



QUANTIFICATION OF SEISMIC RESILIENCE

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ABSTRACT

The concept of seismic resilience needs a unified terminology and a common reference frame for quantitative evaluation. The evaluation can be based on non-dimensional analytical functions related to variations of losses within a specified “recovery period”. The resilience must refer to both direct and indirect losses. The path to recovery usually depends on available resources and may take different shapes which can be estimated by proper recovery functions. The loss functions have major uncertainties due to the uncertain nature of the earthquake and structural behavior as well as due to uncertain description of functionality limits. Therefore losses can be described as functions of fragility of systems’ components. These fragility functions can be determined through the use of multidimensional performance limit thresholds, which allow considering simultaneously different mechanical-physical variables such as forces, velocities, displacements and accelerations along with other functional limits. A procedure which defines resilience as function of losses and loss recovery based on multidimensional system fragility is formulated and an example is presented for a typical California hospital considering direct and indirect losses in its physical system and in the population served by the system.

Introduction

Recent events have shown that essential facilities, such as hospitals, are vulnerable to extreme events such as earthquakes or other disasters. In order to reduce the losses in these essential facilities the emphasis has shifted to mitigations and preventive actions before the extreme event happens. Mitigation actions can reduce the vulnerability of such facilities. However, in case of insufficient mitigation actions, or in case that the events exceed expectations, damage occurs and a recovery process is necessary in order to continue to have a functional community. Seismic resilience, as defined in this paper, describes the loss and loss recovery required to maintain the function of the system with minimal disruption. While

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mitigation may emphasize use of technologies and implementation of policies to reduce losses, the resilience considers also the recovery process including behavior of individuals and organizations in the post disaster phase. A wealth of information is available on specific actions, policies or scenarios that can be adopted to reduce the direct and indirect economic losses attributable to earthquakes, but there is little information on procedures on how to quantify these actions and policies. Seismic resilience can compare losses and different pre and post event measures verifying if these strategies and actions can reduce or eliminate disruptions in presence of earthquake events.

Bruneau et al (2003) offered a very broad definition of resilience to cover all actions that reduces losses from hazard, including mitigation and more rapid recovery. The above paper defined the community earthquake resilience as “the ability of social units (e.g. organizations, communities) to mitigate hazards, contain the effects of disasters when they occur, and carry out recovery activities in ways to minimize social disruption and mitigate the effectors of further earthquakes”. The authors suggested that resilience can be conceptualized along four interrelated dimensions: technical, organizational, social and economic (TOSE). The first two components are more related to the resilience of critical physical systems such as water systems and hospitals. The last two components are more related to the affected community. The above paper defined a fundamental framework for evaluating community resilience without any actual quantification and implementation. Chang et al. (2004) proposed a series of quantitative measures of resilience and demonstrated them in a case study of an actual community, the seismic mitigation of Memphis water system. This paper attempts to provide a quantitative definition of resilience through the use of an analytical function which allows identification of quantitative measures of resilience.

Definitions and formulations

To establish a common frame of reference, the fundamental concepts of resilience are analyzed, a unified terminology is proposed and an application to health care facilities is presented:

Definition 1: *Resilience is defined as a normalized function indicating capability to sustain a level of functionality or performance for a given building, bridge, lifeline, networks or community over a period of time T_{LC} (life cycle, life span etc. etc) including the recovery period after damage in an extreme event.*

The time T_{LC} includes the building recovery time T_{RE} and the business interruption time that is usually smaller compared to the other one.

Definition 2: *The recovery time T_{RE} is the time necessary to restore the functionality of a community or a critical infrastructure system (water supply, electric power, hospital etc.) to a desired level below, same or better than the original, allowing proper operation of the system*

The recovery time $T_{RE}(I,location)$ is a random variable with high uncertainties. It typically depends on the earthquake intensities the type of area considered, the availability of resources such as capital, materials and labor following major seismic event. For these reasons this is the most difficult quantity to predict in the resilience function. Porter et al. (2001) attempted to make distinction between downtime and repair time and he tries to quantify the latter. In his work he

combines damage states with repair duration, and probability distributions to estimate assembly repair durations.

