Cooperative Communication Protocol Designs Based on Optimum Power and Time Allocation

Zijian Mo, *Student Member, IEEE*, Weifeng Su, *Senior Member, IEEE*, Stella Batalama, *Senior Member, IEEE*, and John D. Matyjas, *Senior Member, IEEE*

***Abstract*—Cooperative communication has emerged as a new wireless network communication concept, in which parameter optimization such as power budget and time allocation plays an important role in cooperative relaying protocol designs. While most existing works on cooperative relaying protocol designs considered equal-time allocation scenario, i.e., equal time duration is assigned to each source and each relay, in this work we intend to design and optimize cooperative communication protocols by exploring all possible variations in time and power domains. We consider a cooperative relaying network in which no channel state information (CSI) is available at the transmitter side and the protocol optimization is based on channel statistics (i.e., mean and variance) and it does not depend on instantaneous channel information. First, we consider an ideal cooperative relaying protocol where the system can use arbitrary re-encoding methods at the relay and adjust time allocation arbitrarily. We obtain an optimum strategy of power and time allocations to minimize the outage probability of the ideal cooperative protocol. Specifically, for any given time allocation, we are able to determine the corresponding optimum power allocation analytically with a closed-form expression. We also show that to minimize the outage probability of the protocol, one should always allocate more energy and time to the source than the relay. Second, with more realistic consideration, we design a practical cooperative relaying protocol based on linear mapping, i.e., using linear mapping as the re-encoding method at the relay and considering integer time slots in the two phases. The theoretical results from the ideal cooperative protocol serve as a guideline and benchmark in the practical cooperative protocol design. We also develop an optimum linear mapping to minimize the outage probability of the linear-mapping based cooperative protocol. Extensive numerical and simulation studies illustrate our theoretical developments and show that the performance of the proposed cooperative relaying protocol based on the optimum linear mapping is close to the performance benchmark of the ideal cooperative protocol.**

***Index Terms*—Cooperative communications, decode-and-forward (DF) relaying, optimum time and power allocation, outage probability, wireless networks.**

Manuscript received August 20, 2013; revised January 24, 2014 and March 25, 2014; accepted March 25, 2014. Date of publication April 6, 2014; date of current version August 8, 2014. The material in this paper will be presented in part at IEEE International Conference on Communications, Sydney, Australia, June 2014. This work was supported in part by the U.S. Air Force Research Laboratory (AFRL) under Grants FA87501110015, FA87501110016 and FA87501410074. Security clearance approval for public release (reference number: 88ABW-2013-4002). The associate editor coordinating the review of this paper and approving it for publication was M. McKay.

Z. Mo, W. Su, and S. Batalama are with the Department of Electrical Engineering, State University of New York, Buffalo, NY 14260 USA (e-mail: zijianmo@buffalo.edu; weifeng@buffalo.edu; batalama@buffalo.edu).

J. D. Matyjas is with the Air Force Research Laboratory/RIT, Rome, NY 13441 USA (e-mail: John.Matyjas@us.af.mil).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TWC.2014.2315813

I. Introduction

Cooperative communication has received significant attention recently as an emerging communication concept for wireless networks [1], [2]. Due to the broadcast nature of wireless transmissions, cooperative communications enables neighboring network nodes to share resources and cooperate to send information to an intended node. Distributed transmissions from source and relay nodes provide spatial diversity (as well as multiplexing gain in some designs) for information detection at a destination node. Cooperative communications can significantly improve system performance and robustness of wireless networks especially in severe fading environment. Cooperative relaying techniques have been considered in some latest communication standards, for example, in IEEE 802.16j WiMAX standard [3] and 3GPP's Long Term Evolution (LTE)-Advanced standard [4].

The idea of cooperative communications can be traced back to 1970s [5] and information-theoretic studies have been extensively carried out since then (see [6]–[10] and the references therein). In recent years, besides information-theoretic studies, many efforts have been shifted to design practical cooperative communication protocols for wireless networks with specific system constraints and quality-of-service (QoS) requirements. Various cooperative relaying protocols such as decode-and-forward (DF) relaying protocol and amplify-and-forward (AF) relaying protocol were proposed for wireless networks (see, for examples, [11]–[14]) and substantial performance gains of such relaying protocols have been demonstrated compared to conventional non-cooperative transmission approach. Cooperative relaying protocols have been generalized to multi-relay networks with either parallel or sequential multiple relays to further improve network performance with price of higher complexity and more overhead [11], [15]. Cooperative communication protocols have also been integrated into conventional QoS control mechanisms such as automatic-repeat-request (ARQ) protocols to enhance reliability and robustness of wireless networks [16], [17]. Cooperative relaying techniques have been applied to multimedia wireless broadcast and multicast applications with substantial data rate increase and power saving [18], [19]. More thorough discussions on basic theories, protocols, and applications for cooperative communications can be found in [1], [2].

Resource allocation in cooperative communication protocols such as power budget and time allocation to source and relays plays an important role in the overall performance of the protocols. Note that most existing works on cooperative relaying protocol designs considered equal-time allocation scenario, i.e., equal time duration is assigned to each source and each relay [11]–[22]. For example, with equal-time allocation, in [20] source and relay power allocation was optimized such that the outage probability of the cooperative relaying protocol is minimized. In [21], also with equal-time allocation, optimum power allocation was determined in which symbol error rate (SER) performances of both DF and AF relaying protocols were optimized, respectively. However, there are rather limited studies on *non-equal* time allocation scenario and further on joint optimization of power allocation and time allocation in cooperative relaying protocol designs. In [23], by assuming that instantaneous channel state information (CSI) of source-destination, source-relay and relay-destination links are available, joint power and time allocation were optimized to minimize the outage probability of the cooperative communication system. In [24], it was shown that the problem of minimizing the outage probability with respect to joint power and time allocation at source and relays is a multi-variable convex optimization problem for nonorthogonal cooperative communication systems where the source and the relay are allowed to transmit simultaneously. However, numerical method was considered in [24] to solve the convex optimization problem without analytical solutions. It is hard to obtain insight understanding of the cooperative relaying protocols based on the numerical results. We note that in both [23] and [24], independent codebooks were used for source and relays in the cooperative relaying protocol designs.

In this paper, we intend to design and optimize cooperative communication protocols by exploring all possible variations in time and power domains. We assume that no channel state information (CSI) is available at transmitter side and the protocol optimization is based on channel statistics (i.e., mean and variance) and it does not depend on instantaneous channel information. We consider an orthogonal cooperative communication scenario, in which the source transmits signals in Phase I and the relay decodes the signals and forwards them to the destination in Phase II. First, we analyze an ideal cooperative communication protocol where the system can use arbitrary re-encoding methods at the relay and adjust time allocation arbitrarily between Phases I and II. We develop an asymptotically tight approximation for the outage probability of the cooperative protocol, then based on the asymptotically tight approximation of outage probability, we are able to obtain an optimum strategy of power and time allocations in the ideal cooperative protocol. Specifically, for any given time allocation, we are able to determine the corresponding optimum power allocation at the source and relay analytically with a closed-form expression. We also show theoretically that to minimize the outage probability of the protocol, one should always allocate more energy and time to Phase I than that to Phase II in the protocol.

Note that in the ideal cooperative protocol in which there is no constraint on the re-encoding methods and time allocation, it may not be easy or feasible to implement it in practical systems. So, next we would like to propose a practical cooperative communication protocol design based on linear mapping. The theoretical results from the ideal cooperative protocol will serve as guideline and benchmark in the practical cooperative protocol design. More specifically, we design a cooperative communication protocol by considering linear mapping as the re-encoding method at the relay, where the protocol uses integer time slots in Phases I and II. It is much easier to implement the linear mapping forwarding method with the time allocation of integer time slots in the two phases in practical systems. We also develop an optimum linear mapping to minimize the outage probability of the linear-mapping based cooperative protocol. Interestingly, simulation results show that the performance of the proposed cooperative protocol based on the optimum linear mapping is close to the performance benchmark of the ideal cooperative protocol.

The rest of the paper is organized as follows. In Section II, we introduce a general system model for cooperative communication protocol design with flexibility in time/power allocation and arbitrary re-encoding methods. In Section III, we study the ideal cooperative protocol and determine the corresponding optimum time and power allocations analytically. In Section IV, we design the practical cooperative communication protocol based on linear mapping and optimize the linear mapping function to minimize the outage probability of the protocol. In Section V, extensive numerical and simulation studies are presented to verify our theoretical development. Finally, conclusions are made in Section VI.

The paper uses the following notation. Bold letters in uppercase and lowercase denote matrices and vectors, respectively. $(\cdot)^{\mathcal{H}}$, $det(\cdot)$ and $Tr(\cdot)$ represent Hermitian transpose, determinant and trace operators, respectively. $\mathbf{I}_L$ is an $L \times L$ identity matrix, and $\mathbf{diag}(\lambda_1, \ldots, \lambda_L)$ is an $L \times L$ diagonal matrix with diagonal elements $\lambda_1, \ldots, \lambda_L$.

II. System Model

We consider a cooperative wireless network that consists of one source, one destination and one relay using DF relaying protocol. The cooperative strategy is described as followed with two phases. In Phase I, the source transmits information signal to the destination, and the signal is also received by the relay as well. In Phase II, if the relay is able to fully decode the information signal, it helps forwarding the information to the destination via certain re-encoding/transform methods. Throughout this paper, we consider narrowband transmissions in the wireless network in which channel between any two nodes is subject to the effects of frequency nonselective Rayleigh fading and additive white Gaussian noise (AWGN). We assume that the channel state information is available only at the receivers, not at the transmitters. Nodes in the network work in a half-duplex mode where they cannot transmit and receive simultaneously in a same frequency band.

More specifically, in Phase I, the source broadcasts its information signal $x_s(t)$ to both the destination and the relay during time interval $(0, T_1]$. The received signals $y_{s,d}(t)$ and $y_{s,r}(t)$ at the destination and at the relay can be modeled as

$$y_{s,d}(t) = \sqrt{P_1} h_{s,d} x_s(t) + n_{s,d}(t), \quad 0 < t \leq T_1, \qquad (1)$$

$$y_{s,r}(t) = \sqrt{P_1} h_{s,r} x_s(t) + n_{s,r}(t), \quad 0 < t \leq T_1, \qquad (2)$$

where P_1 is the transmission power, $x_s(t)$ is normalized with power 1, and $n_{s,d}(t)$ and $n_{s,r}(t)$ are corresponding received noise at the destination and the relay. In (1) and (2), $h_{s,d}$ and $h_{s,r}$ are the channel coefficients from the source to the destination and the relay, respectively. In Phase II, if the relay fully decodes the information from the source, then the relay re-encodes the information and forwards it to the destination with power P_2, otherwise the relay remains idle. We denote the time duration of Phase II as T_2. Then, the received signal $y_{r,d}(t)$ at the destination in Phase II can be modeled as

$$y_{r,d}(t) = \sqrt{\tilde{P}_2} h_{r,d} x_r(t) + n_{r,d}(t), \quad T_1 < t \le T_1 + T_2 \quad (3)$$

where $\tilde{P}_2 = P_2$ if the relay correctly decodes the information signal $x_s(t)$, otherwise $\tilde{P}_2 = 0$, $h_{r,d}$ is the channel coefficient from the relay to the destination, and $n_{r,d}(t)$ is received noise at the destination in Phase II. In (3), $x_r(t) \triangleq \mathcal{M}(x_s(t))$ is a re-encoded version of the original information signal $x_s(t)$ and it is normalized with average power 1. We note that theoretically arbitrary re-encoding function $\mathcal{M}(\cdot)$ may be considered in the protocol design.

