Numerical comparison of decomposition algorithms for non-convex sum-utility problems

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Jan. 11, 2013

This technical report is a supporting document to the paper


In the following we will use the same notation as introduced in the above paper.

In this report we provide a detailed comparison of the following algorithms SJBR[1], SCALE (SCALE one step)[2], WMMSE[3], and MDP[4] under different settings and scenarios. We start focusing on frequency-selective channels, see Sec. 1; then we consider MIMO flat-fading channels, see Sec. 2.

1 Sum-Rate Maximization over SISO ICs

We compare the performance of SJBR[1], SCALE (SCALE one step)[2], WMMSE[3], and MDP[4] algorithms, in terms of convergence speed and achievable sum-rate.

1.1 Numerical simulation setup

We simulated SISO frequency channels with $N = 64$ subcarriers; the channels are generated as FIR filters of order $L = 10$, whose taps are i.i.d. Gaussian random variables with zero mean and variance $1/d_{ij}^3$, where $d_{ij}$ is the distance between the transmitter $j$ and the receiver $i$. For the sake of simplicity, we set $d_{ij} = d_{ji}$ and $d_{ii} = d_{jj}$ for all $i$ and $j \neq i$. We compare the aforementioned algorithms in both low interference ($d \triangleq d_{ij}/d_{ii} = 3$) and high interference ($d = 1$) scenarios. We assume that all the users have the same power budget $P$, noise variance $\sigma^2$, and $SNR = P/\sigma^2 = 3$dB. All the algorithms are initialized by choosing the uniform power allocation, and are terminated when (the absolute value) of the sum-rate error in two consecutive rounds becomes smaller than a given accuracy criterion. Here we consider two different termination accuracies, namely, $10^{-3}$ and $10^{-6}$. In all the algorithms under consideration there is a bisection loop; the accuracy in the bisection loops is set to be one magnitude higher than the termination accuracy (i.e. $10^{-4}$ and $10^{-7}$, respectively). For the proposed algorithm, the SJBR algorithm, we use the step size rule (Rule#1 [1, eq. (19)])

$$\gamma^n = \gamma^{n-1} (1 - \epsilon \gamma^{n-1}) , \ n = 1, 2, \cdots$$

(1)

where $\epsilon$ is set to $10^{-2}$. The average is taken over 100 independent channel realizations.

1.2 Figures

In this subsection, we show the performance of the aforementioned algorithms for different network sizes, i.e. Number of users equals to 5, 10, 15, 30, 50, 75, 100.
1.2.1 Termination accuracy $10^{-3}$
- $d = 3$ (Low interference scenario)

- $d = 1$ (High interference scenario)

1.2.2 Termination accuracy $10^{-6}$
- $d = 3$ (Low interference scenario)

- $d = 1$ (High interference scenario)
1.2.3 Conclusions

The 'Sum-rate vs Number of Users' figures show that all the algorithms reach the similar averaged sum-rate in all scenarios and under different termination criteria. But the 'Iterations vs Number of Users' figures clearly show that the proposed algorithm, the SJBR, is much faster than all the others. The iteration gap between SJBR and all the other algorithms, ranges from three times to one order of magnitude. Moreover, the higher the termination accuracy the larger the gap. Note that SJBR, WMMSE, SCALE and SCALE one step are simultaneous-based schemes, while MDP is a sequential algorithm.

1.2.4 A further test on iterations number

The figures above suggest that the iterations needed for SJBR algorithm to converge do not increase significantly as the number of users increases. To further test this property, we simulated in the next plots where the Number of Users are chosen from 5 up to 200.

**Termination accuracy $10^{-3}$**

- $d = 3$ (Low interference scenario)

  ![Convergence Speed](image1)
  ![Achievable Sum Rate](image2)

- $d = 1$ (High interference scenario)

  ![Convergence Speed](image3)
  ![Achievable Sum Rate](image4)

**Termination accuracy $10^{-6}$**

- $d = 3$ (Low interference scenario)

  ![Convergence Speed](image5)
  ![Achievable Sum Rate](image6)
• $d = 1$ (High interference scenario)

The above figures show that indeed the iterations number of SJBR algorithm does not increase significantly when the network size increases.

2 Sum-Rate Maximization over MIMO ICs

We focus now on a MIMO scenario and compare the performance of the MIMO SJBR[1], WMMSE[5], and MDP[6]. The setup we simulated is described next.