Definition 3: *Disaster resilient community is a community that can withstand an extreme event, natural or man made event, with a tolerable level of losses and can take mitigation action consistent with achieving that level of protection.* (Mileti, 1999, p5)

Another useful concept to define is “a disaster resilient community”. This Resilience is defined graphically as the normalized shaded area underneath the function shown in Figure 1 where in the x-axis there is the time range considered to calculate resilience while in the Y-axis there is the **functionality** $Q(t)$ of the system measured as a non dimensional quantity. Analytically $Q(t)$ is a non stationary stochastic process and each ensemble it is a piecewise continuous function as the one shown in Figure 1. Mathematically the resilience can be expressed by equation (1):

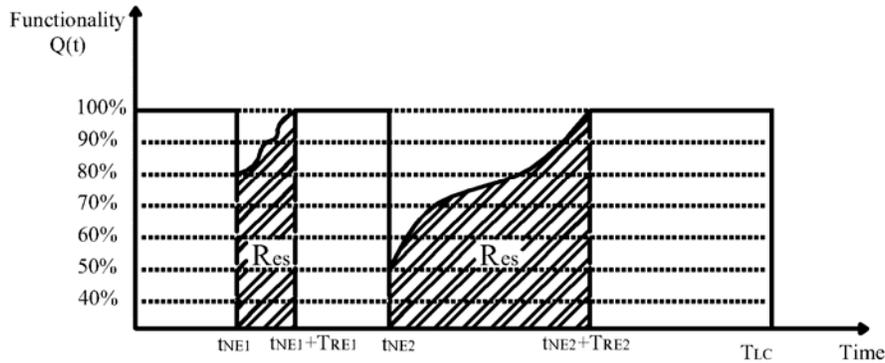


Figure 1. Uncoupled Resilience

$$\bar{R} = \frac{1}{N_I} \sum_{I=1}^{N_I} \left\{ \frac{1}{N_E} \cdot \sum_{E=1}^{N_E} \frac{1}{T_{RE}} \cdot \int_{t_{0E}}^{t_{0E}+T_{RE}} \left\{ 1 - L(I, T_{RE}) \cdot \begin{bmatrix} H(t-t_{0E}) + \\ -H(t-(t_{0E} + T_{RE})) \end{bmatrix} \cdot \alpha_R \cdot f_{REC}(t, t_{0E}, T_{RE}) \right\} \cdot dt \cdot p_E(0, T_{LC}) \right\} \cdot P(I) \quad (1)$$

Where N_E is the number extreme events expected during the lifespan (or control period) T_{LC} of the system, N_I is the number of different extreme events intensities expected during the lifespan (or control period) T_{LC} of the system; T_{RE} is the recovery time from event E ; t_{0E} is the time of occurrence of event E ; $L(I, T_{RE})$ is the normalized loss function; $f_{REC}(t, t_{0E}, T_{RE})$ is the recovery function; $P(I)$ is the Probability that an event I of given intensities happens in a given time interval T_{LC} ; $p_E(0, T_{LC})$ is the probability that an event happens E times in a given time interval T_{LC} ; α_R is a recovery factor and $H(t_0)$ is the Heaviside step function. In equation 1 there are the loss function $L(I, T_{RE})$, the recovery function $f_{REC}(t, t_{0E}, T_{RE})$ and the fragility function that does not appear explicitly, but it is included in the loss function that will be defined in the following sections.

Loss function

The estimation of losses and in particular the losses associated with extreme events requires first of all some damage descriptors that must be translated into monetary losses and other units like the number of people requiring hospitalization. This type of losses are highly uncertain and they are different for every specific scenario considered, but considering all the possible cases some common parameters that influence the losses can be identified. In fact the

loss function $L(I, T_{RE})$ can be expressed as a function of earthquake intensity I and recovery time T_{RE} (downtime). The losses are divided in two groups: **Structural losses** [L_S] and **Non Structural losses** [L_{NS}]. The non structural losses can be divided in four contributions: (i) Direct economic losses $L_{NS,DE}$ (Contents losses); (ii) Direct Casualties losses $L_{NS,DC}$; (iii) Indirect economic losses $L_{NS,IE}$ (Business interruption losses); (iv) Indirect Casualties losses $L_{NS,IC}$. The physical structural losses are expressed as ratio of building repair and replacement costs and it is expressed as:

$$L_S(I) = \sum_{j=1}^5 \left[\frac{C_{S,j}}{I_S} \cdot \prod_{i=1}^{t_i} \frac{(1+\delta_i)}{(1+r_i)} \right] \cdot P_j(R_j \geq d_{S,j}/I) \quad (2)$$