We assume that the channel coefficients $h_{s,d}$, $h_{s,r}$, and $h_{r,d}$ are modeled as zero-mean complex Gaussian random variables with variances $\delta_{s,d}^2$, $\delta_{s,r}^2$, and $\delta_{r,d}^2$, respectively. The noise terms $n_{s,d}(t)$, $n_{s,r}(t)$, and $n_{r,d}(t)$ are modeled as zero-mean AWGN with variance $\mathcal{N}_0$. We denote the total time duration of each transmission period as $T \triangleq T_1 + T_2$ and assume that the fading channels are quasi-static within each transmission period. We denote the ratio of the time allocation in Phase I over the whole period as $\alpha \triangleq T_1/T \in (0, 1)$. If the average transmission power of the protocol in each transmission period is P, then the source and relay transmission powers P_1 and P_2 should satisfy the constraint

$$P_1 T_1 + P_2 T_2 = PT \quad (4)$$

or equivalently $\alpha P_1 + (1-\alpha) P_2 = P$. For convenience, we further denote β as the ratio of the energy consumed in Phase I over the total energy consumption in each transmission period, i.e., $\beta \triangleq P_1 T_1 / PT$, so we have $\beta = \alpha P_1 / P$.

We note that the system model specified above for cooperative relaying protocol designs has two variable domains to optimize: the power domain of allocating P_1 and P_2 and the time domain of allocating T_1 and T_2. With given power budget P, we intend to optimize the power variables and the time variables such that the outage probability of the cooperative relaying protocol is minimized. The outage probability is defined as the probability that the maximum mutual information $\mathcal{I}$ of the transceiver is smaller than a predetermined target transmission rate R_T. If $\mathcal{I} > R_T$, we assume that the receiver can decode the message correctly with negligible error probability. In the following, we first study an ideal cooperative relaying protocol where there is no constraint on the re-encoding function $\mathcal{M}(\cdot)$ and time allocation. The corresponding theoretical analysis will serve as guideline and benchmark in following practical cooperative communication protocol design. Then, with more realistic consideration for implementation, we propose a practical cooperative relaying protocol by considering linear mapping technique as the re-encoding function $\mathcal{M}(\cdot)$, and interestingly, the performance of the proposed practical protocol design is very close to the performance benchmark of the ideal cooperative protocol.

III. Ideal Cooperative Communication Protocol

In this section, we focus on joint optimization of power allocation and time allocation in an ideal cooperative protocol where the system can use arbitrary re-encoding function $\mathcal{M}(\cdot)$ at the relay and adjust time allocation arbitrarily between Phases I and II. It has been shown that the relay and the source can use independent codebooks to achieve maximum rate in cooperative communication protocols [23]. In the ideal cooperative protocol, we assume that the source and the relay perfectly cooperate by using two independent codebooks. We also assume that the system can arbitrarily allocate time duration to both phases, i.e., T_1 and T_2 can be arbitrary positive numbers ($T_1 + T_2 = T$). In the following, we first calculate the outage probability of the ideal cooperative protocol. Then, we obtain optimum power and time allocation to minimize the outage probability of the ideal cooperative protocol. For any give time allocation $\alpha \in (0, 1)$, we determine the optimum power allocation at the source and the relay analytically with closed-form expression. Finally, we show that to minimize the outage probability of the protocol, one should allocate more energy and time to Phase I than Phase II.

A. Maximum Mutual Information and Outage Probability

First, we would like to derive the outage probability of the ideal cooperative relaying protocol. In Phase I, with independently and identically distributed (i.i.d.) circularly symmetric complex Gaussian input signals, the maximum mutual information between the source and the destination is given by

$$\mathcal{I}_{s,d} = \alpha \log_2 \left(1 + \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0} \right) \quad (5)$$

in which the time allocation ratio α shows the fact that Phase I occupies time duration T_1 in each transmission period $(0, T]$. Since $h_{s,d} \sim \mathcal{CN}(0, \delta_{s,d}^2)$, then $|h_{s,d}|^2$ is an exponential random variable with parameter $\lambda_{s,d} = 1/\delta_{s,d}^2$. Thus, the probability that the destination decodes incorrectly in Phase I can be calculated as

$$Pr[\mathcal{I}_{s,d} < R_T] = 1 - \exp\left\{ -\frac{\mathcal{N}_0}{P_1 \delta_{s,d}^2} \left(2^{\frac{R_T}{\alpha}} - 1 \right) \right\}. \quad (6)$$

Similarly, the maximum mutual information $\mathcal{I}_{s,r}$ between the source and the relay in Phase I is

$$\mathcal{I}_{s,r} = \alpha \log_2 \left(1 + \frac{P_1 |h_{s,r}|^2}{\mathcal{N}_0} \right) \quad (7)$$

and the outage probability that the relay fails to decode the message in Phase I is

$$Pr[\mathcal{I}_{s,r} < R_T] = 1 - \exp\left\{ -\frac{\mathcal{N}_0}{P_1 \delta_{s,r}^2} \left(2^{\frac{R_T}{\alpha}} - 1 \right) \right\}. \quad (8)$$

If the relay fully decodes the message from the source, then the relay forwards the message to the destination by using an independent codebook. With two independent codebooks at the source and relay, the channels of source-destination and relay-destination can be viewed as a pair of parallel channels, thus the joint maximum mutual information from the source to the destination in the two phases is given by

$$\mathcal{I}_{joint} = \alpha \log_2 \left(1 + \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0}\right) + (1-\alpha) \log_2 \left(1 + \frac{P_2 |h_{r,d}|^2}{\mathcal{N}_0}\right). \quad (9)$$

Since $h_{r,d} \sim \mathcal{CN}(0, \delta_{r,d}^2)$, $|h_{r,d}|^2$ is an exponential random variable with parameter $\lambda_{r,d} = 1/\delta_{r,d}^2$. The probability density function (pdf) of the random variable $|h_{r,d}|^2$ is $f_{|h_{r,d}|^2}(z_{r,d}) = (1/\delta_{r,d}^2)\exp\{-(z_{r,d}/\delta_{r,d}^2)\}$ with $z_{r,d} \geq 0$. Similarly, the pdf of the random variable $|h_{s,d}|^2$ is given by $f_{|h_{s,d}|^2}(z_{s,d}) = (1/\delta_{s,d}^2)\exp\{-(z_{s,d}/\delta_{s,d}^2)\}$ with $z_{s,d} \geq 0$. Then, with the relay signal, the probability that the destination decodes incorrectly is given by (10), shown at the bottom of the page. Note that the outage events of the cooperative relaying protocol have two possibilities: 1) both the destination and the relay fail to decode the message in phase I; and 2) the destination fails to decode the message jointly in Phase II even when the relay fully decodes the message and helps forwarding the message to the destination. Therefore, the overall outage probability $\mathcal{P}_{out}$ of the ideal cooperative protocol is given by (11), shown at the bottom of the page.

B. Optimum Power and Time Allocations

In this subsection, we would like to minimize the outage probability in (11) to determine optimum power and time allocations for the ideal cooperative relaying protocol. However, the closed-form expression in (11) is not tractable for analytical purpose. Therefore, in the following, we first derive an approximation of the outage probability (11) which is asymptotically tight, then we determine the optimum time and power allocations based on the asymptotically tight approximation of the outage probability.

We would like to use the first-order Taylor series approximation, i.e., $\exp(x) \approx 1 + x$ for x close to 0, to simplify the two terms $Pr[\mathcal{I}_{s,d} < R_T]$ and $Pr[\mathcal{I}_{s,r} < R_T]$ in (11). With high SNR where $P_1/\mathcal{N}_0$ is large, we have

$$Pr[\mathcal{I}_{s,d} < R_T] \approx \frac{\mathcal{N}_0 (2^{\frac{R_T}{\alpha}} - 1)}{P_1 \delta_{s,d}^2}, \quad (12)$$

$$Pr[\mathcal{I}_{s,r} < R_T] \approx \frac{\mathcal{N}_0 \left(2^{\frac{R_T}{\alpha}} - 1\right)}{P_1 \delta_{s,r}^2}. \quad (13)$$

Thus, the product of $Pr[\mathcal{I}_{s,d} < R_T]$ and $Pr[\mathcal{I}_{s,r} < R_T]$ can be approximated as

$$Pr[\mathcal{I}_{s,d} < R_T] \cdot Pr[\mathcal{I}_{s,r} < R_T] \approx \frac{\mathcal{N}_0^2}{P_1^2 \delta_{s,d}^2} A(\alpha) \quad (14)$$

where

$$A(\alpha) = \frac{\left(2^{\frac{R_T}{\alpha}} - 1\right)^2}{\delta_{s,r}^2}. \quad (15)$$

$$Pr[\mathcal{I}_{joint} < R_T] = \int_0^{\frac{\mathcal{N}_0}{P_1}\left(2^{\frac{R_T}{\alpha}}-1\right)} f_{|h_{s,d}|^2}(z_{s,d}) \left(\int_0^{\frac{\mathcal{N}_0}{P_2}\left(2^{\frac{R_T - \alpha \log_2(1+P_1 z_{s,d}/\mathcal{N}_0)}{1-\alpha}}-1\right)} f_{|h_{r,d}|^2}(z_{r,d}) dz_{r,d} \right) dz_{s,d}$$
$$= \int_0^{\frac{\mathcal{N}_0}{P_1}\left(2^{\frac{R_T}{\alpha}}-1\right)} \left(1 - \exp\left\{-\frac{\mathcal{N}_0}{P_2 \delta_{r,d}^2}\left(2^{\frac{R_T - \alpha \log_2(1+P_1 z_{s,d}/\mathcal{N}_0)}{1-\alpha}} - 1\right)\right\}\right) \frac{\exp\left\{-\frac{z_{s,d}}{\delta_{s,d}^2}\right\}}{\delta_{s,d}^2} dz_{s,d} \quad (10)$$

$$\mathcal{P}_{out} = Pr[\mathcal{I}_{s,d} < R_T] Pr[\mathcal{I}_{s,r} < R_T] + (1 - Pr[\mathcal{I}_{s,r} < R_T]) Pr[\mathcal{I}_{joint} < R_T]$$
$$= \left(1 - \exp\left\{-\frac{\mathcal{N}_0}{P_1 \delta_{s,d}^2}\left(2^{\frac{R_T}{\alpha}} - 1\right)\right\}\right)\left(1 - \exp\left\{-\frac{\mathcal{N}_0}{P_1 \delta_{s,r}^2}\left(2^{\frac{R_T}{\alpha}} - 1\right)\right\}\right) + \exp\left\{-\frac{\mathcal{N}_0}{P_1 \delta_{s,r}^2}\left(2^{\frac{R_T}{\alpha}} - 1\right)\right\}$$
$$\times \int_0^{\frac{\mathcal{N}_0}{P_1}\left(2^{\frac{R_T}{\alpha}}-1\right)} \left(1 - \exp\left\{-\frac{\mathcal{N}_0}{P_2 \delta_{r,d}^2}\left(2^{\frac{R_T - \alpha \log_2(1+P_1 z_{s,d}/\mathcal{N}_0)}{1-\alpha}} - 1\right)\right\}\right) \frac{1}{\delta_{s,d}^2} \exp\left\{-\frac{z_{s,d}}{\delta_{s,d}^2}\right\} dz_{s,d} \quad (11)$$

Note that $A(\alpha) > 0$ and its first-order differential is continuous for any $\alpha \in (0,1)$. Furthermore, we can see in (13) that the term $Pr[\mathcal{I}_{s,r} < R_T] \ll 1$ in the high SNR region, so we have $1 - Pr[\mathcal{I}_{s,r} < R_T] \approx 1$ with large $P_1/\mathcal{N}_0$.

To obtain an asymptotically tight approximation for the term $Pr[\mathcal{I}_{joint} < R_T]$, we need the following lemma that was developed in [17].

