On Optimum Selection Relaying Protocols in Cooperative Wireless Networks

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***Abstract*—In this letter, the outage probabilities of selection relaying protocols are analyzed and compared for cooperative wireless networks. It is assumed that both source and relay use equal allocated time in transmission. Depending on the quality of the source-relay channel, the relay may choose either Decode-and-Forward (DF), Amplify-and-Forward (AF), or Direct-Transmission (DT) to forward signals. It turns out that in terms of outage probability, two selection relaying schemes are better than others: selecting between DF and AF protocols (DF-AF) or selecting between DF and DT protocols (DF-DT). It is shown that with an equal power allocation, both of the DF-AF and DF-DT selection relaying protocols have the same asymptotic outage probability. However, with an optimum power allocation strategy, the DF-AF selection scheme is in general better than the DF-DT selection scheme. Note that the optimum power allocations depend on channel variances, not on instantaneous channel gains. When the quality of the relay-destination link is much better than that of the source-relay link, observed from simulation, the outage probability of the DF-AF selection protocol with its optimum power allocation is 1.5dB better than that of the DF-DT selection with its own optimum power allocation. Extensive simulations are presented to validate the analytical results.**

***Index Terms*—Cooperative communications, amplify-and-forward (AF) relaying, decode-and-forward (DF) relaying, selection relaying, optimum power allocation, wireless network.**

I. Introduction and System Model

Recently, an exciting concept of cooperative communications and networking [1]–[6] was proposed that allows different users in a wireless network to share resources and cooperate through distributed transmissions. Various cooperative communication protocols have been proposed for wireless networks to exploit diversity. Specifically, in [1], [2], when a user/node helps others to forward information, it serves as a relay. The relay may first decode the received information and then forward the decoded symbol to the destination, which results in a *decode-and-forward* (DF) cooperation protocol, or the relay may simply amplify and forward the received signal, which results in an *amplify-and-forward* (AF) cooperation protocol. It was shown in [7] that the DF and AF relaying protocols are comparable in terms of outage probability under a worst-case scenario that the channel between cooperating users may be unreliable for exchanging information. Moreover, it was shown in [2] that in term of outage probability performance, a selection scheme by switching between the DF and Direct-Transmission (DT) protocols is much better than the fixed DF and AF relaying schemes if the relay is allowed to choose a forwarding scheme according to the source-relay ($S \to R$) channel condition. Since the relay may choose any of the three protocols (DF, AF or DT) to forward information, a natural question is which selection relaying scheme performs the best.

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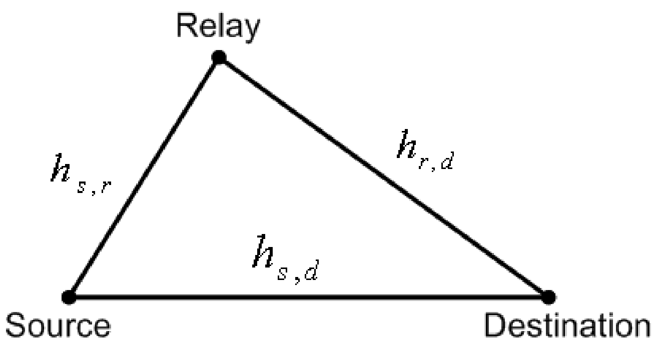

Fig. 1. A simplified cooperative communication model.

In this letter, we analyze and compare all possible selection combinations in term of outage probability performance. We consider a cooperative communication model as shown in Fig. 1, which can be implemented in two phases. It is assumed that both source and relay use equal allocated time in transmission. In Phase 1, the source broadcasts its information and the information is received by both the relay and the destination. The received signals at the relay and destination can be modeled as $y_r = \sqrt{P_1} h_{s,r} x + n_{s,r}$ and $y_{d,1} = \sqrt{P_1} h_{s,d} x + n_{s,d}$, where P_1 is the transmitted power by the source, x is the transmitted signal with unit mean power, $n_{s,r}$ and $n_{s,d}$ represent the additive white noise, $h_{s,r}$ and $h_{s,d}$ are channel coefficients of the $S \to R$ and $S \to D$ links, respectively. In Phase 2, the relay may forward the received information to the destination in different ways that result in different cooperation protocols [2].

