Problem 7.9  For a wave characterized by the electric field

\[ E(z,t) = \hat{x} a_x \cos(\omega t - kz) + \hat{y} a_y \cos(\omega t - kz + \delta) \]

identify the polarization state, determine the polarization angles \((\gamma, \chi)\), and sketch the locus of \(E(0,t)\) for each of the following cases:

(a)  \(a_x = 3\) V/m, \(a_y = 4\) V/m, and \(\delta = 0\)
(b)  \(a_x = 3\) V/m, \(a_y = 4\) V/m, and \(\delta = 180^\circ\)
(c)  \(a_x = 3\) V/m, \(a_y = 3\) V/m, and \(\delta = 45^\circ\)
(d)  \(a_x = 3\) V/m, \(a_y = 4\) V/m, and \(\delta = -135^\circ\)

Solution:

\[ \begin{array}{c}
\text{Figure P7.9: Plots of the locus of } E(0,t) .
\end{array} \]
\[ \psi_0 = \tan^{-1}(a_y/a_x), \quad \text{[Eq. (7.60)]}, \]
\[ \tan 2\gamma = (\tan 2\psi_0) \cos \delta \quad \text{[Eq. (7.59a)]}, \]
\[ \sin 2\chi = (\sin 2\psi_0) \sin \delta \quad \text{[Eq. (7.59b)]}. \]

<table>
<thead>
<tr>
<th>Case</th>
<th>(a_x)</th>
<th>(a_y)</th>
<th>(\delta)</th>
<th>(\psi_0)</th>
<th>(\gamma)</th>
<th>(\chi)</th>
<th>Polarization State</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>53.13°</td>
<td>53.13°</td>
<td>0</td>
<td>Linear</td>
</tr>
<tr>
<td>(b)</td>
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<td>4</td>
<td>180°</td>
<td>53.13°</td>
<td>-53.13°</td>
<td>0</td>
<td>Linear</td>
</tr>
<tr>
<td>(c)</td>
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<td>3</td>
<td>45°</td>
<td>45°</td>
<td>45°</td>
<td>22.5°</td>
<td>Left elliptical</td>
</tr>
<tr>
<td>(d)</td>
<td>3</td>
<td>4</td>
<td>-135°</td>
<td>53.13°</td>
<td>-56.2°</td>
<td>-21.37°</td>
<td>Right elliptical</td>
</tr>
</tbody>
</table>

(a) \( E(z,t) = \hat{x}3 \cos(\omega t - kz) + \hat{y}4 \cos(\omega t - kz) \).
(b) \( E(z,t) = \hat{x}3 \cos(\omega t - kz) - \hat{y}4 \cos(\omega t - kz) \).
(c) \( E(z,t) = \hat{x}3 \cos(\omega t - kz) + \hat{y}3 \cos(\omega t - kz + 45°) \).
(d) \( E(z,t) = \hat{x}3 \cos(\omega t - kz) + \hat{y}4 \cos(\omega t - kz - 135°) \).
Problem 7.10  The electric field of a uniform plane wave propagating in free space is given by

\[ \mathbf{E} = (\hat{x} + j\hat{y})30e^{-j\pi z/6} \text{ (V/m)} \]

Specify the modulus and direction of the electric field intensity at the \( z = 0 \) plane at \( t = 0, 5, \) and \( 10 \) ns.

Solution:

\[
\mathbf{E}(z,t) = \Re[\mathbf{E}e^{j\omega t}]
= \Re[(\hat{x} + j\hat{y})20e^{-j\pi z/6}e^{j\omega t}]
= \Re[(\hat{x} + \hat{y}e^{j\pi/2})20e^{-j\pi z/6}e^{j\omega t}]
= \hat{x}20\cos(\omega t - \pi z/6) + \hat{y}20\cos(\omega t - \pi z/6 + \pi/2)
= \hat{x}20\cos(\omega t - \pi z/6) - \hat{y}20\sin(\omega t - \pi z/6) \text{ (V/m)},
\]

\[
|\mathbf{E}| = \left[ E_x^2 + E_y^2 \right]^{1/2} = 20 \text{ (V/m)},
\]

\[
\psi = \tan^{-1}\left( \frac{E_y}{E_x} \right) = -\left( \omega t - \pi z/6 \right).
\]

From

\[
f = \frac{c}{\lambda} = \frac{kc}{2\pi} = \frac{\pi/6 \times 3 \times 10^8}{2\pi} = 2.5 \times 10^7 \text{ Hz},
\]

\[
\omega = 2\pi f = 5\pi \times 10^7 \text{ rad/s}.
\]

