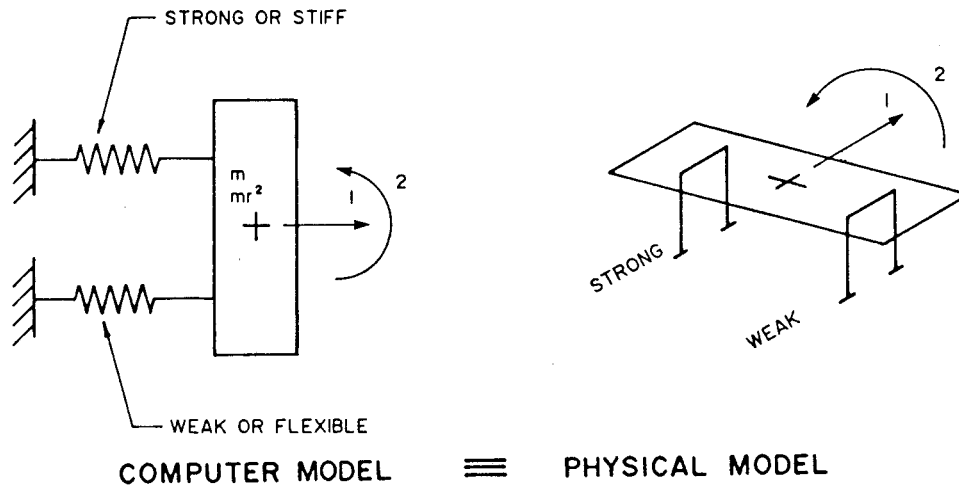


Figure 2. Proposed simple general purpose model for the study of inelastic torsional coupling.

are assumed to be infinitely rigid in their own plane, elements in the orthogonal direction are ignored for the sake of simplicity (i.e. $\sum K_{jy} x_j^2 = 0$) and lateral load resisting elements are assumed to be equidistant from the centre of mass. Nonetheless, this simple general purpose model of minimal complexity must be able to capture the essence of the particular behavioral features of seismic inelastic torsional coupling in order to be acceptable.

CASE COMPARISON OF STATIC, DYNAMIC ELASTIC AND DYNAMIC INELASTIC RESPONSE

In an ideal design process, where the engineer has unrestrained freedom on the structural layout and dimensioning, the LLRSEs can be proportioned such that the centre of resistance will coincide with the centre of mass (unless the centre of mass is not contained between the resisting elements, as would be the case for a building with a single eccentric core). In the special case where only two structural elements are provided for the lateral resistance system in the x direction, the resulting structure is statically determinate, and, in consequence, the lateral shear force must be distributed to the elements solely by the laws of equilibrium. In this case, a static lateral force applied at the centre of mass will be distributed to the lateral load resisting elements by geometric relations, and independently of the LLRSEs stiffness. Unfortunately, this may not always be possible, either due to imposed architectural constraints or to the detrimental structural consequences of added non-structural components ignored during conception, like for example, the addition of non-structural masonry infill in a steel frame structure.

Assessments of the consequence of a larger than anticipated stiffness can be misleading if performed by traditional linear elastic analysis methods. Evaluation of the changed condition by static analysis, as shown in Figure 3, reveals that the displacements, while dramatically different than for the previous symmetric state, are now of equal or lesser magnitude, indicating that the design remains safe. The fallacy of this perception can be exposed by the simple example following where linear elastic and non-linear inelastic analyses were conducted for the same structure having two LLRSEs.

For these analyses, the N-S component of the 1940 El Centro earthquake record is scaled such that the symmetric two-element system reaches a ductility of exactly four from an inelastic step-by-step dynamic analysis. Then, the stiffness and strength of one of the elements is increased by 50% as on Figure 3. Elements are modeled

