Cooperative Decode-and-Forward ARQ Relaying: Performance Analysis and Power Optimization

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***Abstract*—In this paper we develop a new analytical methodology for the evaluation of the outage probability of cooperative decode-and-forward (DF) automatic-repeat-request (ARQ) relaying under packet-rate fading (fast fading or block fading) channels, where the channels remain fixed within each ARQ transmission round, but change independently from one round to another. We consider a single relay forwarding Alamouti-based retransmission signals in the cooperative ARQ scheme. In particular, (i) we derive a closed-form asymptotically tight (as SNR $\rightarrow \infty$) approximation of the outage probability; (ii) we show that the diversity order of the DF cooperative ARQ relay scheme is equal to $2L-1$, where L is the maximum number of ARQ (re)transmissions; and (iii) we develop the optimum power allocation for the DF cooperative ARQ relay scheme. The closed-form expression clearly shows that the achieved diversity is partially due to the DF cooperative relaying and partially due to the fast fading nature of the channels (temporal diversity). With respect to power allocation, it turns out that the proposed optimum allocation scheme depends only on the link quality of the channels related to the relay, and compared to the equal power allocation scheme it leads to SNR performance gains of more than 1 dB. Numerical and simulation studies illustrate the theoretical developments.**

***Index Terms*—Automatic-repeat-request (ARQ) protocol, cooperative decode-and-forward (DF) relaying, outage probability, wireless networks.**

I. Introduction

Conventional wireless networks involve point-to-point communication links and for that reason do not guarantee reliable transmissions over severe fading channels. On the other hand, cooperative wireless networks exhibit increased network reliability due to the fact that information can be delivered with the cooperation of other users in networks [1]–[14]. In particular, in cooperative systems each user utilizes other cooperative users to create a virtual antenna array and exploit spatial diversity that minimizes the effects of fading and improves overall system performance. Cooperative communications, also known as relay channels, was first introduced in [9] in which a three-way channel was analyzed based on the capacity region. In [10]–[12], relay channels have been analyzed from an information-theoretic point of view. With respect to practical/realistic cooperative communication protocols for wireless networks, past literature includes, but is not limited to, the work in [1]–[7], [13], [14] and the references therein.

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Automatic-repeat-request (ARQ) protocols for wireless communications have been studied extensively in the past and proved themselves as efficient control mechanisms for reliable data packet transmissions at the data link layer [15]–[21]. The basic idea of ARQ protocols is that a receiver requests retransmission when a packet is not correctly received. Recently, in an effort to increase network reliability over poor quality channels, ARQ protocols were studied in the context of cooperative relay networks [22]–[27]. In particular, [22] was among the first such studies to present a general framework of cooperative ARQ relay networks. It was shown that cooperative ARQ relay networks have great advantages in terms of throughput, delay, and energy consumption compared to conventional multihop ARQ networks in which point-to-point ARQ links are concatenated to form network routes. In [23], [24], information-theoretic analysis was developed and upper bounds for the diversity order of a decode-and-forward (DF) cooperative ARQ relay scheme were characterized for both slow and fast fading channels as a means to study the diversity-multiplexing-delay tradeoff. In [25], [26], a closed-form expression of the outage probability of the DF cooperative ARQ relay scheme was obtained for slow fading channels, but, unfortunately, the introduced approach cannot be extended to fast fading channels.

Outage probability is, arguably, a fundamental performance metric for wireless ARQ relay schemes and so is the diversity order. In this paper, we develop a new analytical methodology for the treatment of DF cooperative ARQ relay networks in fast fading (packet-rate fading or block fading) channels, in which each relay forwards Alamouti-based retransmission signals. The analysis leads, for the first time, to a closed-form asymptotically tight (as SNR $\rightarrow \infty$) approximation of the outage probability. The closed-form expression shows that the overall diversity order of the DF cooperative ARQ relay scheme is equal to $2L-1$, where L is the maximum number of ARQ (re)transmissions. The achieved diversity is partially due to the DF cooperative relaying and partially due to the fast fading nature of the channels (temporal diversity due to (re)transmissions over independent fading channels). We note that the diversity of the direct ARQ scheme (without relaying) is only L and it is due to the fast fading nature of the channels. Based on the asymptotically tight approximation of the outage probability, we are able to determine the optimum power that needs to be allocated at the source and at the relay of the DF cooperative ARQ relay scheme for any given total transmission power budget. The optimum power allocation depends on the variance values of the channels involved and the maximum number of (re)transmissions allowed by the protocol. It turns out that the conventional equal-power allocation strategy is not optimum, in general, and the optimum power allocation relies heavily on the link quality of the channels related to the relay. Extensive numerical and simulation results included in this paper illustrate and validate the theoretical developments.

The paper is organized as follows. In Section II, we describe briefly the DF cooperative ARQ relay scheme and the fast (packet-rate) fading channel model. In Section III, we first develop two useful lemmas which form the basis of our analytical approach. Then we derive our asymptotically tight approximation of the outage probability of DF cooperative ARQ relay scheme. In this section, we also include the outage probability expression of the direct ARQ transmission scheme for comparison purposes. In Section IV, we determine the optimum power allocation for the DF cooperative ARQ relay scheme, and in Section V we present numerical and simulation studies. Finally, some conclusions are drawn in Section VI.

II. System Model

We consider a cooperative ARQ relay scheme with one source, one relay and one destination as illustrated in Fig. 1. The DF cooperative ARQ relay scheme works as follows. First, a data packet of b bits is encoded into a sequence of length LT, where L is the maximum number of ARQ (re)transmission rounds allowed in the protocol and T is the duration of a single ARQ (re)transmission. Then, the sequence comprises L different blocks each of length T. In each ARQ (re)transmission round a block of the message is sent, so the transmission rate is $R = b/T$. When the source transmits a block of the message to the destination, it is also received by the relay. The destination indicates success or failure of receiving the message by feeding back a single bit of acknowledgement (ACK) or negtive-acknowledgement (NACK). The feedback is assumed to be detected reliably both by the source and by the relay. If an ACK is received or the retransmission reaches the maximum number of rounds, the source stops transmitting the current message and starts transmitting a new message. If a NACK is received and the retransmission has not reached the maximum number of rounds, the source sends another block of the same message. If the relay decodes successfully before the destination is able to, the relay starts cooperating with the source by transmitting corresponding blocks of the message to the destination by using a space-time transmission scheme [23] (e.g. the Alamouti scheme [28]). The destination combines the received signal in current round and those in previous rounds to jointly decode the data packet. After L ARQ (re)transmission rounds, if the destination still cannot decode the data packet, an outage is declared which means that the mutual information of the DF cooperative ARQ relay channel is below the transmission rate.