At \( z = 0, \)

\[
\psi = -\omega t = -5\pi \times 10^7 t = \begin{cases} 0 & \text{at } t = 0, \\ -0.25\pi = -45^\circ & \text{at } t = 5 \text{ ns}, \\ -0.5\pi = -90^\circ & \text{at } t = 10 \text{ ns}. \end{cases}
\]

Therefore, the wave is LHC polarized.
**Problem 7.12** The electric field of an elliptically polarized plane wave is given by

\[
E(z, t) = [-\hat{x}10 \sin(\omega t - kz - 60^\circ) + \hat{y}30 \cos(\omega t - kz)] \text{ (V/m)}
\]

Determine the following:

(a) The polarization angles \((\gamma, \chi)\).
(b) The direction of rotation.

**Solution:**

(a) \[
E(z, t) = [-\hat{x}10 \sin(\omega t - kz - 60^\circ) + \hat{y}30 \cos(\omega t - kz)] = 10 \sin(\omega t - 60^\circ) \hat{x} + 30 \cos(\omega t - kz) \hat{y} \text{ (V/m)}.
\]

Phasor form:

\[
\tilde{E} = (\hat{x}10 e^{30^\circ} + \hat{y}30) e^{-jkz}.
\]

Since \(\delta\) is defined as the phase of \(E_y\) relative to that of \(E_x\),

\[
\delta = -30^\circ,
\]

\[
\psi_0 = \tan^{-1}\left(\frac{30}{10}\right) = 71.56^\circ,
\]

\[
\tan 2\gamma = (\tan 2\psi_0) \cos \delta = -0.65 \quad \text{or} \quad \gamma = 73.5^\circ,
\]

\[
\sin 2\chi = (\sin 2\psi_0) \sin \delta = -0.40 \quad \text{or} \quad \chi = -8.73^\circ.
\]

(b) Since \(\chi < 0\), the wave is right-hand elliptically polarized.
**Problem 7.17** In a medium characterized by $\varepsilon_r = 9$, $\mu_r = 1$, and $\sigma = 0.1 \text{ S/m}$, determine the phase angle by which the magnetic field leads the electric field at $100 \text{ MHz}$.

**Solution:** The phase angle by which the magnetic field leads the electric field is $-\theta_\eta$, where $\theta_\eta$ is the phase angle of $\eta_c$.

$$\frac{\sigma}{\omega \varepsilon} = \frac{0.1 \times 36\pi}{2\pi \times 10^8 \times 10^{-9} \times 9} = 2.$$ 

Hence, quasi-conductor.

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_r}} \left( 1 - j \frac{\varepsilon''}{\varepsilon'} \right)^{-1/2} = \frac{120\pi}{\sqrt{\varepsilon_r}} \left( 1 - j \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^{-1/2} = 125.67 (1 - j2)^{-1/2} = 71.49 + j44.18 = 84.04 \angle 31.72^\circ.$$ 

Therefore $\theta_\eta = 31.72^\circ$.

Since $\mathbf{H} = (1/\eta_c)\hat{k} \times \mathbf{E}$, $\mathbf{H}$ leads $\mathbf{E}$ by $-\theta_\eta$, or by $-31.72^\circ$. In other words, $\mathbf{H}$ lags $\mathbf{E}$ by $31.72^\circ$. 

---
Problem 7.22  The electric field of a plane wave propagating in a nonmagnetic medium is given by

\[ E = \hat{z} 2.5 e^{-30x} \cos(2\pi \times 10^9 t - 40x) \quad (V/m) \]

Obtain the corresponding expression for \( \mathbf{H} \).

Solution: From the given expression for \( E \),

\[ \omega = 2\pi \times 10^9 \quad \text{(rad/s)}, \]
\[ \alpha = 30 \quad \text{(Np/m)}, \]
\[ \beta = 40 \quad \text{(rad/m)}. \]

From (7.65a) and (7.65b),

\[ \alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon' = -\frac{\omega^2}{c^2} \varepsilon_r', \]
\[ 2\alpha\beta = \omega^2 \mu \varepsilon'' = \frac{\omega^2}{c^2} \varepsilon_r''. \]

