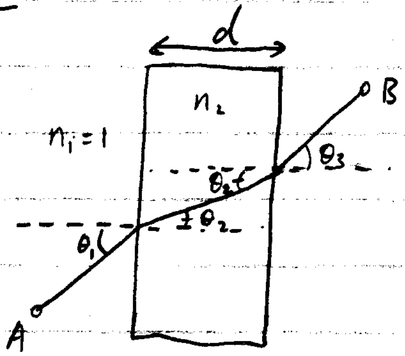
1.2.1

(a)



$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (1)$$

$$n_1 \sin\theta_3 = n_2 \sin\theta_2 \qquad (2)$$

$$\therefore \quad n_1 \sin\theta_1 = n_1 \sin\theta_3$$

$$\sin\theta_1 = \sin\theta_3$$

$$\theta_1 = \theta_3 \qquad \underline{\text{Q.E.D.}}$$

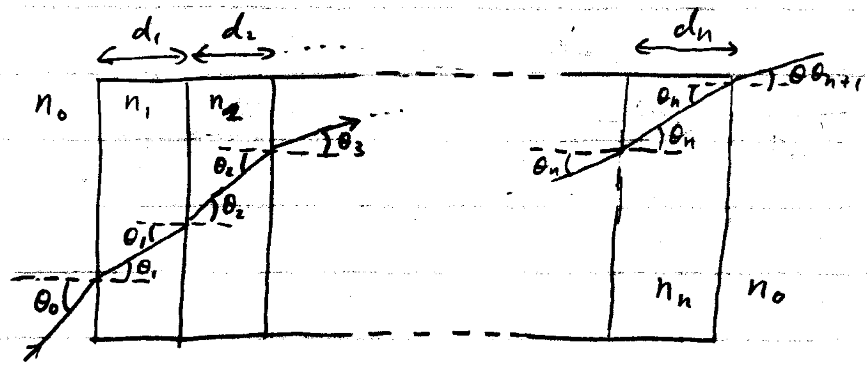
Fermat's Principle:

Light travels along the path of least time.

Looking at other alternatives from $A \to B$

$\Rightarrow$ Snell's law gives minimum time

(b)



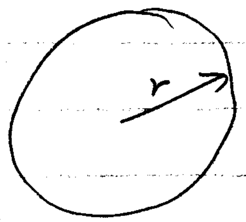
$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_2 \sin\theta_2 = \cdots = n_n \sin\theta_n = n_0 \sin\theta_{n+1}$$

$$\Rightarrow \quad n_0 \sin\theta_0 = n_0 \sin\theta_{n+1}$$

$$\sin\theta_0 = \sin\theta_{n+1}$$

$$\theta_0 = \theta_{n+1} \qquad \underline{\text{Q.E.D.}}$$

* 2.2-2



$$P = \int_A I(\vec{r},t)\, dA$$

$$I(\vec{r},t) = I(r)$$, i.e. I is isotropic and it does not change with time

$$\therefore \quad P = I(r,t) \int_A dA = I(r,t)\, A$$

where A is the surface area of a sphere of radius r.

$$A = 4\pi r^2$$

$$\therefore \quad P = I(r,t)\, 4\pi r^2 = 4\pi r^2 I(r,t)$$

$$\therefore \quad \boxed{I(r) = \frac{P}{4\pi r^2}}$$

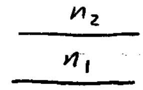
At $r = 1\text{m}$ for $P = 100\text{W}$,

$$I = \frac{100\text{W}}{4\pi (1\text{m})^2} = 7.96\ \text{W/m}^2$$

1.2-3 NA of claddless fiber

$n_1 = 1.46$

$n_2 = 1$



Definition : $NA = \sin\theta_a = (n_1^2 - n_2^2)^{1/2}$

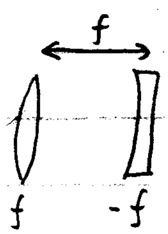
$$= (1.46^2 - 1^2)^{1/2}$$

$$= 1.064$$

$NA > 1 \Rightarrow \theta_a = 90^\circ$

i.e. it accepts any angle !

1.4-1



$$M = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

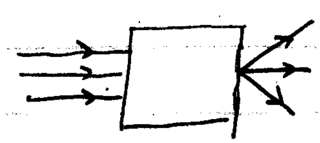
$$= \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} 0 & f \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & f \\ -1/f & 2 \end{pmatrix}$$

Properties :