Lemma 1 [17]: Assume that u_{s_1} and v_{s_2} be two independent scalar random variables. If their cumulative distribution functions (CDF) satisfy the following properties

$$\lim_{s_1\to\infty} s_1 \cdot Pr[u_{s_1} < R] = a \cdot f(R),$$

$$\lim_{s_2\to\infty} s_2 \cdot Pr[v_{s_2} < R] = b \cdot g(R)$$

where a and b are constants, $f(R)$ and $g(R)$ are monotonically increasing functions, and the derivative of $g(R)$, denoted as $g'(R)$, is integrable, then the CDF of the sum of the two independent random variables has the following property:

$$\lim_{s_1,s_2\to\infty} s_1 s_2 \cdot Pr[u_{s_1}+v_{s_2}<R] = ab\cdot\int_0^R f(r)g'(R-r)dr. \quad (16)$$

To use Lemma 1, we observe that $\mathcal{I}_{joint}$ in (9) includes two independent random variables which can be written as $\mathcal{I}_{joint} = u_{s_1} + u_{s_2}$, where $u_{s_1} = \alpha \log_2(1 + (P_1|h_{s,d}|^2/\mathcal{N}_0))$ and $u_{s_2} = (1-\alpha)\log_2(1 + (P_2|h_{r,d}|^2/\mathcal{N}_0))$. Let $s_1 = P_1/\mathcal{N}_0$ and $s_2 = P_2/\mathcal{N}_0$, and since $|h_{s,d}|^2$ and $|h_{r,d}|^2$ are exponential random variables with parameter $1/\delta_{s,d}^2$ and $1/\delta_{r,d}^2$, so we have

$$\lim_{s_1\to\infty} s_1 \cdot Pr[u_{s_1}<R_T] = \lim_{s_1\to\infty} s_1 \cdot Pr\left[|h_{s,d}|^2 < \frac{2^{\frac{R_T}{\alpha}}-1}{s_1}\right] = \frac{1}{\delta_{s,d}^2}\underbrace{\left(2^{\frac{R_T}{\alpha}}-1\right)}_{\triangleq f(R_T)}, \quad (17)$$

$$\lim_{s_2\to\infty} s_2 \cdot Pr[u_{s_2}<R_T] = \lim_{s_2\to\infty} s_2 \cdot Pr\left[|h_{r,d}|^2 < \frac{2^{\frac{R_T}{1-\alpha}}-1}{s_2}\right] = \frac{1}{\delta_{r,d}^2}\underbrace{\left(2^{\frac{R_T}{1-\alpha}}-1\right)}_{\triangleq g(R_T)}. \quad (18)$$

We can see that in (17) and (18), both $f(R_T)$ and $g(R_T)$ are monotonically increasing functions, and furthermore we have $g'(R_T) = (\ln 2/(1-\alpha))2^{R_T/(1-\alpha)}$. By applying Lemma 1, we obtain an approximation of $Pr[\mathcal{I}_{joint} < R_T]$ at the high SNR region as

$$\begin{aligned} Pr[\mathcal{I}_{joint}<R_T] &\approx \frac{\mathcal{N}_0^2}{P_1P_2\delta_{s,d}^2\delta_{r,d}^2}\int_0^{R_T} f(r)g'(R_T-r)dr \\ &= \frac{\mathcal{N}_0^2}{P_1P_2\delta_{s,d}^2\delta_{r,d}^2}\int_0^{R_T}\left(2^{\frac{r}{\alpha}}-1\right)\frac{\ln 2}{1-\alpha}2^{\frac{R_T-r}{1-\alpha}}dr \\ &= \frac{\mathcal{N}_0^2}{P_1P_2\delta_{s,d}^2}\underbrace{\frac{\ln 2}{\delta_{r,d}^2(1-\alpha)}2^{\frac{R_T}{1-\alpha}}\int_0^{R_T}\left[2^{\frac{r(1-2\alpha)}{\alpha(1-\alpha)}}-2^{\frac{r}{\alpha-1}}\right]dr}_{\triangleq B(\alpha)}. \quad (19)\end{aligned}$$

In (19), we observe that $B(\alpha) > 0$ and its first-order differential is continuous for any $\alpha \in (0,1)$. To further simplify (19), we calculate the integral in $B(\alpha)$ by considering two cases: $\alpha = 1/2$ and $\alpha \neq 1/2$. When $\alpha = 1/2$, $B(\alpha)$ can be derived as

$$\begin{aligned} B(\alpha) &= \frac{2\ln 2}{\delta_{r,d}^2}2^{2R_T}\int_0^{R_T}\left[1-2^{-2r}\right]dr \\ &= \frac{1}{\delta_{r,d}^2}(2R_T\ln 2\cdot 2^{2R_T} - 2^{2R_T}+1). \quad (20)\end{aligned}$$

When $\alpha \neq 1/2$, since $\int_0^{R_T} 2^{cr}dr = (1/c\ln 2)[2^{cR_T}-1]$ for any constant $c > 0$, we have

$$\begin{aligned} B(\alpha) &= \frac{\ln 2}{\delta_{r,d}^2(1-\alpha)}2^{\frac{R_T}{1-\alpha}}\left[\int_0^{R_T}2^{\frac{r(1-2\alpha)}{\alpha(1-\alpha)}}dr - \int_0^{R_T}2^{\frac{r}{\alpha-1}}dr\right] \\ &= \frac{\ln 2}{\delta_{r,d}^2(1-\alpha)}2^{\frac{R_T}{1-\alpha}}\left[\frac{\alpha(1-\alpha)}{(1-2\alpha)\ln 2}\left(2^{\frac{R_T(1-2\alpha)}{\alpha(1-\alpha)}}-1\right) - \frac{\alpha-1}{\ln 2}\left(2^{\frac{R_T}{\alpha-1}}-1\right)\right] \\ &= \frac{1}{\delta_{r,d}^2}\left(\frac{\alpha}{1-2\alpha}2^{\frac{R_T}{\alpha}} + \frac{\alpha-1}{1-2\alpha}2^{\frac{R_T}{1-\alpha}}+1\right). \quad (21)\end{aligned}$$

Therefore, $B(\alpha)$ in (19) can be given explicitly as

$$B(\alpha) = \begin{cases} \frac{1}{\delta_{r,d}^2}(2R_T\ln 2\cdot 2^{2R_T} - 2^{2R_T}+1), & \alpha = \frac{1}{2}; \\ \frac{1}{\delta_{r,d}^2}\left(\frac{\alpha}{1-2\alpha}2^{\frac{R_T}{\alpha}} + \frac{\alpha-1}{1-2\alpha}2^{\frac{R_T}{1-\alpha}}+1\right), & \text{otherwise.}\end{cases} \quad (22)$$

We summarize the above discussion on the approximation of the outage probability in the following theorem.

Theorem 1: In the ideal cooperative protocol, the outage probability can be approximated as

$$\mathcal{P}_{out} \approx \tilde{\mathcal{P}}_{out} \triangleq \frac{\mathcal{N}_0^2 A(\alpha)}{\delta_{s,d}^2 P_1^2} + \frac{\mathcal{N}_0^2 B(\alpha)}{\delta_{s,d}^2 P_1 P_2} \quad (23)$$

where $A(\alpha)$ and $B(\alpha)$ are specified in (15) and (22), respectively, and the approximation is asymptotically tight at high SNR.

In Fig. 1, we compare the exact value of the outage probability which was calculated based on (11) and the asymptotic approximation of the outage probability in (23) in three scenarios with different channel variances: (i) $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1,1,1\}$; (ii) $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1,10,1\}$; and (iii) $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1,1,10\}$. In the comparison, we assume that $\alpha = 1/2$, $R_T = 2$, $\mathcal{N}_0 = 1$, and $P_1 = P_2 = P$.

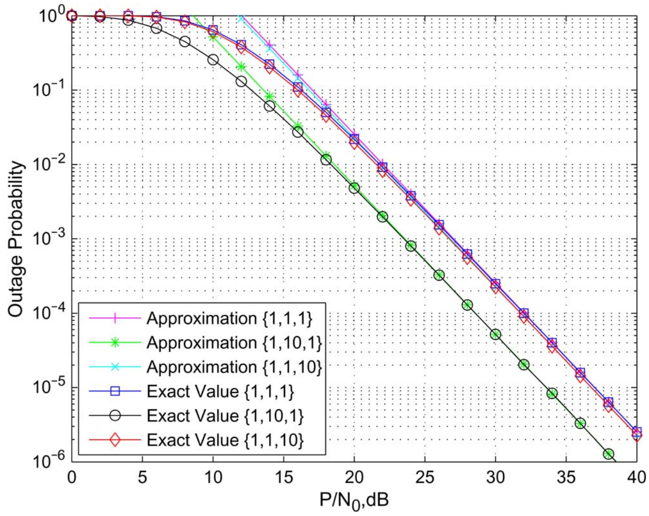


Fig. 1. Comparison of the exact and approximation of the outage probability in three cases: $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1,1,1\}$, $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1,10,1\}$ and $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1,1,10\}$. Assume that $\alpha = 1/2$, $R_T = 2$, $\mathcal{N}_0 = 1$, and $P_1 = P_2 = P$.

From the figure, we can see that the approximation of the outage probability is tight at reasonable high SNR for various channel conditions. The curve of the outage probability approximation merges with the curve of the exact calculation at an outage probability of 10^{-2} in each case. From Fig. 1, we also observe that when the SNR is lower than 18 dB, the approximation of the outage probability is not tight in which, however, the corresponding outage probability is around 10^{-1} and it may not be acceptable in many practical applications. Thus, for practical systems with reasonable high SNR requirement, the asymptotically tight approximation of the outage probability in (23) is useful to get some insight understanding of the system performance. More comparisons of the approximation and exact value of the outage probability can be found in Figs. 2–4 with $\alpha \neq 1/2$.

Next, we would like to jointly optimize power and time allocations for the ideal cooperative protocol based on the asymptotically tight approximation of the outage probability developed in Theorem 1. Note that the time allocation ratio $\alpha = T_1/T$ may take any number in the range of (0, 1), while the power parameters P_1 and P_2 should satisfy the power constraint in (4), i.e., $\alpha P_1 + (1-\alpha)P_2 = P$. The problem of optimizing time and power can be specified as follows:

$$\begin{aligned} \min_{\alpha, P_1, P_2} \quad & \tilde{\mathcal{P}}_{out}(\alpha, P_1, P_2) \triangleq \frac{\mathcal{N}_0^2 A(\alpha)}{\delta_{s,d}^2 P_1^2} + \frac{\mathcal{N}_0^2 B(\alpha)}{\delta_{s,d}^2 P_1 P_2} \\ \text{s.t.} \quad & \alpha P_1 + (1-\alpha)P_2 = P, \\ & 0 < \alpha < 1, \quad P_1 > 0, \quad P_2 > 0. \end{aligned} \tag{24}$$

We found that for any given time allocation ratio $\alpha \in (0,1)$, we are able to express the corresponding optimum powers P_1 and P_2 in terms of the time allocation ratio α with closed-form expressions, which are denoted as $P_1^*(\alpha)$ and $P_2^*(\alpha)$, respectively. Moreover, we found that for any time allocation ratio $\alpha \in (0,1)$, the protocol should allocate more energy to Phase I than that to Phase II (i.e., $\beta = (\alpha P_1^*(\alpha)/P) > 1/2$) to minimize the outage probability of the protocol. The results are summarized in the following theorem and the proof is included in Appendix I.

Theorem 2: In the ideal cooperative protocol, for any given time allocation ratio $\alpha \in (0,1)$, the corresponding optimum powers $P_1^*(\alpha)$ and $P_2^*(\alpha)$ are given by

$$P_1^*(\alpha) = \frac{1}{\alpha} \cdot \frac{1+\sqrt{1+8\left[\alpha A(\alpha)\right] / \left[(1-\alpha)B(\alpha)\right]}}{3+\sqrt{1+8\left[\alpha A(\alpha)\right] / \left[(1-\alpha)B(\alpha)\right]}} P, \tag{25}$$

$$P_2^*(\alpha) = \frac{1}{1-\alpha} \cdot \frac{2}{3+\sqrt{1+8\left[\alpha A(\alpha)\right] / \left[(1-\alpha)B(\alpha)\right]}} P, \tag{26}$$

where $A(\alpha)$ and $B(\alpha)$ are specified in (15) and (22), respectively. Moreover, the resulting energy allocation ratio $\beta = \alpha P_1^*(\alpha)/P$ is strictly larger than 1/2, which means that, the protocol should allocate more energy to Phase I than that to Phase II to minimize the outage probability.