Where P_j is the probability of exceeding a performance limit state j conditional an extreme event of intensity I happens (the fragility function); $C_{S,j}$ are the building repair costs associate to a j damage state; I_S are the replacement building costs; r is the discount annual rate; t_i is the time range in years between the initial investments and the time occurrence of the extreme event; δ_i is the depreciation annual rate. The equation (2) assumes that the initial value of the building is affected by the discount rate, but it also decreases through the time according to the depreciation rate δ_i that is variable through the time. A similar formulation is used for non structural direct economic losses $L_{NS,DE,k}(I)$ where an identical term to equation (2) is obtained for every non structural component k used inside the affected system. This term can be much higher than the structural losses in essential facilities like hospitals or research laboratory. The total non structural losses are obtained with a weight average expressed as:

$$L_{NS,DE}(I) = \left(\sum_{k=1}^{N_{ns}} w_k \cdot L_{NS,k}(I) \right) / N_{ns} \quad (3)$$

Where N_{ns} is the number of non structural components inside the buildings; w_k is the weight factor associated to every non structural component inside the building. Non structural components such as the ceilings, elevators, mechanical and electrical equipments, piping, partitions, glasses etc. are considered. The **direct casualties losses** L_{DC} are expressed as ratio between the number of person injured N_{in} over the total number of occupants N_{tot} :

$$L_{NS,DC}(I) = \frac{N_{in}}{N_{tot}} \quad (4)$$

The number of injured people N_{in} in fatal and nonfatal manner depends on multiple factors like, the *time of the day of the earthquake*, the *age of the population* and the *number and proximity of available hospitals*. The time of the day when the earthquake happens determines the number of people exposed to injury, so the probability of having a large number of people injured varies during the day. The age of population is also very important as indicated by Peek-Asa et al. (1998) who found that during the 1994 Northridge earthquake the predominant number of people injured were elder people. In fact 31.2% of fatalities and 75.8% of hospitalized were people over the age of 65. Even though these data are related only to Northridge earthquake it is possible to conclude that older people move less quickly to evacuate damaged buildings and to avoid falling objects, and they are more vulnerable to traumatic injuries. The number and the proximity of available hospitals determines the number of serious injured that prove fatal. In order to make risk estimates through resilience it is necessary to make empirical predictions of casualties that must be based on structural damage or ground motion. Table 1 reports the four casualties' severity levels of HAZUS as function of ground motion intensities. Peek-Asa found

that for 1994 Northridge earthquake the ground motion levels (MMI) were better predictor of casualties' rates than building damage because the number of people injured in location where structural damage occurred is only a small amount of the total number of people injured; in fact many injuries have other causes than structural damage or collapse. For example minor injuries results from being struck by objects and from falling. So the MMI allows a ruff estimation of casualties rates based on the population that is subjected to various intensities levels. It is important to recognize that in the table are not taken in account the type of constructions and the severity of injuries. However this ratio in the table is only representative of the injuries treatment at a hospital because minor injuries that does not requires hospitalization can be very numerous.

Table 1 Casualty rate as a function of MMI
[from (Peek-Asa et al. 2000)]

MMI Level	Casualties Rate per 100,000 Population
<VI	0.03
VI	0.16
VII	2.1
VIII	5.1
IX	44

The **indirect economic losses** $L_{NS,IE}(I, T_{RE})$ are time dependent compared to all the previous losses considered. They are the most difficult post-earthquake losses to quantify, because of the different forms they can assume, so at the moment there is no equation for this term. They can be generated by *business interruptions, relocation expenses, rental income losses, etc.* The losses of revenue either permanent or temporary can be caused by damage to structures and contents and this is important for manufacturing and retail facilities, but also for lifeline because damage to facilities can mean less ability to deliver resources and services like electricity, water, natural gas, transportations. For example structural damage like collapse of a span of a bridge generates direct losses, but also indirect losses due to the loss of revenue from bridge tolls and they can be significant. In other cases even if structural damage and loss of contents are minimal, they may be some indirect losses due to the disruption of some services such as water and power and the losses can be more significant than direct losses. So the losses due to business interruption should be modeled considering both the amount of structural losses L_S , and the time necessary to repair the structure T_{RE} . These two quantities are not independent, but are related because the time of recovery T_{RE} increases when the amount of structural damage $L_S(I)$ increases. So seismic resilience R_{es} is able to estimate the recovery of losses during the period of repair T_{RE} , using different type of recovery functions that fit to the type of area considered and to the intensities of the earthquake. Also the indirect casualties losses L_{IC} belong to this section. For the case of a hospital this can be expressed as ratio between the number of persons injured N_{in} outside the hospital over the total population around N_{tot} :

$$L_{NS,IC}(I) = \frac{N_{in}}{N_{tot}} \quad (5)$$

Analytically the total direct losses $L_{NS,D}$ and the total indirect losses $L_{NS,I}$ and the total non structural losses L_{NS} are expressed as:

$$L_{NS,D} = L_{NS,DE}^{\alpha_{DE}} \cdot (1 + L_{NS,DC}^{\alpha_{DC}}); \quad L_{NS,I} = L_{NS,IE}^{\alpha_{IE}} \cdot (1 + L_{NS,IC}^{\alpha_{IC}}); \quad L_{NS} = L_{NS,D} \cdot (1 + L_{NS,I}^{\alpha_I}) \quad (6)$$

