6.1.2 SOLUTION:

Assume the rotation angle of the coordinate system is α,

So, the linearly polarized wave $\begin{bmatrix}\cos\theta_1\\ \sin\theta_1\end{bmatrix}$ should be expressed as $\begin{bmatrix}\cos(\theta_1+\alpha)\\ \sin(\theta_1+\alpha)\end{bmatrix}$

$$T\cdot J^{L}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos(\theta_1+\alpha)\\ \sin(\theta_1+\alpha)\end{bmatrix}=\begin{bmatrix}\cos(\theta_1+\alpha+\theta)\\ \sin(\theta_1+\alpha+\theta)\end{bmatrix}$$

Which is the same with output wave in the new coordinate system

$$\begin{bmatrix}\cos(\theta_1+\theta)\\ \sin(\theta_1+\theta)\end{bmatrix}\xrightarrow{\text{in the new coordinate system}}\begin{bmatrix}\cos(\theta_1+\theta+\alpha)\\ \sin(\theta_1+\theta+\alpha)\end{bmatrix}$$

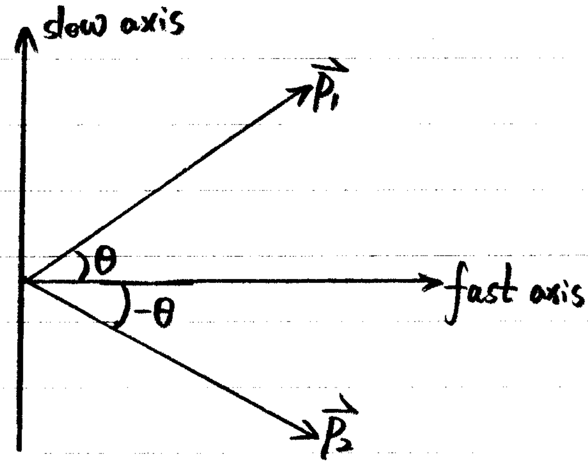
or, $\cancel{T'=R(\theta)\cdot T\cdot R(-\theta)=T}$ $T'=R(-\alpha)\cdot T\cdot R(\alpha)=T$, where $R(\alpha)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

6.1.3 SOLUTION:

The half-wave retarder can be represented by the matrix

$$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

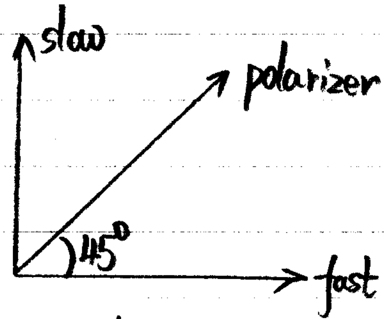
So, $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}=\begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix}=\begin{bmatrix}\cos(-\theta)\\ \sin(\theta)\end{bmatrix}$



That means the transmitted light is rotated by an angle 2θ.

Because for different polarization direction the transmitted light is rotated by different angles, the half-wave retarder can not be equivalent to a polarization rotator.

6.1.6 SOLUTION:



The quarter-wave retarder can be represented by the matrix

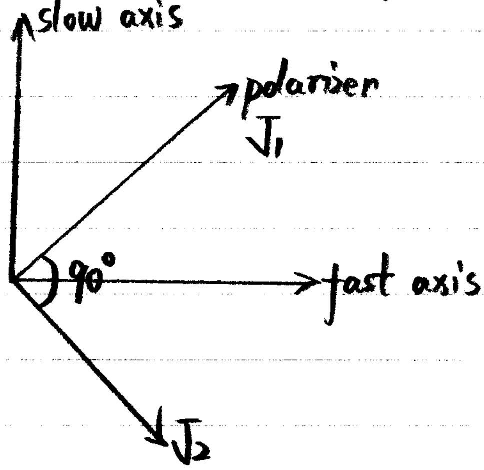
$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix}$$

The light wave transmitted through the polarizer, can be represented by Jones vector $J_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$, so, after transmitting quarter wave retarder, reflected by mirror, and transmitting quarter wave retarder again, it can be expressed as

$$J_2 = T_1 \cdot T_1 \cdot J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

The polarization direction has been rotated 90°, which is perpendicular to the polarizer direction.

So, it can not ~~pas~~ transmitt the polarizer again.



6.23 SOLUTION:

For TE waves, the phase change $\delta_\perp$ can be represented by

$$\tan\frac{\delta_\perp}{2} = \frac{\sqrt{\sin^2\theta_1 - n'^2}}{\cos\theta_1} \qquad n' = \frac{1}{n} = \frac{1}{1.5}$$

For TM waves, the phase change $\delta_{\parallel}$ can be represented by

$$\tan\frac{\delta_{\parallel}}{2} = \frac{\sqrt{\sin^2\theta_1 - n'^2}}{n'^2\cos\theta_1} \qquad n' = \frac{1}{n} = \frac{1}{1.5}$$

So, the phase retardation between TE and TM waves can be represented by

$$\tan\frac{\delta_{\parallel}-\delta_\perp}{2} = \frac{\cos\theta_1\sqrt{\sin^2\theta_1 - n'^2}}{\sin^2\theta_1} \qquad n' = \frac{1}{n} = \frac{1}{1.5}$$

$$\Rightarrow \delta_{\parallel} - \delta_\perp = 2\cdot \operatorname{arctg}\left[\frac{\cos\theta_1\sqrt{\sin^2\theta_1 - n'^2}}{\sin^2\theta_1}\right]$$