Based on Theorem 2, we substitute the optimum power solutions $P_1^*(\alpha)$ and $P_2^*(\alpha)$ into the optimization problem (24) to find the optimum time allocation ratio, i.e.,

$$\begin{aligned} \min_{\alpha} \quad & \tilde{\mathcal{P}}_{out}(\alpha) \triangleq \frac{\mathcal{N}_0^2 A(\alpha)}{\delta_{s,d}^2 P_1^*(\alpha)^2} + \frac{\mathcal{N}_0^2 B(\alpha)}{\delta_{s,d}^2 P_1^*(\alpha) P_2^*(\alpha)} \\ \text{s.t.} \quad & 0 < \alpha < 1. \end{aligned} \tag{27}$$

We can see that the optimization in (27) has only a single variable α, and we can apply numerical search of the single variable α over the interval (0, 1) to obtain the optimum time allocation ratio α^* that minimizes the asymptotic outage probability $\tilde{\mathcal{P}}_{out}$. The objective function in (27) is continuous in terms of the variable α, thus we are able to obtain the optimum value of α with sufficiently small searching step size. With the optimum time allocation ratio α^*, based on Theorem 2 we can get the corresponding optimum source transmission power $P_1^*(\alpha^*)$ and the optimum relay transmission power $P_2^*(\alpha^*)$. We note that the above two-step optimization procedure [i.e., using Theorem 2 to reduce the three-variable optimization problem in (24) into the single-variable optimization problem in (27)] is guaranteed to find the global optimal solution for the joint optimization with the three variables α, P_1 and P_2. Let us denote $(\alpha^*, P_1^*(\alpha^*), P_2^*(\alpha^*))$ as the solution of optimum power and time allocation for the ideal cooperative protocol. We have the following result regarding the optimum time allocation α^*.

Theorem 3: In the ideal cooperative protocol, the optimum time allocation ratio α^* is strictly larger than 1/2, i.e., $\alpha^* \in (1/2, 1)$, which means that the protocol should allocate more time to Phase I than to Phase II to minimize the outage probability of the protocol.

The proof of Theorem 3 is included in Appendix II. Theorem 3 shows that the equal-time allocation strategy in most existing cooperative relaying protocol (allocating equal time among the two phases) is not optimal in general.

IV. Practical Cooperative Communication Protocol Design Based on Linear Mapping

It is difficult, sometimes may be infeasible, to implement the ideal cooperative protocol as the re-encoding function $\mathcal{M}(\cdot)$

and time allocation can be arbitrary. In this section, with more realistic consideration, we design a cooperative communication protocol based on linear mapping, where the relay uses linear mapping as the re-encoding function $\mathcal{M}(\cdot)$. Also, the practical cooperative protocol allocates integer time slots in Phases I and II. It is much easier to implement the linear mapping forwarding method with the time allocation of integer time slots in both phases. The theoretical results from the ideal cooperative protocol in the previous section will serve as guideline and benchmark for the proposed linear-mapping based cooperative protocol design. Interestingly, the practical cooperative protocol based on optimum linear mapping performs closely to the performance benchmark of the ideal cooperative protocol. In the following, we first specify the practical cooperative communication protocol design based on linear mapping. Then we optimize the linear mapping of the protocol such that the outage probability of the proposed protocol is minimized.

A. Protocol Design

We intend to design a practical cooperative relaying protocol with L time slots in Phase I and K time slots in Phase II. We assume that each time slot has time duration T_s. In Phase I, the source broadcasts a block of symbols $\mathbf{x}_s(\mathbf{x}_s = (x_s[1], \ldots, x_s[L])^T)$ using L time slots, in which each element of $\mathbf{x}_s$ has unit power. In Phase II, if the relay successfully decodes the message, then the relay takes K time slots to forward a re-encoded message $\mathbf{x}_r(\mathbf{x}_r = (x_r[1], \ldots, x_r[K])^T)$, where $\mathbf{x}_r = \mathbf{G}\mathbf{x}_s$ and $\mathbf{G}$ is a $K \times L$ matrix representation of the linear mapping. In the protocol, the total time duration of Phase I is $T_1 = LT_s$ and the time duration of Phase II is $T_2 = KT_s$, thus the time allocation ratio α of the protocol is

$$\alpha = \frac{LT_s}{LT_s + KT_s} = \frac{L}{L+K}. \tag{28}$$

Based on the theoretical results in the previous section, we know that we should allocate more time to Phase I than that to Phase II to minimize the outage probability (i.e., $1/2 < \alpha < 1$), which means that we should choose $K < L$ in the practical cooperative protocol design.

The general system model in Section II can be modified for the linear-mapping based cooperative protocol in a discrete version as follows:

$$\mathbf{y}_{s,d} = \sqrt{P_1}\mathbf{H}_{s,d}\mathbf{x}_s + \mathbf{n}_{s,d}, \tag{29}$$

$$\mathbf{y}_{s,r} = \sqrt{P_1}\mathbf{H}_{s,r}\mathbf{x}_s + \mathbf{n}_{s,r}, \tag{30}$$

$$\mathbf{y}_{r,d} = \sqrt{\tilde{P}_2}\mathbf{H}_{r,d}\mathbf{x}_r + \mathbf{n}_{r,d}, \tag{31}$$

where $\tilde{P}_2 = P_2$ if the relay correctly decodes the information signal $x_s(t)$, otherwise $\tilde{P}_2 = 0$ (the power parameters P_1 and P_2 are selected based on the designed guideline from the previous section), $\mathbf{H}_{s,d} = h_{s,d}\mathbf{I}_L$, $\mathbf{H}_{s,r} = h_{s,r}\mathbf{I}_L$, $\mathbf{H}_{r,d} = h_{r,d}\mathbf{I}_K$, $\mathbf{y}_{s,d}$ and $\mathbf{y}_{s,r}$ are the signal vectors of size $L \times 1$ received at the destination and the relay, respectively during Phase I, $\mathbf{y}_{r,d}$ is the signal vector of size $K \times 1$ received at the destination during Phase II, $\mathbf{n}_{s,d}$ and $\mathbf{n}_{s,r}$ are the noise vectors of size $L \times 1$ at the destination and the relay, respectively in Phase I, and $\mathbf{n}_{r,d}$ is the noise vector of size $K \times 1$ at the destination in Phase II. In (29)–(31), the elements of the noise vectors $\mathbf{n}_{s,d}$, $\mathbf{n}_{s,r}$ and $\mathbf{n}_{r,d}$ are independent Gaussian random variables with mean zero and variance $\mathcal{N}_0$. The channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are modeled as zero-mean complex Gaussian random variables with variances $\delta_{s,d}^2$, $\delta_{s,r}^2$ and $\delta_{r,d}^2$, respectively. We note that the overall power constraint in (4) should be satisfied in the linear-mapping based cooperative protocol. There may be power saving when relay not forwarding information ($\tilde{P}_2 = 0$), however the probability of relay not forwarding is small and the power saving is negligible.

At the destination, it combines the received signals $\mathbf{y}_{s,d}$ and $\mathbf{y}_{r,d}$ from both phases to jointly detect the original message $\mathbf{x}_s$. The combined signal at the destination can be expressed as

$$\mathbf{y}_d \triangleq \begin{pmatrix} \mathbf{y}_{s,d} \\ \mathbf{y}_{r,d} \end{pmatrix} = \begin{pmatrix} \sqrt{P_1}\mathbf{H}_{s,d}\mathbf{x}_s \\ \sqrt{\tilde{P}_2}\mathbf{H}_{r,d}\mathbf{x}_r \end{pmatrix} + \begin{pmatrix} \mathbf{n}_{s,d} \\ \mathbf{n}_{r,d} \end{pmatrix}. \tag{32}$$

Since $\mathbf{x}_r = \mathbf{G}\mathbf{x}_s$, the combined signal $\mathbf{y}_d$ can be further simplified as

$$\mathbf{y}_d = \mathbf{H}_d\mathbf{x}_s + \mathbf{n}_d \tag{33}$$

where $\mathbf{H}_d \triangleq \begin{pmatrix} \sqrt{P_1}\mathbf{H}_{s,d} \\ \sqrt{\tilde{P}_2}\mathbf{H}_{r,d}\mathbf{G} \end{pmatrix}$, $\mathbf{n}_d \triangleq \begin{pmatrix} \mathbf{n}_{s,d} \\ \mathbf{n}_{r,d} \end{pmatrix}$ and $\mathbf{n}_d \sim \mathcal{CN}(0, \mathcal{N}_0\mathbf{I}_{L+K})$.

B. Optimum Linear Mapping Design

In this subsection, we optimize the linear mapping matrix $\mathbf{G}$ of size $K \times L (K < L)$ such that the outage probability of the proposed linear-mapping based cooperative protocol is minimized.

First, we derive the outage probability of the linear-mapping based cooperative protocol. From (29), we observe that the channel from the source to the destination in Phase I is a multi-input multi-output complex Gaussian channel. Thus, with i.i.d. circular symmetric complex Gaussian inputs, the maximum mutual information between the source and the destination in Phase I is

$$\begin{aligned} \mathcal{I}_{s,d} &= \frac{1}{L+K}\log_2\left[\det\left(\mathbf{I}_L + \frac{P_1}{\mathcal{N}_0}\mathbf{H}_{s,d}\mathbf{H}_{s,d}^{\mathcal{H}}\right)\right] \\ &= \frac{L}{L+K}\log_2\left(1 + \frac{P_1}{\mathcal{N}_0}|h_{s,d}|^2\right), \end{aligned} \tag{34}$$

where the factor $1/L+K$ in the first equality is due to the fact that the cooperative protocol uses $L+K$ time slots in Phases I and II. Similarly, the maximum mutual information between the source and the relay in Phase I is given by

$$\mathcal{I}_{s,r} = \frac{L}{L+K}\log_2\left(1 + \frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2\right). \tag{35}$$

In Phase II, if the relay decodes the message from the source correctly (i.e., $\tilde{P}_2 = P_2$), the destination can utilize the combined signal $\mathbf{y}_d$ from two phases to detect the original message $\mathbf{x}_s$. From (33), we can see that the channel between the output $\mathbf{y}_d$ and the input $\mathbf{x}_s$ is an equivalent multi-input, multi-output complex Gaussian channel. Therefore, the maximum mutual information of the joint detection in Phase II is

$$\begin{aligned}\mathcal{I}_d &= \frac{1}{L+K}\log_2\det\left(\mathbf{I}_{L+K}+\frac{\mathbf{H}_d\mathbf{H}_d^{\mathcal{H}}}{\mathcal{N}_0}\right)\\ &= \frac{1}{L+K}\log_2\det\left[\left(1+\frac{P_1|h_{s,d}|^2}{\mathcal{N}_0}\right)\mathbf{I}_L+\frac{P_2|h_{r,d}|^2}{\mathcal{N}_0}\mathbf{G}^{\mathcal{H}}\mathbf{G}\right]\end{aligned} \tag{36}$$

where the second equality is due to the fact that $\det(\mathbf{I}_{L+K}+(\mathbf{H}_d\mathbf{H}_d^{\mathcal{H}}/\mathcal{N}_0))=\det(\mathbf{I}_L+(\mathbf{H}_d^{\mathcal{H}}\mathbf{H}_d/\mathcal{N}_0))$ and $\mathbf{H}_d^{\mathcal{H}}\mathbf{H}_d=(P_1|h_{s,d}|^2/\mathcal{N}_0)\mathbf{I}_L+(P_2|h_{r,d}|^2/\mathcal{N}_0)\mathbf{G}^{\mathcal{H}}\mathbf{G}$. Thus, for any given target transmission rate R_T, the outage probability of the proposed linear-mapping based cooperative protocol is

$$\begin{aligned}\mathcal{P}_{out} &= Pr[\mathcal{I}_{s,d}<R_T]Pr[\mathcal{I}_{s,r}<R_T]\\ &\quad +(1-Pr[\mathcal{I}_{s,r}<R_T])\,Pr[\mathcal{I}_d<R_T],\end{aligned} \tag{37}$$

in which $\mathcal{I}_{s,d}$, $\mathcal{I}_{s,r}$ and $\mathcal{I}_d$ are specified in (34)–(36), respectively.