- For the DF protocol, the relay decodes the received signal, and then forwards the decoded information to the destination. The received signal at the destination can be modeled as $y_{d,2} = \sqrt{P_2} h_{r,d} \tilde{x} + n_{r,d}$, where P_2 is the transmitted power by the relay, $\tilde{x}$ is a decoded symbol based on the received signal at the relay, and $n_{r,d}$ represents the additive white noise at the destination.
- For the AF protocol, the relay simply amplifies the received signal and forwards it to the destination. The received signal at the destination in this case can be modeled as $y_{d,2} = \sqrt{P_2} h_{r,d} x_r + n_{r,d}$, where $x_r = \beta y_r$ and $\beta = \frac{1}{\sqrt{|h_{s,r}|^2 P_1 + \mathcal{N}_0}}$ [2] is an amplification parameter.

- For the repetition based DT protocol, the information is transmitted by the source through the $S \to D$ link in Phase 2 without involving the relay. The received signal at the destination in this case can be modeled as $y_{d,2} = \sqrt{P_2}h_{s,d}x + n_{s,d}$.

The channel coefficients $h_{s,r}$, $h_{s,d}$ and $h_{r,d}$ are modeled as independent, zero-mean complex Gaussian random variables with variances $\delta^2_{s,r}$, $\delta^2_{s,d}$ and $\delta^2_{r,d}$, respectively. We assume that the channel coefficients are known at the receiver side, but unknown at the transmitter side. The noises $n_{s,r}$, $n_{s,d}$ and $n_{r,d}$ are modeled as zero-mean circularly-symmetric complex Gaussian random variables with variance $\mathcal{N}_0$.

A selection protocol of switching between the DF and DT schemes has been analyzed in [2] in terms of outage probability. Specifically, for a given transmission rate R, if the $S \to R$ link with an instantaneous channel coefficient $h_{s,r}$ is able to support the transmission rate R, i.e., $\frac{1}{2}\log_2\left(1 + \frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2\right) \geq R$, or equivalently $|h_{s,r}|^2 \geq \Delta = \frac{(2^{2R}-1)\mathcal{N}_0}{P_1}$, then, the relay selects the DF protocol to forward information. On the other hand, if $|h_{s,r}|^2 < \Delta$, then the source sends the original information to the destination again without using the relay, i.e, a repetition based DT protocol is used in Phase 2. We observe that in the DF-DT selection relaying protocol, the relay remains idle in Phase 2 if the channel $|h_{s,r}|^2$ is below the threshold Δ. In fact, the relay may choose the AF protocol to forward signal in this case, which leads to a DF-AF selection relaying protocol. There are totally nine selection combinations as the relay may choose one of the three protocols (DF, AF or DT) when the channel $|h_{s,r}|^2$ is below or above the threshold Δ. An interesting question to be addressed is which selection relaying protocol is the best in terms of minimizing outage probability?

II. Outage Probability Analysis for Selection Relaying Protocols

When the instantaneous channel coefficient $|h_{s,r}|^2$ is above the threshold, i.e., $|h_{s,r}|^2 \geq \Delta$, for a given rate R, we determine the outage events for the DT, AF and DF protocols respectively as follows [2]

$$\text{DT}: \quad \left\{|h_{s,r}|^2 \geq \Delta\right\} \cap \left\{\frac{P_1+P_2}{\mathcal{N}_0}|h_{s,d}|^2 < g(R)\right\}, \quad (1)$$

$$\text{AF}: \quad \left\{|h_{s,r}|^2 \geq \Delta\right\} \cap \left\{\frac{P_1}{\mathcal{N}_0}|h_{s,d}|^2 + \frac{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2}{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2 + \frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2 + 1} < g(R)\right\}, \quad (2)$$

$$\text{DF}: \quad \left\{|h_{s,r}|^2 \geq \Delta\right\} \cap \left\{\frac{P_1}{\mathcal{N}_0}|h_{s,d}|^2 + \frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2 < g(R)\right\}, \quad (3)$$

in which $g(R) = 2^{2R} - 1$. In (2), we observe that $\frac{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2}{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2+\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2+1} < \frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2$. Thus, when $|h_{s,r}|^2 \geq \Delta$, the outage probability of the AF protocol is

$$P^{out}_{AF} > Pr\left\{\frac{P_1}{\mathcal{N}_0}|h_{s,d}|^2 + \frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2 < g(R)\right\} = P^{out}_{DF}, \quad (4)$$

which means that the outage probability of the AF protocol is large than that of the DF protocol if the $S \to R$ channel is good enough to support the given transmission rate.

