Problem 8.27 A plane wave in air with
\[ \mathbf{E}^i = \hat{y} \ 20e^{-j(3x+4z)} \ (\text{V/m}) \]
is incident upon the planar surface of a dielectric material, with \( \varepsilon_r = 4 \), occupying the half-space \( z \geq 0 \). Determine:

(a) The polarization of the incident wave.

(b) The angle of incidence.

(c) The time-domain expressions for the reflected electric and magnetic fields.

(d) The time-domain expressions for the transmitted electric and magnetic fields.

(e) The average power density carried by the wave in the dielectric medium.

Solution:

(a) \( \mathbf{E}^i = \hat{y} \ 20e^{-j(3x+4z)} \ V/m \).

Since \( \mathbf{E}^i \) is along \( \hat{y} \), which is perpendicular to the plane of incidence, the wave is perpendicularly polarized.

(b) From Eq. (8.48a), the argument of the exponential is
\[ -jk_1(x \sin \theta_i + z \cos \theta_i) = -j(3x + 4z). \]

Hence,
\[ k_1 \sin \theta_i = 3, \quad k_1 \cos \theta_i = 4, \]

from which we determine that
\[ \tan \theta_i = \frac{3}{4} \quad \text{or} \quad \theta_i = 36.87^\circ, \]

and
\[ k_1 = \sqrt{3^2 + 4^2} = 5 \quad (\text{rad/m}). \]

Also,
\[ \omega = u_p k = ck = 3 \times 10^8 \times 5 = 1.5 \times 10^9 \quad (\text{rad/s}). \]

(c)
\[ \eta_1 = \eta_0 = 377 \ \Omega, \]
\[ \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{2} = 188.5 \ \Omega, \]
\[ \theta_i = \sin^{-1} \left[ \frac{\sin \theta_i}{\sqrt{\varepsilon_r}} \right] = \sin^{-1} \left[ \frac{\sin 36.87^\circ}{\sqrt{4}} \right] = 17.46^\circ, \]
\[ \Gamma_{\perp} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_i} = -0.41. \]
\[ \tau_\perp = 1 + \Gamma_\perp = 0.59. \]

In accordance with Eq. (8.49a), and using the relation \( E^r_0 = \Gamma_\perp E^l_0 \),

\[
\bar{E}^r = -\hat{y} \cdot 8.2 \cdot 10^6 e^{-j(3x-4z)}, \\
\bar{H}^r = -\left(\hat{x} \cos \theta_i + \hat{z} \sin \theta_i\right) \frac{8.2}{\eta_0} e^{-j(3x-4z)},
\]

where we used the fact that \( \theta_i = \theta_t \) and the \( z \)-direction has been reversed.

\[
E^r = \Re\left[\bar{E}^r e^{j\omega t}\right] = -\hat{y} \cdot 8.2 \cdot 10^6 \cos(1.5 \times 10^9 t - 3x + 4z) \quad \text{(V/m)}, \\
H^r = -\left(\hat{x} \cdot 17.4 + \hat{z} \cdot 13.06\right) \cos(1.5 \times 10^9 t - 3x + 4z) \quad \text{(mA/m)}.
\]

(d) In medium 2,

\[
k_2 = k_1 \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 5 \sqrt{4} = 20 \quad \text{(rad/m)},
\]

and

\[
\theta_i = \sin^{-1}\left[\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin \theta_1\right] = \sin^{-1}\left[\frac{1}{2} \sin 36.87^\circ\right] = 17.46^\circ
\]

and the exponent of \( E^i \) and \( H^i \) is

\[-jk_2(x \sin \theta_i + z \cos \theta_i) = -j 10(x \sin 17.46^\circ + z \cos 17.46^\circ) = -j(3x + 9.54z).\]

Hence,

\[
\bar{E}^i = \hat{y} \cdot 20 \times 10^6 e^{-j(3x+9.54z)}, \\
\bar{H}^i = -\left(\hat{x} \cos \theta_i + \hat{z} \sin \theta_i\right) \frac{20 \times 0.59}{\eta_2} e^{-j(3x+9.54z)}, \\
E^i = \Re\left[\bar{E}^i e^{j\omega t}\right] = \hat{y} \cdot 11.8 \cos(1.5 \times 10^9 t - 3x - 9.54z) \quad \text{(V/m)}, \\
H^i = -\left(\hat{x} \cos 17.46^\circ + \hat{z} \sin 17.46^\circ\right) \frac{11.8}{188.5} \cos(1.5 \times 10^9 t - 3x - 9.54z) \\
= (-\hat{x} \cdot 59.72 + \hat{z} \cdot 18.78) \cos(1.5 \times 10^9 t - 3x - 9.54z) \quad \text{(mA/m)}.
\]