Next, we try to optimize the linear mapping $\mathbf{G}$ to minimize the outage probability in (37). We note that in (37), only the term $\mathcal{I}_d$ depends on the linear mapping $\mathbf{G}$. Thus, to minimize the outage probability $\mathcal{P}_{out}$, it is equivalent to minimize $Pr[\mathcal{I}_d<R_T]$ under the power constraint $Tr(\mathbf{G}\mathbf{G}^{\mathcal{H}})\le K$. The power constraint ensures that the forwarded signals have unit average power. The mutual information $\mathcal{I}_d$ in (36) can be further calculated as

$$\begin{aligned}\mathcal{I}_d &= \frac{1}{L+K}\log_2\left[\left(1+\frac{P_1|h_{s,d}|^2}{\mathcal{N}_0}\right)^L\right.\\ &\qquad\left.\times\det\left(\mathbf{I}_K+\frac{P_2|h_{r,d}|^2/\mathcal{N}_0}{1+P_1|h_{s,d}|^2/\mathcal{N}_0}\mathbf{G}\mathbf{G}^{\mathcal{H}}\right)\right]\\ &\triangleq \frac{1}{L+K}\left(\mathcal{I}_{d,1}+\mathcal{I}_{d,2}\right),\end{aligned} \tag{38}$$

where $\mathcal{I}_{d,1}\triangleq\log_2(1+(P_1|h_{s,d}|^2/\mathcal{N}_0))^L$ and $\mathcal{I}_{d,2}\triangleq\log_2[\det(\mathbf{I}_K+(P_2|h_{r,d}|^2/\mathcal{N}_0/(1+P_1|h_{s,d}|^2/\mathcal{N}_0))\mathbf{G}\mathbf{G}^{\mathcal{H}})]$. We observe that $\mathcal{I}_{d,1}$ does not depend on the linear mapping $\mathbf{G}$. Thus, to maximize $\mathcal{I}_d$, it is equivalent to maximize $\mathcal{I}_{d,2}$ under the power constraint that $Tr(\mathbf{G}\mathbf{G}^{\mathcal{H}})\le K$. To further calculate $\mathcal{I}_{d,2}$, we consider eigen-decomposition $\mathbf{G}\mathbf{G}^{\mathcal{H}}=\mathbf{U}^{\mathcal{H}}\mathbf{\Lambda}\mathbf{U}$, where $\mathbf{U}$ is a unitary matrix, $\mathbf{\Lambda}=\mathbf{diag}(\lambda_1,\ldots,\lambda_K)$ and the eigenvalues $\lambda_1,\ldots,\lambda_K$ are non-negative. Thus, we have

$$\begin{aligned}\mathcal{I}_{d,2} &= \log_2\left[\det\left(\mathbf{I}_K+\frac{P_2|h_{r,d}|^2/\mathcal{N}_0}{1+P_1|h_{s,d}|^2/\mathcal{N}_0}\mathbf{U}^{\mathcal{H}}\mathbf{\Lambda}\mathbf{U}\right)\right]\\ &= \sum_{i=1}^{K}\log_2\left(1+\frac{P_2|h_{r,d}|^2/\mathcal{N}_0}{1+P_1|h_{s,d}|^2/\mathcal{N}_0}\lambda_i\right).\end{aligned} \tag{39}$$

Since $\sum_{i=1}^{K}\lambda_i=Tr(\mathbf{G}\mathbf{G}^{\mathcal{H}})\le K$, the problem of optimizing the linear mapping G is specified as

$$\begin{aligned}\max_{\lambda_1,\ldots,\lambda_K}\quad &\sum_{i=1}^{K}\log_2\left(1+\frac{P_2|h_{r,d}|^2/\mathcal{N}_0}{1+P_1|h_{s,d}|^2/\mathcal{N}_0}\lambda_i\right)\\ \text{s.t.}\quad &\sum_{i=1}^{K}\lambda_i\le K.\end{aligned} \tag{40}$$

We can solve the problem by using Lagrange multipliers. Consider an Lagrange multiplier μ and an Lagrange function

$$\mathcal{L}(\lambda_1,\ldots,\lambda_K)=\sum_{i=1}^{K}\log_2\left(1+\frac{P_2|h_{r,d}|^2/\mathcal{N}_0}{1+P_1|h_{s,d}|^2/\mathcal{N}_0}\lambda_i\right)+\mu\sum_{i=1}^{K}\lambda_i. \tag{41}$$

Differentiating the Lagrange function with respect to λ_i, we have

$$\frac{\partial\mathcal{L}(\lambda_1,\ldots,\lambda_K)}{\partial\lambda_i}=\frac{1}{\lambda_i+\frac{1+P_1|h_{s,d}|^2/\mathcal{N}_0}{P_2|h_{r,d}|^2/\mathcal{N}_0}}+\mu. \tag{42}$$

Let $\partial\mathcal{L}(\lambda_1,\ldots,\lambda_K)/\partial\lambda_i=0$, we obtain an expression of optimum λ_i as

$$\lambda_i=\max\left(0,-\frac{1}{\mu}-\frac{1+P_1|h_{s,d}|^2/\mathcal{N}_0}{P_2|h_{r,d}|^2/\mathcal{N}_0}\right). \tag{43}$$

Since $\sum_{i=1}^{K}\lambda_i=K$, we have

$$\mu=-\frac{P_2|h_{r,d}|^2/\mathcal{N}_0}{1+P_1|h_{s,d}|^2/\mathcal{N}_0+P_2|h_{r,d}|^2/\mathcal{N}_0}. \tag{44}$$

By substituting (44) into (43), we have

$$\begin{aligned}\lambda_i &= \frac{1+P_1|h_{s,d}|^2/\mathcal{N}_0+P_2|h_{r,d}|^2/\mathcal{N}_0}{P_2|h_{r,d}|^2/\mathcal{N}_0}-\frac{1+P_1|h_{s,d}|^2/\mathcal{N}_0}{P_2|h_{r,d}|^2/\mathcal{N}_0}\\ &= 1,\end{aligned} \tag{45}$$

i.e., the optimum solution for the problem (40) is $\lambda_i=1$ for $i=1,\ldots,K$. Therefore, the linear mapping $\mathbf{G}$ minimizes the outage probability in (37) if and only if $\mathbf{G}\mathbf{G}^{\mathcal{H}}=\mathbf{I}_K$. We summarize the above discussion in the following theorem.

Theorem 4: In the linear-mapping based cooperative protocol, any $K\times L$ $(K<L)$ linear mapping G with power constraint $Tr(\mathbf{G}\mathbf{G}^{\mathcal{H}})\le K$ minimizes the outage probability of the protocol if and only if it satisfies $\mathbf{G}\mathbf{G}^{\mathcal{H}}=\mathbf{I}_K$. Consequently, we may select any K rows of an $L\times L$ unitary matrix to form the optimum linear mapping $\mathbf{G}$.

V. Simulation Results and Discussion

In this section, we provide some numerical studies and simulation results to illustrate the performance benchmark of the ideal cooperative protocol and the performance of the practical cooperative protocol based on the optimum linear mapping. In all numerical studies and simulations, we assume that the target transmission rate is $R_T=2$ bits/s/Hz, the noise variance is $\mathcal{N}_0=1$, and the variance of the source-destination channel is normalized as $\delta_{s,d}^2=1$. We consider three exemplary scenarios with various channel variances: (i) $\{\delta_{s,r}^2,\delta_{r,d}^2\}=\{1,1\}$; (ii) $\{\delta_{s,r}^2,\delta_{r,d}^2\}=\{10,1\}$; and (iii) $\{\delta_{s,r}^2,\delta_{r,d}^2\}=\{1,10\}$.

In Figs. 2–4, we plot the outage probability of the ideal cooperative protocol in terms of different time allocation ratio $\alpha\in(0,1)$ for the three channel conditions (i)–(iii), respectively. In the study, we assume $P/\mathcal{N}_0=30$ dB. For any given time allocation ratio $\alpha\in(0,1)$, the corresponding optimum power allocation $P_1=P_1^*(\alpha)$ and $P_2=P_2^*(\alpha)$ are calculated based on Theorem 2. In each figure, we plot the outage probability approximation based on (23) as well as the exact outage probability calculated based on (11) for comparison. In the

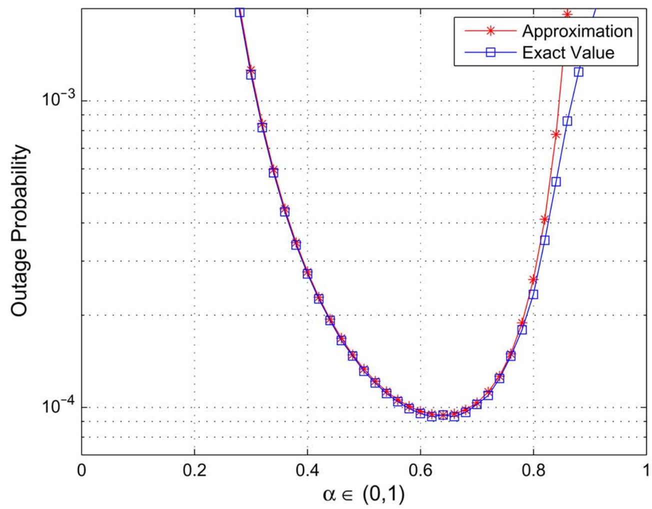


Fig. 2. Outage probability of the ideal cooperative protocol with different time allocation ratio $\alpha \in (0,1)$ under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1,1\}$. Assume that $P_1 = P_1^*(\alpha)$ and $P_2 = P_2^*(\alpha)$ for any given $\alpha \in (0,1)$ based on Theorem 2, and $P/\mathcal{N}_0 = 30$ dB.

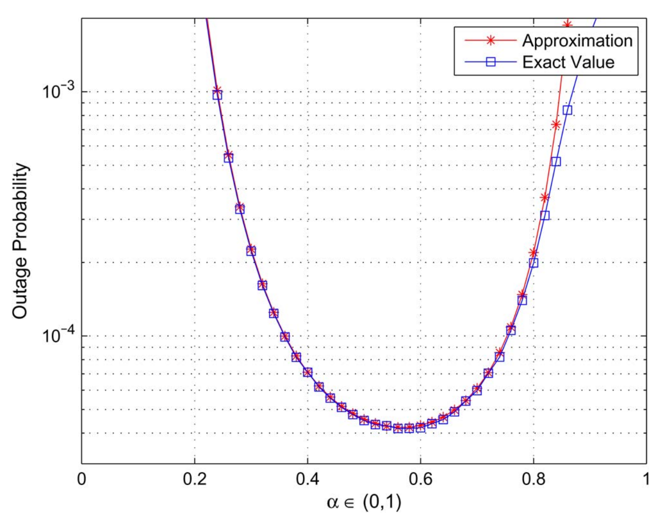


Fig. 3. Outage probability of the ideal cooperative protocol with different time allocation ratio $\alpha \in (0,1)$ under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{10,1\}$. Assume that $P_1 = P_1^*(\alpha)$ and $P_2 = P_2^*(\alpha)$ for any given $\alpha \in (0,1)$ based on Theorem 2, and $P/\mathcal{N}_0 = 30$ dB.

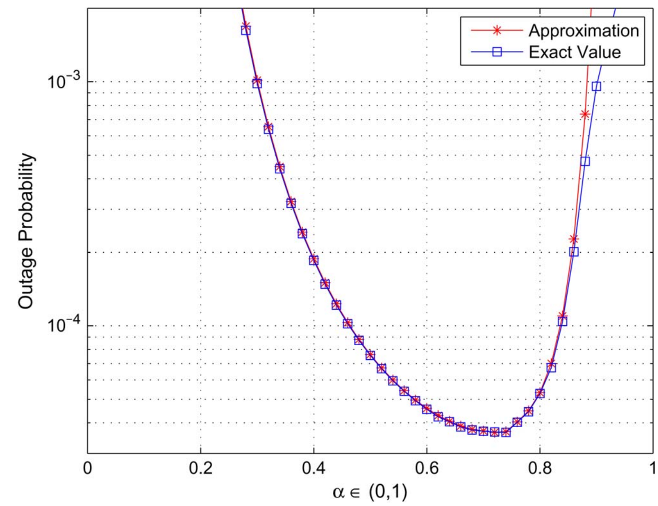


Fig. 4. Outage probability of the ideal cooperative protocol with different time allocation ratio $\alpha \in (0,1)$ under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1,10\}$. Assume that $P_1 = P_1^*(\alpha)$ and $P_2 = P_2^*(\alpha)$ for any given $\alpha \in (0,1)$ based on Theorem 2, and $P/\mathcal{N}_0 = 30$ dB.

