EE310 – Chapter 1

Introduction to Electronics

Lecture Slides

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1.1 Signal Source

(a) Thévenin Form

(b) Norton Form

1.1 For the signal-source representations shown in Figs. 1.1(a) and 1.1(b), what are the open-circuit output voltages that would be observed? If, for each, the output terminals are short-circuited (i.e., wired together), what current would flow? For the representations to be equivalent, what must the relationship be between \(v_i\), \(i_s\), and \(R_s\)?

\[\text{Sol:}\]

1) Open Circuit Voltage:
   \(V_{oc} = v_s\)
   \(V_{oc} = i_s R_s\)

2) Short circuit current:
   \(i_{sc} = \frac{v_s}{R_s}\)
   \(i_{sc} = i_s\)

3) For Thévenin (a) and Norton (b) to represent the same source: \(V_{oc}\) and \(i_{sc}\) must be the same.

\[\therefore V_{oc} = v_s = i_s R_s\]
\[i_{sc} = \frac{v_s}{R_s} = i_s\]

1.2 A signal source has an open-circuit voltage of 10 mV and a short-circuit current of 10 \(\mu\)A. What is the source resistance?

\[\text{Sol:}\]

\[V_{oc} = 10 \text{ mV} \quad i_{sc} = 10 \text{ mA} \quad R_s = ?\]

\[i_{sc} = \frac{v_s}{R_s} = \frac{10 \text{ mV}}{R_s} = 10 \text{ mA} \quad \therefore R_s = \frac{10 \text{ mV}}{10 \text{ mA}} = 1 \text{ k}\Omega\]
1.2 Frequency Spectrum

\[ v_a(t) = V_a \sin(\omega t) \]

**FIGURE 1.3** Sine-wave voltage signal of amplitude \( V_a \) and frequency \( f = 1/T \) Hz. The angular frequency \( \omega = 2\pi f \text{ rad/s} \).

**FIGURE 1.4** A symmetrical square-wave signal of amplitude \( V \).

**Fourier Expansion:**

\[ v(t) = \frac{4V}{\pi} \left( \sin\omega_0 t + \frac{1}{3}\sin3\omega_0 t + \frac{1}{5}\sin5\omega_0 t + \cdots \right) \]

\[ \omega_0 = \frac{2\pi}{T} \]

**FIGURE 1.5** The frequency spectrum (also known as the line spectrum) of the periodic square wave of Fig. 1.4.
1.6 When the square-wave signal of Fig. 1.4, whose Fourier series is given in Eq. (1.2), is applied to a resistor, the total power dissipated may be calculated directly using the relationship \( P = \frac{1}{T} \int_0^T \frac{v^2}{R} \, dt \) or indirectly by summing the contribution of each of the harmonic components, that is, \( P = P_1 + P_3 + P_5 + \cdots \), which may be found directly from rms values. Verify that the two approaches are equivalent. What fraction of the energy of a square wave is in its fundamental? In its first five harmonics? In its first seven? First nine? In what number of harmonics is 90% of the energy? (Note that in counting harmonics, the fundamental at \( \omega_0 \) is the first, the one at 2\( \omega_0 \) is the second, etc.)

\[ \text{Solution:} \quad P = \frac{1}{T} \int_0^T \frac{v^2}{R} \, dt, \quad v^2 = \left( \frac{4V}{\pi} \right)^2 \left( \sin^2 \omega_0 t + \frac{1}{9} \sin^2 3 \omega_0 t \right. \\
\left. \quad + \frac{1}{25} \sin^2 5 \omega_0 t + \cdots \right) \]

\[ = \frac{1}{T} \left( \frac{4V}{\pi} \right)^2 \frac{T}{2R} \left( 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots \right) \]

\[ = \frac{8}{\pi^2} \frac{V^2}{R} \sum_{m=1}^{\infty} \frac{1}{m^2} \]

\[ = \frac{8}{\pi^2} \sum_{m=1}^{M} \frac{1}{m^2} \approx \frac{\pi^2}{6} \]