(e)

\[
S_w^\prime = \frac{|E^i_0|^2}{2\eta_2} = \frac{(11.8)^2}{2 \times 188.5} = 0.36 \quad \text{(W/m}^2)\).
**Problem 8.29**  A plane wave in air with

\[ \vec{E}^i = (\hat{x} \ 9 - \hat{y} \ 4 - \hat{z} \ 6)e^{-j(2x+3z)} \:\text{(V/m)} \]

is incident upon the planar surface of a dielectric material, with \( \varepsilon_r = 2.25 \), occupying the half-space \( z \geq 0 \). Determine

(a) The incidence angle \( \theta_i \).
(b) The frequency of the wave.
(c) The field \( \vec{E}^r \) of the reflected wave.
(d) The field \( \vec{E}^t \) of the wave transmitted into the dielectric medium.
(e) The average power density carried by the wave into the dielectric medium.

**Solution:**

(a) From the exponential of the given expression, it is clear that the wave direction of travel is in the \( x-z \) plane. By comparison with the expressions in (8.48a) for perpendicular polarization or (8.65a) for parallel polarization, both of which have the same phase factor, we conclude that:

\[ k_1 \sin \theta_i = 2, \]
\[ k_1 \cos \theta_i = 3. \]
Hence,

\[ k_1 = \sqrt{2^2 + 3^2} = 3.6 \text{ (rad/m)} \]
\[ \theta_i = \tan^{-1}\left(\frac{2}{3}\right) = 33.7^\circ. \]

Also,

\[ k_2 = k_1 \sqrt{\varepsilon_{t2}} = 3.6 \sqrt{2.25} = 5.4 \text{ (rad/m)} \]
\[ \theta_2 = \sin^{-1}\left[\sin\theta_1 \sqrt{\frac{1}{2.25}}\right] = 21.7^\circ. \]

(b) \[
\begin{align*}
  k_1 &= \frac{2\pi f}{c} \\
  f &= \frac{k_1 c}{2\pi} = \frac{3.6 \times 3 \times 10^8}{2\pi} = 172 \text{ MHz}.
\end{align*}
\]

(c) In order to determine the electric field of the reflected wave, we first have to determine the polarization of the wave. The vector argument in the given expression for \( \vec{E}^i \) indicates that the incident wave is a mixture of parallel and perpendicular polarization components. Perpendicular polarization has a \( \hat{y} \)-component only (see 8.46a), whereas parallel polarization has only \( \hat{x} \) and \( \hat{z} \) components (see 8.65a). Hence, we shall decompose the incident wave accordingly:

\[ \vec{E}^i = \vec{E}^i_\perp + \vec{E}^i_\parallel \]

with

\[ \vec{E}^i_\perp = -\hat{y} 4e^{-j(2x + 3z)} \text{ (V/m)} \]
\[ \vec{E}^i_\parallel = (\hat{x} 9 - \hat{z} 6)e^{-j(2x + 3z)} \text{ (V/m)} \]

From the above expressions, we deduce:

\[ E^i_{\perp 0} = -4 \text{ V/m} \]
\[ E^i_{\parallel 0} = \sqrt{9^2 + 6^2} = 10.82 \text{ V/m}. \]

Next, we calculate \( \Gamma \) and \( \tau \) for each of the two polarizations:

\[ \Gamma_\perp = \frac{\cos \theta_i - \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2 \theta_i}} \]
Using \( \theta_i = 33.7^\circ \) and \( \varepsilon_2/\varepsilon_1 = 2.25/1 = 2.25 \) leads to:

\[
\Gamma_\perp = -0.25 \\
\tau_\perp = 1 + \Gamma_\perp = 0.75.
\]

Similarly,

\[
\Gamma_\perp = \frac{-(\varepsilon_2/\varepsilon_1) \cos \theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2 \theta_i}}{(\varepsilon_2/\varepsilon_1) \cos \theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2 \theta_i}} = -0.15,
\]

\[
\tau_\parallel = (1 + \Gamma_\parallel) \frac{\cos \theta_i}{\cos \theta_i} = (1 - 0.15) \frac{\cos 33.7^\circ}{\cos 21.7^\circ} = 0.76.
\]