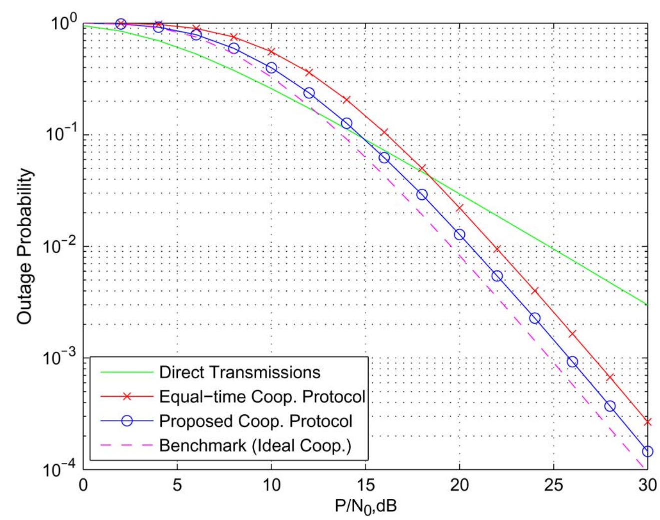


Fig. 5. Performance of the proposed practical cooperative protocol with the optimum linear mapping under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1,1\}$.

case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$, Fig. 2 shows that the optimum time allocation ratio is $\alpha^* = 0.66$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.0971P$ and $P_2^*(\alpha^*) = 0.8115P$ based on Theorem 2, in which the energy allocation ratio $\beta = 0.7192$. In case of $\delta_{s,r}^2 = 10$ and $\delta_{r,d}^2 = 1$, Fig. 3 shows that the optimum time allocation ratio is $\alpha^* = 0.59$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.0196P$ and $P_2^*(\alpha^*) = 0.9718P$ based on Theorem 2 and the energy allocation ratio $\beta = 0.5969$. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, Fig. 4 shows that the optimum time allocation ratio is $\alpha^* = 0.74$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.1112P$ and $P_2^*(\alpha^*) = 0.6835P$ and the energy allocation ratio $\beta = 0.8122$.

Figs. 2–4 show that for all three channel conditions, the optimum time allocation ratio α^* is strictly larger than 1/2 which is consistent to the theoretical development in Theorem 3, and the corresponding energy allocation ratio β is strictly larger than 1/2 which is consistent to the result in Theorem 2. Moreover, we observe that the larger the ratio of the relay-destination link quality over the source-relay link quality $(\delta_{r,d}^2/\delta_{s,r}^2)$ is, the larger the optimum time allocation ratio α^* and the corresponding energy allocation ratio β are. The phenomenon can be understood as follows: when the relay is closer to the destination and the source-relay channel link is relatively weak, in this case the system should allocate more time to Phase I for the relay to receive enough information signals. If the received signals at the relay are too weak, the relay is not able to decode the signals and the signal forwarding in Phase II is not necessary although the relay-destination link is strong in this case. In the three figures, we can see that the approximation of the outage probability based on (23) matches the exact value of the outage probability based on (11) very well.

Figs. 5–7 show the outage probability performance of the proposed practical cooperative relaying protocol based on the

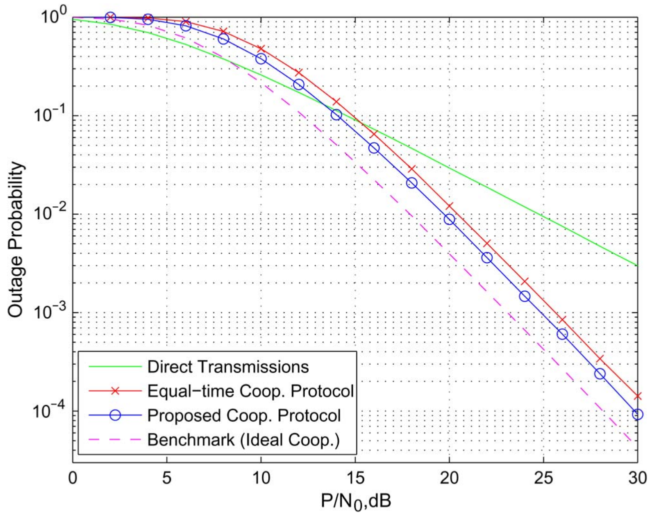


Fig. 6. Performance of the proposed practical cooperative protocol with the optimum linear mapping under the channel condition $\{\delta^2_{s,r}, \delta^2_{r,d}\} = \{10, 1\}$.

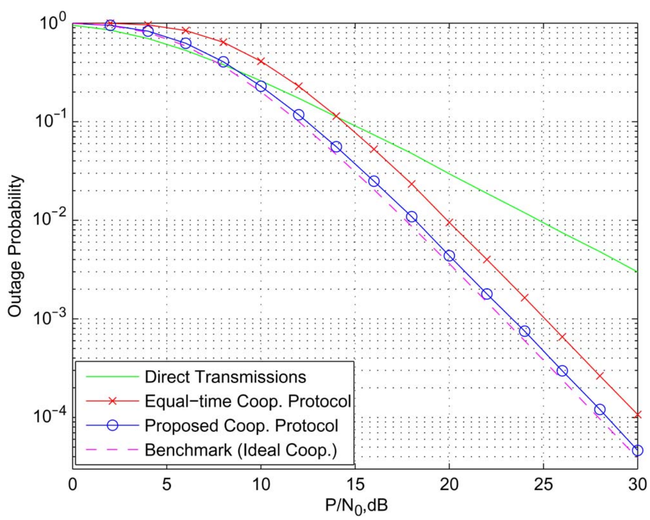


Fig. 7. Performance of the proposed practical cooperative protocol with the optimum linear mapping under the channel condition $\{\delta^2_{s,r}, \delta^2_{r,d}\} = \{1, 10\}$.

optimum linear mapping for three channel conditions (i)–(iii), respectively. For comparison, the figures also show the performance of the direct transmissions, the performance of the cooperative protocol based on equal time allocation, and the performance benchmark from the ideal cooperative protocol. Table I specifies time and power parameters (α, P_1, P_2) for each cooperative protocol under the three channel conditions, respectively. In the linear-mapping based cooperative protocol, the time ratio and power parameter (α, P_1, P_2) are selected based on the design guideline from the ideal case in Section III. For the selection of parameters K and L, we try to select smaller integers such that the ratio $L/(L+K)$ is close to the optimum time allocation ratio α^*. For example, when $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$, the optimum time allocation ratio is $\alpha^* = 0.66$, thus we consider small integers $L = 2$ and $K = 1$ such that the resulting ratio is close to the optimum time allocation ratio. When $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$, Fig. 5 shows that the proposed practical cooperative protocol outperforms the equal-time based cooperative protocol with performance improvement about 1.5 dB, and the difference between the performances of the practical linear-mapping based cooperative protocol and the ideal benchmark is less than 1 dB. When $\delta^2_{s,r} = 10$ and $\delta^2_{r,d} = 1$, we can see in Fig. 6 that there is a 2 dB difference between the proposed practical cooperative protocol and the ideal performance benchmark, but compared to the equal-time based cooperative protocol, the performance difference is less than 1 dB. In this case, the performance gain over the equal-time based cooperative protocol is narrowed because the equal-time allocation ratio is close to the optimum time ratio which is $\alpha^* = 0.59$ in this case. When $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 10$, Fig. 7 shows that the performance of the proposed practical cooperative protocol has over 2.5 dB improvement compared to the equal-time based cooperative protocol. In this case, the performance of the proposed linear-mapping based cooperative protocol is very close to the ideal performance benchmark (less than 0.25 dB). From the figures, we can see that the equal-time based cooperative protocol is not optimum in general, and the proposed practical cooperative protocol with the optimum linear mapping has performance gain in all three different channel conditions, which varies according to channel conditions. Moreover, we observe that when the ratio of the relay-destination link quality over the source-relay link quality $(\delta^2_{r,d}/\delta^2_{s,r})$ becomes larger, the gap between the performance of the proposed practical linear-mapping based cooperative protocol and the ideal benchmark becomes smaller.

VI. Conclusion

In this paper, we designed and optimized cooperative relaying protocols by exploring possible variations in time and power domains. First, we analyzed and optimized the ideal cooperative communication protocol where the system can use arbitrary re-encoding function $\mathcal{M}(\cdot)$ at the relay and adjust time allocation arbitrarily between Phases I and II. Based on the asymptotically tight approximation of the outage probability, we obtained the optimum strategy of power and time allocations to minimize the outage probability of the ideal cooperative protocol. For any given time allocation $\alpha \in (0, 1)$, we determined the corresponding optimum power allocation at the source and the relay analytically with a closed-form expression. We also showed theoretically that to minimize the outage probability of the protocol, one should always allocate more energy and time to Phase I than Phase II in the protocol. We note that in the ideal cooperative protocol in which there is no constraint on the re-encoding methods and time allocation, it may not be easy/feasible to implement it in practical systems. Therefore, with more realistic consideration, we proposed a practical cooperative relaying protocol design based on linear mapping, where the protocol considers linear mapping forwarding method at the relay and uses integer time slots in Phases I and II. The theoretical results from the ideal cooperative protocol served as guideline and benchmark in the practical cooperative protocol design. We also developed an optimum linear mapping to minimize the outage probability of the linear-mapping based cooperative protocol. Simulation results show that the practical cooperative relaying protocol based on the

TABLE I
TIME AND POWER PARAMETERS FOR DIFFERENT CHANNEL CONDITIONS

$\{\delta_{s,r}^2, \delta_{r,d}^2\}$	Resource Allocation (α, P_1, P_2)		
	Ideal Coop.	Proposed Coop.	Equal-time Coop.
$\{1, 1\}$	$(0.66, 1.0971P, 0.8115P)$	$(\frac{2}{3}, 1.0788P, 0.8423P)$	$(\frac{1}{2}, 1.6328P, 0.3672P)$
$\{10, 1\}$	$(0.59, 1.0196P, 0.9718P)$	$(\frac{3}{5}, 0.9948P, 1.0079P)$	$(\frac{1}{2}, 1.2946P, 0.7054P)$
$\{1, 10\}$	$(0.74, 1.1112P, 0.6835P)$	$(\frac{3}{4}, 1.0829P, 0.7513P)$	$(\frac{1}{2}, 1.8560P, 0.1440P)$

optimum linear mapping outperforms the existing cooperative protocol with equal time allocation, and more interestingly, the performance of the practical linear-mapping based cooperative relaying protocol is close to the performance benchmark of the ideal cooperative protocol. We observed that when the ratio of the relay-destination link quality over the source-relay link quality $(\delta_{r,d}^2/\delta_{s,r}^2)$ becomes larger, the gap between the performance of the proposed linear-mapping based cooperative protocol and the ideal benchmark becomes smaller.

APPENDIX I
PROOF OF THEOREM 2

For simplicity, in this proof we drop the parameter α in $A(\alpha)$ and $B(\alpha)$, and denote them as A and B, respectively. Based on the power constraint $\alpha P_1 + (1-\alpha)P_2 = P$, the relay transmission power P_2 can be written as

$$P_2 = \frac{P - \alpha P_1}{1-\alpha}. \tag{46}$$

Since $P_2 > 0$, (46) implies that $\alpha P_1 < P$. Therefore, for any given time allocation ratio $\alpha \in (0,1)$, the problem (24) can be reduced as

$$\min_{P_1} \quad \tilde{\mathcal{P}}_{out}(P_1) = \frac{\mathcal{N}_0^2 A}{\delta_{s,d}^2 P_1^2} + \frac{\mathcal{N}_0^2(1-\alpha)B}{\delta_{s,d}^2 P_1(P-\alpha P_1)}$$
$$\text{s.t.} \quad 0 < \alpha P_1 < P. \tag{47}$$

To find the optimal power P_1 in the problem (47), we proceed in two steps.

