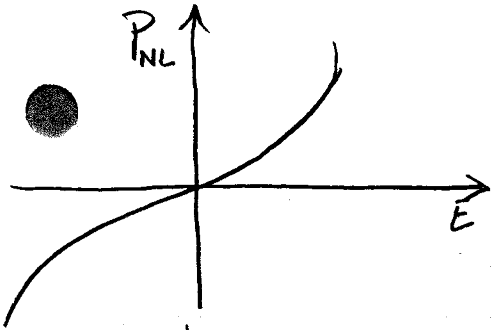
Third Harmonic Generation and Self-phase Modulation 11/3/96 4



$$E(t) = \mathrm{Re}\{E(\omega)\exp(j\omega t)\}$$

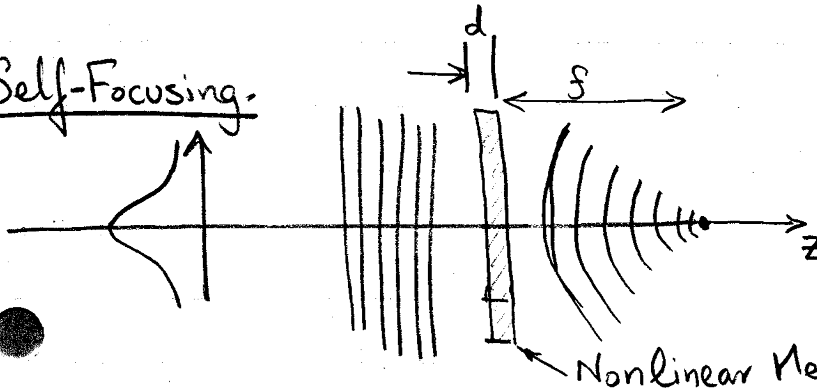
$$P_{NL}(\omega) = 3\chi^{(3)}|E(\omega)|^2 E(\omega)$$
$$P_{NL}(3\omega) = \chi^{(3)} E^3(\omega).$$

Optical Kerr Effect $\quad n(I) = n + n_2 I \qquad n_2 = \frac{3\eta_0}{n^2\epsilon_0}\chi^{(3)}$

Self-phase Modulation.

$$\Delta n \Rightarrow \Delta\phi = \frac{2\pi n_2 L}{\lambda_0 A} P. \qquad P_\pi = \frac{\lambda_0 A}{2 L n_2}.$$

Self-Focusing.



Third-order nonlinear medium acts as a lens. ⇒ f depends on intensity.

Spatial Solitons - intense beam creates its own waveguide.

If the intensity of the beam has the same spatial distribution in the transverse plane as one of the modes of the waveguide that the beam creates ⇒ self-consistent propagate without changing its spatial distribution.

→ Diffraction compensated by nonlinear effect.
→ beam is confined in its self-created waveguide.
- Self-guided beams are called Spatial Solitons.

$$\left[\nabla^2 + n^2(I)\,k_0^2\right] E = 0 \qquad n(I) = n + n_2 I \,,\; k_0 = \frac{\omega}{c_0} \qquad I = \frac{|E|^2}{2\eta}.$$

Nonlinear differential equation in E $\qquad E = A\exp(-jkz)$

$k = nk_0$ $\quad$ assume $A = A(x,z)$ and varies slowly in the z direction in comparison to $\lambda = \frac{2\pi}{k}$.

$$\Rightarrow \frac{\partial^2}{\partial z^2}\left[A\exp(-jkz)\right] \simeq \left(-2jk\frac{\partial A}{\partial z} - k^2 A\right)\exp(-jkz).$$

$$\Rightarrow \frac{\partial^2 A}{\partial x^2} - 2jk\frac{\partial A}{\partial z} + k_0^2\left[n^2(I) - n^2\right]A = 0.$$

typically $n_2 I \ll n$.

$$\Rightarrow n^2(I) - n^2 = (n(I)-n)(n(I)+n) \simeq (n_2 I)(2n) = \frac{2n_2 n|A|^2}{2\eta} = \underline{\underline{\frac{n^2 n_2}{\eta_0}|A|^2}}$$

$$\Rightarrow \underbrace{\frac{\partial^2 A}{\partial x^2} + \frac{n_2}{\eta_0}k^2|A|^2 A = 2jk\frac{\partial A}{\partial z}.}_{\text{nonlinear Schrödinger Equation.}}$$