\[ \sum_{m=1}^{M} \frac{1}{m^2} \approx \frac{\pi^2}{6} \]

\[ P = P_1 + P_3 + P_5 + P_7 + \cdots \]

\[ \frac{P_i}{P} = \frac{1}{\sum \frac{1}{m^2}} = \frac{1}{\pi^2/6} = 0.81 \]

\[ \frac{P_1 + P_3 + P_5}{P} = \frac{1 + \frac{1}{9} + \frac{1}{25}}{\sum \frac{1}{m^2}} = \frac{1.151}{\pi^2/6} = 0.93 \]

\[ \frac{P_1 + P_3}{P} = \frac{1 + \frac{1}{9}}{\pi^2/6} = 0.90 \]
Gain:
\[ v_o(t) = A \cdot v_i(t) \]

**FIGURE 1.11** (a) A voltage amplifier fed with a signal \( v_i(t) \) and connected to a load resistance \( R_L \).
(b) Transfer characteristic of a linear voltage amplifier with voltage gain \( A_v \).

\[
A_v = \frac{v_o}{v_i} \quad \left[ \frac{V}{V} \right] \leftrightarrow \quad 20 \log |A_v| \quad [\text{dB}]
\]

\[
A_i = \frac{i_o}{i_i} \quad \left[ \frac{A}{A} \right] \leftrightarrow \quad 20 \log |A_i| \quad [\text{dB}]
\]

\[
A_p = \frac{P_o}{P_i} \quad \left[ \frac{W}{W} \right] = \frac{v_o \cdot i_o}{v_i \cdot i_i} \leftrightarrow \quad 10 \log |A_p| \quad [\text{dB}]
\]

1.8 An amplifier has a voltage gain of 100 V/V and a current gain of 1000 A/A. Express the voltage and current gains in decibels and find the power gain.

**Sol.**
\[
A_v = 100 \quad \frac{V}{V} \leftrightarrow \quad 20 \log 100 = 40 \text{ dB}
\]
\[
A_i = 1000 \quad \frac{A}{A} \leftrightarrow \quad 20 \log 1000 = 60 \text{ dB}
\]
\[
A_p = A_v \cdot A_i = 10^2 \cdot 10^3 \quad \frac{W}{W} \leftrightarrow \quad 10 \log 10^5 = 50 \text{ dB}
\]
1.9 An amplifier operating from a single 15-V supply provides a 12-V peak-to-peak sine-wave signal to a 1-kΩ load and draws negligible input current from the signal source. The dc current drawn from the 15-V supply is 8 mA. What is the power dissipated in the amplifier, and what is the amplifier efficiency?

\[ P_{dc} = V_1 I_1 + V_2 I_2 \]
\[ P_{dc} + P_I = P_L + P_{dissipated} \]

Solution: DC supply: \( V_{dc} = 15-V \) \( I_{dc} = 8 \text{ mA} \)

Input: \( i_i \approx 0 \)

Output: \( v_o = 6V \sin \omega t \) \( R_L = 1-\text{k}\Omega \)

\[ P_{diss} = P_{dc} + P_I - P_L \]

\[ P_{dc} = V_{dc} I_{dc} = 15V \times 8 \text{ mA} = 120 \text{ mW} \]

\[ P_I \approx 0 \]

\[ P_L = \frac{V_{o,rms}^2}{R_L} = \frac{(6/\sqrt{2})^2}{1-\text{k}\Omega} = 18 \text{ mW} \]

\[ \therefore P_{diss} = 120 + 0 - 18 = 102 \text{ mW} \]

Efficiency \( \eta = \frac{P_L}{P_{dc}} \times 100 = \frac{18}{120} \times 100 = 15\% \)
1.5 Amplifier Circuit Models

- Voltage Amp

Open Circuit: \( v_o = A_v o v_i \)

\[ \therefore \text{Gain} = \frac{v_o}{v_i} = A_v o \]