Where α_{DE} is the weighting factor related to construction losses in economic terms; α_{IE} is the weighting factor related to business interruption, relocation expenses, rental income losses, etc. etc.; α_{IE} , α_{IC} are the weighting factors related to occupancy (es. School, critical facilities, density of population); α_I is the weighting factor related to indirect losses (i.e. importance of the facilities for the community, influence of the facilities versus other system, etc). These weighting factors are determined by socio-political criteria (cost benefit analysis, emergency functions, social factor, etc.). This subject is usually covered jointly by engineers, economists, and social scientists. Finally L_S and L_{NS} are summed together to obtain the total loss function $L(I, T_{RE})$.

Recovery functions

Different kind of recovery functions can be chosen depending on system and society response. Three recovery functions are shown in equation (7): linear, exponential and trigonometric:

$$\begin{aligned} f_{rec}(t, T_{RE}) &= \left(1 - \frac{t - t_{0E}}{T_{RE}}\right) \\ f_{rec}(t) &= \exp\left[-(t - t_{0E}) * (\ln 200) / T_{RE}\right] \\ f_{rec}(t) &= 0.5 * \left\{1 + \cos\left[\pi(t - t_{0E}) / T_{RE}\right]\right\} \end{aligned} \quad (7)$$

The simplest form is a linear recovery function that is generally used when there is no information regarding the society response. The exponential recovery function is used where the society response to an extreme event is very fast driven by an initial inflow of resources, but then the rapidity of recovery decreases. Trigonometric recovery function is used when the society response to a drastic event is very slow initially. This could be due to lack of organization and/or resources. As soon the community organizes himself, thanks for example to the help of other communities, then the recovery system starts operating and the rapidity of recovery increases.

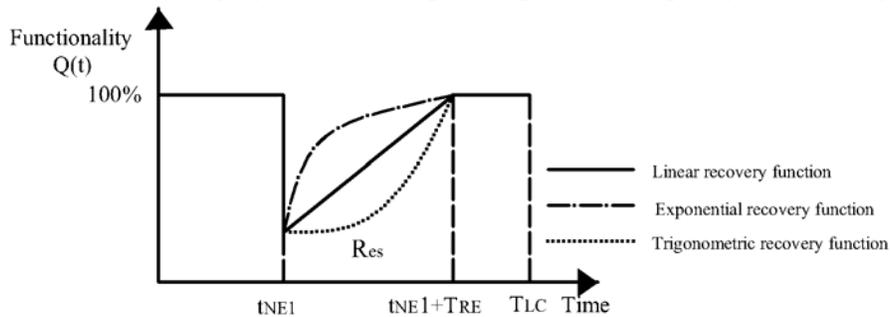


Figure 2. Recovery functions.

Fragility function

The calculation of seismic resilience imply the determination of fragility, or reliability of a given system, which can be a whole building, a non structural component, a lifeline system, a

community, etc. Fragility curves are functions that represent the conditional probability that the response of a system subjected to various seismic excitations exceeds a given performance limit state. Theoretically fragility represents the probability that the response $R=[R_1, \dots, R_n]$ of a specific structure (or family of structures) exceeds a given performance threshold $r_{lim}=[r_{lim1}, \dots, r_{limn}]$ associated with a limit state, conditional on earthquake intensity parameter I . This definition in N dimensional form can be expressed by the following equation:

$$Fragility = P\{R_1 \geq r_{lim1} \cup R_2 \geq r_{lim2} \dots \cup R_N \geq r_{limN} / I\} = P\left\{\bigcup_{i=1}^N R_i \geq r_{limi} / I\right\} \quad (8)$$

where R_i is the response parameter related to a certain quantity (deformation, force, velocity, etc.); r_{limi} is the response threshold parameter related to a certain quantity that is correlated with the performance level. The fragility explicitly appears in the expression of the loss function (2) where the normalized value of the losses is multiplied by $P_j(R_j \geq d.s./I)$, the probability of exceeding a given performance level conditional an event of intensity I happens. This value can be obtained by the fragility function knowing the intensity I of the event. The definition of fragility in equation (8) requires implicitly the definition of the performance limit state thresholds r_{limi} that are discussed in the following section.