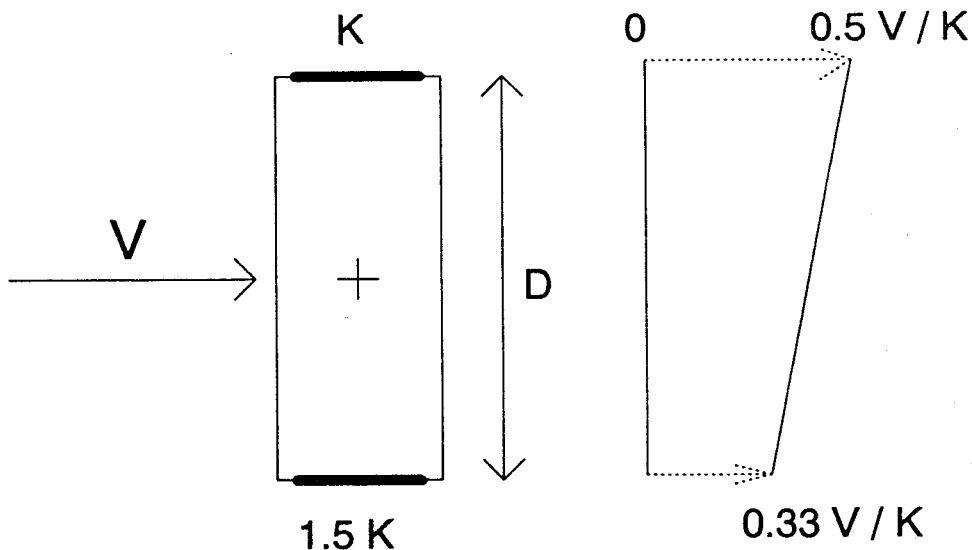


Figure 3. Effect of changed stiffness conditions using linear elastic stiffness analysis.

as bi-linear hysteretic with 0.5% strain hardening. The results for both the elastic and inelastic analyses are presented in Figure 4. These results demonstrate the large amplification of edge displacement produced by the inelastic torsional coupling of the structure.

The large corresponding ductility demand on the weaker element obtained can not be predicted from either the static analysis or the elastic dynamic step-by-step analysis. This increase in ductility demand can be explained by examining the instantaneous state of the equation of motion.

INELASTIC STATUS OF EQUATIONS OF MOTION

During response to an earthquake excitation, the initiation of yielding in one of the LLRSEs will affect the instantaneous properties of the physical system, such that the equations of motion for the monosymmetric structure can be re-written as:

$$\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{bmatrix} \ddot{v}_x \\ \ddot{v}_\theta \end{bmatrix} + \begin{bmatrix} K'_x & K'_x e' \\ K'_x e' & K'_\theta \end{bmatrix} \begin{bmatrix} v_x \\ v_\theta \end{bmatrix} = \begin{bmatrix} -m\ddot{v}_{gx} \\ 0 \end{bmatrix} \quad (9)$$

where all variables remain as previously defined with the exception that the primes (') are used to represent instantaneous properties. It is noteworthy that the stiffness

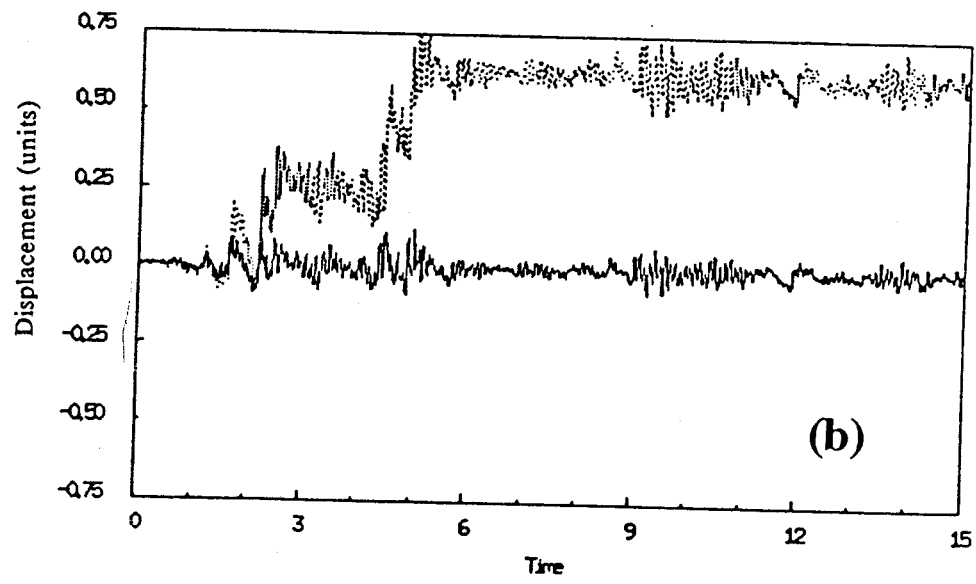
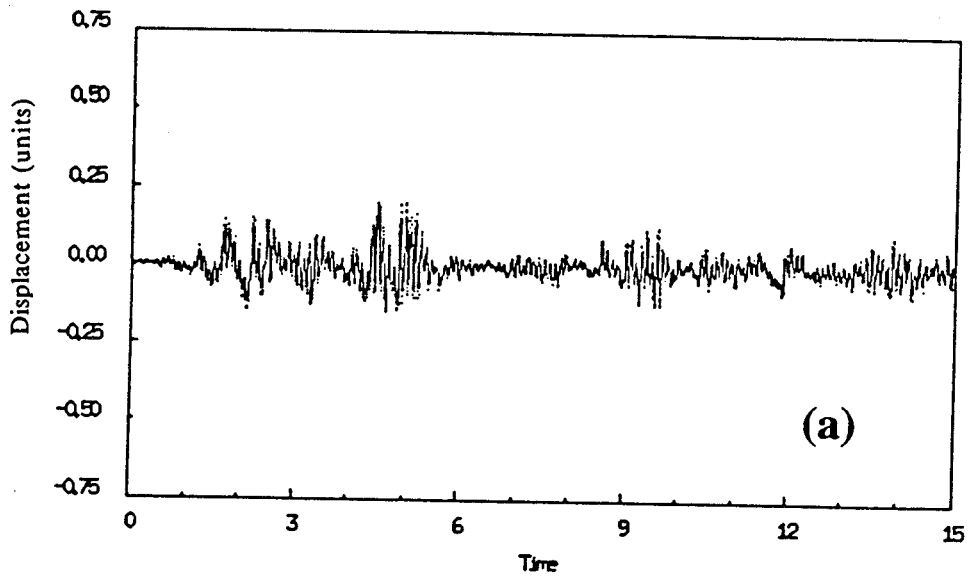