- Plane Wave (rays are para-axial)

$$\begin{pmatrix} 0 & f \\ -1/f & 2 \end{pmatrix} \begin{pmatrix} r_{in} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{r_{in}}{f} \end{pmatrix}$$



plane wave to point source

2.2-5

$\boxed{\text{plane wave}}$

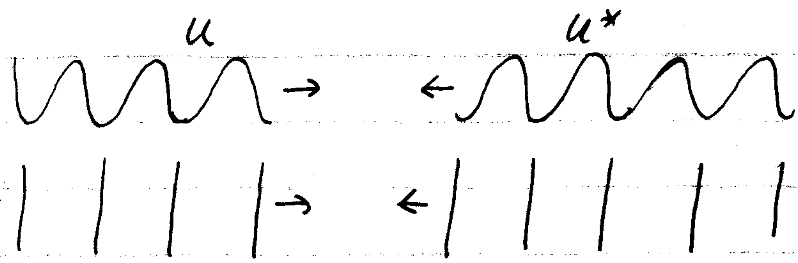
$$U(\vec r) = A \exp\left(-j \frac{k(x+y)}{\sqrt{2}}\right)$$

$$U^*(\vec r) = A \exp\left(+j \frac{k(x+y)}{\sqrt{2}}\right)$$

intensity: $I(\vec r) = |U(\vec r)|^2 = \boxed{U^*(\vec r)\, U(\vec r) = U(\vec r)\, U^*(\vec r)}$

$\therefore$ they have the same intensity

wavefront:



The curvature of their wavefronts are the same
The wavefront normal are opposite

$\boxed{\text{Spherical wave}}$

$$U(\vec r) = \frac{A}{r} \exp(-jkr)$$

$$U^*(\vec r) = \frac{A}{r} \exp(jkr)$$

intensity: $I(\vec r) = U^*(\vec r)\, U(\vec r) = U(\vec r)\, U^*(\vec r)$

again the intensities are the same

wavefront: magnitude $= \frac{A}{r}$ for both, so same wavefronts.

Again the wavefront normal are oppsite,
one away from the center, the other away
from the center of the spheres.

2.4-2

$$t(x,y) = \exp(-jnk_o d)$$

is the complex transmittance of a plane wave travels normal to a plate of thickness d of index n.

For the multi-layered plate,

$$t_{total} = t_1 t_2 t_3 \dots t_N$$

$$= \prod_{q=1}^{N} t_q$$

$$= \prod_{q=1}^{N} \exp(-j n_q k_o d_q)$$

$$= \exp\left(\sum_{q=1}^{N} -j n_q k_o d_q\right)$$

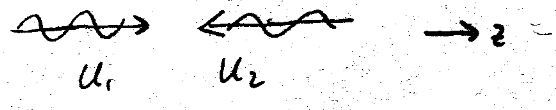
$$\boxed{t_{total} = \exp\left(-j k_o \sum_{q=1}^{N} n_q d_q\right)}$$

For the same result in free space

$$t = \exp(-j n_o k_o d.) = \exp(-j k_o d) = t_{total}$$

$$\therefore \boxed{d = \sum_{q=1}^{N} n_q d_q = \text{OPL}}$$

.5-1

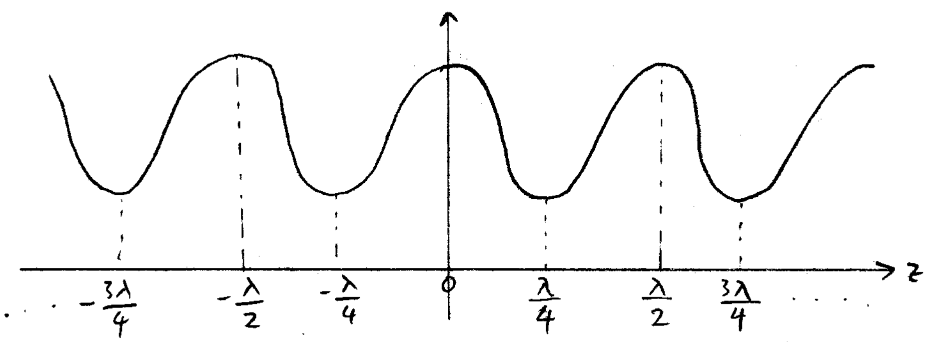


$$U_1 = A_1 e^{j(kz+\omega t)}$$
$$U_2 = A_2 e^{j(-kz+\omega t)}$$

$$k = \frac{2\pi}{\lambda}$$

$$I = |U_1 + U_2|^2$$
$$= (U_1 + U_2)(U_1 + U_2)^*$$
$$= (U_1 + U_2)(U_1^* + U_2^*)$$
$$= U_1^2 + U_2^2 + U_1^* U_2 + U_1 U_2^*$$
$$= A_1^2 + A_2^2 + A_1 A_2 e^{-jkz-j\omega t} e^{-jkz+j\omega t} + A_1 A_2 e^{jkz+j\omega t} e^{jkz-j\omega t}$$
$$= A_1^2 + A_2^2 + A_1 A_2 \left[e^{-j2kz} + e^{j2kz} \right]$$
$$= A_1^2 + A_2^2 + 2A_1 A_2 \cos 2kz$$
$$= I_1 + I_2 + 2A_1 A_2 \cos \frac{4\pi z}{\lambda}$$

