APPLICATION OF MULTIDISCIPLINARY DESIGN OPTIMIZATION TO RACECAR DESIGN AND ANALYSIS

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Abstract
Multidisciplinary design instances arise when the performance of large-scale, complex systems can be affected through the optimal design of several smaller functional units or subsystems. In this paper, we describe the use of multidisciplinary design optimization to resolve system-level tradeoffs during racecar design. Our implementation involves three design variables: weight distribution, aerodynamic downforce distribution, and roll stiffness distribution. The objective is to determine the racecar configuration that minimizes lap time around a skidpad of constant radius while satisfying a yaw balance constraint. The force and aerodynamic components of the design optimization problem provide the multidisciplinary setting in which Collaborative Optimization is implemented and compared with previous results obtained from a traditional optimization formulation.

Keywords: multidisciplinary design optimization, Collaborative Optimization

Nomenclature
A' normalized weight distribution  
C' normalized aero downforce distribution  
K' normalized roll stiffness distribution  
u lap speed  
β vehicle sideslip angle  
δ vehicle wheel steer angle  
et lap time  
Fx tractive force  
Fy lateral force  
Fz normal force  
ID Yaw yaw force  
YawBal yaw force balance

I. Introduction
Multidisciplinary design optimization (MDO) approaches the design problem through a decomposition of the system into its constituent subsystems. These subsystems are intrinsically linked through design, function, and performance. There have been numerous approaches proposed for analyzing such design problems. Multidisciplinary design optimization methods employ individual analyses for each subsystem. These analyses are then aggregated by a system-level coordination procedure that ensures compatibility of the subsystems. Reviews of the fundamental approaches to multidisciplinary design optimization can be found in Refs. 1, 2. In contrast, traditional optimization methods collapse the analysis of the subsystems into the single-level design problem of optimizing the behavior of a conglomerated system-level performance.

Collaborative Optimization is a popular multidisciplinary design and optimization framework. Collaborative Optimization provides added design flexibility by using a system-level optimizer to act on an overall design objective subject to the subsystem compatibility constraints. Applications of Collaborative Optimization include launch vehicle design, aircraft wing design, lunar ascent trajectory, underwater vehicle design, and the design of a combustion chamber for an internal combustion engine. Extensions to Collaborative Optimization include the multiobjective approaches of Tappeta and Renaud and McAllister, et al. and implementations accounting for uncertainty in design. Implementations of Collaborative Optimization are computationally expensive due to large numbers of iterations required to seek attainment of the compatibility constraints, which enforce equality of shared variables. In some applications, convergence issues may arise. DeMiguel and Murray describe a penalty method that has been shown to effectively mitigate these convergence difficulties.
The focus of our work is to pose an existing engineering design scenario in the context of multidisciplinary design and optimization. This implementation will then be available to explore such issues as (i) the utility of using surrogate approximations to computationally expensive subsystem analyses, (ii) the impact of uncertainty on design variables and performance measures, (iii) solution rates and convergence properties of different solution algorithms, and (iv) multidimensional data visualization. These efforts will establish a new test problem for MDO researchers.

The remainder of this paper is organized as follows. In the next section, we discuss the racecar design problem in the context of MDO. Section III presents the traditional and Collaborative Optimization formulations with the corresponding results given in Section IV. Final remarks are offered in Section V.

II. Application

As discussed by Kasprzak, racecar design provides a rich environment in which to apply multidisciplinary design optimization techniques. Racecar configuration and analysis involves knowledge of aerodynamics, structural mechanics, tire performance, and vehicle dynamics. This information is attained from disciplinary experts who have different opinions and control over the performance of the vehicle. The range of adjustment on the design variables may be limited during the racing season (e.g., center of gravity location), and sanctioning bodies limit the amount of on-track testing that can be conducted. As a result, vehicle simulations must be used to optimize a racecar before it is constructed. Advantages gained through simulation increase the vehicle’s potential, and when combined with a talented driver, translate into an increase in on-track performance.

During a lap on a particular racetrack, a driver is faced with a number of different types of corners andstraights. Designing a racecar to perform well across turns of all radii on a single track involves a set of conflicting tradeoffs. Each segment of the racetrack has its own optimal vehicle characteristics. The optimal racecar for tight cornering is vastly different than one for sweeping, large-radii curves. Kasprzak, et al. and Hacker, et al. use multiobjective optimization to maximize racecar performance across multiple tracks of different radii.

Our racecar model is based on the classic bicycle model of Milliken and Milliken, which has been expanded to include four individual wheels. Equations of motion are written for lateral acceleration, longitudinal acceleration, and yaw acceleration. The tires, which may be different for front and rear, are modeled using tabular tire data including representations of nonlinearities such as load sensitivity and slip angle saturation. Wheel loads are calculated based on static load, aerodynamic downforce, and lateral load transfer. Figure 1 illustrates a simplified sketch of the racecar model. There are three primary design variables: roll stiffness distribution ($K'$), weight distribution ($A'$), and aerodynamic downforce distribution ($C'$). All three design variables are normalized quantities between 0 and 1.

