Problem 2.4  A 1-GHz parallel-plate transmission line consists of 1.2-cm-wide copper strips separated by a 0.15-cm-thick layer of polystyrene. Appendix B gives $\mu_c = \mu_0 = 4\pi \times 10^{-7}$ (H/m) and $\alpha_c = 5.8 \times 10^7$ (S/m) for copper, and $\varepsilon_r = 2.6$ for polystyrene. Use Table 2-1 to determine the line parameters of the transmission line. Assume $\mu = \mu_0$ and $\sigma \simeq 0$ for polystyrene.

Solution:

$$R' = \frac{2R_s}{w} = \frac{2}{w} \sqrt{\frac{\pi f \mu_c}{\alpha_c}} = \frac{2}{1.2 \times 10^{-2}} \left( \frac{\pi \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7} \right)^{1/2} = 1.38 \text{ (\Omega/m)},$$

$$L' = \frac{\mu d}{w} = \frac{4\pi \times 10^{-7} \times 1.5 \times 10^{-3}}{1.2 \times 10^{-2}} = 1.57 \times 10^{-7} \text{ (H/m)},$$

$$G' = 0 \text{ because } \sigma = 0,$$

$$C' = \frac{\varepsilon w}{d} = \frac{\varepsilon_0 \varepsilon_r w}{d} = \frac{10^{-9}}{36\pi} \times 2.6 \times \frac{1.2 \times 10^{-2}}{1.5 \times 10^{-3}} = 1.84 \times 10^{-10} \text{ (F/m)}.$$
Problem 2.6 A coaxial line with inner and outer conductor diameters of 0.5 cm and 1 cm, respectively, is filled with an insulating material with $\varepsilon_r = 4.5$ and $\sigma = 10^{-3}$ S/m. The conductors are made of copper.

(a) Calculate the line parameters at 1 GHz.
(b) Compare your results with those based on CD Module 2.2. Include a printout of the screen display.

Solution: (a) Given

$$a = (0.5/2) \text{ cm} = 0.25 \times 10^{-2} \text{ m},$$
$$b = (1.0/2) \text{ cm} = 0.50 \times 10^{-2} \text{ m},$$

combining Eqs. (2.5) and (2.6) gives

$$R' = \frac{1}{2\pi} \sqrt{\frac{\pi f \mu_c}{\sigma_c}} \left( \frac{1}{a} + \frac{1}{b} \right)$$
$$= \frac{1}{2\pi} \sqrt{\frac{\pi(10^9 \text{ Hz})(4\pi \times 10^{-7} \text{ H/m})}{5.8 \times 10^7 \text{ S/m}}} \left( \frac{1}{0.25 \times 10^{-2} \text{ m}} + \frac{1}{0.50 \times 10^{-2} \text{ m}} \right)$$
$$= 0.788 \Omega/\text{m}.$$

From Eq. (2.7),

$$L' = \frac{\mu}{2\pi} \ln \left( \frac{b}{a} \right) = \frac{4\pi \times 10^{-7} \text{ H/m}}{2\pi} \ln 2 = 139 \text{ nH/m}.$$

From Eq. (2.8),

$$G' = \frac{2\pi \sigma}{\ln (b/a)} = \frac{2\pi \times 10^{-3} \text{ S/m}}{\ln 2} = 9.1 \text{ mS/m}.$$

From Eq. (2.9),

$$C' = \frac{2\pi \varepsilon}{\ln (b/a)} = \frac{2\pi \varepsilon \varepsilon_0}{\ln (b/a)} = \frac{2\pi \times 4.5 \times (8.854 \times 10^{-12} \text{ F/m})}{\ln 2} = 362 \text{ pF/m}.$$

(b) Solution via Module 2.2:
Module 2.2  Coaxial Cable

\[ \sigma = 0.0010 \text{ S/m} \]
\[ \varepsilon_r = 4.5 \]

\[ f = 1.0 \text{ [GHz]} \]