Figure 4. (a) Elastic and (b) Inelastic time history analyses of a two LLRSE structure with $T_x = 0.1$ seconds, $\mu_{SDOF} = 4$, $\Omega = 1.6$, and element stiffnesses of k (dotted line) and $1.5 k$ (solid line); Yield displacement is 0.12 units.

matrix is equivalent to a tangent stiffness in which not only the translational and rotational stiffnesses (K_x' and K_θ') are modified by the initiation of yielding, but also the value of the static eccentricity of the system (e'). Damping, if introduced in this instantaneous equation of motion, would remain unaffected if selected of a Rayleigh type, i.e., as a function of the mass and initial stiffness matrices, both unchanged as well.

Re-arranging the equations, as per the previous section, leads to instantaneous values of Ω' , ω_x' and (e'/r). Obviously, the aforementioned conditions necessary to obtain equivalent elastic response v_x and rv_θ at the centre of mass are not sufficient to ensure the same match in the inelastic domain. For two structures to share identical inelastic response, their tangent stiffness properties must match throughout a given earthquake excitation. This additional very restrictive condition makes the equivalence of complex structures with simpler models, or structures with different types of LLRSE models, virtually impossible, hence the incentive to select a simple general purpose model which will lead to conservative assessments of the seismic inelastic response of torsionally coupled structures. Toward that goal, selecting a two LLRSE system will ensure very large reductions in K_x' and K_θ' , coupled with large increases in e' , when yielding occurs in one of the LLRSE. Depending on its inelastic hysteretic model stiffness characteristics, such a structure would nonetheless recover its original properties when both LLRSE are yielding simultaneously.

Thus, a minimal structural system with two LLRSE can apparently be useful in acquiring a fundamental understanding of how various parameters are affecting the inelastic torsional response. Again, conservatively, LLRSE oriented perpendicularly to the unidirectional earthquake excitation can be neglected; it is the authors' experience that those are not always located favourably to improve the torsional resistance.

It is noteworthy that for linear elastic symmetric structures ($e = 0$), the equations of motion become uncoupled, and the torsional response would typically not be calculated due to the absence of torsional excitation in the above formulation. This uncoupling would persist until dissymmetric yielding of the LLRSEs occurs, at which point a transient state of torsional coupling is established. This special case has been discussed elsewhere [4].

GEOMETRIC EQUIVALENCE OF NON-LINEAR INELASTIC STRUCTURES

In spite of the very restrictive aforementioned conditions necessary to produce equivalent torsionally coupled structures through their inelastic response, some simple geometric constraints can still be established to define equivalent structures of similar

plan layout but of different scale. For example, equivalent structures each with two LLRSE of equal strength are shown in Figure 5.