$$\frac{4\pi z}{\lambda} = \pi$$
$$z = \frac{\lambda}{4}$$



3.1-1 Beam Parameters

Nd:YAG : $\lambda = 1.06\,\mu m$

Gaussian Beam

$P = 1\,W$

$2\theta_0 = 1$ mrad

Beam waist: $$\theta_0 = \frac{2}{\pi}\frac{\lambda}{2W_0} \quad (3.1\text{-}20)$$

$$\therefore \; W_0 = \frac{2}{\pi}\frac{\lambda}{2\theta_0}$$

$$= \frac{2}{\pi}\frac{1.06\,\mu m}{2\times 0.001\,rad}$$

$$\boxed{W_0 = 3.37\times10^{-2}\ cm}$$

Depth of focus $= 2z_0$:

$$2z_0 = \frac{2\pi W_0^2}{\lambda} \quad (3.1\text{-}21)$$

$$= \frac{2\pi \times (3.37\times10^{-2}\,cm)^2}{\lambda}$$

$$\boxed{D.O.F. = 67.3\ cm}$$

Maximum intensity :

$$P = \frac{1}{2} I_0 (\pi W_0^2) \quad (3.1\text{-}14)$$

$$\therefore \; I_0 = \frac{2P}{\pi W_0^2}$$

$$= \frac{2\times 1W}{\pi\times(3.37\times10^{-2}\,cm)^2}$$

$$\boxed{I_0 = 560\ W/cm^2}$$

Intensity on beam axis at $z = 100$ cm:

$$I(0,z) = \frac{I_0}{1 + (z/z_0)^2} \qquad (3.1\text{-}13)$$

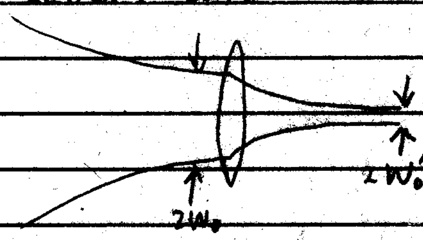
$$I(0, 100\text{ cm}) = \frac{I_0}{1 + \left(\frac{100\text{ cm}}{z_0}\right)^2}$$

$$= \frac{560\text{ W/cm}^2}{1 + \left(\frac{100\text{ cm}}{67.3\text{cm}/2}\right)^2}$$

$$\boxed{I(0, 100\text{ cm}) = 57\text{ W/cm}^2}$$

3.21 Beam Focusing

Take the extream case



$\lambda = 488$ nm

$W_0 = 0.5$ mm

$W_0' = 100\,\mu$m

So we can use Eq. 3.2-13

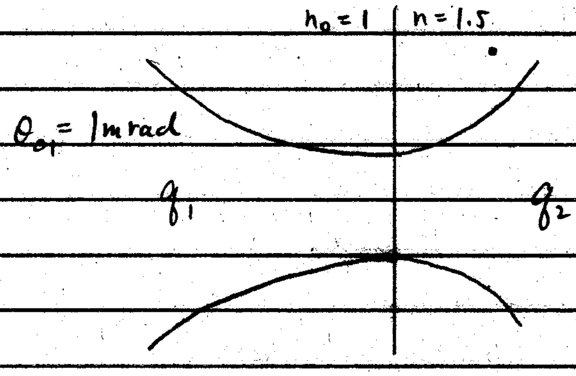
$$W_0' = \frac{W_0}{\left[1 + (z_0/f)^2\right]^{1/2}}$$

$$f = \frac{z_0}{\left[\left(\frac{W_0}{W_0'}\right)^2 - 1\right]^{1/2}}$$

$$z_0 = \frac{\pi W_0^2}{\lambda} = \frac{\pi\,(0.05\text{ cm})^2}{488\times10^{-8}\text{ cm}} = 160.9\text{ cm}$$

$$f = \frac{160.9\text{cm}}{\left[\left(\frac{500\,\mu m}{100\,\mu m}\right)^2 - 1\right]^{1/2}} = \underline{\underline{32.8\text{ cm}}}$$

*3.2-3 Beam Refraction



ABCD matrix of the boundary $= \begin{pmatrix} 1 & 0 \\ 0 & 1/n \end{pmatrix}$

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} = nq_1$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi W^2(z)}$$

$$\therefore \quad \frac{1}{q_2} = \frac{1}{nq_1}$$

Compare the imaginary part of $\frac{1}{q_1}$ and $\frac{1}{q_2}$ gives

$$\frac{\lambda_2}{\pi W_{0_2}^2} = \frac{\lambda_1}{\pi n W_{0_1}^2}$$

$$W_{0_1} = W_{0_2} = W_0 \Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \frac{1}{n}$$

For beam divergence, $\theta_0 = \frac{2}{\pi}\frac{\lambda}{2W_0}$

$$\therefore \quad \frac{\theta_{02}}{\theta_{01}} = \frac{\lambda_2}{\lambda_1} = \frac{1}{n} \quad \Rightarrow \quad \theta_{02} = \frac{\theta_{01}}{n} = 0.667\,\text{mrad}$$