Input

- Inner radius \( a = 2.5 \text{ [mm]} \)
- Shield radius \( b = 5 \text{ [mm]} \)
- Frequency \( f = 1.0E9 \text{ [Hz]} \)

Output

Select: Impedance vs. Radius \( b \)

Real Part of Characteristic Impedance

\[ Z_0 = 19.605065 + j0.03034369 \text{ [\Omega]} \]
\[ C' = 360.67376 \text{ [pF/m]} \]
\[ L' = 138.629436 \text{ [nH/m]} \]
\[ R' = 0.787839 \text{ [\Omega/m]} \]
\[ G' = 0.009065 \text{ [S/m]} \]

\[ \lambda_0 = 0.3 \text{ [m]} \text{ in vacuum} \]
\[ \lambda = 0.1414 \text{ [m]} \text{ in guide} \]

\[ \alpha = 0.10895 \text{ [Np/m]} \]
\[ \beta = 44.428883 \text{ [rad/m]} \]
Problem 2.7  Find $\alpha$, $\beta$, $u_p$, and $Z_0$ for the two-wire line of Problem 2.2. Compare results with those based on CD Module 2.1. Include a printout of the screen display.

Solution: From Problem 2.2:

\[
R' = 3.71 \, \Omega/m, \\
L' = 1.36 \times 10^{-6} \, \text{H/m}, \\
G' = 1.85 \times 10^{-6} \, \text{S/m}, \\
C' = 2.13 \times 10^{-11} \, \text{F/m}.
\]

At 2 GHz:

\[
\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
= 0.0076 + j67.54.
\]

Hence

\[
\alpha = 0.0076 \, \text{Np/m}, \\
\beta = 67.54 \, \text{rad/m}. \\
u_p = \frac{\omega}{\beta} = \frac{2\pi \times 2 \times 10^9}{67.54} = 1.86 \times 10^8 \, \text{m/s}, \\
Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = 253 \, \Omega.
\]
Module 2.1 Two-Wire Line

Input
- Wire Diameter \( d = 2.0 \) [mm]
- Centers distance \( D = 30.0 \) [mm]
- Frequency \( f = 2.0E9 \) [Hz]
- \( \varepsilon_r = 2.6 \)
- \( \sigma = 2.0E-6 \) [S/m]
- \( \sigma_c = 5.8E7 \) [S/m]

Output
- \( f = 2.0 \) [GHz]

Structure Data
- \( d = 2.0 \) [mm]
- \( D = 30.0 \) [mm]
- \( D / d = 15.0 \)
- \( Z_0 = 253.037142 - j0.026617 \) [Ω]
- \( C' = 21.241303 \) [pF/m]
- \( L' = 1.360034 \) [µH/m]
- \( R' = 3.713907 \) [Ω/m]
- \( G' = 2.0E-6 \) [S/m]
- \( \lambda_0 = 0.15 \) [m] in vacuum
- \( \lambda = 9.3026 \) [cm] in guide
- \( \alpha = 0.007572 \) [Np/m]
- \( \beta = 67.542213 \) [rad/m]
Problem 2.13  In addition to not dissipating power, a lossless line has two important features: (1) it is dispertionless ($\mu_p$ is independent of frequency) and (2) its characteristic impedance $Z_0$ is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G'$$  (distortionless line).

Such a line is called a distortionless line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R' \sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \quad \beta = \omega \sqrt{L'C'}, \quad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \sqrt{L'C'} \left( \frac{R'}{L'} + j\omega \right) \left( \frac{G'}{C'} + j\omega \right) = \sqrt{L'C'} \left( \frac{R'}{L'} + j\omega \right) = R' \sqrt{\frac{C'}{L'}} + j\omega \sqrt{L'C'}.$$

Hence,

$$\alpha = \text{Re}(\gamma) = R' \sqrt{\frac{C'}{L'}}, \quad \beta = \text{Im}(\gamma) = \omega \sqrt{L'C'}, \quad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}} \sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$
Problem 2.16  A transmission line operating at 125 MHz has $Z_0 = 40 \ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75 \ \text{rad/m}$. Find the line parameters $R'$, $L'$, $G'$, and $C'$. 