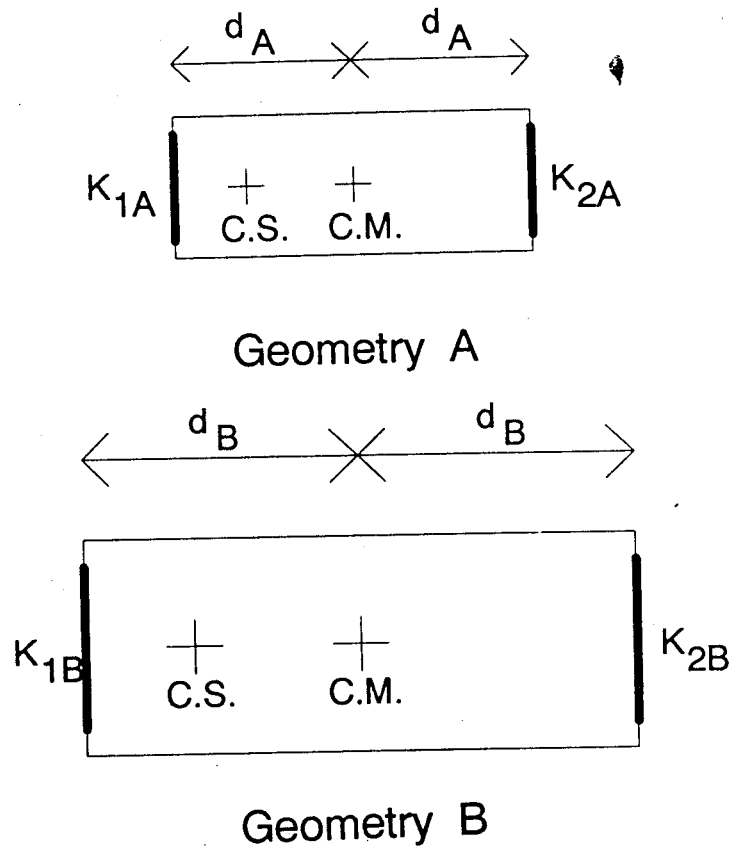


Figure 5. Geometrically equivalent non-linear inelastic structures

Assuming that LLRSEs have a hysteretic bi-linear model with no strain hardening (elastic-perfectly plastic), for those two structure of different geometry to share the same ω_x , ω_x' , ω_θ , ω_θ' , (e/r) and (e'/r) , at all times, it implies that:

$$\omega_{xA}^2 = \frac{K_{1A} + K_{2A}}{M_A} = \omega_{xB}^2 = \frac{K_{1B} + K_{2B}}{M_B} \quad (10)$$

$$\omega_{xA}^2 = K_{1A} / M_A = \omega_{xB}^2 = K_{1B} / M_B \quad (11)$$

Thus

$$K_{1A} / M_A = K_{1B} / M_B \quad (12)$$

and

$$K_{2A} / M_A = K_{2B} / M_B \quad (13)$$

Also $\omega_{\theta A} = \omega_{\theta B}$ implies

$$[(K_{1B} + K_{2B}) / M_B] [d_B / r_B]^2 = [(K_{1A} + K_{2A}) / M_A] [d_A / r_A]^2 \quad (14)$$

and therefore

$$d_A / r_A = d_B / r_B = D = \text{constant} \quad (15)$$

where the subscripts A and B correspond to geometry A and B on Figure 5, and d is the distance from a LLRSE to the centre of mass (equidistant elements in this example). Thus, two LLRSE systems with the same ω_x , Ω , (e/r) , and same ratio $(d/r) = D$ can be of different geometry and still have the same element response, i.e., identical element time histories and ductility demands can be obtained from a wider structure with a larger radius of gyration as long as the geometric ratio (d/r) is preserved. This ratio fixes the proportional geometric configuration of a structure as scaled by its radius of gyration. It is more restrictive than the (e/r) ratio. Numerical examples of this special case are presented elsewhere [4].

Should both LLRSE be of same stiffness and strength, but not equidistant to the centre of mass, different values of $(d_i/r) = D_i$ will exist, i being index for i -th element. Whereas it is possible to have structures with various types of eccentricity sharing the same ω_x , Ω and (e/r) values, if their D_i 's are different, their inelastic response will not be the same. Since (e'/r) values will not agree either in this case, the same conclusion could have been reached without considering the (d_i/r) ratios. Nonetheless, D_i values being calculated from initial properties only, are simpler to evaluate than all possible values of e' .