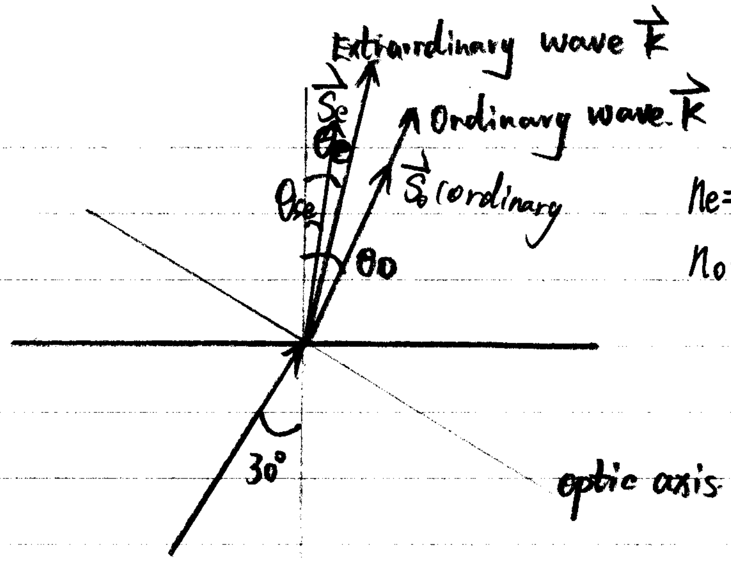
$n = 1.5$, $\sin\theta_c = \frac{1}{1.5} \Rightarrow \theta_c = 0.7297$

$\theta_1 = 1.20\theta_c = 0.8757$

So, $\sin\theta_1 = 0.768$, $\cos\theta_1 = 0.6405$

So, $\delta_{\parallel} - \delta_\perp = 2\cdot\operatorname{arctg}(\cancel{0.2593}\ 0.414) = \cancel{0.5074 = 29.07°}\ 44.98°$

6.33 SOLUTION:

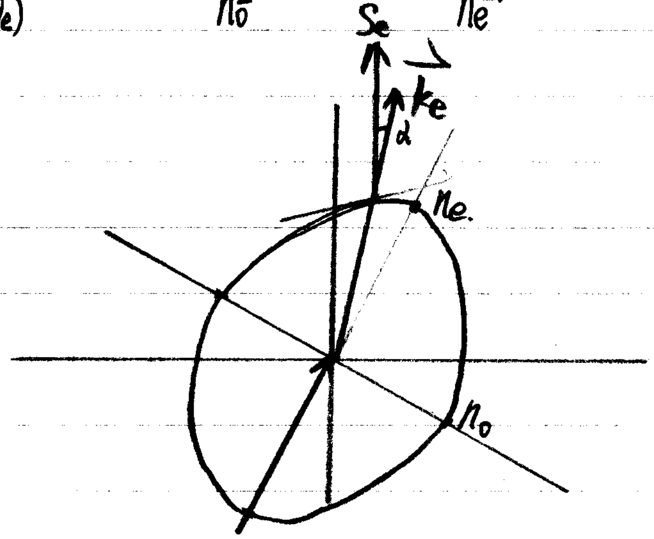


$n_e = 1.553$
$n_o = 1.544$

For, O light wave, $\sin\theta_1 = n_o \cdot \sin\theta_o \Rightarrow \sin\theta_o = \frac{1}{2\times1.544} \Rightarrow \theta_o = 18.89^\circ$

For, e light wave, $\sin\theta_1 = n(\theta_e)\cdot\sin\theta_e$

where $\frac{1}{n^2(\theta_e)} = \frac{\cos^2(60^\circ+\theta_e)}{n_o^2} + \frac{\sin^2(60^\circ+\theta_e)}{n_e^2} \Rightarrow n(\theta_e) = \sqrt{\frac{n_o^2 n_e^2}{n_e^2\cdot\cos^2(60^\circ+\theta_e) + n_o^2\sin^2(60^\circ+\theta_e)}}$



where, $\tan(60^\circ+\theta_{se}) = \tan(60^\circ+\theta_e)\left(\frac{n_o}{n_e}\right)^2$.

$\alpha = \theta_e - \theta_{se}$

where $\tan\alpha = \frac{1}{2}\frac{n_e^2 - n_o^2}{n_o^2\sin^2(60^\circ+\theta_e) + n_e^2\cdot\cos^2(60^\circ+\theta_e)}\cdot\sin2(60^\circ+\theta_e)$

6.6.1 $T = \frac{1}{2}\begin{bmatrix} 1, & -i \\ +i, & 1 \end{bmatrix}$

can coverts Light with any state of polarization into right circularly polarized Light.

For example, $J = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

$$J' = T \cdot J = \frac{1}{2}\begin{bmatrix} \cos\theta \mp i \cdot \sin\theta \\ +i \cdot \cos\theta + \sin\theta \end{bmatrix} = \frac{1}{2}\begin{bmatrix} e^{-i\theta} \\ +i \cdot e^{-i\theta} \end{bmatrix} = \frac{1}{2} \cdot e^{-i\theta}\begin{bmatrix} 1 \\ +i \end{bmatrix}$$