The electric fields of the reflected and transmitted waves for the two polarizations are given by (8.49a), (8.49c), (8.65c), and (8.65e):

\[
\tilde{\mathbf{E}}^r_\perp = \hat{y} E^r_{\perp,0} e^{-jk_1(x \sin \theta_t - z \cos \theta_t)}
\]

\[
\tilde{\mathbf{E}}^t_\perp = \hat{y} E^t_{\perp,0} e^{-jk_2(x \sin \theta_t + z \cos \theta_t)}
\]

\[
\tilde{\mathbf{E}}^r_\parallel = (\hat{x} \cos \theta_t + \hat{z} \sin \theta_t) E^r_{\parallel,0} e^{-jk_1(x \sin \theta_t - z \cos \theta_t)}
\]

\[
\tilde{\mathbf{E}}^t_\parallel = (\hat{x} \cos \theta_t - \hat{z} \sin \theta_t) E^t_{\parallel,0} e^{-jk_2(x \sin \theta_t + z \cos \theta_t)}
\]

Based on our earlier calculations:

\[
\theta_t = \theta_i = 33.7^\circ
\]

\[
\theta_t = 21.7^\circ
\]

\[
k_1 = 3.6 \text{ rad/m}, \quad k_2 = 5.4 \text{ rad/m},
\]

\[
E^r_{\perp,0} = \Gamma_\perp E^1_{\perp,0} = (-0.25) \times (-4) = 1 \text{ V/m}.
\]

\[
E^t_{\perp,0} = \tau_\perp E^1_{\perp,0} = 0.75 \times (-4) = -3 \text{ V/m}.
\]

\[
E^r_{\parallel,0} = \Gamma_\parallel E^1_{\parallel,0} = (-0.15) \times 10.82 = -1.62 \text{ V/m}.
\]

\[
E^t_{\parallel,0} = \tau_\parallel E^1_{\parallel,0} = 0.76 \times 10.82 = 8.22 \text{ V/m}.
\]

Using the above values, we have:

\[
\tilde{\mathbf{E}}^r = \tilde{\mathbf{E}}^r_\perp + \tilde{\mathbf{E}}^r_\parallel
\]

\[
= (\hat{x} E^r_{\parallel,0} \cos \theta_t + \hat{y} E^r_{\perp,0} + \hat{z} E^r_{\parallel,0} \sin \theta_t)e^{-j(2x-3z)}
\]

\[
= (-\hat{x} 1.35 + \hat{y} - 20.90)e^{-j(2x-3z)} \quad (\text{V/m}).
\]
(d) \[
\bar{E}^t = \bar{E}_{\perp}^t + \bar{E}_{\parallel}^t
\]
\[= (\hat{x} 7.65 - \hat{y} 3 - 23.05) e^{-j(2\pi + \frac{5}{2})} \text{ (V/m)}.\]

(e) \[
S^t = \frac{|E_0^t|^2}{2\eta_2}
\]
\[|E_0^t|^2 = (7.65)^2 + 3^2 + (3.05)^2 = 76.83\]
\[\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{t_2}}} = \frac{377}{1.5} = 251.3 \text{ \Omega} \]
\[S^t = \frac{76.83}{2 \times 251.3} = 152.86 \text{ (mW/m}^2).\]
Problem 8.31  A parallel-polarized plane wave is incident from air onto a dielectric medium with $\epsilon_r = 9$ at the Brewster angle. What is the refraction angle?

Solution:

For nonmagnetic materials, Eq. (8.72) gives

$$\theta_1 = \theta_B = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \tan^{-1} 3 = 71.57^\circ.$$  

But

$$\sin \theta_2 = \frac{\sin \theta_1}{\sqrt{\epsilon_2}} = \frac{\sin 71.57^\circ}{3} = 0.32,$$

or $\theta_2 = 18.44^\circ$. 
Problem 8.35  A parallel-polarized beam of light with an electric field amplitude of 10 (V/m) is incident in air on polystyrene with \( \mu_r = 1 \) and \( \varepsilon_r = 2.6 \). If the incidence angle at the air–polystyrene planar boundary is 50°, determine the following:

(a) The reflectivity and transmissivity.

(b) The power carried by the incident, reflected, and transmitted beams if the spot on the boundary illuminated by the incident beam is 1 m² in area.