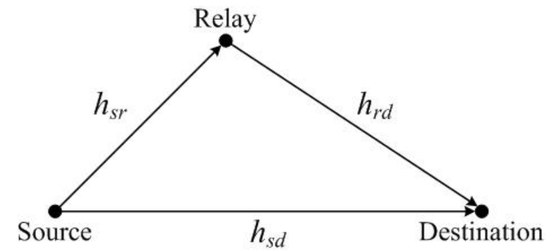

Fig. 1. Illustration of the cooperative ARQ relay scheme with one source, one relay and one destination.

The DF cooperative ARQ relay scheme can be modeled as follows. The received signal $y_{r,m}$ at the relay at the m-th $(1 \leq m \leq L)$ ARQ (re)transmission round can be modeled as

$$y_{r,m} = \sqrt{P_s} h_{sr,m} x_s + \eta_{r,m}, \tag{1}$$

where P_s is the transmitted power of the source signal x_s, $h_{sr,m}$ is the coefficient of the source-relay channel at the m-th ARQ (re)transmission round, and $\eta_{r,m}$ is the additive noise. If the relay is not involved in forwarding, the received signal $y_{d,m}$ at the destination at the m-th ARQ (re)transmission round is

$$y_{d,m} = \sqrt{P_s} h_{sd,m} x_s + \eta_{d,m}, \tag{2}$$

where $h_{sd,m}$ is the source-destination channel coefficient at the m-th ARQ (re)transmission round. If the relay receives the data packet from the source successfully, it helps in forwarding the packet to the destination using the Alamouti scheme. Specifically, each block x_s of the data sequence can be considered as having two parts, $x_{s,1}$, and $x_{s,2}$ (i.e. $x_s = [x_{s,1}\ x_{s,2}]$). The relay forwards the block in the form of $x_r = \left[-x_{s,2}^*\ x_{s,1}^*\right]$. The received signal $y_{d,m}$ at the destination at the m-th ARQ (re)transmission round can be written as

$$y_{d,m} = \sqrt{P_s} h_{sd,m} x_s + \sqrt{P_r} h_{rd,m} x_r + \eta_{d,m}, \tag{3}$$

where P_r is the transmitted power at the relay and $h_{rd,m}$ is the channel coefficient from the relay to the destination at the m-th ARQ (re)transmission round. At the destination, the message block x_s can be recovered based on the orthogonal structure of the Alamouti code [23], [28]. The channel coefficients $h_{sd,m}$, $h_{sr,m}$ and $h_{rd,m}$ are modeled as independent, zero-mean complex Gaussian random variables with variance σ_{sd}^2, σ_{sr}^2 and σ_{rd}^2, respectively. We consider a fast fading scenario, i.e. the channels remain fixed within one ARQ (re)transmission round, but change independently from one round to another (packet-rate fading or block fading). The channel state information is assumed to be known at the receiver and unknown at the transmitter. The noise terms $\eta_{r,m}$ and $\eta_{d,m}$ are modeled as zero-mean complex Gaussian random variables with variance $\mathcal{N}_0$.

III. Outage Probability Analysis

A. Two Lemmas

First we develop two lemmas that will play a key role in analyzing the outage probability of the DF cooperative ARQ relay scheme at high SNR. The first lemma reveals asymptotic behavior for the cumulative distribution function of the sum of two independent random variables, which are related to the mutual information of channel links in the cooperative ARQ scheme. The second lemma studies a key function which will be used to characterize the outage probability of the cooperative ARQ scheme.

Lemma 1: Assume that $\mathsf{u}_{s_1,...,s_M}$ and $\mathsf{v}_{s_1,...,s_M}$ be two independent scalar random variables. If their cumulative distribution functions (CDF) satisfy the following properties

$$\lim_{\substack{s_i\to\infty\\1\le i\le M}} \prod_{i=1}^{M} s_i^{d_1} \cdot \Pr\left[\mathsf{u}_{s_1,...,s_M} < t\right] = a \cdot f(t),$$
$$\lim_{\substack{s_i\to\infty\\1\le i\le M}} \prod_{i=1}^{M} s_i^{d_2} \cdot \Pr\left[\mathsf{v}_{s_1,...,s_M} < t\right] = b \cdot g(t),$$

where d_1, d_2, a and b are constants, $f(t)$ and $g(t)$ are monotonically increasing functions, and the derivative of $f(t)$ (denoted as $f'(t)$) is integrable, then the CDF of the sum of the two independent random variables follows the following property

$$\begin{aligned}\lim_{\substack{s_i\to\infty\\1\le i\le M}} \prod_{i=1}^{M} s_i^{d_1+d_2} \cdot \Pr\left[\mathsf{u}_{s_1,...,s_M} + \mathsf{v}_{s_1,...,s_M} < t\right] \\ = ab \cdot \int_0^t g(x) f'(t-x)\mathrm{d}x. \qquad (4)\end{aligned}$$

A proof of Lemma 1 is included in Appendix A. We note that the special case of Lemma 1 with $M=1$ was presented in [29]. Lemma 1 will be used to approximate the outage probability of the ARQ schemes at high SNR scenario. In the following, we define a key function $F_n(\beta_1, ..., \beta_n; t)$ which will be used to characterize the outage probability of the DF cooperative ARQ relay scheme. For any integer $n \ge 2$ and non-zero constants $\beta_1, \beta_2, \cdots, \beta_n$, define

$$\begin{aligned}&F_n(\beta_1, ..., \beta_n; t)\\ &\triangleq \int_0^t \int_0^{x_n} \cdots \int_0^{x_2} 2^{\beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n} \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_n. \qquad (5)\end{aligned}$$

The following lemma evaluates $F_n(\beta_1, ..., \beta_n; t)$ in closed-form[1], and its proof can be found in Appendix B.