When $|h_{s,r}|^2 \geq \Delta$, the outage probabilities of the DT and DF protocols can be asymptotically approximated as $P^{out}_{DT} \to \frac{g(R)\mathcal{N}_0}{\delta^2_{s,d}(P_1+P_2)}$, and $P^{out}_{DF} \to \frac{1}{2\delta^2_{r,d}\delta^2_{s,d}}\frac{g^2(R)\mathcal{N}_0^2}{P_1P_2}$. Note that when

$$P_1P_2 > \frac{P_1+P_2}{2}\frac{g(R)\mathcal{N}_0}{\delta^2_{r,d}}, \quad (5)$$

we have $P^{out}_{DT} > P^{out}_{DF}$. The mild condition (5) can be easily satisfied in practice when P_1 and P_2 are generally large. Therefore, when $|h_{s,r}|^2 \geq \Delta$, the DF protocol has the lowest outage probability among the three if $\frac{P_1}{\mathcal{N}_0}$ and $\frac{P_2}{\mathcal{N}_0}$ are large. This result narrows the comparisons aiming to the best selection relaying scheme, i.e., when $|h_{s,r}|^2 \geq \Delta$, the DF protocol should be chosen to forward information.

A. Outage Probability for the DF-AF Selection Relaying Protocol

The outage probability of the DF-AF selection protocol can be given by

$$\begin{aligned} P^{out}_{DF-AF} &= Pr\left\{|h_{s,r}|^2 < \Delta\right\} Pr\Bigg\{\frac{P_1}{\mathcal{N}_0}|h_{s,d}|^2 \\ &+ \frac{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2}{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2 + \frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2 + 1} < g(R)\Bigg| |h_{s,r}|^2 < \Delta\Bigg\} \\ &+ Pr\{|h_{s,r}|^2 \geq \Delta\} Pr\left\{\frac{P_1}{\mathcal{N}_0}|h_{s,d}|^2 + \frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2 < g(R)\right\} \end{aligned} \quad (6)$$

To obtain a closed form expression for the outage probability of the DF-AF selection protocol in (6) is quite challenging if not impossible. In the following, we try to find an asymptotic outage probability for the DF-AF selection protocol by determining asymptotic lower and upper bounds.

Lemma 1: When $\frac{P_1}{\mathcal{N}_0}\delta^2_{s,r} \to \infty$ and $\frac{P_2}{\mathcal{N}_0}\delta^2_{r,d} \to \infty$, then for any constant u with $0 < u < 1$, we have (7) (see top of next page).

Proof : First, let us find a lower bound for the conditional probability. (See (8) on next page.) Next, we are going to find an upper bound for the conditional probability. For convenience, denote $v = |h_{s,r}|^2$ and $w = |h_{r,d}|^2$, then the conditional probability can be rewritten as

$$\begin{aligned} &Pr\left\{\frac{1}{v} + \frac{P_1}{P_2w} + \frac{\mathcal{N}_0}{P_2vw} > \frac{P_1}{g(R)u\mathcal{N}_0}\middle| v < \frac{g(R)\mathcal{N}_0}{P_1}\right\} \\ &= Pr\left\{v < \frac{1+\frac{\mathcal{N}_0}{P_2w}}{\frac{P_1}{g(R)u\mathcal{N}_0} - \frac{P_1}{P_2w}}\middle| v < \frac{g(R)\mathcal{N}_0}{P_1}\right\}. \end{aligned} \quad (9)$$

For a given constant $k > 0$, the probability in (9) can be calculated by separating it into two parts as

$$\begin{aligned} &Pr\left\{w < \frac{kg(R)u\mathcal{N}_0}{P_1}, v < \frac{1+\frac{\mathcal{N}_0}{P_2w}}{\frac{P_1}{g(R)u\mathcal{N}_0} - \frac{P_1}{P_2w}}\middle| v < \frac{g(R)\mathcal{N}_0}{P_1}\right\} \\ &+ Pr\left\{w \geq \frac{kg(R)u\mathcal{N}_0}{P_1}, v < \frac{1+\frac{\mathcal{N}_0}{P_2w}}{\frac{P_1}{g(R)u\mathcal{N}_0} - \frac{P_1}{P_2w}}\middle| v < \frac{g(R)\mathcal{N}_0}{P_1}\right\} \quad (10) \\ &\leq Pr\left\{w < \frac{kg(R)u\mathcal{N}_0}{P_1}\right\} \\ &+ Pr\left\{v < \frac{\frac{g(R)u\mathcal{N}_0}{P_1} + \frac{\mathcal{N}_0}{kP_2}}{1 - \frac{P_1}{P_2k}}\middle| v < \frac{g(R)\mathcal{N}_0}{P_1}\right\}. \quad (11) \end{aligned}$$