Using the above values for \( \omega, \alpha, \) and \( \beta \), we obtain the following:

\[ \varepsilon_r' = 1.6, \]
\[ \varepsilon_r'' = 5.47. \]
\[ \eta_c = \sqrt{\frac{\mu}{\varepsilon'} \left( 1 - j \frac{\varepsilon''}{\varepsilon'} \right)^{-1/2}} \]
\[ = \frac{\eta_0}{\sqrt{\varepsilon'}} \left( 1 - j \frac{\varepsilon''}{\varepsilon'} \right)^{-1/2} = \frac{377}{\sqrt{1.6}} \left( 1 - j \frac{5.47}{1.6} \right)^{-1/2} = 157.9 e^{j36.85^\circ} \quad (\Omega). \]
\[ \tilde{E} = \hat{z} 2.5 e^{-30x} e^{-j40x}, \]
\[ \tilde{H} = \frac{1}{\eta_c} \hat{k} \times \tilde{E} = \frac{1}{157.9 e^{j36.85^\circ}} \hat{k} \times \hat{z} 2.5 e^{-30x} e^{-j40x} = -\hat{y} 0.16 e^{-30x} e^{-40x} e^{-j36.85^\circ}, \]
\[ \mathbf{H} = \Re \{ \tilde{H} e^{j\omega t} \} = -\hat{y} 0.16 e^{-30x} \cos(2\pi \times 10^9 t - 40x - 36.85^\circ) \quad (A/m). \]
**Problem 7.24** In a nonmagnetic, lossy, dielectric medium, a 300-MHz plane wave is characterized by the magnetic field phasor

\[ \mathbf{H} = (\mathbf{\hat{x}} - j4\mathbf{\hat{z}})e^{-2\gamma}e^{-j9y} \text{ (A/m)} \]

Obtain time-domain expressions for the electric and magnetic field vectors.

**Solution:**

\[ \mathbf{E} = -\eta_c \mathbf{\hat{k}} \times \mathbf{H}. \]

To find \( \eta_c \), we need \( \varepsilon' \) and \( \varepsilon'' \). From the given expression for \( \mathbf{H} \),

\[ \alpha = 2 \text{ (Np/m)}, \]
\[ \beta = 9 \text{ (rad/m)}. \]

Also, we are given than \( f = 300 \text{ MHz} = 3 \times 10^8 \text{ Hz} \). From (7.65a),

\[ \alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon', \]
\[ 4 - 81 = -(2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \varepsilon'_r \times \frac{10^{-9}}{36\pi}, \]

whose solution gives

\[ \varepsilon'_r = 1.95. \]

Similarly, from (7.65b),

\[ 2\alpha\beta = \omega^2 \mu \varepsilon'', \]
\[ 2 \times 2 \times 9 = (2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \varepsilon''_r \times \frac{10^{-9}}{36\pi}, \]

which gives

\[ \varepsilon''_r = 0.91. \]

\[ \eta_c = \sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j \frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \]
\[ = \eta_0 \sqrt{\frac{\varepsilon}{\varepsilon'_r}} \left(1 - j \frac{0.91}{1.95}\right)^{-1/2} = \frac{377}{\sqrt{1.95}} (0.93 + j0.21) = 256.9 e^{j12.6^\circ}. \]

Hence,

\[ \mathbf{E} = -256.9 e^{j12.6^\circ} \mathbf{\hat{y}} \times (\mathbf{\hat{x}} - j4\mathbf{\hat{z}})e^{-2\gamma}e^{-j9y} \]
\[ = (\mathbf{\hat{x}} j4 + \mathbf{\hat{z}}) 256.9 e^{-2\gamma}e^{-j9y} e^{j12.6^\circ} \]
\[ = (\mathbf{\hat{x}} e^{j\pi/2} + \mathbf{\hat{z}}) 256.9 e^{-2\gamma}e^{-j9y} e^{j12.6^\circ}, \]
\( \mathbf{E} = \Re \{ \tilde{\mathbf{E}} e^{j\omega t} \} \)
\[ = \hat{x} 1.03 \times 10^3 e^{-2y} \cos(\omega t - 9y + 102.6^\circ) \]
\[ + \hat{z} 256.9 e^{-2y} \cos(\omega t - 9y + 12.6^\circ) \] (V/m),
\( \mathbf{H} = \Re \{ \tilde{\mathbf{H}} e^{j\omega t} \} \)
\[ = \Re \{ (\hat{x} + j\hat{z}) e^{-2y} e^{-j9y} e^{j\omega t} \} \]
\[ = \hat{x} e^{-2y} \cos(\omega t - 9y) + \hat{z} 4 e^{-2y} \sin(\omega t - 9y) \] (A/m).
Problem 8.2  A plane wave traveling in medium 1 with \( \varepsilon_1 = 2.25 \) is normally incident upon medium 2 with \( \varepsilon_2 = 4 \). Both media are made of nonmagnetic, nonconducting materials. If the electric field of the incident wave is given by

\[
E_i = 8 \cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(V/m)}.
\]

(a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.