Lemma 2: For any integer $n \ge 2$ and non-zero constants $\beta_1, \beta_2, \cdots, \beta_n$, the function $F_n(\beta_1, ..., \beta_n; t)$ can be calculated as follows

$$\begin{aligned}&F_n(\beta_1, ..., \beta_n; t)\\ &= \sum_{\substack{\delta_1,...,\delta_{n-1}\\ \in\{0,1\}}} \frac{(-1)^{n+\delta_1+\cdots+\delta_{n-1}} (\ln 2)^{-n}}{\prod_{m=1}^{n} \left[\sum_{l=1}^{m} i_{m,l}(\boldsymbol{\delta})\beta_l\right]} \left(2^{\left[\sum_{l=1}^{n} i_{n,l}(\boldsymbol{\delta})\beta_l\right]t} - 1\right), \qquad (6)\end{aligned}$$

where the variables $\delta_1, \delta_2, ..., \delta_{n-1} \in \{0,1\}$, $\boldsymbol{\delta} \triangleq \{\delta_1, \delta_2, ..., \delta_{n-1}\}$, and the coefficients $\{i_{m,l}(\boldsymbol{\delta}) : 1 \le m \le n, 1 \le l \le m\}$ are specified as

$$i_{1,1}(\boldsymbol{\delta}) = i_{2,2}(\boldsymbol{\delta}) = \cdots = i_{n,n}(\boldsymbol{\delta}) = 1,$$

and, for any $m = 2, 3, ..., n$,

$$i_{m,l}(\boldsymbol{\delta}) = \delta_{m-1} \cdot i_{m-1,l}(\boldsymbol{\delta}), \quad l = 1, 2, ..., m-1.$$

[1]Arguably, the significance of Lemma 2 goes beyond the problem considered in this paper.

B. Outage Probability of the Direct ARQ Transmission Scheme

For comparison purposes, in this subsection, we evaluate the outage probability of the direct ARQ transmission scheme. The destination of a direct ARQ transmission scheme receives information from the source directly, without involving the relay. The mutual information between the source and the destination in the m-th round of the direct ARQ transmission scheme is[2]

$$I_{sd,m} = \log_2\left(1 + \frac{P_s}{\mathcal{N}_0}|h_{sd,m}|^2\right). \qquad (7)$$

The total mutual information after L ARQ rounds is $I_{sd}^{tot} = \sum_{m=1}^{L} I_{sd,m}$. Thus, the outage probability of the direct ARQ scheme after L ARQ rounds is

$$\begin{aligned}P^{out,L} &= \Pr\left[I_{sd}^{tot} < R\right]\\ &= \Pr\left[\sum_{m=1}^{L} \log_2\left(1 + \frac{P_s}{\mathcal{N}_0}|h_{sd,m}|^2\right) < R\right]. \qquad (8)\end{aligned}$$

A closed-form expression of (8) is not tractable. However, an approximation of the outage probability for high-SNR can be obtained as an application of Lemma 1. Specifically, let $u_m = \log_2\left(1 + \frac{P_s}{\mathcal{N}_0}|h_{sd,m}|^2\right)$ and $s = \frac{P_s}{\mathcal{N}_0}$. Since $|h_{sd,m}|^2$ is an exponential random variable with parameter σ_{sd}^{-2}, we have

$$\begin{aligned}\lim_{s\to\infty} s \cdot \Pr\left[u_m < t\right] &= \lim_{s\to\infty} s \cdot \Pr\left[|h_{sd,m}|^2 < \frac{2^t - 1}{s}\right]\\ &= \frac{1}{\sigma_{sd}^2}\left(2^t - 1\right). \qquad (9)\end{aligned}$$

Since u_m, $1 \le m \le L$, are independent random variables, by applying Lemma 1 with $M=1$ recursively, we have an approximation of the outage probability at high-SNR as

$$P^{out,L} \approx g_L(R)\left(\frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s}\right)^L, \qquad (10)$$

where $g_L(\cdot)$ is defined as

$$g_n(t) = \int_0^t g_{n-1}(x) f'(t-x)\mathrm{d}x, \quad n \ge 1, \qquad (11)$$

with $g_0(t) = 1$ and $f(t) = 2^t - 1$. Expressions (10) and (11) appeared also in [29]. However, the calculation of the coefficient $g_L(R)$ that involves L recursive integrals was not treated in [29]. In this paper, we develop a closed-form expression for the function $g_n(t)$ for any $n \ge 1$. Specifically, since $g_n(t) = \int_0^t g_{n-1}(x) f'(t-x)\mathrm{d}x$ and $f'(t) = 2^t \ln 2$, we

[2]We assume that the transmission time in each round is long enough for the validity of the average mutual information in (7).

have

$$\begin{aligned} g_n(t) &= \int_0^t\int_0^{x_{n-1}}\cdots\int_0^{x_2} g_1(x_1)f'(x_2-x_1)f'(x_3-x_2)\cdots \\ &\quad\times f'(x_{n-1}-x_{n-2})f'(t-x_{n-1})\,\mathrm{d}x_1\mathrm{d}x_2\cdots\mathrm{d}x_{n-1} \\ &= \int_0^t\int_0^{x_{n-1}}\cdots\int_0^{x_2}(2^{x_1}-1)(\ln 2)^{n-1}2^{x_2-x_1}2^{x_3-x_2}\cdots \\ &\quad\times 2^{x_{n-1}-x_{n-2}}2^{t-x_{n-1}}\mathrm{d}x_1\mathrm{d}x_2\cdots\mathrm{d}x_{n-1} \\ &= 2^t(\ln 2)^{n-1}\int_0^t\int_0^{x_{n-1}}\cdots\int_0^{x_2}(1-2^{-x_1})\mathrm{d}x_1\mathrm{d}x_2\cdots\mathrm{d}x_{n-1} \\ &= 2^t\frac{(\ln 2)^{n-1}}{(n-2)!}\int_0^t x^{n-2}(1-2^{-t+x})\,\mathrm{d}x. \end{aligned}$$

Since $\int_0^t x^{n-2}\,\mathrm{d}x = \frac{1}{n-1}t^{n-1}$, and from ([30], Eqn. 2.321) we also have

$$\begin{aligned} \int_0^t x^{n-2}2^x\,\mathrm{d}x &= -2^t\sum_{m=1}^{n-1}\frac{(n-2)!}{(-\ln 2)^m(n-m-1)!}t^{n-m-1} \\ &\quad+\frac{(n-2)!}{(-\ln 2)^{n-1}}, \end{aligned}$$

therefore, a closed-form expression of $g_n(t)$ can be obtained as follows

$$\begin{aligned} g_n(t) &= 2^t\frac{(t\cdot\ln 2)^{n-1}}{(n-1)!} \\ &\quad+2^t\sum_{m=1}^{n-1}\frac{(-1)^m(t\cdot\ln 2)^{n-m-1}}{(n-m-1)!}+(-1)^n \\ &= 2^t\sum_{m=1}^{n}\frac{(-1)^{n-m}}{(m-1)!}(t\cdot\ln 2)^{m-1}+(-1)^n, \end{aligned} \tag{12}$$

which can be calculated efficiently.