Solution:  Given an arbitrary transmission line, $f = 125 \ \text{MHz}$, $Z_0 = 40 \ \Omega$, $\alpha = 0.02 \ \text{Np/m}$, and $\beta = 0.75 \ \text{rad/m}$. Since $Z_0$ is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.13, $\beta = \omega \sqrt{L'/C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \ \text{nH/m}.$$  

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \ \text{nH/m}}{40^2} = 23.9 \ \text{pF/m}.$$  

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'} \sqrt{\frac{R'}{G'}} = \sqrt{R'G'} \sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \ \text{Np/m} \times 40 \ \Omega = 0.6 \ \Omega/m$$  

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \ \text{Np/m})^2}{0.8 \ \Omega/m} = 0.5 \ \text{mS/m}.$$
Problem 2.21  On a 150-Ω lossless transmission line, the following observations were noted: distance of first voltage minimum from the load = 3 cm; distance of first voltage maximum from the load = 9 cm; \( S = 3 \). Find \( Z_L \).

Solution: Distance between a minimum and an adjacent maximum = \( \lambda /4 \). Hence,

\[
9 \text{ cm} - 3 \text{ cm} = 6 \text{ cm} = \frac{\lambda}{4},
\]

or \( \lambda = 24 \text{ cm} \). Accordingly, the first voltage minimum is at \( d_{\text{min}} = 3 \text{ cm} = \frac{\lambda}{8} \).

Application of Eq. (2.71) with \( n = 0 \) gives

\[
\theta_t = 2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = -\pi,
\]

which gives \( \theta_t = -\pi/2 \).

\[
|\Gamma| = \frac{S - 1}{S + 1} = \frac{3 - 1}{3 + 1} = \frac{2}{4} = 0.5.
\]

Hence, \( \Gamma = 0.5 e^{-j\pi/2} = -j0.5 \).

Finally,

\[
Z_L = Z_0 \left[ \frac{1 + \Gamma}{1 - \Gamma} \right] = 150 \left[ \frac{1 - j0.5}{1 + j0.5} \right] = (90 - j120) \Omega.
\]

---
**Problem 2.22** Using a slotted line, the following results were obtained: distance of first minimum from the load = 4 cm; distance of second minimum from the load = 14 cm; voltage standing-wave ratio = 1.5. If the line is lossless and $Z_0 = 50 \, \Omega$, find the load impedance.

**Solution:** Following Example 2.6: Given a lossless line with $Z_0 = 50 \, \Omega$, $S = 1.5$, $d_{\text{min}(0)} = 4 \, \text{cm}$, $d_{\text{min}(1)} = 14 \, \text{cm}$. Then

\[
d_{\text{min}(1)} - d_{\text{min}(0)} = \frac{\lambda}{2}
\]

or

\[
\lambda = 2 \times (d_{\text{min}(1)} - d_{\text{min}(0)}) = 20 \, \text{cm}
\]

and

\[
\beta = \frac{2\pi}{\lambda} = \frac{2\pi \, \text{rad/cycle}}{20 \, \text{cm/cycle}} = 10\pi \, \text{rad/m}.
\]

From this we obtain

\[
\theta = 2\beta d_{\text{min}(n)} - (2n + 1)\pi \, \text{rad} = 2 \times 10\pi \, \text{rad/m} \times 0.04 \, \text{m} - \pi \, \text{rad}
= -0.2\pi \, \text{rad} = -36.0^\circ.
\]

Also,

\[
|\Gamma| = \frac{S - 1}{S + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.
\]