The second term in (11) is obtained by replacing w in the probability (10) with $\frac{kg(R)u\mathcal{N}_0}{P_1}$ since the probability (10) is

$$Pr\left\{\frac{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2}{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2+\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2+1} < g(R)u \,\middle|\, |h_{s,r}|^2<\Delta\right\} \to u \tag{7}$$

$$\begin{aligned} & Pr\left\{\frac{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2}{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2+\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2+1} < g(R)u \,\middle|\, \frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2<g(R)\right\} \\ \geq\ & Pr\left\{\min\left(\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2, \frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2\right) < g(R)u \,\middle|\, \frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2<g(R)\right\} \\ =\ & 1 - Pr\left\{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2 \geq g(R)u \,\middle|\, \frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2<g(R)\right\} Pr\left\{\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2 \geq g(R)u\right\} \\ =\ & 1 - \frac{e^{-\frac{1}{\delta_{s,r}^2}\frac{g(R)u\mathcal{N}_0}{P_1}} - e^{-\frac{1}{\delta_{s,r}^2}\frac{g(R)\mathcal{N}_0}{P_1}}}{1-e^{-\frac{1}{\delta_{s,r}^2}\frac{g(R)\mathcal{N}_0}{P_1}}} e^{-\frac{1}{\delta_{r,d}^2}\frac{g(R)u\mathcal{N}_0}{P_2}} \to u \ (\frac{P_1}{\mathcal{N}_0}\delta_{s,r}^2\to\infty, \frac{P_2}{\mathcal{N}_0}\delta_{r,d}^2\to\infty) \end{aligned} \tag{8}$$

$$Pr\left\{w < \frac{g(R)u\mathcal{N}_0}{\sqrt{P_2}}\right\} + Pr\left\{v < \frac{\frac{g(R)u\mathcal{N}_0}{P_1}+\frac{\mathcal{N}_0}{P_1\sqrt{P_2}}}{1-\frac{1}{\sqrt{P_2}}} \,\middle|\, v<\frac{g(R)\mathcal{N}_0}{P_1}\right\} \tag{12}$$

non-increasing in terms of increasing w. Since k is arbitrary, we observe that if $k = \frac{P_1}{\sqrt{P_2}}$, substituting k into (11), we have (12). Let $\frac{P_2}{\mathcal{N}_0}\delta_{r,d}^2 \to \infty$, then the first term in (12) goes to zero, and the second term converges to the following probability

$$\begin{aligned} & Pr\left\{v < \frac{g(R)uN_0}{P_1} \,\middle|\, v<\frac{g(R)\mathcal{N}_0}{P_1}\right\} \\ =\ & \frac{1-e^{-\frac{1}{\delta_{s,r}^2}\frac{g(R)u\mathcal{N}_0}{P_1}}}{1-e^{-\frac{1}{\delta_{s,r}^2}\frac{g(R)\mathcal{N}_0}{P_1}}}. \end{aligned} \tag{13}$$

Let $\frac{P_1}{\mathcal{N}_0}\delta_{s,r}^2 \to \infty$, then (13) converges to u asymptotically. From (9)-(13), we can see that the conditional probability in (7) is asymptotically upper-bounded by u. □

With the result in Lemma 1, we are able to determine an asymptotic outage probability for the DF-AF selection relaying protocol as follows.

Theorem 1: When $\frac{P_1}{\mathcal{N}_0}\delta_{s,r}^2 \to \infty$ and $\frac{P_2}{\mathcal{N}_0}\delta_{r,d}^2 \to \infty$, the outage probability of the DF-AF selection protocol can be asymptotically approximated as

$$P_{DF-AF}^{out} \to \frac{g^2(R)\mathcal{N}_0^2}{2\delta_{s,d}^2}\left(\frac{1}{\delta_{s,r}^2P_1^2}+\frac{1}{\delta_{r,d}^2P_1P_2}\right). \tag{14}$$

Proof : The calculation of the outage probability of the DF-AF selection protocol in (6) can be separated as two parts by following the two events that the $S \to R$ channel quality is below or above the threshold. Specifically, the probabilities of the events that the instantaneous channel $|h_{s,r}|^2$ is below or above the threshold Δ can be determined respectively as $Pr\left\{|h_{s,r}|^2<\Delta\right\} = 1-e^{-\frac{1}{\delta_{s,r}^2}\frac{g(R)\mathcal{N}_0}{P_1}}$, and $Pr\{|h_{s,r}|^2\geq\Delta\} = e^{-\frac{1}{\delta_{s,r}^2}\frac{g(R)\mathcal{N}_0}{P_1}}$. When the channel $|h_{s,r}|^2 \geq \Delta$, the outage probability is

$$\begin{aligned} & Pr\{|h_{s,r}|^2\geq\Delta\}Pr\left\{\frac{P_1}{\mathcal{N}_0}|h_{s,d}|^2+\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2<g(R)\right\} \\ \to\ & \frac{1}{2\delta_{r,d}^2\delta_{s,d}^2}\frac{g^2(R)\mathcal{N}_0^2}{P_1P_2} \quad (\frac{P_1}{\mathcal{N}_0}\delta_{s,r}^2, \frac{P_2}{\mathcal{N}_0}\delta_{r,d}^2\to\infty). \end{aligned} \tag{15}$$