First, by taking derivative of the target function $\tilde{\mathcal{P}}_{out}(P_1)$ in terms of P_1, we have

$$\frac{\partial \tilde{\mathcal{P}}_{out}}{\partial P_1} = \frac{\mathcal{N}_0^2}{\delta_{s,d}^2} \frac{-2A(P-\alpha P_1)^2 - (1-\alpha)B(P-2\alpha P_1)P_1}{P_1^3(P-\alpha P_1)^2}. \tag{48}$$

Let $\partial \tilde{\mathcal{P}}_{out}(P_1)/\partial P_1 = 0$, we have

$$-2A(P-\alpha P_1)^2 - (1-\alpha)B(P-2\alpha P_1)P_1 = 0. \tag{49}$$

By solving the above equation, we have two possible solutions as follows:

$$P_{1,\pm}^* = \frac{[4\alpha A - (1-\alpha)B] \pm \sqrt{8\alpha(1-\alpha)AB + (1-\alpha)^2B^2}}{4\alpha\,[\alpha A - (1-\alpha)B]} P. \tag{50}$$

Second, for the two possible solutions, we would like to show that only the solution $P_{1,-}^*$ can satisfy the constraint in (47). We detail the discussion in two scenarios:

1) When $\alpha A > (1-\alpha)B$, in this case the denominators of $P_{1,-}^*$ and $P_{1,+}^*$ are positive. We have the analysis in Scenario 1, shown at the bottom of the page. We can see that the solution $P_{1,+}^*$ does not satisfy the power constraint and only $P_{1,-}^*$ satisfies the power constraint in this case. Moreover, the corresponding energy allocation ratio $\beta = \alpha P_{1,-}^*/P$ is within (1/2, 1).
2) When $\alpha A < (1-\alpha)B$, in this case the denominators of $P_{1,-}^*$ and $P_{1,+}^*$ in (50) are negative. For convenience, we multiply both their denominators and numerators by -1, then we have the analysis in Scenario 2, shown at the bottom of the next page. We can see that the

Scenario 1: $P_{1,-}^*$:

$$\begin{aligned} \alpha P_{1,-}^* &= \frac{[4\alpha A - (1-\alpha)B] - \sqrt{8\alpha(1-\alpha)AB + (1-\alpha)^2B^2}}{4\,[\alpha A - (1-\alpha)B]} P \\ &> \frac{[4\alpha A - (1-\alpha)B] - \sqrt{4\alpha^2A^2 + 4\alpha(1-\alpha)AB + (1-\alpha)^2B^2}}{4\,[\alpha A - (1-\alpha)B]} P \\ &> \frac{[4\alpha A - (1-\alpha)B] - \sqrt{[2\alpha A + (1-\alpha)B]^2}}{4\,[\alpha A - (1-\alpha)B]} P = \frac{P}{2}, \\ \alpha P_{1,-}^* &< \frac{[4\alpha A - (1-\alpha)B] - \sqrt{9(1-\alpha)^2B^2}}{4\,[\alpha A - (1-\alpha)B]} P = P; \end{aligned}$$

$P_{1,+}^*$:

$$\begin{aligned} \alpha P_{1,+}^* &= \frac{[4\alpha A - (1-\alpha)B] + \sqrt{8\alpha(1-\alpha)AB + (1-\alpha)^2B^2}}{4\,[\alpha A - (1-\alpha)B]} P \\ &> \frac{[4\alpha A - (1-\alpha)B] - \sqrt{9(1-\alpha)^2B^2}}{4\,[\alpha A - (1-\alpha)B]} P > P \end{aligned}$$

solution $P_{1,-}^*$ is the only feasible solution satisfying the power constraint in (47), and the corresponding energy allocation ratio β is within the interval (1/2, 1).

The above discussion shows that $P_{1,-}^*$ is the only solution that satisfies the power constraint and minimize the outage probability. The resulting energy allocation ratio β is always strictly larger than 1/2.

We may rewrite the optimum power $P_{1,-}^*$ as

$$P_{1,-}^* = \frac{1+\sqrt{1+8(\alpha A)/\left[(1-\alpha)B\right]}}{3+\sqrt{1+8(\alpha A)/\left[(1-\alpha)B\right]}}\frac{P}{\alpha}. \quad (51)$$

Note that A and B are shorthands of $A(\alpha)$ and $B(\alpha)$, respectively. For completeness, for any given time allocation ratio $\alpha \in (0,1)$, the optimum source transmission power $P_1^*(\alpha)$ is given by

$$P_1^*(\alpha) = \frac{1+\sqrt{1+8\left[\alpha A(\alpha)\right]/\left[(1-\alpha)B(\alpha)\right]}}{3+\sqrt{1+8\left[\alpha A(\alpha)\right]/\left[(1-\alpha)B(\alpha)\right]}}\frac{P}{\alpha}. \quad (52)$$

Based on (46), the corresponding optimum relay transmission power $P_2^*(\alpha)$ is given by

$$P_2^*(\alpha) = \frac{2}{3+\sqrt{1+8\left[\alpha A(\alpha)\right]/\left[(1-\alpha)B(\alpha)\right]}}\frac{P}{1-\alpha}. \quad (53)$$

Therefore, we prove Theorem 2 completely. □

Appendix II
Proof of Theorem 3

We would like to prove the result by contradiction. If there exists an optimum solution (α^*, P_1^*, P_2^*) with $\alpha^* \le 1/2$ that achieves the minimum outage probability $\tilde{\mathcal{P}}_{out}^*$ in the problem (24), we will find another solution that results in smaller outage probability to contradict the assumption.

With the assumption of the optimum solution (α^*, P_1^*, P_2^*) with $\alpha^* \le 1/2$, the resulting minimum outage probability $\tilde{\mathcal{P}}_{out}^*$ can be expressed as

$$\tilde{\mathcal{P}}_{out}^* = \tilde{\mathcal{P}}_{out}(\alpha^*, P_1^*, P_2^*) = \frac{\mathcal{N}_0^2 A(\alpha^*)}{\delta_{s,d}^2 (P_1^*)^2} + \frac{\mathcal{N}_0^2 B(\alpha^*)}{\delta_{s,d}^2 P_1^* P_2^*}. \quad (54)$$

Let us consider a new family of resource allocation strategy $(\hat{\alpha}, \hat{P}_1, \hat{P}_2) = (\tau\alpha^*, (P_1^*/\tau), ((1-\alpha^*)P_2^*/(1-\tau\alpha^*)))$ for $0 < \tau < 1/\alpha^*$. We can check that the new resource allocation strategy $(\hat{\alpha}, \hat{P}_1, \hat{P}_2)$ satisfies the power constraint in (24). Especially when $\tau = 1$, the new resource allocation solution is reduced to the optimum solution (α^*, P_1^*, P_2^*). With the new resource allocation strategy, the resulting outage probability is

$$\begin{aligned} C(\tau) &\triangleq \tilde{\mathcal{P}}_{out}(\hat{\alpha}, \hat{P}_1, \hat{P}_2) \\ &= \frac{\mathcal{N}_0^2 (\tau\alpha^*)^2 A(\tau\alpha^*)}{\delta_{s,d}^2 (P_1^*\alpha^*)^2} + \frac{\mathcal{N}_0^2 \tau\alpha^*(1-\tau\alpha^*) B(\tau\alpha^*)}{\delta_{s,d}^2 P_1^* P_2^* \alpha^*(1-\alpha^*)}. \end{aligned} \quad (55)$$

Since $(\ln 2/\alpha)\int_0^{R_T} 2^{v/\alpha} dv = 2^{R_T/\alpha} - 1$, we can rewrite the functions $A(\alpha)$ and $B(\alpha)$, defined in (15) and (19), respectively, as follows:

$$A(\alpha) = \frac{(\ln 2)^2}{\delta_{s,r}^2 \alpha^2} \int_0^{R_T}\int_0^{R_T} 2^{\frac{v}{\alpha}} 2^{\frac{r}{\alpha}} dv dr, \quad (56)$$

$$B(\alpha) = \frac{(\ln 2)^2}{\delta_{r,d}^2 \alpha(1-\alpha)} \int_0^{R_T}\int_0^{R_T - r} 2^{\frac{v}{\alpha}} 2^{\frac{r}{1-\alpha}} dv dr. \quad (57)$$

Thus, $C(\tau)$ can be written as

$$\begin{aligned} C(\tau) = {} & c_1 \int_0^{R_T}\int_0^{R_T} 2^{\frac{v}{\tau\alpha^*}} 2^{\frac{r}{\tau\alpha^*}} dv dr \\ & + c_2 \int_0^{R_T}\int_0^{R_T - r} 2^{\frac{v}{\tau\alpha^*}} 2^{\frac{r}{1-\tau\alpha^*}} dv dr, \end{aligned} \quad (58)$$

where $c_1 = \mathcal{N}_0^2(\ln 2)^2/\delta_{s,d}^2\delta_{s,r}^2(P_1^*\alpha^*)^2$ and $c_2 = \mathcal{N}_0^2(\ln 2)^2/\delta_{s,d}^2\delta_{r,d}^2 P_1^* P_2^* \alpha^*(1-\alpha^*)$ are positive constants.

$$\begin{aligned} \underline{\textbf{Scenario 2}}: \quad P_{1,-}^*: \quad \alpha P_{1,-}^* &= \frac{-\left[4\alpha A - (1-\alpha)B\right] + \sqrt{8\alpha(1-\alpha)AB + (1-\alpha)^2B^2}}{-4\left[\alpha A - (1-\alpha)B\right]} P \\ &> \frac{-\left[4\alpha A - (1-\alpha)B\right] + \sqrt{4\alpha^2A^2 + 4\alpha(1-\alpha)AB + (1-\alpha)^2B^2}}{-4\left[\alpha A - (1-\alpha)B\right]} P \\ &> \frac{-\left[4\alpha A - (1-\alpha)B\right] + \sqrt{\left[2\alpha A + (1-\alpha)B\right]^2}}{-4\left[\alpha A - (1-\alpha)B\right]} P = \frac{P}{2}, \\ \alpha P_{1,-}^* &< \frac{-\left[4\alpha A - (1-\alpha)B\right] + \sqrt{9(1-\alpha)^2B^2}}{-4\left[\alpha A - (1-\alpha)B\right]} P = P; \\ P_{1,+}^*: \quad \alpha P_{1,+}^* &= \frac{-\left[4\alpha A - (1-\alpha)B\right] - \sqrt{8\alpha(1-\alpha)AB + (1-\alpha)^2B^2}}{-4\left[\alpha A - (1-\alpha)B\right]} P \\ &< \frac{-\left[4\alpha A - (1-\alpha)B\right] - (1-\alpha)B}{-4\left[\alpha A - (1-\alpha)B\right]} P < 0 \end{aligned}$$

In the following, we would like to show that the differential of the function $C(\tau)$ at $\tau = 1$ is negative, i.e., $\partial C(\tau)/\partial\tau|_{\tau=1} < 0$. By taking derivative over the function $C(\tau)$, we have

$$\left.\frac{\partial C(\tau)}{\partial \tau}\right|_{\tau=1} = c_1 \ln 2 \int_0^{R_T}\int_0^{R_T} \left(-\frac{v+r}{\alpha^*} 2^{\frac{v+r}{\alpha^*}}\right) dvdr + c_2\alpha^* \ln 2 \int_0^{R_T}\int_0^{R_T-r} 2^{\frac{v}{\alpha^*}} 2^{\frac{r}{1-\alpha^*}} \times \left(-\frac{v}{(\alpha^*)^2} + \frac{r}{(1-\alpha^*)^2}\right) dvdr. \quad (59)$$