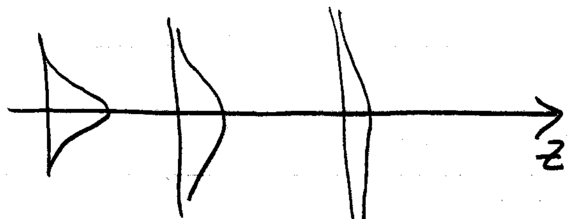
$$A(x,z) = A_0\,\mathrm{sech}\left(\frac{x}{W_0}\right)\exp\left(-j\frac{z}{4z_0}\right). \qquad W_0 \text{ is a constant}$$

$\mathrm{sech}(\cdot)$ hyperbolic-secant function.

A_0 satisfies $n_2\left(\frac{A_0^2}{2\eta_0}\right) = \frac{1}{k^2 W_0^2}$ $\qquad \underbrace{z_0 = \frac{1}{2}kW_0^2 = \frac{\pi W_0^2}{\lambda}.}_{\text{Raleigh Range.}}$

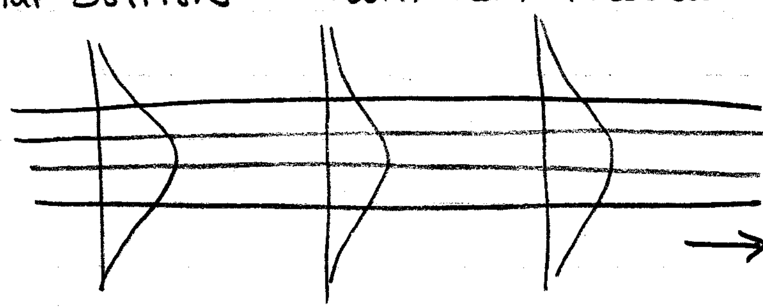
$$I(x,z) = \frac{|A(x,z)|^2}{2\eta} = \frac{A_0^2}{2\eta}\,\mathrm{sech}^2\!\left(\frac{x}{W_0}\right). \quad \text{indepent of } z\,!$$

Gaussian beam in linear medium:



spreading w/ z

Spatial Soliton in nonlinear meddium.



same for all z

$A(x,z)$ corresponds to a mode of a waveguide w.

$$n + n_2 I = n\left[1 + \frac{1}{k^2 W_0^2}\,\mathrm{sech}^2\!\left(\frac{x}{W_0}\right)\right].$$

$E = A\exp(-jkz)$ propagat constant $k + \frac{1}{4z_0} = k\left(1 + \frac{\lambda^2}{8\pi^2 W_0^2}\right)$

$$c = \frac{c_0}{n\left(1 + \frac{\lambda^2}{8\pi^2 W_0^2}\right)}$$

For localized beams W_0 small $\Rightarrow$ smaller velocity c.
larger beams W_0 large $\Rightarrow$ velocity $\sim c$.

Raman Gain. $\chi^{(3)}$ actually is complex.

$$\chi^{(3)} = \chi_R^{(3)} + j\,\chi_I^{(3)}$$

$$\left[\; n_2 = \frac{3\eta\,\chi^{(3)}}{\epsilon_0 n} \;\right]$$

$$\therefore \quad \Delta\phi \simeq 2\pi \frac{n_2 L}{\lambda_0 A} P = \frac{6\pi\eta_0}{\epsilon_0}\,\chi^{(3)}\,\frac{L}{\lambda_0 A}\,P.$$

also complex.

Propagation phase factor $\exp(-j\phi)$ is a phase shift

$$\Delta\phi = \left(\frac{6\pi\eta_0}{\epsilon_0}\right)\frac{\chi_R^{(3)}}{n^2}\left(\frac{L}{\lambda_0 A}\right)P.$$

and gain coefficient

$$\exp\left(\tfrac{1}{2}\gamma L\right).$$

$$\gamma = \frac{12\pi\eta_0}{\epsilon_0}\,\frac{\chi_I^{(3)}}{n^2}\,\frac{1}{\lambda_0 A}\,P. \quad \leftarrow \text{proportional to } P.$$

called the Raman gain.

coupling of light to the high-frequency vibrational modes of the medium. $\Rightarrow$ an energy source providing the gain.