For multi-LLRSE systems, the geometric non-dimensional form derived above remains applicable, and again, all constants $(d_i/r) = D_i$ must be the same between two structures for their response to be similar. From those observations, all structures

with the same D_i 's, ω_x , Ω , and (e/r) , and implicitly sharing damping, earthquake excitations, element hysteretic models and model inter-relation between different elements, will have the same element inelastic response.

For a structure with two LLRSE equidistant to the centre of mass and given ω_x , Ω , and (e/r) , only one geometric pattern D is possible, but for a multi-LLRSE system, there are many possible values for D_i 's, even if the elements are equidistant to the centre of mass in pairs. For example, in that last case, in a plan layout with four LLRSEs, structures with very stiff inside LLRSEs and very weak outside LLRSEs can be elastically equivalent to more balanced structures while inelastically dissimilar. Similar relationships could be developed for an infinity of plan layouts without benefiting much the design process. While it is important to realize the additional constraints necessary for equivalencing inelastic structures, it is instead more productive to demonstrate that findings on the behaviour of relatively simple structures can be safely generalized to more complex structures. This approach has been adopted elsewhere [7] using the simple model formulated here and in the companion paper [13].

CONSIDERATION OF ROTATIONAL INERTIA

The influence of the rotational inertia on the inelastic torsional response is best described when considering initially symmetric structures, i.e., structures where the normalized eccentricity (e/r) is zero. In this case, it is the Ω factor that reflects the significance of r , the radius of gyration of the floor plan, here taken around the centre of mass. This dimensional parameter, a physical representation of the mass-distribution around the centre of mass, is related to the selected floor plan configuration; although this property in practice is mostly inalterable by the engineer, the effects of variation in radius of gyration on the response of the structures at hand are of interest.

For a given floor translational mass m , a reduction in r will reduce the mass moment of inertia, mr^2 , and will simultaneously produce an increase in Ω , as

$$\Omega = \omega_\theta / \omega_x = (K_\theta m) / (mr^2 K_x) = K_\theta / (K_x r^2) \quad (16)$$

An initially symmetric structure will respond in a purely translational manner until yielding of one of the LLRSE, at which time the mass moment of inertia of the floor plan will provide an effective inertia (or resistance) against the introduction of torsional

movement during that interval when the instantaneous physical properties of the structure provide a temporary mismatch between the centre of stiffness and centre of mass. If the mass moment of inertia is very small (large Ω), it is easy to produce a rotational movement as there is little resistance to the induction of angular motion. In the opposite fashion, if the mass moment of inertia is large (small Ω), considerable inertial resistance to angular motion exists and very little of it may develop.

This phenomenon is illustrated in Figures 6(a) to 6(g). In that case, a number of initially symmetric structures of uncoupled translational frequency $T_x = 0.1$ seconds, having one LLRSE yielding at 80% of the reference yield strength F_y , and the other at F_y , have been analyzed under the N-S component of the 1940 El Centro earthquake record scaled such as to produce a ductility demand of 4 on the equivalent SDOF. The bilinear hysteretic element model with 0.5% strain hardening was selected in this study, and Rayleigh-type damping was set to be 2% at each of the fundamental periods of the selected structures. Time history translational, Figure 6(b)-6(d), and rotational, Figures 6(e)-6(g), responses are presented for structures having Ω values of 0.4, 1.0 and 1.6, along with the time history response of the equivalent SDOF, Figure 6(a), for comparison purposes. Note that plots of angular motion time histories are at different amplitude scales.

Not only is it obvious from this Figure that the structures with lower radii of gyration (thus lower mr^2 and higher Ω) are excited into a larger angular motion, but the transient nature of the inelastic torsional response of initially symmetric structures is well illustrated. Torsional motions are rapidly damped out once the structure returns to a state of balanced yielding or non-yielding, and the torsional frequency and damping characteristics of the structure greatly influence the rapidity with which those movements are attenuated.

LIMITATIONS OF THE PROPOSED MODEL

In most cases, the simple general purpose two LLRSE model proposed will provide a conservative estimate of the response, partly because of the absence of plan redundancy and partly because LLRSEs in the direction perpendicular to the earthquake excitation have been neglected. While a normalization procedure presented elsewhere [4,7] addresses this first concern satisfactorily, it remains that perpendicular LLRSE, when properly located, can indeed contribute positively to the torsional resistance. An assessment of this favourable contribution would require further research.