(b) Determine the average power densities of the incident, reflected and transmitted waves.

Solution:

(a)

\[
E_i = 8 \cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(V/m)},
\]

\[
\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_1}} = \frac{\eta_0}{\sqrt{2.25}} = \frac{377}{1.5} = 251.33 \ \Omega,
\]

\[
\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_2}} = \frac{\eta_0}{\sqrt{4}} = \frac{377}{2} = 188.5 \ \Omega,
\]

\[
\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1/2 - 1/1.5}{1/2 + 1/1.5} = -0.143,
\]

\[
\tau = 1 + \Gamma = 1 - 0.143 = 0.857,
\]

\[
E_f = \Gamma E_i = -1.14 \times 8 \cos(6\pi \times 10^9 t + 30\pi x) \quad \text{(V/m)}.
\]

Note that the coefficient of \( x \) is positive, denoting the fact that \( E_f \) belongs to a wave traveling in \(-x\)-direction.

\[
E_1 = E_i + E_f = \hat{y} [8 \cos(6\pi \times 10^9 t - 30\pi x) - 1.14 \cos(6\pi \times 10^9 t + 30\pi x)] \quad \text{(A/m)},
\]

\[
H_i = \hat{z} \frac{8}{\eta_1} \cos(6\pi \times 10^9 t - 30\pi x) = \hat{z} 31.83 \cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(mA/m)},
\]

\[
H_f = \hat{z} \frac{1.14}{\eta_1} \cos(6\pi \times 10^9 t + 30\pi x) = \hat{z} 4.54 \cos(6\pi \times 10^9 t + 30\pi x) \quad \text{(mA/m)},
\]

\[
H_1 = H_i + H_f = \hat{z} [31.83 \cos(6\pi \times 10^9 t - 30\pi x) + 4.54 \cos(6\pi \times 10^9 t + 30\pi x)] \quad \text{(mA/m)}.
\]

Since \( k_1 = \omega \sqrt{\mu \varepsilon_1} \) and \( k_2 = \omega \sqrt{\mu \varepsilon_2} \),

\[
k_2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} k_1 = \sqrt{\frac{4}{2.25}} 30\pi = 40\pi \quad \text{(rad/m)},
\]
\( \mathbf{E}_2 = \mathbf{E}^t = \hat{\mathbf{y}} 8\pi \cos(6\pi \times 10^9 t - 40\pi x) = \hat{\mathbf{y}} 6.86 \cos(6\pi \times 10^9 t - 40\pi x) \) (V/m),

\( \mathbf{H}_2 = \mathbf{H}^t = \hat{z} \frac{8\pi}{\eta_2} \cos(6\pi \times 10^9 t - 40\pi x) = \hat{z} 36.38 \cos(6\pi \times 10^9 t - 40\pi x) \) (mA/m).

(b)

\[
S_{av}^i = \frac{\hat{x} \frac{8^2}{2\eta_1}}{2} = \frac{64}{2 \times 251.33} = \hat{x} 127.3 \text{ (mW/m}^2),
\]

\[
S_{av}^r = -|\Gamma|^2 S_{av}^i = -\hat{x} (0.143)^2 \times 0.127 = -\hat{x} 2.6 \text{ (mW/m}^2),
\]

\[
S_{av}^t = \frac{|E_0|^2}{2\eta_2}
\]

\[
= \frac{\hat{x} \frac{3(8)^2}{2\eta_2}}{2\eta_2} = \frac{\hat{x} (0.86)^2 \times 64}{2 \times 188.5} = \hat{x} 124.7 \text{ (mW/m}^2).
\]

Within calculation error, \( S_{av}^i + S_{av}^r = S_{av}^t \).
Problem 8.6  A 50-MHz plane wave with electric field amplitude of 50 V/m is normally incident in air onto a semi-infinite, perfect dielectric medium with $\varepsilon_r = 36$. Determine the following:

(a) $\Gamma$

(b) The average power densities of the incident and reflected waves.

(c) The distance in the air medium from the boundary to the nearest minimum of the electric field intensity, $|E|$.

Solution:

(a) 
\[
\eta_1 = \eta_0 = 120\pi \, (\Omega), \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{120\pi}{\sqrt{\varepsilon_2}} = \frac{120\pi}{6} = 20\pi \, (\Omega),
\]
\[
\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{20\pi - 120\pi}{20\pi + 120\pi} = -0.71.
\]
Hence, $|\Gamma| = 0.71$ and $\theta_\eta = 180^\circ$.