![Figure 1. Sketch of the Racecar Model](image)

III. Formulation

This section outlines the equations that govern racecar design. The analysis begins with the calculation of parameters and concludes with an iterative analysis to solve for lateral forces given the center of gravity and roll stiffness.

Table 1 presents the design variables under consideration for the racecar optimization. All design variables have lower and upper bounds of 0.3 and 0.6, respectively.

```
<table>
<thead>
<tr>
<th>Var. Description</th>
<th>Init. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A'$ Weight</td>
<td>0.4</td>
</tr>
<tr>
<td>$C'$ Aero downforce</td>
<td>0.4</td>
</tr>
<tr>
<td>$K'$ Roll stiffness</td>
<td>0.3</td>
</tr>
</tbody>
</table>
```

Table 2 indicates the fixed racecar and track parameters used in this study. For instance, we considered a racecar with a wheelbase of 9.67 feet and mass of 41.7 slugs traveling on a 400-foot radius curve.
Figure 2 illustrates the relationships between lateral forces and slip angles. As indicated, the center of gravity defines the origin of the coordinate system, and clockwise moments are positive.

Equations (1 and 2) are used to compute the front and rear lift coefficients, $CLF$ and $CLR$, respectively, based on the aerodynamic down-force distribution, $C'$.  

\[
CLF = -0.5 \times C' 
\]  

\[
CLR = -1 \times (5 + (-5 \times C')) 
\]  

Equation (3) calculates half the weight of the car, $halfwt$, where $g$ is the acceleration due to gravity.

\[
halfwt = mass \times g/2 
\]  

Equations (4-6) determine the coefficients for front and rear downforce, $FDwnfc$ and $RDwnfc$, and aerodynamic drag, $Dragc$, where $Den$ is the atmospheric density.

\[
FDwnfc = -((Den \times CLF \times RefArea)/2) 
\]  

\[
RDwnfc = -((Den \times CLR \times RefArea)/2) 
\]  

\[
Dragc = -(Den \times CD \times RefArea)/2 
\]  

Table 3 indicates the parameters that must be initialized before proceeding with the iterative analysis to solve for the lateral forces.

Equations (7-9) determine the aerodynamic forces, where positive quantities indicate downforce. The aerodynamic force acting on the front and rear wheels is represented by $AeroFzF$ and $AeroFzR$, respectively. $AeroFx$ is an aerodynamic force that opposes forward motion.

\[
AeroFzF = FDwnfc \times uOld^2 
\]  

\[
AeroFzR = RDwnfc \times uOld^2 
\]  

\[
AeroFx = Dragc \times uOld^2 
\]  

Equation (10) indicates the required tractive effort, $FxReq$, which is always positive.

\[
FxReq = AeroFx + |FyF \times \sin(MaxAlphaF)| 
\]  

\[
+ |FyR \times \sin(MaxAlphaR)| 
\]  

Front and rear wheel loads, $FLT$ and $RLT$, are given by Equations (11 and 12).

\[
FLT = (Fy \times h/tF) \times K' 
\]  

\[
RLT = (Fy \times h/tR) \times (1-K') 
\]  

Equations (13-16) determine the downforce on each of the four wheels. For instance, $FzRF$, is the downforce acting on the right front wheel.

\[
FzLF = (1-A') \times halfwt + FLT + AeroFzF/2 
\]  

\[
FzRF = (1-A') \times halfwt - FLT + AeroFzF/2 
\]  

\[
FzLR = A' \times halfwt + RLT + AeroFzR/2 
\]  

\[
FzRR = A' \times halfwt - RLT + AeroFzR/2 
\]  

Based on the installed tires with tabulated lateral forces due to normal load and slip angle, quadratic approximation is used to determine maximum slip angles, $MaxAlphaF$ and $MaxAlphaR$, and lateral forces, $FyF$ and $FyR$, for the front and rear axles.

Equations (17 and 18) check the lateral forces on the rear wheels, $FyLR$ and $FyRR$, and, if required, reduce these forces due to the friction ellipse effect.
Equation (19) calculates the total rear lateral force, $F_yR$, as a sum of lateral forces acting on each of the two rear wheels.

$$F_yR = F_yLR + F_yRR$$  \hspace{1cm} (19)$$

Equations (20 and 21) determine the total yaw force, $YawBal$.

$$IDYaw = (F_yRF - F_yLF) \times tF \times \sin(MaxAlphaF) + (F_yRR - F_yLR) \times tR \times \sin(MaxAlphaR)$$  \hspace{1cm} (20)$$

$$YawBal = [A' \times F_yF \times \cos(MaxAlphaF)] - [B' \times F_yR \times \cos(MaxAlphaR)] + IDYaw$$  \hspace{1cm} (21)$$