C. Outage Probability of the DF Cooperative ARQ Relay Scheme

In this subsection, we derive the outage probability of the DF cooperative ARQ relay scheme under packet-rate fading conditions. In the DF cooperative ARQ relay scheme, if the relay decodes the message from the source correctly, say, at the k-th round, then at the $(k+1)$-th round, the relay starts forwarding appropriate ARQ blocks to the destination. Let $\{T_r = k\}$ denote the event of successful message decoding by the relay at the k-th round and subsequent ARQ block forwarding at the $(k+1)$-th round for any $k = 1, 2, \cdots, L-1$. Especially, let $\{T_r = L\}$ denote the event that the relay decodes unsuccessfully in the first $L-1$ rounds (in this case, no matter the relay decodes successfully or not at the L-th round, it has no chance to help in forwarding).

Furthermore, let us use $P^{out}_{T_r=k}$ to denote the conditional probability that the destination decodes the message unsuccessfully after L ARQ (re)transmission rounds given that the event $\{T_r = k\}$ occurred. In other words, $P^{out}_{T_r=k}$ denote the outage probability at the destination despite the fact that the relay started forwarding at the $(k+1)$-th round for any $k = 1, 2, \cdots, L-1$. Therefore, the outage probability of the DF cooperative ARQ relay scheme after L ARQ (re)transmission rounds can be written as

$$P^{out,L} = \sum_{k=1}^{L} P^{out}_{T_r=k}\cdot \Pr\left[T_r = k\right]. \tag{13}$$

First, we calculate the probability of the event $\{T_r = k\}$, i.e. $\Pr[T_r = k]$ for any $k = 1, 2, \cdots, L$. The mutual information between the source and the relay in the m-th ARQ round is

$$I_{sr,m} = \log_2\left(1+\frac{P_s}{\mathcal{N}_0}|h_{sr,m}|^2\right). \tag{14}$$

We note that the channels change independently over each ARQ (re)transmission round in a fast fading scenario, so the mutual information of fading channels can be viewed as a sum of independent random variables. The probability that the relay decodes the message successfully at the first round $(T_r = 1)$ is

$$\Pr[T_r = 1] = \Pr[I_{sr,1}\geq R] = \exp\left(-\frac{2^R-1}{\sigma^2_{sr}}\cdot\frac{\mathcal{N}_0}{P_s}\right). \tag{15}$$

For any $T_r = k$, $k = 2, 3, ..., L-1$, we have

$$\begin{aligned} \Pr[T_r = k] &= \Pr\left[\sum_{m=1}^{k-1} I_{sr,m} < R, \sum_{m=1}^{k} I_{sr,m}\geq R\right] \\ &= \Pr\left[R - I_{sr,k}\leq\sum_{m=1}^{k-1} I_{sr,m} < R\right] \\ &= \Pr\left[\sum_{m=1}^{k-1} I_{sr,m} < R\right] - \Pr\left[\sum_{m=1}^{k} I_{sr,m} < R\right] \\ &\approx g_{k-1}(R)\left(\frac{\mathcal{N}_0}{\sigma^2_{sr}P_s}\right)^{k-1} - g_k(R)\left(\frac{\mathcal{N}_0}{\sigma^2_{sr}P_s}\right)^{k}, \end{aligned} \tag{16}$$

where $g_{k-1}(\cdot)$ and $g_k(\cdot)$ are specified in (12) in the previous subsection. The approximation in (16) is obtained by applying Lemma 1 with $M = 1$ recursively. Finally, if $T_r = L$, we have

$$\Pr[T_r = L] = \Pr\left[\sum_{m=1}^{L-1} I_{sr,m} < R\right]\approx g_{L-1}(R)\left(\frac{\mathcal{N}_0}{\sigma^2_{sr}P_s}\right)^{L-1}. \tag{17}$$

To summarize, the probability $\Pr[T_r = k]$ in (13) can be given by the following branch function

$$\Pr[T_r = k]\approx \begin{cases} \exp\left(-\frac{2^R-1}{\sigma^2_{sr}}\cdot\frac{\mathcal{N}_0}{P_s}\right), & k = 1; \\ g_{k-1}(R)\left(\frac{\mathcal{N}_0}{\sigma^2_{sr}P_s}\right)^{k-1} - g_k(R)\left(\frac{\mathcal{N}_0}{\sigma^2_{sr}P_s}\right)^{k}, & 2\leq k\leq L-1; \\ g_{L-1}(R)\left(\frac{\mathcal{N}_0}{\sigma^2_{sr}P_s}\right)^{L-1}, & k = L. \end{cases} \tag{18}$$

Next, we calculate the conditional outage probability $P^{out}_{T_r=k}$ for any $k = 1, 2, \cdots, L$. This is done by the following theorem.

Theorem 1: The conditional outage probability $P^{out}_{T_r=k}$, $1\leq k\leq L$, is given by

$$P^{out}_{T_r=k}\approx\begin{cases} \frac{b_k(R)}{2^{L-k}}\left(\frac{\mathcal{N}_0}{\sigma^2_{sd}P_s}\right)^{L}\left(\frac{\mathcal{N}_0}{\sigma^2_{rd}P_r}\right)^{L-k}, & 1\leq k\leq L-1; \\ g_L(R)\left(\frac{\mathcal{N}_0}{\sigma^2_{sd}P_s}\right)^{L}, & k = L, \end{cases} \tag{19}$$

where $g_k(\cdot)$ are specified in (12), and

$$b_k(t)\triangleq\int_0^t g_k(x)q'_{L-k}(t-x)\mathrm{d}x, \tag{20}$$

in which $q_1(t) = (2^t - 1)^2$ and for any $2 \le n \le L-1$, $q_n(t)$ is given by

$$q_n(t) = (-2\ln 2)^{n-1} \sum_{\substack{\alpha_1,\cdots,\alpha_{n-1} \\ \in\{0,1\}}} (-1)^{\alpha_1+\cdots+\alpha_{n-1}} 2^{(1+\alpha_{n-1})t} \times \{F_{n-1}(1-\alpha_1, \alpha_1-\alpha_2, \cdots, \alpha_{n-2}-\alpha_{n-1}; t) - 2F_{n-1}(-\alpha_1, \alpha_1-\alpha_2, \cdots, \alpha_{n-2}-\alpha_{n-1}; t) + F_{n-1}(-1-\alpha_1, \alpha_1-\alpha_2, \cdots, \alpha_{n-2}-\alpha_{n-1}; t)\}. \tag{21}$$

The function $F_{n-1}(\cdot\,; t)$ is specified in Lemma 2.