(b) 
\[
S^i_{av} = \frac{|E^i_0|^2}{2\eta_1} = \frac{(50)^2}{2 \times 120\pi} = 3.32 \, (W/m^2),
\]
\[
S^r_{av} = |\Gamma|^2 S^i_{av} = (0.71)^2 \times 3.32 = 1.67 \, (W/m^2).
\]

(c) In medium 1 (air),
\[
\lambda_1 = c/f = \frac{3 \times 10^8}{5 \times 10^7} = 6 \, m.
\]
From Eqs. (8.16) and (8.17),
\[
l_{max} = \frac{\theta_\lambda \lambda_1}{4\pi} = \frac{\pi \times 6}{4\pi} = 1.5 \, m,
\]
\[
l_{min} = l_{max} - \frac{\lambda_1}{4} = 1.5 - 1.5 = 0 \, m \text{ (at the boundary)}.
\]
Problem 8.15  A 5-MHz plane wave with electric field amplitude of 10 (V/m) is normally incident in air onto the plane surface of a semi-infinite conducting material with \( \varepsilon_r = 4 \), \( \mu_r = 1 \), and \( \sigma = 100 \) (S/m). Determine the average power dissipated (lost) per unit cross-sectional area in a 2-mm penetration of the conducting medium.

Solution: For convenience, let us choose \( \mathbf{E}^i \) to be along \( \hat{x} \) and the incident direction to be \( +\hat{z} \). With
\[
k_1 = \frac{\omega}{c} = \frac{2\pi \times 5 \times 10^6}{3 \times 10^8} = \frac{\pi}{30} \text{ (rad/m)},
\]
we have
\[
\mathbf{E}^i = \hat{x} 10 \cos \left( \pi \times 10^7 t - \frac{\pi}{30} z \right) \text{ (V/m)},
\]
\[
\eta_1 = \eta_0 = 377 \text{ \Omega}.
\]
From Table 7-1,
\[
\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega \varepsilon_r \varepsilon_0} = \frac{100 \times 36\pi}{\pi \times 10^7 \times 4 \times 10^{-9}} = 9 \times 10^4,
\]
which makes the material a good conductor, for which
\[
\begin{align*}
\alpha_2 &= \sqrt{\frac{\pi \mu \sigma}{\varepsilon_0}} = \sqrt{\frac{\pi \times 5 \times 10^6 \times 4\pi \times 10^{-7} \times 100}{377}} = 44.43 \text{ (Np/m)}, \\
\beta_2 &= 44.43 \text{ (rad/m)}, \\
\eta_c &= (1 + j) \frac{\alpha_2}{\sigma} = (1 + j) \frac{44.43}{100} = 0.44 (1 + j) \text{ \Omega}.
\end{align*}
\]
According to the expression for \( S_{av2} \) given in the answer to Exercise 8.3,
\[
S_{av2} = \hat{z} |\tau|^2 \frac{|E_0^i|^2}{2} e^{-2\alpha_2 z} \Re \left( \frac{1}{\eta_{c_2}^*} \right).
\]
The power lost is equal to the difference between \( S_{av2} \) at \( z = 0 \) and \( S_{av2} \) at \( z = 2 \text{ mm} \). Thus,
\[
P' = \text{power lost per unit cross-sectional area} \\
= S_{av2}(0) - S_{av2}(z = 2 \text{ mm}) \\
= |\tau|^2 \frac{|E_0^i|^2}{2} \Re \left( \frac{1}{\eta_{c_2}^*} \right) [1 - e^{-2\alpha_2 z_1}]
\]
where \( z_1 = 2 \text{ mm} \).
\[
\tau = 1 + \Gamma \\
= 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 1 + \frac{0.44 (1 + j) - 377}{0.44 (1 + j) + 377} \approx 0.0023 (1 + j) = 3.3 \times 10^{-3} e^{j45^\circ}.
\]
\[ \text{Re} \left( \frac{1}{\eta_{c2}} \right) = \text{Re} \left( \frac{1}{0.44 \left( 1 + j \right)^*} \right) \\
= \text{Re} \left( \frac{1}{0.44 \left( 1 - j \right)} \right) = \frac{1 + j}{0.44 \times 2} = \frac{1}{0.88} = 1.14, \]

\[ P' = \left( 3.3 \times 10^{-3} \right)^2 \frac{10^2}{2} \times 1.14 \left[ 1 - e^{-2 \times 44.43 \times 2 \times 10^{-3}} \right] = 1.01 \times 10^{-4} \text{ (W/m}^2\text{)}. \]