Also, under very large earthquake excitations, both LLRSE of the simple proposed structure can yield almost simultaneously, with only a small transition time during

which the instantaneous coupling properties hold their most critical values; this particular effect is further studied in the companion paper [13].

It is noteworthy that the proposed simple general purpose model has been used by the authors in a comprehensive investigation of the behaviour of torsionally coupled structures [4,7] conducted according to a special normalization procedure. Although the details of that study are beyond the scope of this paper, extensions of that parametric study to monosymmetric structures having a large number of LLRSE in plan, or to more complex LLRSE hysteretic models, such as the elastic buckling brace element model and the physical brace element model, have confirmed that a structure with two LLRSE can be used reliably to predict the inelastic torsionally coupled response of structures sharing identical hysteretic element model.

Finally, the proposed model being of single story, may be unsuitable for the study of complex multistory structures without some modifications, especially in the case where ductility demand tends to concentrate on a few weaker stories; Analogous limitations also exist for ideal-symmetric structures. Additional research is needed.

CONCLUSION

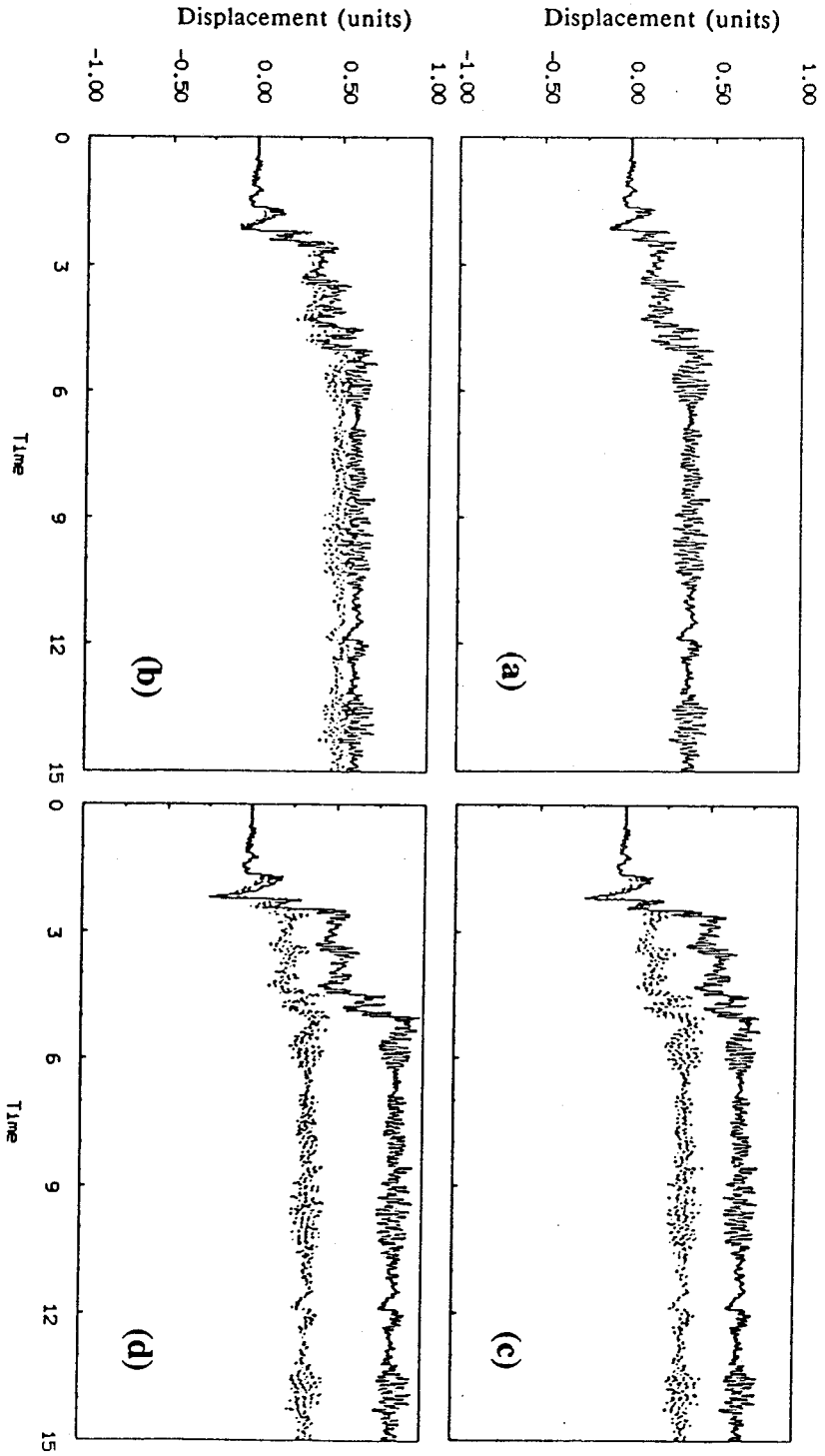
A simple general purpose model has been proposed for the study of seismic inelastic torsional coupling. The building code indirect influence in lessening torsional redundancy, as well as the necessity to satisfactorily capture the non-linear inelastic behaviour of torsionally coupled structures, have dictated the minimal characteristics required of this model. A simple model to conservatively estimate structural response is an acceptable alternative considering the stringent and impractical geometric and parametric requirements required if identical structural response is to be ensured in the non-linear inelastic range. The proposed two LLRSE structure is used to illustrate the major influence of the rotational inertia on the inelastic torsional response. The influence of other modelling considerations will be addressed in a companion paper [13].