Now, with \( R_L \):

\( v_o = A_v o v_i \frac{R_L}{R_L + R_o} \)

\[ \therefore \text{Gain} = \frac{v_o}{v_i} = A_v o \frac{R_L}{R_L + R_o} = A_v \]

For a large \( A_v \), make \( R_L \ll R_o \)

1.12 The output voltage of a voltage amplifier has been found to decrease by 20\% when a load resistance of 1 kΩ is connected. What is the value of the amplifier output resistance?

\[ \text{Sol)} \text{ After } R_L = 1 \text{k}Ω \text{ is connected,} \]

\[ A_v = A_v o \frac{1k}{1k + R_o} = 0.8A_v o \]

\[ \therefore R_o = 0.25k \]
1.13 An amplifier with a voltage gain of +40 dB, an input resistance of 10 kΩ, and an output resistance of 1 kΩ is used to drive a 1-kΩ load. What is the value of $A_{vo}$? Find the value of power gain in dB.

Amp: $40 \text{ dB}, \quad R_i = 10 \text{ kΩ}, \quad R_o = 1 \text{ kΩ}$

Load: $R_L = 1 \text{ kΩ}$

**Solution:**

$40 \text{ dB} = 20 \log |AVo| \quad \Rightarrow \quad AVo = 100 \frac{V_o}{V_i}$

**Power Gain:**

$$P_L = \frac{V_o^2}{R_L}$$

$$P_i = \frac{V_i^2}{R_i}$$

$$\therefore \quad A_p = \frac{P_L}{P_i} = \left( \frac{V_o}{V_i} \right)^2 \frac{R_i}{R_L}$$

$$\frac{V_o}{V_i} = AVo \frac{R_L}{R_L + R_o} = 100 \frac{1\text{k}}{1\text{k} + 1\text{k}} = 50$$

$$A_p = (50) \left( \frac{10\text{k}}{1\text{k}} \right) = 25,000 \frac{W}{W} = 10 \log (25,000) \text{ dB} = 43.98 \text{ dB}$$

**Overall Voltage Gain:**

$$\frac{V_o}{V_s} = V_s - to - V_o$$

$$V_i = V_s \frac{R_i}{R_i + R_s}$$

$$\frac{V_o}{V_s} = \frac{V_o}{V_i} \frac{V_i}{V_s} = A_V \frac{R_i}{R_i + R_s} = AVo \frac{R_L}{R_L + R_o} \frac{R_i}{R_i + R_s}$$
1.16 (a) Model the three-stage amplifier of Example 1.3 (without the source and load) using the voltage amplifier model. What are the values of $R_i$, $A_{vo}$, and $R_o$?

**Solution**

$$R_i = R_{i1} = 1 \text{M} \Omega$$

$$R_0 = R_{o3} = 10 \Omega$$

$$A_{vo} = \frac{V_o}{V_i} = \frac{V_{o3}}{V_{i1}}$$

$$= \frac{V_{o3}}{V_{i3}} \cdot \frac{V_{i2}}{V_{i1}}$$

$$= 1 \times 90.9 \times 9.9$$

$$= 900.1 \text{ V/V}$$

From Example 1.3

$$\frac{V_{o3}}{V_{i3}} = 1 \text{ V/V}$$

$$\frac{V_{i3}}{V_{i2}} = 100 \cdot \frac{100k}{10k+1k} = 90.9 \text{ V/V}$$

$$\frac{V_{i2}}{V_{i1}} = 10 \cdot \frac{100k}{100k+1k} = 9.9 \text{ V/V}$$
Consider the transconductance amplifier whose model is shown in the third row of Table 1.1. Let a voltage signal-source $v_i$ with a source resistance $R_s$ be connected to the input and a load resistance $R_L$ be connected to the output. Show that the overall voltage-gain is given by

$$\frac{v_o}{v_i} = G_m \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

**Solution**

Transconductance Amplifier with Load, Source

$$V_i = V_S \frac{R_i}{R_i + R_s} \quad V_o = G_m V_i \cdot (R_o \parallel R_L)$$