When the instantaneous channel coefficient $|h_{s,r}|^2$ is below the threshold, the outage probability is determined by the related AF protocol. For simplicity, denote $r = \frac{P_1}{\mathcal{N}_0}|h_{s,d}|^2$ and $s = \frac{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2}{\frac{P_1}{\mathcal{N}_0}|h_{s,r}|^2+\frac{P_2}{\mathcal{N}_0}|h_{r,d}|^2+1}$, then we have (16), see top of next page, in which we change variable $r' = \frac{r}{g(R)}$ to get the last equation. Since $0<1-r'<1$, according to Lemma 1, when $\frac{P_1}{\mathcal{N}_0}\delta_{s,r}^2\to\infty$, $\frac{P_2}{\mathcal{N}_0}\delta_{r,d}^2\to\infty$, (16) can be asymptotically approximated as

$$\frac{g(R)\mathcal{N}_0}{\delta_{s,d}^2P_1}\int_0^1(1-r')dr' = \frac{1}{2\delta_{s,d}^2}\frac{g(R)\mathcal{N}_0}{P_1}. \tag{17}$$

Note that $Pr\left\{|h_{s,r}|^2<\Delta\right\} = 1-e^{-\frac{1}{\delta_{s,r}^2}\frac{g(R)\mathcal{N}_0}{P_1}}$ which can be approximated as $\frac{g(R)\mathcal{N}_0}{\delta_{s,r}^2P_1}$, thus

$$\begin{aligned} & Pr\left\{|h_{s,r}|^2<\Delta\right\}Pr\{r+s<g(R)\,\big|\,|h_{s,r}|^2<\Delta\} \\ & \to \frac{g^2(R)\mathcal{N}_0{}^2}{2\delta_{s,d}^2\delta_{s,r}^2P_1^2}. \end{aligned} \tag{18}$$

By combining the results in (15) and (18), we have the asymptotic outage probability approximation for the DF-AF selection protocol as shown in Theorem 1. □

When SNR is high, the outage probability of the DF-DT selection protocol can be given as [2]

$$P_{DF-DT}^{out} \to \frac{g^2(R)N_0^2}{2\delta_{s,d}^2}\left(\frac{2}{\delta_{s,r}^2P_1(P_1+P_2)}+\frac{1}{\delta_{r,d}^2P_1P_2}\right), \tag{19}$$

$$Pr\{r+s<g(R)\Big||h_{s,r}|^2<\Delta\} = \int_0^{g(R)} Pr\{s<g(R)-r\Big||h_{s,r}|^2<\Delta\}p_r(r)dr.$$
$$= \frac{g(R)\mathcal{N}_0}{\delta_{s,d}^2 P_1}\int_0^1 Pr\{s<g(R)(1-r')\Big||h_{s,r}|^2<\Delta\}e^{-\frac{g(R)r'\mathcal{N}_0}{\delta_{s,d}^2P_1}}dr' \quad (16)$$

in which we use power P_1 and P_2 in the two phases instead of equal power ($P_1 = P_2$) in [2]. From (14) and (19), we can see that if $P_1 > P_2$, the asymptotic outage probability of the DF-AF selection protocol is less than that of the DF-DT selection protocol. If $P_1 = P_2$, both selection protocols have the same asymptotic outage probabilities. If $P_1 < P_2$, the asymptotic outage probability of the DF-AF protocol is larger than that of the DF-DT selection protocol. However, the case of $P_1 < P_2$ is out of consideration in practice since for better protocol performance, we should always allocate more power at the source and less power at the relay, as shown in the next subsection.