Proof : When the relay cooperates with the source by jointly sending a message block via the Alamouti scheme, the mutual information of the cooperative channels in the m-th ARQ round is given by [23]

$$I_{srd,m} = \log_2\left(1 + \frac{P_s}{\mathcal{N}_0}|h_{sd,m}|^2 + \frac{P_r}{\mathcal{N}_0}|h_{rd,m}|^2\right). \tag{22}$$

Thus, with L ARQ rounds, the total mutual information is

$$I^{tot}_{d,T_r=k} = \begin{cases} \sum_{m=1}^{k} I_{sd,m} + \sum_{m=k+1}^{L} I_{srd,m}, & 1 \le k < L; \\ \sum_{m=1}^{L} I_{sd,m}, & k = L. \end{cases} \tag{23}$$

The above mutual information is based on the assumption that the channels change independently over each ARQ (re)transmission round, so the mutual information of fading channels can be viewed as a sum of independent random variables. We also note that when $T_r = L$, the relay has no chance to cooperate since the source starts sending a new packet.

The conditional outage probability $P^{out}_{T_r=k}$ can be evaluated as

$$P^{out}_{T_r=k} = \Pr\left[I^{tot}_{d,T_r=k} < R\right]. \tag{24}$$

Note that, when $T_r = L$, the relay has no chance to cooperate regardless of whether the relay decodes correctly at the L-th round or not. Thus, in this case, the conditional outage probability $P^{out}_{T_r=L}$ is reduced to the direct ARQ scenario and it is given by

$$P^{out}_{T_r=L} = \Pr\left[I^{tot}_{d,T_r=L} < R\right] \approx g_L(R)\left(\frac{\mathcal{N}_0}{\sigma^2_{sd}P_s}\right)^L. \tag{25}$$

In the following, we calculate the conditional outage probability (24) for any $T_r = k$, $k = 1, 2, ..., L-1$. For simplicity in presentation, we introduce the following notation. Let

$$v_m = \begin{cases} \log_2\left(1 + s_1|h_{sd,m}|^2\right), & 1 \le m \le k; \\ \log_2\left(1 + s_1|h_{sd,m}|^2 + s_2|h_{rd,m}|^2\right), & k+1 \le m \le L, \end{cases} \tag{26}$$

where $s_1 = P_s/\mathcal{N}_0$ and $s_2 = P_r/\mathcal{N}_0$. Then, the total mutual information can be written as

$$I^{tot}_{d,T_r=k} = \sum_{m=1}^{L} v_m. \tag{27}$$

We note that for any $1 \le m \le k$, $|h_{sd,m}|^2$ is an exponential random variable with parameter σ^{-2}_{sd}. Thus,

$$\lim_{s_1\to\infty} s_1 \cdot \Pr[v_m < t] = \lim_{s_1\to\infty} s_1 \cdot \Pr\left[|h_{sd,m}|^2 < \frac{2^t-1}{s_1}\right] = \frac{1}{\sigma^2_{sd}}(2^t - 1), \quad m = 1, 2\cdots, k. \tag{28}$$

Since v_m, $1 \le m \le k$, are independent random variables, by applying Lemma 1 with $M = 1$ recursively, we have

$$\lim_{s_1\to\infty} s_1^k \cdot \Pr\left[\sum_{m=1}^{k} v_m < t\right] = \left(\frac{1}{\sigma^2_{sd}}\right)^k g_k(t), \tag{29}$$

where $g_k(t)$ is given in (12). For any m, $k+1 \le m \le L$, v_m involves the sum of two independent exponential random variables $|h_{sd,m}|^2$ and $|h_{rd,m}|^2$ with parameters σ^{-2}_{sd} and σ^{-2}_{rd}, respectively, and the distribution of v_m can be specified as

$$\Pr[v_m < t] = \Pr\left[s_1|h_{sd,m}|^2 + s_2|h_{rd,m}|^2 < 2^t - 1\right] = \begin{cases} 1 - \left(1 + \frac{1}{\sigma^2_{sd}}\frac{2^t-1}{s_1}\right)\exp\left(-\frac{1}{\sigma^2_{sd}}\frac{2^t-1}{s_1}\right), & \text{if } \frac{s_1}{\sigma^2_{rd}} = \frac{s_2}{\sigma^2_{sd}}; \\ 1 - \frac{s_1\sigma^2_{sd}}{s_1\sigma^2_{sd} - s_2\sigma^2_{rd}}\exp\left(-\frac{1}{\sigma^2_{sd}}\frac{2^t-1}{s_1}\right) - \frac{s_2\sigma^2_{rd}}{s_2\sigma^2_{rd} - s_1\sigma^2_{sd}}\exp\left(-\frac{1}{\sigma^2_{rd}}\frac{2^t-1}{s_2}\right), & \text{if } \frac{s_1}{\sigma^2_{rd}} \ne \frac{s_2}{\sigma^2_{sd}}, \end{cases} \tag{30}$$

for any $m = k+1, ..., L$. Thus, for any $m = k+1, ..., L$, we have

$$\lim_{\substack{s_i\to\infty \\ 1\le i\le 2}} s_1 s_2 \cdot \Pr[v_m < t] = \frac{1}{2\sigma^2_{sd}\sigma^2_{rd}}(2^t - 1)^2. \tag{31}$$

Let $q_0(t) = 1$ and $p(t) = (2^t - 1)^2$, then $p'(t) = 2(2^{2t} - 2^t)\ln 2$. Since v_m, $k+1 \le m \le L$, are independent to each other, by applying Lemma 1 with $M = 2$ recursively, we can show that for any $n = 1, 2, ..., L-k$,

$$\lim_{\substack{s_i\to\infty \\ 1\le i\le 2}} (s_1 s_2)^n \cdot \Pr\left[\sum_{m=k+1}^{k+n} v_m < t\right] = \left(\frac{1}{2\sigma^2_{sd}\sigma^2_{rd}}\right)^n q_n(t), \tag{32}$$

in which

$$q_n(t) = \int_0^t q_{n-1}(x)p'(t-x)\mathrm{d}x, \quad n = 1, 2, ..., L-k. \tag{33}$$