In low-loss media $\rightarrow$ Raman gain is larger than loss $\Rightarrow$ amplifier.

Amplifier + feedback = laser

Low-loss optical fibers $\Rightarrow$ Fiber Raman lasers.

Four-wave mixing

Three-wave mixing is generally not possible in a third order nonlinear medium.

Three waves ω_1, ω_2, and ω_3 cannot be coupled by the system without the help of a fourth wave.

Four wave mixing in third-order media:

$$E(t) = \mathrm{Re}\{E(\omega_1)\exp(j\omega_1 t)\} + \mathrm{Re}\{E(\omega_2)\exp(j\omega_2 t)\} + \mathrm{Re}\{E(\omega_3)\exp(j\omega_3 t)\}.$$

$$E(t) = \sum_{q=\pm1,\pm2,\pm3} \tfrac{1}{2} E(\omega_q)\exp(j\omega_q t).$$

$\omega_{-q} = -\omega_q$ $\qquad E(-\omega_q) = E^*(\omega_q)$.

$$P_{NL} = 4\chi^{(3)} E^3$$

$$\Rightarrow P_{NL}(t) = \frac{1}{2}\chi^{(3)} \underbrace{\sum_{q,r,l=\pm1,\pm2,\pm3}}_{} E(\omega_q)E(\omega_r)E(\omega_l)\exp(j(\omega_q+\omega_r+\omega_l)t)$$

# of terms $6^3 = 216$ terms.

P_{NL} has $\omega_1, \dots 3\omega_1, \dots, 2\omega_1 \pm \omega_2, \dots, \pm\omega_1 \pm \omega_2 \pm \omega_3$.

$P_{NL}(\omega_q+\omega_r+\omega_L)$ corresponds to $\omega_q+\omega_r+\omega_L$

Add permutations of q, r, l. $\qquad P_{NL}(\omega_3+\omega_4-\omega_1)$ involves six permutations.

$\omega_3+\omega_4-\omega_1$. (how many permutations ?).

$\omega_p+\omega_q-\omega_r$.

p	q	r
3	4	-1
4	3	-1
-1	3	4
-1	4	3
3	-1	4
4	-1	3

6 ways.

$\Rightarrow P_{NL}(\omega_3+\omega_4-\omega_1)$.

$$= 6\chi^{(3)} E(\omega_3)E(\omega_4)E^*(\omega_1).$$

$$\Rightarrow P_{NL}(\omega_2) = 6\chi^{(3)} E(\omega_3)E(\omega_4)E^*(\omega_1).$$

$\boxed{\omega_2 = \omega_3+\omega_4-\omega_1} \Rightarrow$ frequency matching.

Waves 1, 3, 4 are plane waves of wavevectors $\vec{k}_1, \vec{k}_3$ and $\vec{k}_4$
$E(\omega_q) \propto \exp(-j\vec{k}_q\cdot\vec{r})$ $q=1,3,4$ then.

$$P_{NL}(\omega_2) \propto \exp(-j\vec{k}_3\cdot\vec{r})\exp(-j\vec{k}_4\cdot\vec{r})\exp(j\vec{k}_1\cdot\vec{r}) = \exp(-j(\vec{k}_3+\vec{k}_4-\vec{k}_1)\cdot\vec{r}).$$

$\boxed{\vec{k}_2 = \vec{k}_3+\vec{k}_4-\vec{k}_1} \Rightarrow$ phase-matching condition. for four wave mixing.

Optical Phase Conjugation.

Frequency-matching condition is satisfied when:

$\omega_1=\omega_2=\omega_3=\omega_4=\omega$. <u>degenerate</u> <u>four-wave</u> <u>mixing</u>.

Suppose waves 3 and 4 are uniform plane waves traveling in opposite directions.

$E_3(\vec{r}) = A_3\exp(-j\vec{k}_3\cdot\vec{r})$. $\quad E_4(\vec{r}) = A_4\exp(-j\vec{k}_4\cdot\vec{r})$.