B. Optimum Power Allocation

First, let us determine an optimum power allocation scheme for the DF-AF selection protocol according to the asymptotic outage probability in (14), in which we intend to minimize the overall power consumption of $P_1 + P_2$. By taking derivative of the expression (14) with respect to P_1 and with any given total power $P_1 + P_2 = 2P$, we can get an optimum power allocation for the DF-AF selection protocol in high SNR range as follows: (Note that there is only one local minimum in the optimization and the boundaries of $P_1 = 0$ and $2P$ are trivial.)

$$P_1 = \frac{2\delta_{s,r} + 2\sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}}P, \quad (20)$$

$$P_2 = \frac{4\delta s,r}{3\delta_{s,r} + \sqrt{\delta_{s,r}^2 + 8\delta_{r,d}^2}}P. \quad (21)$$

Substituting it into (14), we obtain the outage probability of the DF-AF selection protocol as

$$P_{DF-AF}^{out,opt} = \frac{g^2(R)\mathcal{N}_0{}^2}{4\delta_{s,d}^2\delta_{r,d}^2\delta_{s,r}^2P^2} \frac{\sqrt{\delta_{s,r}^2+8\delta_{r,d}^2}(\delta_{s,r}^3+8\delta_{s,r}\delta_{r,d}^2)+8\delta_{r,d}^4-\delta_{s,r}^4+20\delta_{s,r}^2\delta_{r,d}^2}{16\delta_{r,d}^2}, \quad (22)$$

which indicates the best possible performance for using the DF-AF selection protocol with power allocation between the source and the relay.

We can also determine an optimum power allocation scheme for the DF-DT selection protocol according to the asymptotic outage probability as follows:

$$P_1 = \frac{2\delta_{r,d}^2 + \delta_{s,r}^2 - \sqrt{\delta_{s,r}^4 + 2\delta_{r,d}^2\delta_{s,r}^2}}{\delta_{r,d}^2}P, \quad (23)$$

$$P_2 = \frac{\sqrt{\delta_{s,r}^4 + 2\delta_{r,d}^2\delta_{s,r}^2} - \delta_{s,r}^2}{\delta_{r,d}^2}P. \quad (24)$$

Substituting the optimum power allocation result into (19), we have

$$P_{DF-DT}^{out,opt} = \frac{g^2(R)\mathcal{N}_0{}^2}{4\delta_{s,d}^2\delta_{r,d}^2\delta_{s,r}^2P^2}\left(\delta_{s,r}^2+\delta_{r,d}^2+\sqrt{\delta_{s,r}^4+2\delta_{s,r}^2\delta_{r,d}^2}\right), \quad (25)$$

which shows the best performance for the DF-DT selection protocol with possible power allocation between the source and the relay.

From both the optimum power allocation results, we observe that $1 \leq P_1/P \leq 2$, $0 \leq P_2/P \leq 1$, and $P_1 > P_2$, i.e., we should allocate more power at the source and less power at the relay. When $\delta_{s,r}^2 \gg \delta_{r,d}^2$, for both of the DF-DT and DF-AF selection protocols, the optimum power allocation is reduced to the equal power allocation (i.e., $P_1 = P_2 = P$). In this case, it seems that the source and the relay are almost at the same position and thus it is reasonable to allocate equal power between them. On the other hand, when $\delta_{s,r}^2 \ll \delta_{r,d}^2$, P_1 goes to $2P$ and P_2 goes to zero which means almost all the power should be allocated to the source. We can see that in general the equal power allocation is not optimum for both the DF-AF and DF-DT selection protocols. We also observe that for both the DF-DT and DF-AF selection protocols, the asymptotic optimum power allocations do not rely on the $S \rightarrow D$ link. To give an intuitive explanation, we inspect the asymptotic outage probability expressions for the DF-AF and DF-DT selections in (14) and (19), which show that the $S \rightarrow D$ link can contribute diversity order 1 as long as the transmitted power at the source is high. However, a second order diversity resulting from the source-relay-destination link cannot be guaranteed unless the $S \rightarrow R$ and $R \rightarrow D$ links are appropriately balanced (power allocation). The phenomenon was also reported in [8], [9] in which an optimum power allocation was addressed based on the symbol error rate performance analysis.

We compare the outage probability of the DF-AF and DF-DT selection protocols, each with its own optimum power allocation as follows.

Theorem 2: Each with optimum power allocation, the outage probability of the DF-AF selection protocol is less than that of the DF-DT selection protocol, i.e., $P_{DF-AF}^{out,opt} < P_{DF-DT}^{out,opt}$.