2.1 Numerical simulation setup

We simulated uncorrelated fading channel model, where the coefficients are Gaussian distributed with zero mean and variance $1/d_{ij}$ ($d_{ij} = d_{ji}$ and $d_{ii} = d_{jj}$ for all $i$ and $j \neq i$). All the transmitters/receivers are equipped with 4 antennas. We compare the MIMO algorithms in three scenarios: the normalized distance $d \triangleq d_{ij}/d_{ii} = 3$, $d = 2$, and $d = 1$. In the simulations, we assume that all the users have the same power budget $P$, noise covariance matrix $R_n = \sigma^2 I$, and $SNR = P/\sigma^2 = 3dB$. All the algorithms are initialized by choosing a uniform power allocation among the antennas, and are terminated when (the absolute value) of the sum-rate error in two consecutive rounds becomes smaller than a given accuracy. Similarly to the SISO case, we consider two different termination accuracies namely $10^{-3}$, $10^{-6}$. The accuracy in the bisection loops is one magnitude higher than the termination accuracy (i.e. $10^{-4}$ and $10^{-7}$, respectively). In SJBR algorithm we use the step size rule (1) with $\epsilon = 10^{-5}$. The average is taken over 100 independent channel realizations.

2.2 Figures

The figures in this subsection show the performance of the aforementioned MIMO algorithms in seven network sizes (Number of users equals to 5, 10, 15, 30, 50, 75, 100), under different normalized distances and accuracies.

2.2.1 Termination accuracy $10^{-3}$

• $d = 3$ (Low interference scenario)
• $d = 2$ (Medium interference scenario)

![Graphs showing convergence speed and achievable sum rate for $d = 2$]

• $d = 1$ (High interference scenario)

![Graphs showing convergence speed and achievable sum rate for $d = 1$]

2.2.2 Termination accuracy $10^{-6}$

• $d = 3$ (Low interference scenario)

![Graphs showing convergence speed and achievable sum rate for $d = 3$]

• $d = 2$ (Medium interference scenario)

![Graphs showing convergence speed and achievable sum rate for $d = 2$]
- $d = 1$ (High interference scenario)

2.2.3 Conclusions

The figures show that all the algorithms reach almost the same averaged sum-rate in all simulated scenarios. As in the SISO case, in the MIMO setting, the proposed algorithm, SJBR, is faster than all the others; depending on the specific scenario (and termination accuracy) the iteration gap ranges from few iterations to one order of magnitude. We recall here that SJBR and WMMSE are simultaneous-based schemes, whereas MDP is a sequential algorithm.

2.2.4 A further test on iterations number

We plot here similar curves as before, but increasing the network size till 200 users. All algorithms reach the same sum rate (and thus the Sum Rate figures are omitted). The curves show that the convergence speed of the proposed algorithm, the SJBR, does not increase significantly with respect to the number of users.

Termination accuracy $10^{-3}$

Termination accuracy $10^{-6}$
2.3 Choices of step-size rules and parameters for SJBR in MIMO ICs

The convergence of SJBR algorithm with a diminishing step size is guaranteed, if the step-size rule satisfies the conditions in Theorem 3 [1]. Many step-size rules satisfy such conditions. A natural question is then to understand how the performance of the SJBR are affected by the choice of the step-size rule, which is the goal of this section. More specifically, here we compare the performance of SJBR algorithm using the following feasible diminishing step-size rules

- Step-size rule 1: \(\gamma^n = \gamma^{n-1} (1 - \epsilon \gamma^{n-1})\), \(\epsilon \in (0, 1), n = 1, 2, \cdots\)
- Step-size rule 2: \(\gamma^n = \frac{\gamma^{n-1} + \alpha}{1 + \beta n}\), \(\alpha, \beta \in (0, 1), \alpha \leq \beta, n = 1, 2, \cdots\)
- Step-size rule 3: \(\gamma^n = \frac{\gamma^{n-1} + \alpha \log(n)}{1 + \beta \sqrt{n}}\), \(\alpha, \beta \in (0, 1), \alpha \leq \beta, n = 1, 2, \cdots\)

### 2.3.1 Termination accuracy \(10^{-3}\)

When the termination accuracy is low, these three step-size rules lead to very similar performance (at least of the given choice of the free parameters, namely: \(\epsilon = 10^{-5}, \alpha = 10^{-6}, \beta = 3 \times 10^{-3}\)).

### 2.3.2 Termination accuracy \(10^{-6}\)

In “High interference” scenario (see the figure corresponding to \(d = 1\)), the situation is different: rule 3 seems to need an ad-hoc tuning, which should depend on the network size. Rules 1 and 2 instead keep working quite good. The performance of SJBR becomes more sensitive to the step size choices when the termination accuracy increases, especially in the high interference scenario.