So

\[
Z_L = Z_0 \left( \frac{1 + \Gamma}{1 - \Gamma} \right) = 50 \left( \frac{1 + 0.2e^{-j36.0^\circ}}{1 - 0.2e^{-j36.0^\circ}} \right) = (67.0 - j16.4) \, \Omega.
\]
Problem 2.26  A 50-Ω lossless transmission line is connected to a load composed of a 75-Ω resistor in series with a capacitor of unknown capacitance (Fig. P2.26). If at 10 MHz the voltage standing wave ratio on the line was measured to be 3, determine the capacitance $C$.

\[ Z_0 = 50 \, \Omega \]

\[ R_L = 75 \, \Omega \]

\[ C = ? \]

**Figure P2.26:** Circuit for Problem 2.26.

**Solution:**

\[
|\Gamma| = \left| \frac{S - 1}{S + 1} \right| = \frac{3 - 1}{3 + 1} = \frac{2}{4} = 0.5
\]

\[
Z_L = R_L - jX_C, \quad \text{where} \quad X_C = \frac{1}{\omega C}.
\]

\[
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}
\]

\[
|\Gamma|^2 = \left[ \frac{Z_L - Z_0}{Z_L + Z_0} \right] \left( \frac{Z_L^* - Z_0}{Z_L^* + Z_0} \right)
\]

\[
|\Gamma|^2 = \frac{Z_LZ_L^* + Z_0^2 - Z_0(Z_L + Z_L^*)}{2Z_LZ_0^* + Z_0^2 + Z_0(Z_L + Z_L^*)}
\]

Noting that:

\[
Z_LZ_L^* = (R_L - jX_C)(R_L + jX_C) = R_L^2 + X_C^2,
\]

\[
Z_0(Z_L + Z_L^*) = Z_0(R_L - jX_C + R_L + jX_C) = 2Z_0R_L.
\]

\[
|\Gamma|^2 = \frac{R_L^2 + X_C^2 + Z_0^2 - 2Z_0R_L}{2R_L^2 + X_C^2 + Z_0^2 + 2Z_0R_L},
\]

Upon substituting $|\Gamma_L| = 0.5$, $R_L = 75 \, \Omega$, and $Z_0 = 50 \, \Omega$, and then solving for $X_C$, we have

\[ X_C = 66.1 \, \Omega. \]

Hence

\[
C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 10^7 \times 66.1} = 2.41 \times 10^{-10} = 241 \, \text{pF}.
\]
**Problem 2.28** A lossless transmission line of electrical length \( l = 0.35\lambda \) is terminated in a load impedance as shown in Fig. P2.28. Find \( \Gamma \), \( S \), and \( Z_{\text{in}} \). Verify your results using CD Modules 2.4 or 2.5. Include a printout of the screen’s output display.

![Figure P2.28: Circuit for Problem 2.28.](image)

**Solution:** From Eq. (2.59),

\[
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(60 + j30) - 100}{(60 + j30) + 100} = 0.307e^{j132.5^\circ}.
\]

From Eq. (2.73),

\[
S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.307}{1 - 0.307} = 1.89.
\]

From Eq. (2.79)

\[
Z_{\text{in}} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) = 100 \left[ \frac{(60 + j30) + j100\tan\left(\frac{2\pi \text{ rad}}{\lambda}0.35\lambda\right)}{100 + j(60 + j30)\tan\left(\frac{2\pi \text{ rad}}{\lambda}0.35\lambda\right)} \right] = (64.8 - j38.3) \Omega.
\]
**Problem 2.29** Show that the input impedance of a quarter-wavelength–long lossless line terminated in a short circuit appears as an open circuit.

**Solution:**

\[
Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right).
\]

For \( l = \frac{\lambda}{4} \), \( \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \). With \( Z_L = 0 \), we have

\[
Z_{in} = Z_0 \left( \frac{jZ_0 \tan \pi/2}{Z_0} \right) = j\infty \quad \text{(open circuit)}.
\]