Proof : From (22) and (25), we can see that in order to prove $P_{DF-AF}^{out,opt} < P_{DF-DT}^{out,opt}$, it is sufficient to show

$$\frac{\sqrt{\delta_{s,r}^2+8\delta_{r,d}^2}(\delta_{s,r}^3+8\delta_{s,r}\delta_{r,d}^2)+8\delta_{r,d}^4-\delta_{s,r}^4+20\delta_{s,r}^2\delta_{r,d}^2}{16\delta_{r,d}^2} < \delta_{s,r}^2+\delta_{r,d}^2+\sqrt{\delta_{s,r}^4+2\delta_{s,r}^2\delta_{r,d}^2},$$

or equivalently,

$$\sqrt{\delta_{s,r}^2+8\delta_{r,d}^2}(\delta_{s,r}^3+8\delta_{s,r}\delta_{r,d}^2) - 16\delta_{r,d}^2\delta_{s,r}\sqrt{\delta_{s,r}^2+2\delta_{r,d}^2}$$

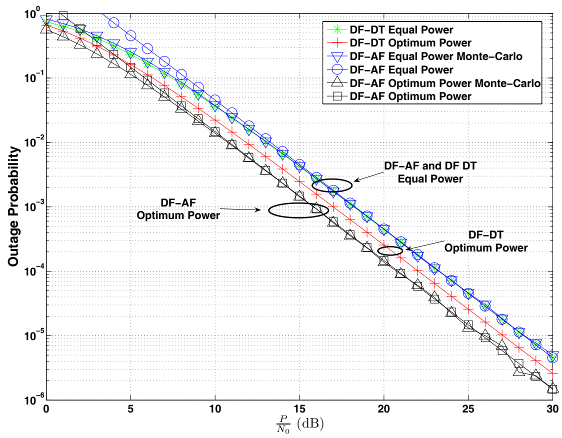

Fig. 2. Outage probability comparison of the DF-AF and DF-DT selection protocols with equal or optimum power allocations. Assume $\delta^2_{s,d} = 1$, $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 100$, R=1.

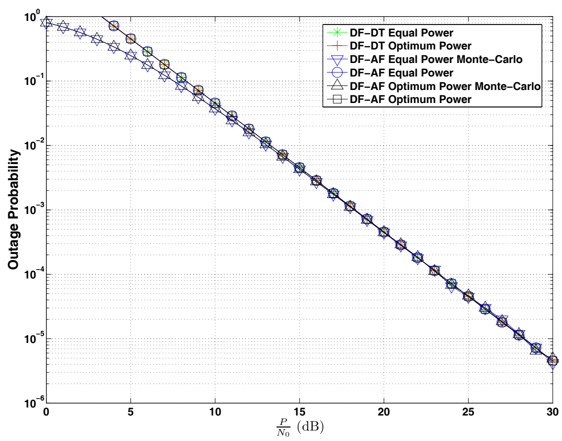

Fig. 3. Outage probability comparison of the DF-AF and DF-DT selection protocols with equal or optimum power allocations. Assume $\delta^2_{s,d} = 1$, $\delta^2_{s,r} = 100$ and $\delta^2_{r,d} = 1$, R=1.

$$< 8\delta^4_{r,d} + \delta^4_{s,r} - 4\delta^2_{s,r}\delta^2_{r,d}. \tag{26}$$

It is obvious that the right-hand side of the inequality (26) is non-negative. Therefore, if the left-hand side of the inequality is negative, the result in the theorem holds. In the sequel, we assume that the left-hand side of the inequality is non-negative, i.e., $\sqrt{\delta^2_{s,r} + 8\delta^2_{r,d}}(\delta^2_{s,r} + 8\delta^2_{r,d}) \geq 16\delta^2_{r,d}\delta_{s,r}\sqrt{\delta^2_{s,r} + 2\delta^2_{r,d}}$. By solving the inequality, we have $\delta^2_{s,r} > (4\sqrt{13} - 12)\delta^2_{r,d}$.

With the assumption that the left-hand side of (26) is non-negative, we are going to prove another form of (26) by squaring both sides of the inequality, or equivalently, to prove the following inequality

$$\begin{aligned} &\delta^6_{s,r}\delta^2_{r,d} + 13\delta^4_{s,r}\delta^4_{r,d} + 34\delta^2_{s,r}\delta^6_{r,d} - 2\delta^8_{r,d} \\ &< \delta^2_{s,r}\delta^2_{r,d}\sqrt{(\delta^2_{s,r} + 8\delta^2_{r,d})(\delta^2_{s,r} + 2\delta^2_{r,d})}(\delta^2_{s,r} + 8\delta^2_{r,d}). \end{aligned} \tag{27}$$

We observe that the right-hand side of (27) is non-negative. The left-hand side of (27) can be lower-bounded by $46\delta^8_{r,d}$ which is also non-negative. Therefore, by squaring both sides of (27), we have

$$\begin{aligned} &(\delta^4_{r,d} - \delta^4_{s,r})(4\delta^4_{r,d} + 3\delta^4_{s,r}) \\ < \;& \delta^2_{s,r}\delta^2_{r,d}(136\delta^4_{r,d} - 81\delta^2_{s,r}\delta^2_{r,d} + 16\delta^4_{s,r}). \end{aligned} \tag{28}$$