Multidimensional performance limit state function

The calculation of fragility has been performed using a generalized formula that describes the multidimensional performance limit state threshold (MPLT) and allows considering multiple limit states related to different quantities in the same formulation (Cimellaro et al. 2005). The multidimensional performance limit state function $L(R_1, \dots, R_n)$ for N -dimensional case, when N different types of limit states are considered simultaneously, is the following:

$$L(R_1, \dots, R_n) = \sum_{i=1}^n \left(\frac{R_i}{r_{limi}} \right)^{Ni} - 1 \quad (9)$$

This model can be used to build the fragility curve of a single non structural component, or also to obtain the overall fragility curve of the entire building with non structural components, because it allows to control different response parameters (Force, displacement, velocity, accelerations etc. etc.) in the building and combine together in a unique fragility curve. The different limit states can be modeled as deterministic or random variables and they can be considered either linear, non linear dependent or independent using an opportune choice of the parameters that appear in Eq. (9). In a 3D non-dimensional space when the multidimensional performance threshold considers only three response parameters, equation (9) assumes the shape as shown in Figure 3a.

In the bi-dimensional case the response of the system can be visualized in a space where in the X axis there are the spectral displacements while in the Y -axis there are the pseudo spectral accelerations and in the Z - axis there is the probability that the given value happens. The shape of the response curve of the system in this space is similar to a “bell surface” (Bruneau et al., 2004) while the multidimensional performance threshold MPLT in this space is represented by a cylindrical non linear function that relates acceleration performance threshold A_{LS} to displacement performance threshold D_{LS} (see Fig.3(b)). The probability that the response exceeds a specific performance threshold can be directly calculated from the volume under the surface distribution exceeding the specified limit represented in Fig. 3(b) by a dotted line.

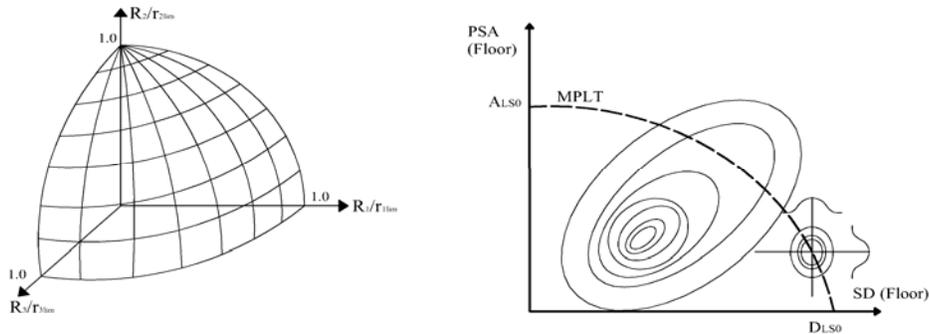


Figure 3. Multidimensional threshold performance limit. (a)3D; (b) 2D.

Case Study: Demonstration Hospital

The methodology described above has been applied to an essential facility in the San Fernando Valley in Southern California. The hospital [W70] was constructed in the early 1970s to meet the seismic requirements of the 1970 Uniform Building Code (ICBO, 1970) (see Yuan, Whittaker et al. 2003). The lateral force resisting system is comprised of four moment-resisting frames in the north-south direction and two perimeter moment-resisting frames in the east-west direction. The computer program IDARC2D (Reinhorn et al. 2004) has been used to perform the non linear time history analysis of the hospital using a bi-dimensional inelastic MDOF model. A series of 100 synthetic near fault ground motions, defined as “MCEER series” (Wanitkorkul, A., Filiatrault, A., 2005) which correspond to different return periods (250, 500, 1000 and 2500 years), has been used to build the fragility curves of the building using the procedure described in Cimellaro et al (2005). The values for the loss estimation have been taken from HAZUS evaluation. The structural losses for this type of building have been taken equal to 0.2%, 1.4%, 7.0% 14.0% of the building replacement costs for the case of slight, moderate, extensive and complete damage, respectively. A discount annual rate of 4% and a depreciation annual rate of 1% have been used. The non structural losses have been taken equal to 1.8%, 8.6%, 32.8% 86% of the building replacement costs for the same damage states. The number of people injured compared to the 4000 people assumed inside the hospital, and the 1000 outside the hospital are for different damage state equal to 0.05%, 0.23% 1.1% 6.02% 75% of the occupants (FEMA2001). Severity of the casualties was not differentiated. Other losses like the relocation costs, rental income losses and the loss of income have been also considered using the procedure described in HAZUS for this type of building. Finally Fig 4 shows the functionality curves related to the four different hazard level considered for different types of recovery functions.