Therefore, the overall voltage gain is

$$\frac{V_o}{V_S} = \frac{V_o}{V_i} \cdot \frac{V_i}{V_S} = G_m (R_o \parallel R_s) \cdot \frac{R_i}{R_i + R_s}$$
1.6 FREQUENCY RESPONSE OF AMPLIFIERS

Transfer Function $T(\omega)$

$|T(\omega)| = \frac{V_o}{V_i}$

$\angle T(\omega) = \phi$

1.6.2 Amplifier Bandwidth

$T(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$

OR,

$T(s) = \frac{V_o(s)}{V_i(s)}$, where

$s \equiv j\omega = \text{Complex Freq.}$
1.6.4 Single-Time-Constant Networks

(a) Low-Pass: \[ \frac{V_o}{V_i} = \frac{1/SC}{1/SC + R} = \frac{1}{1 + SCR} = \frac{1}{1 + \frac{s}{w_0}} \]

where, \( w_0 = \frac{1}{CR} \)

To account for DC gain, use \( K \):

\[ T(s) = \frac{K}{1 + \frac{s}{w_0}} \]

ex) \( T(w = 0) = K \quad T(w = \infty) = 0 \quad \therefore \text{LP} \)

(b) High-Pass: \[ T(s) = \frac{V_o}{V_i} = \frac{K}{1 + \frac{s}{w_0}} \]

ex) \( T(w = 0) = 0 \quad T(w = \infty) = K \quad \therefore \text{HP} \)
Consider a voltage amplifier having a frequency response of the low-pass STC type with a dc gain of 60 dB and a 3-dB frequency of 1000 Hz. Find the gain in dB at \( f = 10 \text{ Hz} \), \( 10 \text{ kHz} \), \( 100 \text{ kHz} \), and \( 1 \text{ MHz} \).

**Solution:** \( K = 60 \text{ dB} = 1000 \% \), \( f_{3\text{dB}} = 1 \text{ kHz} \), LP

**Gain at \( f = 10 \), \( 10 \text{ k} \), \( 100 \text{ k} \), and \( 1 \text{ MHz} \) ?**

**LP:** \( T(s) = \frac{K}{1 + \frac{s}{w_0}} \)

At \( w = w_0 \),

\[ \left| T(s) \right| = \frac{K}{\sqrt{2}} \text{, and} \]

\[ 20 \log \left| T(s) \right| = 20 \log K - 3 \text{ dB} \]

\[ \therefore w_0 = 2\pi f_{3\text{dB}} \]

For \( |s| = w \gg w_0 \),

\[ \left| T(s) \right| \approx \frac{K}{1 + \frac{s}{w_0}} = K \frac{w_0}{w} \]

Gain \( \approx 20 \log \left| T(s) \right| \approx 20 \log K - 20 \log \frac{w}{w_0} \)

\[ \text{ex: } w \rightarrow 10 \times \ 100 \times \ 1000 \times \]

Gain \( \rightarrow -20 \text{ dB} \quad -40 \text{ dB} \quad -60 \text{ dB} \)

For \( |s| = w \ll w_0 \), \( T(s) = K \)

\[ \therefore f = 10 \text{ Hz} \ll 1 \text{ k} \ll 10 \text{ k} \quad 100 \text{ k} \quad 1 \text{ MHz} \]

Gain \( \approx 60 \text{ dB} \ 57 \text{ dB} \ 40 \text{ dB} \ 20 \text{ dB} \ 0 \text{ dB} \)

**Note:** Unity Gain Frequency \( w_T \): Gain = 0 dB at \( w_T \)

Gain = \( K \frac{w_0}{w_T} = 1 \quad \therefore w_T = K w_0 \)

Check \( f_T = K f_{3\text{dB}} = 1000 \frac{f_0}{1 \text{ MHz}} = 1000 \text{ kHz} \)