We can see that the right-hand side of (28) is non-negative. With the assumption that $\delta^2_{s,r} > (4\sqrt{13}-12)\delta^2_{r,d}$, the left-hand side of (28) is negative, which validates the inequality (28). Therefore, the inequality in (26) also holds if the left-hand side of the inequality is non-negative. □

The result in Theorem 2 can be intuitively explained as follows. When the $S \to R$ link quality is bad (below the threshold), the received signal at the relay will be discarded in the DF-DT selection protocol while in the DF-AF selection protocol, the signal will be amplified and forwarded to the destination. Although the received signal at the relay may be weak, by forwarding it to the destination with the AF protocol one can still gain some spatial diversity advantage. For example, when $\delta^2_{s,r} \ll \delta^2_{r,d}$, the outage probability of the DF-AF selection protocol with its optimum power allocation is less than half that of the DF-DT selection with its own optimum power allocation.

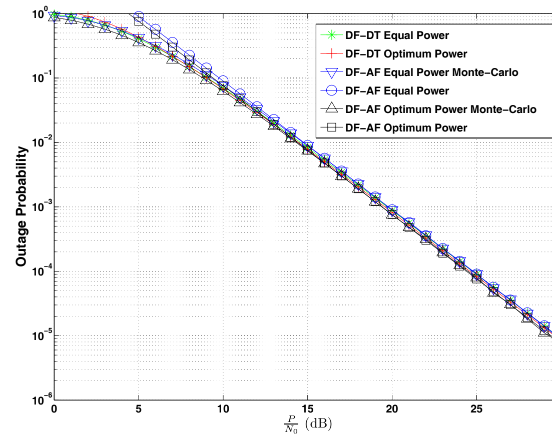

Fig. 4. Outage probability comparison of the DF-AF and DF-DT selection protocols with equal or optimum power allocations. Assume $\delta^2_{s,d} = 1$, $\delta^2_{s,r} = 1$ and $\delta^2_{r,d} = 1$, R=1.

III. Simulations and Concluding Remarks

We performed some simulations to validate the theoretical analysis by simulating three scenarios in which we normalize the variance of $S \to D$ link as $\delta^2_{s,d} = 1$. Fig. 2 shows simulation results for $\delta^2_{r,d} = 100$ and $\delta^2_{s,r} = 1$. We can see that with the equal power allocation scheme ($P_1 = P_2 = P$), the DF-AF and DF-DT selection protocols have comparable performances. Note that the DF-AF selection with an optimum power allocation is 3dB better than that with an equal power allocation, in which the optimum power allocation ratio is $\frac{P_2}{P_1} = 0.0683$. The DF-DT selection protocol with its optimum power scheme is about 1.5dB better than that with an equal power allocation, in which the corresponding optimum power allocation ratio is $\frac{P_2}{P_1} = 0.0705$. We observe that each with its own optimum power allocation, the performance of the DF-AF selection protocol is 1.5dB better than that of the DF-DT selection protocol. Fig. 3 shows simulation results when

$\delta^2_{r,d} = 1$ and $\delta^2_{s,r} = 100$. In this case, the optimum power allocation ratio is $\frac{P_2}{P_1} = 0.9901$ for the DF-DT selection and 0.9808 for the DF-AF selection, which are almost the same as the equal power allocation. Fig. 4 shows simulation results when $\delta^2_{s,r} = \delta^2_{r,d} = 1$. The optimum power is $\frac{P_2}{P_1} = 0.5774$ for the DF-DT selection and 0.5 for the DF-AF selection, and the performances of the two selection protocols are comparable in this case.

In the work, we analyze and compare the outage probabilities for the selection relaying cooperative protocols. First, we show that if the $S \to R$ channel quality is good enough to support a given transmission rate R, i.e., $|h_{s,r}|^2 \geq \Delta$, then the DF protocol is the best choice for the relay to forward signal.Then, we develop explicitly an asymptotic outage probability for the DF-AF selection protocol and determine an optimum power allocation accordingly. Finally, we compare the outage performances of the two selection schemes with or without optimum power allocation, respectively. It turns out that with an equal power allocation, both of the DF-AF and DF-DT selection protocols have the same asymptotic outage probability. However, if both consider optimum power allocation, then the DF-AF selection is better than the DF-DT selection.

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