We can see that when $n = 1$, $q_1(t) = (2^t - 1)^2$. But for larger n, the calculation of $q_n(t)$ is involved. Based on Lemma 2, a closed-form expression for $q_n(t)$ can be obtained as follows:

$$\begin{aligned} q_n(t) &= \int_0^t\int_0^{x_{n-1}}\cdots\int_0^{x_2} q_1(x_1)p'(x_2 - x_1)p'(x_3 - x_2)\cdots \\ &\quad \times p'(x_{n-1} - x_{n-2})p'(t - x_{n-1})\mathrm{d}x_1\mathrm{d}x_2\cdots\mathrm{d}x_{n-1} \\ &= \int_0^t\int_0^{x_{n-1}}\cdots\int_0^{x_2}(2^{x_1} - 1)^2(2\ln 2)^{n-1}\prod_{m=1}^{n-2}\left(2^{x_{m+1}-x_m} - 1\right) \\ &\quad \times (2^{t-x_{n-1}} - 1)2^{t-x_1}\mathrm{d}x_1\mathrm{d}x_2\cdots\mathrm{d}x_{n-1} \\ &= (-2\ln 2)^{n-1}\sum_{\substack{\alpha_1,\ldots,\alpha_{n-1} \\ \in\{0,1\}}}(-1)^{\alpha_1+\cdots+\alpha_{n-1}}2^{(1+\alpha_{n-1})t} \\ &\quad \times \int_0^t\int_0^{x_{n-1}}\cdots\int_0^{x_2}(2^{x_1} - 1)^2 \cdot 2^{-\alpha_1 x_1} \end{aligned}$$

$$\times \prod_{m=2}^{n-1} 2^{(\alpha_{m-1}-\alpha_m)x_m} \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_{n-1}$$
$$= (-2\ln 2)^{n-1} \sum_{\substack{\alpha_1,\ldots,\alpha_{n-1}\\ \in\{0,1\}}} (-1)^{\alpha_1+\cdots+\alpha_{n-1}} 2^{(1+\alpha_{n-1})t}$$
$$\times \big\{ F_{n-1}(1-\alpha_1,\beta_2,...,\beta_{n-1};t) - 2F_{n-1}(-\alpha_1,\beta_2,...,\beta_{n-1};t) + F_{n-1}(-1-\alpha_1,\beta_2,...,\beta_{n-1};t) \big\}, \quad (34)$$

where $\beta_2 = \alpha_1 - \alpha_2$, $\beta_3 = \alpha_2 - \alpha_3$, ..., $\beta_{n-1} = \alpha_{n-2} - \alpha_{n-1}$, and $F_{n-1}(\,\cdot\,;t)$ is defined in (6) (a closed-form expression for $F_{n-1}(\,\cdot\,;t)$ is given in Lemma 2). We note that in the case where β_i is zero for some i, the non-zero condition in Lemma 2 is not satisfied. In such case, we may evaluate $F_{n-1}(\cdot\,;t)$ by applying Lemma 2 with $\beta_i = \varepsilon_i$ where ε_i is sufficiently small (i.e. $\varepsilon_i \to 0$) (Since the function $F_n(\beta_1,...,\beta_n;t)$ defined in (6) is continuous in terms of β_i, $2 \le i \le n-1$, so is the closed-form expression in Lemma 2).

According to the result in (32) with $n = L-k$, we have

$$\lim_{\substack{s_i\to\infty\\ 1\le i\le 2}} (s_1 s_2)^{L-k} \cdot \Pr\left[\sum_{m=k+1}^{L} v_m < t\right] = \left(\frac{1}{2\sigma_{sd}^2 \sigma_{rd}^2}\right)^{L-k} q_{L-k}(t), \quad (35)$$

where $q_{L-k}(t)$ can be calculated specifically based on (34). Combining (29) and (35), and applying Lemma 1, we obtain

$$\lim_{\substack{s_i\to\infty\\ 1\le i\le 2}} s_1^L s_2^{L-k} \cdot \Pr\left[\sum_{m=1}^{k} I_{sd,m} + \sum_{m=k+1}^{L} I_{srd,m} < R\right] = b_k(R)\left(\frac{1}{\sigma_{sd}^2}\right)^{L}\left(\frac{1}{2\sigma_{rd}^2}\right)^{L-k}, \quad (36)$$

where

$$b_k(t) = \int_0^t g_k(x) q'_{L-k}(t-x)\mathrm{d}x. \quad (37)$$

Since $s_1 = P_s/\mathcal{N}_0$ and $s_2 = P_r/\mathcal{N}_0$, so for any $T_r = k$, $k = 1,2,...,L-1$, the conditional probability (24) can be asymptotically approximated as

$$P^{out}_{T_r=k} = \Pr\left[\sum_{m=1}^{k} I_{sd,m} + \sum_{m=k+1}^{L} I_{srd,m} < R\right] \approx \frac{b_k(R)}{2^{L-k}}\left(\frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s}\right)^{L}\left(\frac{\mathcal{N}_0}{\sigma_{rd}^2 P_r}\right)^{L-k}, \quad (38)$$

which completes the proof of the theorem. □

Finally, based on the probability $\Pr[T_r = k]$ in (18) and the conditional outage probability $P^{out}_{T_r=k}$ in Theorem 1, we can obtain the outage probability for the DF cooperative ARQ relay scheme as follows

$$P^{out,L} \approx \sum_{k=1}^{L-1} \frac{b_k(R)}{2^{L-k}}\left[g_{k-1}(R) - g_k(R)\frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s}\right] \times \left(\frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s}\right)^{L}\left(\frac{\mathcal{N}_0}{\sigma_{rd}^2 P_r}\right)^{L-k}\left(\frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s}\right)^{k-1} + g_L(R) g_{L-1}(R)\left(\frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s}\right)^{L}\left(\frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s}\right)^{L-1}, \quad (39)$$

where $g_k(R)$ and $b_k(R)$ are specified in (12) and (37), respectively. Furthermore, we note that the term $g_k(R)\frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s}$ is much smaller than $g_{k-1}(R)$ at high SNR $\frac{P_s}{\mathcal{N}_0}$, which can be ignored in the outage probability, so the asymptotic outage probability in (39) can be further simplified as

$$P^{out,L} \approx \sum_{k=1}^{L} \frac{b_k(R) g_{k-1}(R)}{2^{L-k}} \times \left(\frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s}\right)^{L}\left(\frac{\mathcal{N}_0}{\sigma_{rd}^2 P_r}\right)^{L-k}\left(\frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s}\right)^{k-1}, \quad (40)$$

where $b_L(R) = g_L(R)$. Simulation studies presented later in this paper will illustrate the tightness of the outage probability at high SNR.

Based on the above asymptotic outage probability, we observe that the term $\left(\frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s}\right)^L$ in (40) contributes a diversity order L in the asymptotic outage performance, which is due to the fast fading or block fading nature of the channels. The term $\left(\frac{\mathcal{N}_0}{\sigma_{rd}^2 P_r}\right)^{L-k}\left(\frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s}\right)^{k-1}$ contributes an overall diversity order $(L-k)+(k-1) = L-1$ which is due to the cooperative relaying. Thus, the asymptotic outage probability of the DF cooperative ARQ relay scheme has an overall diversity order $2L-1$. In the case of the equal power allocation, i.e. $P_s = P_r = P$, the contribution of the diversity order in the outage probability is more evident. We recall that the diversity order of the direct ARQ transmission scheme is only L, which is due to the fast fading or block fading nature of the channels, and it is much smaller than that of the DF cooperative ARQ relay scheme.