$\vec{k}_4 = -\vec{k}_3$.

$$\Rightarrow P_{NL}(\omega_2) = 6\chi^{(3)} E(\omega_3)\, E(\omega_4)\, E^*(\omega_1)$$

$$P_{NL}(\omega) = 6\,\chi^{(3)} A_3 A_4\, E_1^*(\vec{r}).$$

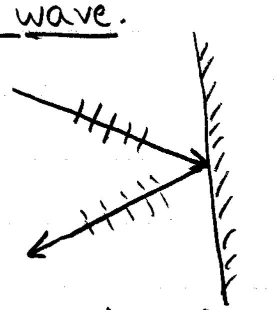
$$\Rightarrow E_2(\vec{r}) \propto \underbrace{A_3 A_4}_{\text{constants}}\ \underbrace{E_1^*(\vec{r})}_{\text{conjugated version of wave 1}}.$$

Device serves as phase conjugator

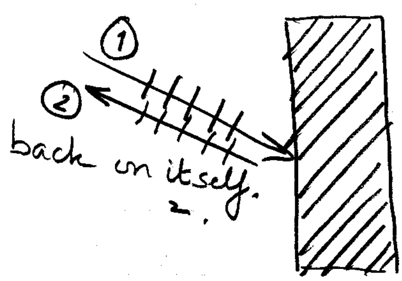
Waves 3 and 4 are called pump waves
Wave 1 – probe wave
Wave 2 – conjugate wave.

Phase conjugator is a special mirror that reflects the wave back onto itself without altering its wavefronts.

Plane wave.

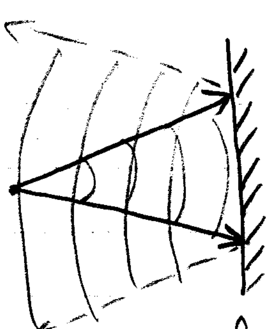


Normal mirror

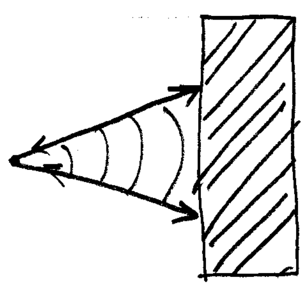


phase conjugator

Spherical wave.



normal mirror.



phase conjugator.

Phase conjugation is analogous to time reversal.

$$E_2(\vec{r},t) = Re\{E_2(r)\exp(j\omega t)\} \propto Re\{E_1^*(\vec{r})\exp(j\omega t)\}.$$

$$E_2(\vec{r},t) = Re\{E_1(\vec{r})\exp(-j\omega t)\}.$$

(since $Re(A) = Re(A^*)$.)

$t \to -t$.

$$E_1(\vec{r},t) = Re\{E_1(\vec{r})\exp(j\omega t)\}.$$

(looks like time reversed version of probe wave.

Notice also that $E_2(\vec{r}) \propto \underbrace{A_3 A_4}_{} E_1^*(r)$.

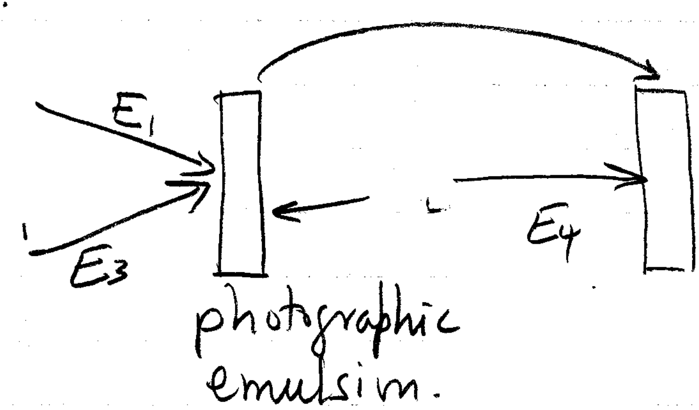
can be used to amplify signal.

$\Rightarrow$ "amplifying mirror".