Equations (22 and 23) are used to enforce yaw balance, $YawBal = 0$. If $YawBal < 0$, Equation (22) provides the necessary adjustment, while Equation (23) is used to correct for $YawBal > 0$.

$$F_yF = \frac{((1-A') \times F_yF \times \cos(MaxAlphaF)) - IDYaw}{A' \times \cos(MaxAlphaF)}$$  \hspace{1cm} (22)$$

$$F_yR = \frac{(A' \times F_yR \times \cos(MaxAlphaF)) + IDYaw}{B' \times \cos(MaxAlphaR)}$$  \hspace{1cm} (23)$$

Equation (24) calculates total lateral force, $F_y$, as a sum of front and rear lateral forces. Then, Equations (25 and 26) are used to determine the corresponding speed, $u$, and lap time, $et$, respectively.

$$F_y = F_yF + F_yR$$  \hspace{1cm} (24)$$

$$u = \sqrt{\frac{F_y \times Radius}{mass}}$$  \hspace{1cm} (25)$$

Equations (1-27) establish the traditional optimization formulation for the racecar design problem. Formulating this as a multidisciplinary CO design problem, Figure 3, we define two disciplinary subspaces: (1) aerodynamics and (2) force analysis. Incorporated in the aerodynamic analysis are Equations (1-9) while the force analysis contains Equations (10-24). The system-level coordinator minimizes lap time, Equation 26, and establishes corresponding targets for design variables $A'$, $C'$, and $K'$ and linking variables $AeroFzF$, $AeroFzR$, and $FxReq$. The goal of each subsystem is to minimize deviation from these established targets to ensure the compatibility dictated by a multi-level formulation.

Note that a pure Collaborative Optimization formulation would only establish targets for shared variables. However, in this implementation, the minimum lap time objective cannot be decomposed, due to the iterative loop of Equations (7-26), for analysis by the aerodynamics and forces subsystems as required to enable the subsystems to determine the values of local variables exclusively.

Our traditional and Collaborative Optimization implementations use the compromise Decision Support Problem (DSP)\textsuperscript{17} to model the racecar design problem. The compromise DSP is a multiobjective mathematical programming formulation used to determine the values of the design variables that satisfy a set of constraints and achieve a set of potentially conflicting goals as closely as possible. The compromise DSP is solved using the Adaptive Linear Programming (ALP) algorithm, which is an extension of sequential linear programming and is part of the DSIDES (Decision Support in Designing Engineering Systems) software. A comprehensive discussion of deviation variables, deviation functions, system constraints, goals, bounds, and the ALP algorithm can be found in Ref. 17.
IV. Results

Table 4 summarizes the results for the traditional and Collaborative Optimization formulations. Both achieve identical designs, within a small tolerance. Solution times are reported for a Sun Ultra 60 with a single 450 MHz processor. The increased computational burden of Collaborative Optimization arises from the significant system and subsystem iterations required to achieve system-level compatibility through equality constraints on linking and shared variables.

Table 4. Racecar Optimization Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Traditional Optimization</th>
<th>Collaborative Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A'$</td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>$C'$</td>
<td>0.574</td>
<td>0.574</td>
</tr>
<tr>
<td>$K'$</td>
<td>0.300</td>
<td>0.301</td>
</tr>
<tr>
<td>$et$</td>
<td>14.879 sec</td>
<td>14.884 sec</td>
</tr>
<tr>
<td>Solution Time</td>
<td>3 min</td>
<td>6 hours</td>
</tr>
</tbody>
</table>

Figure 3. Racecar Design by Collaborative Optimization

Figure 4 shows a convergence history for the traditional optimization formulation. After approximately 50 iterations, the optimizer begins cycling between the reported optimum ($et = 14.879$ sec) and infeasible regions with negative wheel loads. The model assumes all four wheels remain in contact with the track. Hence, negative wheel loads are penalized as evidenced by the large vertical spikes in Figure 4.

Figure 4. Traditional Optimization Convergence
Figure 5 illustrates the system-level convergence history for the Collaborative Optimization formulation. Compared to traditional optimization, Figure 4, Collaborative Optimization largely avoids the infeasible region and begins cycling about feasible solutions after approximately 225 iterations.

V. Closing Remarks
An application of multidisciplinary design optimization to racecar design and analysis has been presented. Both traditional and Collaborative Optimization formulations attain identical optimum solutions. The effectiveness of Collaborative Optimization is offset by the increased computation time necessary to enforce the equality constrained system-level compatibility requirement. However, the Collaborative Optimization formulation more accurately represents the disciplinary organization encountered in conceptual design. We are currently applying the racecar model in a probabilistic setting to explore uncertainty propagation in bi-level formulations.

Acknowledgments
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