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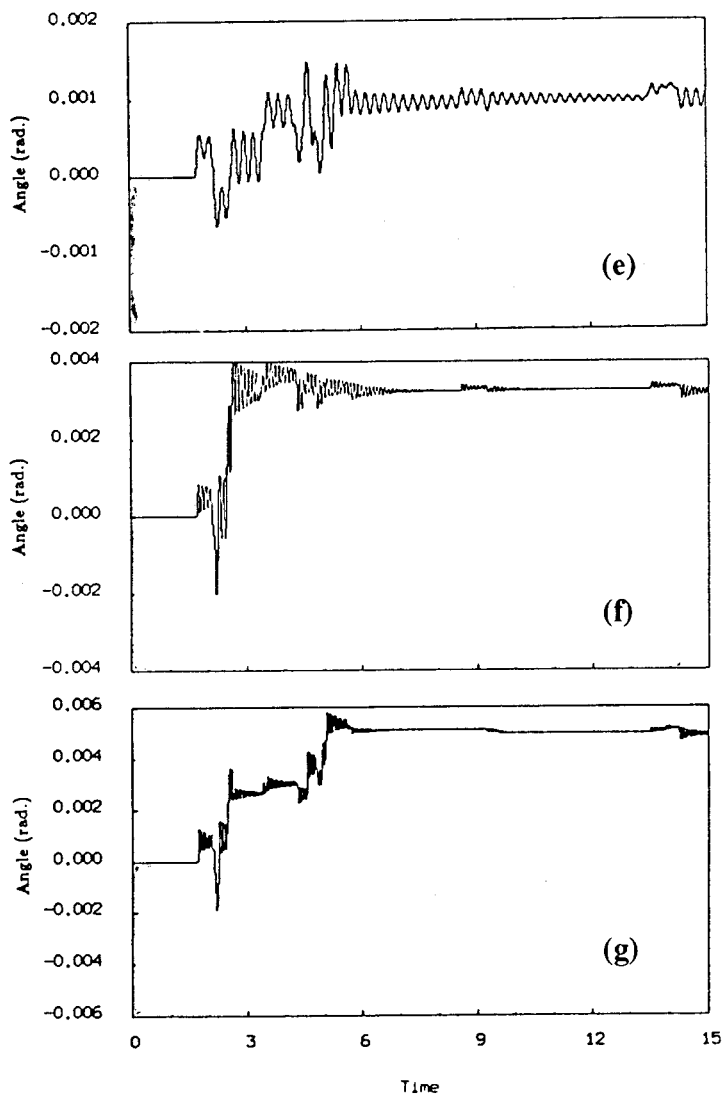


Figure 6. Translational (a-d) and rotational (e-g) time history inelastic responses for (a) a SDOF system, and initially symmetric systems having LLRSE's strength combination of 0.8 and 1.0 F_y , and $\Omega = 0.4$ (b and e), $\Omega = 1.0$ (c and f), $\Omega = 1.6$ (d and g). For all systems, $T_x = 0.1$ seconds, and $\mu_{SDOF} = 4$; Yield displacements are 0.096 units for strong element, and 0.12 units for weak element.

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