### 2.3.3 Parameter choices of Step size rule 1 \((\gamma^n = \gamma^{n-1} (1 - \epsilon \gamma^{n-1}), \epsilon \in (0, 1), n = 1, 2, \cdots)\)

Since Rule 1 worked quite well in all the scenarios that we simulated (and it has only one tuning parameter), it seems to be a good winner candidate. It is then interesting to further study the behavior of the SJBR algorithm based on this rule. In particular, we are interested to understand how sensitive the performance of SJBR is with...
respect to the free parameter $\epsilon$ in (1). In the following we report the numerical results for the following choices of $\epsilon$: $10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}$. The following tables show the iterations number and the achievable sum-rate of the SJBR algorithm.

### Termination accuracy $10^{-3}$

- **$d = 3$ (Low interference scenario)**

<table>
<thead>
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<th># of users</th>
<th>$\epsilon = 10^{-6}$ iteration</th>
<th>sum-rate</th>
<th>$\epsilon = 10^{-5}$ iteration</th>
<th>sum-rate</th>
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- **$d = 2$ (Medium interference scenario)**

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- **$d = 1$ (High interference scenario)**

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### Termination accuracy $10^{-6}$

- **$d = 3$ (Low interference scenario)**

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From the above experiment results, we observe that the performance of SJBR algorithm is not too sensitive to the choice of the parameter $\epsilon$, which makes the proposed algorithm very appealing in practical scenarios.

Description of the results for the WMMSE algorithm and SJBR algorithm in two scenarios, namely the interference broadcasting channel (IBC) and the peer-to-peer interference channel (IC). Both scenarios as well as the WMMSE are simulated using the matlab codes provided by Wei-Cheng Liao.

3 Scenario 1 (IBC)

3.1 Environment Setting

- Multicell cellular systems composed of 7 cell with 1 BS and $K$ randomly generated MTs per cell, where BS = Base Station and MT = Mobile Terminal.
- number of transmit antennas = number of receive antennas = 4
- $\text{SNR} \triangleq P_t/\sigma^2$ for all the links, where $P_t =$ transmit power and $\sigma^2 = 1$ is the variance of the AWGN noise.
- Termination accuracy on the sum-rate = $1e^{-2}$ (nats/s/Hz)
- tolerance of the bisection loop = $1e^{-5}$
- parameters of SJBR:
  - step-size rule: $\gamma^n = \gamma^{n-1} \left(1 - \epsilon \gamma^{n-1}\right)$, with $\gamma^0 = 1$ and $\epsilon = 1e^{-3}$
  - $\tau = 0$
- the initial point is generated randomly and set the same for both algorithms, i.e. $V^{(0)}$ for WMMSE and $Q^{(0)} = V^{(0)} (V^{(0)})^H$ for SJBR
3.2 Results

We report next the following results for the two algorithms: i) average cpu-time required to reach convergence (within the prescribed accuracy) vs. # of MTs in the system; ii) average # of iteration to reach convergence vs. # of MTs in the system; and iii) average sum-rate vs. # of MTs in the system.

The average is taken over 1490 independent channel/users realizations. The above results are given for three different values of the SNR, namely: SNR = 0dB, SNR = 3dB, and SNR = 10dB.

- **SNR = 0dB**

- **SNR = 3dB**

- **SNR = 10dB**
4 Scenario 2 (IC)

4.1 Environment setting

- Multicell cellular systems composed of 7 cells; in each cell there are $K$ active pairs BS-MT randomly placed.
- number of transmit antennas = number of receive antennas = 4
- $\text{SNR} \triangleq \frac{P_t}{\sigma^2}$ for all the links, where $P_t$ = transmit power and $\sigma^2 = 1$ is the variance of the AWGN noise.
- Termination accuracy on the sum-rate $= 1e - 2$ (nats/s/Hz)
- tolerance of the bisection loop $= 1e - 5$
- parameters of SJBR:
  - step-size rule: $\gamma^n = \gamma^{n-1} (1 - \epsilon \gamma^{n-1})$, with $\gamma^0 = 1$ and $\epsilon = 1e - 3$
  - $\tau = 0$
- the initial point is generated randomly and set the same for both algorithms, i.e. $V^{(0)}$ for WMMSE and $Q^{(0)} = V^{(0)} (V^{(0)})^H$ for SJBR

4.2 Results

We report next the following results for the two algorithms: i) average cpu-time required to reach convergence (within the prescribed accuracy) vs. # of BS-MT pairs in the system; ii) average # of iteration to reach convergence vs. # of BS-MT pairs in the system; and iii) average sum-rate vs. # of BS-MT pairs in the system.

The average is taken over 500 independent channel/users realizations. The above results are given for three different values of the SNR, namely: SNR = 0dB, SNR = 3dB, and SNR = 10dB.

- SNR = 0dB

- SNR = 3dB
• SNR = 10dB

References