We can see that in (59), the first term is strictly less than 0. Next, we would like to show that the second term in (59) is non-positive. We denote the integrand as $b(v, r; \alpha^*) = 2^{v/\alpha^*} 2^{r/(1-\alpha^*)}(-(v/(\alpha^*)^2) + (r/(1-\alpha^*)^2))$ and the corresponding integration domain as $\Delta = \{(v, r) \in \mathbb{R}_+^2 : v + r < R_T\}$, then we can rewrite the second term of (59) as $c_2\alpha^* \ln 2 \int\int_\Delta b(v, r; \alpha^*)dvdr$. The symmetric property of the integration domain Δ implies that if $(v, r) \in \Delta$, then $(r, v) \in \Delta$ as well. Thus, the second term of (59) can be given by

$$c_2\alpha^* \ln 2 \iint_\Delta b(v, r; \alpha^*)dvdr = \frac{c_2\alpha^* \ln 2}{2} \iint_\Delta \left[b(v, r; \alpha^*) + b(r, v; \alpha^*)\right] dvdr. \quad (60)$$

To prove the second term of (59) is non-positive, it is sufficient to prove that the sum of $b(v, r; \alpha^*)$ and $b(r, v; \alpha^*)$ is non-positive for any $(v, r) \in \Delta$. Since $\alpha^* \leq 1/2$, we can see that only if $v < r$, we may have $b(v, r; \alpha^*) > 0$, which means that $b(v, r; \alpha^*)$ and $b(r, v; \alpha^*)$ cannot be positive simultaneously. So, there are only two possible cases to consider. When both $b(v, r; \alpha^*)$ and $b(r, v; \alpha^*)$ are non-positive, the proof is trivial. When either $b(v, r; \alpha^*)$ or $b(r, v; \alpha^*)$ is positive, without loss of generality, we assume that $b(v, r; \alpha^*) > 0$ and $b(r, v; \alpha^*) \leq 0$, then we have

$$\begin{aligned} b(v, r; \alpha^*) &+ b(r, v; \alpha^*) \\ &= 2^{\frac{v}{\alpha^*}} 2^{\frac{r}{1-\alpha^*}} \left(-\frac{v}{(\alpha^*)^2} + \frac{r}{(1-\alpha^*)^2}\right) \\ &\quad + 2^{\frac{r}{\alpha^*}} 2^{\frac{v}{1-\alpha^*}} \left(-\frac{r}{(\alpha^*)^2} + \frac{v}{(1-\alpha^*)^2}\right) \\ &\leq 2^{\frac{r}{\alpha^*}} 2^{\frac{v}{1-\alpha^*}} \left(-\frac{r+v}{(\alpha^*)^2} + \frac{r+v}{(1-\alpha^*)^2}\right) \leq 0, \end{aligned} \quad (61)$$

where the first inequality is due to the fact that $v < r$. The above result implies that the second term of (59) is non-positive. Therefore, we conclude that $\partial C(\tau)/\partial\tau|_{\tau=1} < 0$.

The result of $\partial C(\tau)/\partial\tau|_{\tau=1} < 0$ implies that we are able to find a τ ($\tau > 1$) such that $C(\tau) < C(1)$. Since $C(1)$ is the outage probability with the optimum allocation strategy (α^*, P_1^*, P_2^*) and $C(\tau)$ is the outage probability with another feasible allocations strategy $(\tau\alpha^*, (P_1^*/\tau), ((1-\alpha^*)P_2^*/1 - \tau\alpha^*))$, the fact that $C(\tau) < C(1)$ for some $\tau > 1$ contradicts the assumption that there exists an optimum solution (α^*, P_1^*, P_2^*) with $\alpha^* \leq 1/2$. Therefore, we prove Theorem 3 completely. □

ACKNOWLEDGMENT

The findings and conclusions expressed in this material are those of the authors and do not necessarily reflect the views of the U.S. Air Force Research Laboratory (AFRL).

REFERENCES

[1] K. J. R. Liu, A. Sadek, W. Su, and A. Kwasinski, *Cooperative Communications and Networking*, 1st ed. Cambridge, U.K.: Cambridge Univ. Press, 2009.

[2] E. Hossain, D. Kim, and V. K. Bhargava, *Cooperative Cellular Wireless Networks*, 1st ed. Camrbridge, U.K.: Cambridge Univ. Press, 2011.

[3] IEEE 802.16 Relay Task Group, The p802.16j Baseline Document for Draft Standard for Local and Metropolitan Area Networks, 802.16j-06/026r4, May 2007. [Online]. Available: http://grouper.ieee.org/groups/802/16/relay/

[4] "Further advancements for E-UTRA; Physical layer aspects," Sophia-Antipolis, France, 3GPP TR 36.814 V9.0.0, Release 9, Mar. 2010.

[5] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Probab.*, vol. 3, no. 1, pp. 120–154, 1971.

[6] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. IT-25, no. 5, pp. 572–584, Sep. 1979.

[7] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.

[8] Y. Ding, J. K. Zhang, and K. M. Wong, "Ergodic channel capacities for the amplify-and-forward half-duplex cooperative systems," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 713–730, Feb. 2009.

[9] B. Sirkeci-Mergen and M. Gastpar, "On the broadcast capacity of wireless networks with cooperative relays," *IEEE Trans. Inf. Theory*, vol. 56, no. 8, pp. 3847–3861, Aug. 2010.

[10] A. Zaidi, S. Shamai, P. Piantanida, and L. Vandendorpe, "Bounds on the capacity of the relay channel with noncausal state at the source," *IEEE Trans. Inf. Theory*, vol. 59, no. 5, pp. 2639–2672, May 2013.

[11] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2525, Oct. 2003.

[12] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[13] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[14] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.

[15] A. K. Sadek, W. Su, and K. J. R. Liu, "Multi-node cooperative communications in wireless networks," *IEEE Trans. Signal Process.*, vol. 55, no. 1, pp. 341–355, Jan. 2007.

[16] B. Zhao and M. C. Valenti, "Practical relay networks: A generalization of hybrid-ARQ," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 1, pp. 7–18, Jan. 2005.

[17] S. Lee, W. Su, S. Batalama, and J. D. Matyjas, "Cooperative decode-and-forward ARQ relaying: Performance analysis and power optimization," *IEEE Trans. Wireless Commun.*, vol. 9, no. 8, pp. 2632–2642, Aug. 2010.

[18] O. Alay, T. Korakis, Y. Wang, E. Erkip, and S. Panwar, "Layered wireless video multicast using omni-directional relays," in *Proc. IEEE ICASSP*, Apr. 2008, pp. 2149–2152.

[19] H. V. Zhao and W. Su, "Cooperative wireless multicast: Performance analysis and power/location optimization," *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 2088–2100, Jun. 2010.

[20] X. Deng and A. M. Haimovich, "Power allocation for cooperative relaying in wireless networks," *IEEE Commun. Lett.*, vol. 9, no. 11, pp. 994–996, Nov. 2005.

[21] W. Su, A. Sadek, and K. J. R. Liu, "Cooperative communication protocols in wireless networks: Performance analysis and optimum power allocation," *Wireless Pers. Commun.*, vol. 44, no. 2, pp. 181–217, Jan. 2008.

[22] K. G. Seddik, A. K. Sadek, W. Su, and K. J. R. Liu, "Outage analysis and optimal power allocation for multi-node relay networks," *IEEE Signal Process. Lett.*, vol. 14, no. 6, pp. 337–380, Jun. 2007.

[23] D. Gunduz and E. Erkip, "Opportunistic cooperation by dynamic resource allocation," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1446–1454, Apr. 2007.

[24] W. Hachem, P. Bianchi, and P. Ciblat, "Outage probability-based power and time optimization for relay networks," *IEEE Trans. Signal Process.*, vol. 57, no. 2, pp. 764–782, Feb. 2009.

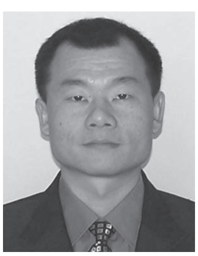
[25] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.

[26] Z. Mo, W. Su, S. N. Batalama, and J. D. Matyjas, "Optimum power and time allocation for cooperative relaying protocol," in *Proc. IEEE Int. Conf. Commun.*, Sydney, N.S.W., Australia, Jun. 2014, pp. 1–7.

[27] Z. Mo, W. Su, S. N. Batalama, and J. D. Matyjas, "Linear-mapping based cooperative relaying protocol design with optimum power and time allocation," in *Proc. IEEE Int. Conf. Commun.*, Sydney, N.S.W., Australia, Jun. 2014, pp. 1–6.

Zijian Mo (S'13) received the B.S. degree in automation engineering from South China University of Technology, Guangzhou, China, in 2009 and the M.S. degree in electrical engineering from the State University of New York, Buffalo, NY, USA, in 2011, where he is currently working toward the Ph.D. degree in electrical engineering.

His research interests include cooperative relaying protocol design and optimization, space-time coding for MIMO wireless communication, and airborne MIMO communication systems.



Weifeng Su (M'03–SM'14) received the B.S. and Ph.D. degrees in mathematics from Nankai University, Tianjin, China, in 1994 and 1999, respectively, and the Ph.D. degree in electrical engineering from the University of Delaware, Newark, DE, USA, in 2002. His research interests span a broad range of areas from signal processing to wireless communications and networking, including space-time coding and modulation for MIMO wireless communications, MIMO-OFDM systems, and cooperative communications for wireless networks.

He joined the Department of Electrical Engineering at the State University of New York, Buffalo, NY, USA in 2005, where he is currently an Associate Professor. From June 2002 to March 2005, he was a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering and the Institute for Systems Research, University of Maryland, College Park, MD, USA. Dr. Su is the recipient of the 2010 IEEE International Conference on Communications (ICC) Best Paper Award. He received the Invention of the Year Award from the University of Maryland in 2005. Dr. Su has been serving as Editor for several IEEE Journals including IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, and IEEE SIGNAL PROCESSING LETTERS. He also co-organized two Special Issues for IEEE Journals in the field of cooperative communications and networking. Dr. Su coauthored the book "Cooperative Communications and Networking" (Cambridge Univ. Press, 2009).

Stella Batalama (S'91–M'94–SM'13) received the Diploma degree in computer engineering and science (5-year program) from the University of Patras, Patras, Greece, in 1989 and the Ph.D. degree in electrical engineering from the University of Virginia, Charlottesville, VA, USA, in 1994.

In 1995, she joined the Department of Electrical Engineering, State University of New York, Buffalo, NY, USA, where she is presently a Professor. From July 2009 to June 2011 she served as the Associate Dean for Research of the School of Engineering and Applied Sciences and since 2010, she is Chair of the Electrical Engineering Department. During the summers of 1997–2002 she was a visiting faculty member of the U.S. Air Force Research Laboratory (AFRL), Rome, NY, USA. From Aug. 2003 to July 2004 she served as the Acting Director of the AFRL Center for Integrated Transmission and Exploitation, Rome, NY, USA.

Her research interests include small-sample-support adaptive filtering and receiver design, cooperative and cognitive communications and networks, covert communications and steganography, robust spread-spectrum communications and adaptive multiuser detection, compressive sampling. Dr. Batalama was an Associate Editor for IEEE COMMUNICATIONS LETTERS (2000–2005) and IEEE TRANSACTIONS ON COMMUNICATIONS (2002–2008).

John D. Matyjas (M'02–SM'13) received the Ph.D. degree in electrical engineering from the State University of New York, Buffalo, NY, USA in 2004. He has since been employed by the Air Force Research Laboratory in Rome, NY, USA, serving as the Connectivity Core Technical Competency Lead. His research interests are in the areas of wireless multiple-access communications and networking, cognitive radio, statistical signal processing and optimization, and neural networks. Additionally, he serves as an adjunct faculty member in the Department of Electrical Engineering, State University of New York Institute of Technology at Utica/Rome. Dr. Matyjas is the recipient of the Mohawk Valley "Engineer of the Year" Award and, as a coauthor, of the 2010 IEEE International Conference on Communications (ICC) "Best Paper Award." He is a member of the IEEE Communications, Information Theory, Computational Intelligence, and Signal Processing Societies, chair of the IEEE Mohawk Valley Chapter Signal Processing Society, and a member of the Tau Beta Pi and Eta Kappa Nu engineering honor societies.