<u>Degenerate Four-wave mixing as a Form of Real-time holography.</u>

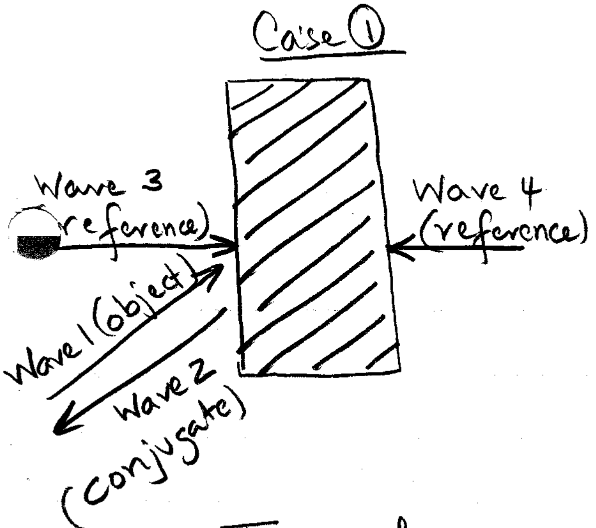
Nonlinear medium permits real-time recording and reconstruction.

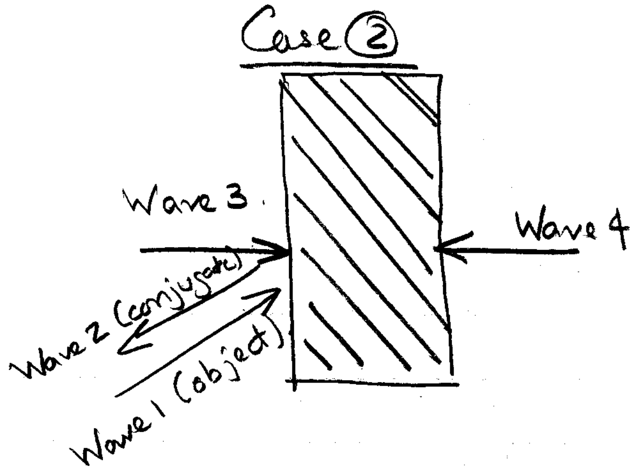
<u>Holography.</u>



$E_4 E_3 E_1^*$

Four waves mixed in a nonlinear medium, each pair of waves interferes and creates a grating, from which the third wave is reflected to produce the fourth wave.





Two reference waves are counter-propagating plane waves.

Two steps of holography. (Case ①).

Step 1. : Object wave 1 is added to reference wave 3 and the intensity of their sum is recorded in the medium in the form of a volume grating (hologram).

Step 2. - Reconstruction wave 4 is Bragg reflected from the grating to create the conjugate wave (wave 2).

Transmission grating.

Case ② - Wave 4 + Wave 1 ⇒ grating. (reflection grating).
Wave 3 reflects to create the conjugate wave 2.

Case ① & ② can both exist, but efficiencies are different.

See page 761 from photocopy.

Coupled-Wave theory of Three-wave mixing.

Quantitative analysis.

Coupled wave equations.

① $\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = -S$. $\quad S = -\mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$.

Suppose $P_{NL} = 2d E^2$.

$$E(t) = \sum_{q=1,2,3} Re\{E_q (\exp(j\omega_q t))\}.$$

$$= \sum_{q=1,2,3} \frac{1}{2}[E_q \exp(j\omega_q t) + E_q^* \exp(-j\omega_q t)].$$

or in compact form

$$E(t) = \sum_{q=\pm1,\pm2,\pm3} \frac{1}{2}E_q \exp(j\omega_q t)$$

$$\boxed{\begin{array}{c}\omega_{-q} = -\omega_q. \\ \hline E_{-q} = E_q^*.\end{array}}$$

36 terms for $P_{NL}(t)$.

$$P_{NL}(t) = \frac{1}{2}d \sum_{q,r=\pm1,\pm2,\pm3} E_q E_r \exp(j(\omega_q+\omega_r)t).$$

and the corresponding radiation source.

$$S = \frac{1}{2}\mu_0 d \sum_{q,r=\pm1,\pm2\pm3} (\omega_q+\omega_r)^2 E_q E_r \exp(j(\omega_q+\omega_r)t).$$

Substitute into ①. If ω_1, ω_2 and ω_3 are distinct.

$$\rightarrow \left.\begin{array}{l}(\nabla^2 + k_1^2) E_1 = -S_1 \\ (\nabla^2 + k_2^2) E_2 = -S_2 \\ (\nabla^2 + k_3^2) E_3 = -S_3.\end{array}\right\} S_q \quad q=1,2,3.$$

S_q is the amplitude of the component S. with $\omega_q, k_q = \frac{n\omega_q}{c_0}$