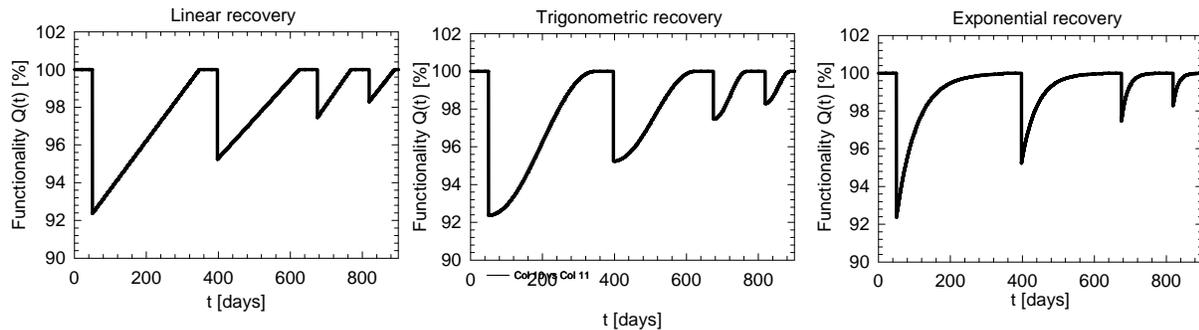


Figure 4. Functionality curves

The values of resilience functions for the four different hazard levels represented by probability of exceedence (PE) in 50 years are reported in figure 5. The resilience of the building is almost constant with the increase of earthquake intensity showing a good behavior of the building. If we compare the functionality values we observe a reduction with the increase of the magnitude as expected due to the increase of the losses and consequentially the effective recovery time.

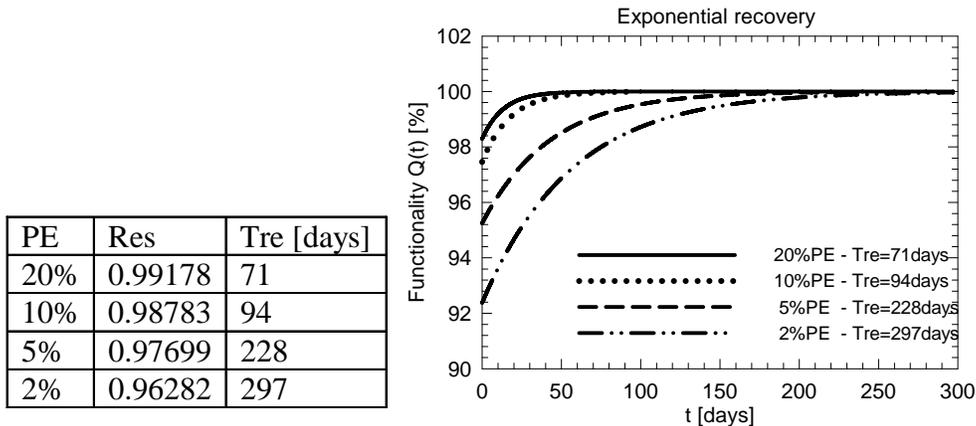


Figure 5. Comparison of functionality curves

Conclusions

The definition of seismic resilience combines information from technical and organizational fields, from seismology and earthquake engineering to social science and economy. So it is clear that many assumptions and interpretations are made during the study of seismic resilience, but the final goal is to integrate the information from these fields in a unique function that reach results that are unbiased by uninformed intuitions or preconceived notions of how large or how small the risk is. The goal of this paper is to provide a quantitative definition of resilience in a rational way through the use of an analytical function that may fit both technical and organizational issues. The fundamental concepts of seismic resilience are analyzed, a common frame of reference is established, a unified terminology is proposed and an application to health care facilities is presented. However, it is important to mention that the assumptions that are made for the case presented are only representative to illustrate the definitions; for other problems users calculating resilience should focus on the assumptions that most influence the problem at hand. Moreover, the formulation is presented for a singular facility; however, the formulation can be easily extended without major changes to a network of distributed facilities, resulting in single functions characterizing the network.

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