Solution:

(a) From Eq. (8.68),

\[
\Gamma_{\parallel} = \frac{-(\varepsilon_2/\varepsilon_1) \cos \theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2 \theta_i}}{(\varepsilon_2/\varepsilon_1) \cos \theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2 \theta_i}}
\]

\[
= \frac{-2.6 \cos 50^\circ + \sqrt{2.6 - \sin^2 50^\circ}}{2.6 \cos 50^\circ + \sqrt{2.6 - \sin^2 50^\circ}} = -0.08,
\]

\[
R_{\parallel} = |\Gamma_{\parallel}|^2 = (0.08)^2 = 6.4 \times 10^{-3},
\]

\[
T_{\parallel} = 1 - R_{\parallel} = 0.9936.
\]

(b)

\[
P_{\parallel}^i = \frac{|E_{\parallel0}^i|^2}{2\eta_1} A \cos \theta_i = \frac{(10)^2}{2 \times 120\pi} \times \cos 50^\circ = 85 \text{ mW},
\]

\[
P_{\parallel}^r = R_{\parallel} P_{\parallel}^i = (6.4 \times 10^{-3}) \times 0.085 = 0.55 \text{ mW},
\]

\[
P_{\parallel}^t = T_{\parallel} P_{\parallel}^i = 0.9936 \times 0.085 = 84.45 \text{ mW}.
\]
Problem 8.36  A 50-MHz right-hand circularly polarized plane wave with an
electric field modulus of 30 V/m is normally incident in air upon a dielectric medium
with $\varepsilon_r = 9$ and occupying the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that
the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the
transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the
boundary and transmitted into the second medium.

Solution:

(a)

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} = \frac{\pi}{3} \text{ rad/m},$$

$$k_2 = \frac{\omega}{c} \sqrt{\varepsilon_r} = \frac{\pi}{3} \sqrt{9} = \pi \text{ rad/m}.$$  

From (7.57), RHC wave traveling in $+z$ direction:

$$\vec{E}_i = a_0(\hat{x} + j\hat{y}) e^{-j\frac{1}{3} \pi} = a_0(\hat{x} - j\hat{y}) e^{-jk_1 z}$$

$$E_i(z, t) = \Re \left[ E e^{j\omega t} \right]$$

$$= \Re \left[ a_0(\hat{x} e^{j(\omega t - k_1 z)} + j\hat{y} e^{j(\omega t - k_1 z) - \frac{1}{2} \pi}) \right]$$

$$= \hat{x} a_0 \cos(\omega t - k_1 z) + \hat{y} a_0 \cos(\omega t - k_1 z - \frac{1}{2} \pi)$$

$$|E_i| = \left[ a_0^2 \cos^2(\omega t - k_1 z) + a_0^2 \sin^2(\omega t - k_1 z) \right]^{1/2} = a_0 = 30 \text{ V/m}.$$  

Hence,

$$\vec{E}_i = 30(x_0 - jy_0)e^{-j\frac{1}{3} \pi} \text{ (V/m)}.$$  

(b)

$$\eta_1 = \eta_0 = 120\pi \text{ (}\Omega\text{)}, \quad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi \text{ (}\Omega\text{).}$$

$$R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 120\pi}{40\pi + 120\pi} = -0.5$$

$$\tau = 1 + R = 1 - 0.5 = 0.5.$$
\[\vec{E}_0 = \Gamma a_0 (\hat{x} - j\hat{y}) e^{jk_1 z}\]
\[= -0.5 \times 30 (\hat{x} - j\hat{y}) e^{jk_1 z}\]
\[= -15 (\hat{x} - j\hat{y}) e^{j\pi z/3} \text{ (V/m)}.\]
\[\vec{E}_1 = \tau a_0 (\hat{x} - j\hat{y}) e^{-jk_2 z}\]
\[= 15 (\hat{x} - j\hat{y}) e^{-j\pi z} \text{ (V/m)}.\]
\[\vec{E}_4 = \vec{E}_1 + \vec{E}_0\]
\[= 30 (\hat{x} - j\hat{y}) e^{-j\pi z/3} - 15 (\hat{x} - j\hat{y}) e^{j\pi z/3}\]
\[= 15 (\hat{x} - j\hat{y}) \left[ 2 e^{-j\pi z/3} - e^{j\pi z/3} \right] \text{ (V/m)}.\]

(d)

% of reflected power = $100 \times |\Gamma|^2 = 100 \times (0.5)^2 = 25\%$

% of transmitted power = $100 |\tau|^2 \frac{n_1}{n_2} = 100 \times (0.5)^2 \times \frac{120\pi}{40\pi} = 75\%$. 

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