IV. Optimum Power Allocation for the DF Cooperative ARQ Relay Scheme

In this section, we derive the asymptotic optimum power allocation strategy for the DF cooperative ARQ relay scheme based on the approximation of the outage probability that was presented in the previous section. Without loss of generality, we denote the total transmission power as $P_s + P_r \triangleq 2P$, where P_s and P_r are the power used by the source and the relay, respectively. Then, for any given total transmission power $2P$, we try to determine optimum power P_s and P_r in order to minimize the asymptotic outage probability.

Let λ denote the ratio of the source power P_s to the total transmission power, i.e. $\lambda = \frac{P_s}{2P}$. Then $0 \le \lambda \le 1$ and $P_r = (1-\lambda)2P$. The asymptotic outage probability of the DF cooperative ARQ relay scheme can be written as

$$P^{out,L} \approx \frac{\sigma_{sr}^2}{(2\sigma_{sd}^2\sigma_{rd}^2)^L}\left(\frac{\mathcal{N}_0}{2P}\right)^{2L-1}\sum_{k=1}^{L} b_k(R) g_{k-1}(R) \times \left(\frac{2\sigma_{rd}^2}{\sigma_{sr}^2}\right)^{k}\frac{1}{\lambda^{L+k-1}(1-\lambda)^{L-k}}. \quad (41)$$

We try to find the optimum power ratio λ $(0 \le \lambda \le 1)$ such that the asymptotic outage probability is minimized. Let $A_k(R) \triangleq b_k(R) g_{k-1}(R)\left(\frac{2\sigma_{rd}^2}{\sigma_{sr}^2}\right)^k$ and

$$G(\lambda) \triangleq \sum_{k=1}^{L}\frac{A_k(R)}{\lambda^{L+k-1}(1-\lambda)^{L-k}}. \quad (42)$$

Then the optimization problem can be formulated as follows

$$\begin{aligned} \min_{\lambda} \quad & G(\lambda) \\ s.t. \quad & 0 < \lambda < 1. \end{aligned} \tag{43}$$

The optimum value of λ (i.e. λ_{opt}) satisfies the following equation

$$\frac{\partial G(\lambda)}{\partial \lambda} = \sum_{k=1}^{L} A_k(R) \left\{ \frac{-(L+k-1)}{\lambda^{L+k}(1-\lambda)^{L-k}} + \frac{L-k}{\lambda^{L+k-1}(1-\lambda)^{L-k+1}} \right\} = 0, \tag{44}$$

or equivalently

$$\sum_{k=1}^{L} A_k(R) \left\{ -(L+k-1)\left(\frac{1-\lambda}{\lambda}\right)^k + (L-k)\left(\frac{1-\lambda}{\lambda}\right)^{k-1} \right\} = 0. \tag{45}$$

Equation (45) can be easily solved by the Newton method. Looking closely at equation (45), we observe the followings. Since $A_k(R)$ is positive, then $\frac{1-\lambda}{\lambda}$ must be less than 1, otherwise the left-hand side of (45) is negative. It is thus implied that $\lambda > \frac{1}{2}$, i.e. $P_s > P$ and $P_r < P$ which means that we should allocate more power at the source and less power at the relay. It also shows that the equal power allocation scheme that assigns equal power to the source and the relay is not optimum in general. On the other hand, for any given transmission rate R, the parameters $A_k(R)$ in (45) depend only on σ_{sr}^2 and σ_{rd}^2 which are the variance values of the source-relay and relay-destination channel links, respectively. Thus, the asymptotic optimum power ratio λ_{opt} depends only on the the variance of the source-relay and relay-destination channels, and not on the source-destination channel link. A similar observation was reported in [7] where the optimum power allocation between the source and the relay was determined based on the analysis of the symbol-error-rate performance.

As an example, we study the case where $L = 2$. In this case, equation (45) is reduced to

$$3A_2(R)\left(\frac{1-\lambda}{\lambda}\right)^2 + 2A_1(R)\left(\frac{1-\lambda}{\lambda}\right) - A_1(R) = 0, \tag{46}$$

so the optimum power ratio is

$$\lambda = \frac{1 + \sqrt{1 + 3\frac{A_2(R)}{A_1(R)}}}{2 + \sqrt{1 + 3\frac{A_2(R)}{A_1(R)}}}. \tag{47}$$

Thus the corresponding optimum power allocation at the source and at the relay is given by

$$P_s = \frac{1 + \sqrt{1 + 3\frac{A_2(R)}{A_1(R)}}}{1 + \frac{1}{2}\sqrt{1 + 3\frac{A_2(R)}{A_1(R)}}} P, \tag{48}$$

$$P_r = \frac{1}{1 + \frac{1}{2}\sqrt{1 + 3\frac{A_2(R)}{A_1(R)}}} P. \tag{49}$$

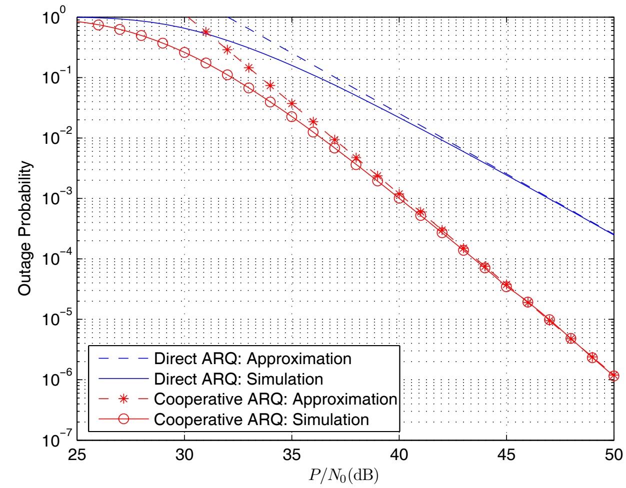

Fig. 2. Outage probability of the direct and DF cooperative ARQ schemes (L=2).

In (48) and (49), $\frac{A_2(R)}{A_1(R)} = \frac{2\, b_2(R) g_1(R)}{b_1(R)} \frac{\sigma_{rd}^2}{\sigma_{sr}^2}$. Thus, the optimum power allocation dependents only on the ratio of the variance of the source-relay channel and the variance of the relay-destination channel, which is consistent with our observation that we based on equation (45). Also, from (48) and (49), we can see that $P < P_s < 2P$ and $0 < P_r < P$, i.e. we should allocate more power at the source and less power at the relay to optimize the overall performance at the destination. Furthermore, (48) and (49) imply that if the relay is located close to the source, i.e. $\sigma_{sr}^2 \gg \sigma_{rd}^2$, P_s goes to $\frac{4}{3}P$ and P_r goes to $\frac{2}{3}P$. On the contrary, if the relay is located close to the destination, i.e. $\sigma_{sr}^2 \ll \sigma_{rd}^2$, P_s goes to $2P$ and P_r goes to 0, which means that, in this case we should allocate most of the power ($2P$) at the source. The latter is reasonable since the cooperative role of the relay is minor in this case.

V. Simulation Results

In this section, we present numerical and simulation studies that compare the performance of the DF cooperative ARQ scheme with that of the direct ARQ scheme. In all studies, the variance of the channel h_{ij} $\{(i,j) \in (s,d),(s,r),(r,d)\}$ is assumed to be $\sigma_{ij}^2 = d_{ij}^{-\mu}$, where d_{ij} is the distance between two nodes and μ is the path loss exponent which is assumed to be $\mu = 3$ in a typical fading environment. We assume that the source-destination distance is $d_{sd} = 10$ m and the relay is located in the midpoint between the source and the destination. We consider a target transmission rate of $R = 2$ bits/s/Hz.

Figs. 2, 3 and 4 illustrate the performance curves when the maximum number of ARQ (re)transmission rounds is $L = 2$, 3 and 4, respectively. In these figures, equal power allocation is assumed, i.e. $P_s = P_r = P$. All figures show that the proposed theoretical approximation of the outage probability of the DF cooperative ARQ relay scheme is tight for high SNR values while it is less tight for low SNR values. For example, in Fig. 2 ($L = 2$), the analytical approximation curve is almost indistinguishable from the simulated curve for all outage performance levels below 10^{-3}. Moreover, the larger the number of ARQ (re)transmission rounds, the

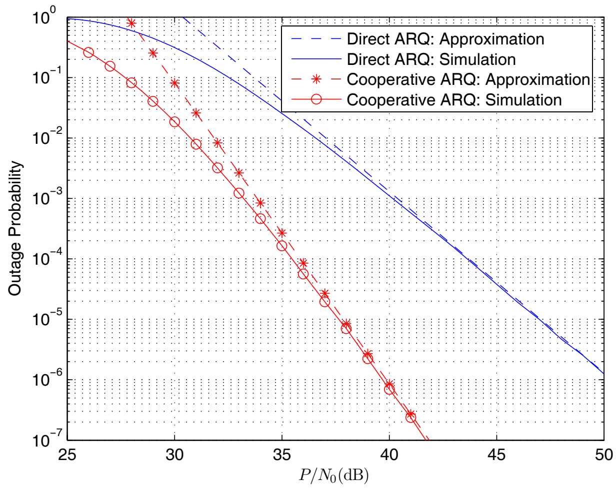

Fig. 3. Outage probability of the direct and DF cooperative ARQ schemes (L=3).

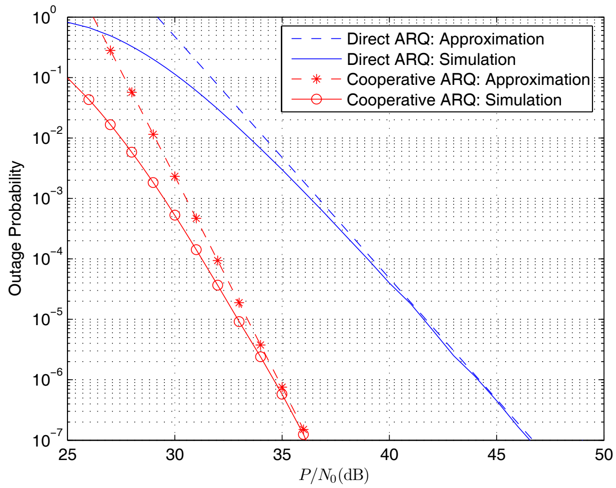

Fig. 4. Outage probability of the direct and DF cooperative ARQ schemes (L=4).

higher the diversity order of the DF cooperative ARQ relay scheme. This observation is consistent with the theoretical result that the diversity order of the DF cooperative ARQ relay scheme increases as the number of ARQ (re)transmission rounds increases.

All Figs. 2–4 show that the DF cooperative ARQ relay scheme significantly outperforms the direct ARQ scheme. At an outage performance of 10^{-4}, the performance of the DF cooperative ARQ relay scheme is about 8dB better than that of the direct ARQ scheme. For the same maximum number of (re)transmission rounds L, the DF cooperative ARQ relay scheme exhibits a higher diversity order than the direct ARQ scheme which is consistent with our theoretical developments, i.e. the DF cooperative ARQ relay scheme has diversity order $2L-1$, while the direct ARQ scheme has diversity order only L.

In Fig. 5, we plot the power allocation optimization function $G(\lambda)$ in terms of λ $(0 \leq \lambda \leq 1)$ for the cases where $L = 2$, 3 and 4, respectively. We assume that the quality of the source-relay link is the same as that of the relay-destination link, i.e. $\sigma_{sr}^2 = \sigma_{rd}^2$. We observe that the optimum power ratio λ is about 0.8 for the three cases. More precisely, from the numerical results, the optimum power ratios are $\lambda = 0.8203$, $\lambda = 0.7969$ and $\lambda = 0.7838$ for $L = 2$, 3 and 4, respectively. It appears that the optimum power ratio decreases gradually when the maximum number of (re)transmission rounds is increased. Furthermore, the optimum power ratio is much larger than $1/2$ which is consistent with our analysis that we should allocate more power at the source and less at the relay.

In Figs. 6–8, we compare the performance of the DF cooperative ARQ relay scheme with optimum power allocation and with equal power allocation for $L = 2$, 3 and 4, respectively. Both simulation and numerical approximation curves are included. In Fig. 6 ($L = 2$), the optimum power allocated at the source and the relay is set to the numerical values obtained from Fig. 5, i.e. $P_s/2P = 0.8203$ and $P_r/2P = 0.1797$, respectively. We observe that the performance of the DF cooperative ARQ relay scheme with the optimum power allocation is about 1.5dB better than that of the scheme with the equal power allocation. In Fig. 7 ($L = 3$), the optimum power allocation is $P_s/2P = 0.7969$ and $P_r/2P = 0.2031$, and the corresponding curve shows a performance improvement of 1.25dB compared to the equal power allocation case. In Fig. 8 ($L = 4$), the optimum power allocation is $P_s/2P = 0.7838$ and $P_r/2P = 0.2162$. We see that the performance curve of the DF cooperative ARQ relay scheme with optimum power allocation also exhibits performance gains of about 1.25dB compared to the performance of the equal power allocation scheme.

VI. Conclusion

In this paper, we developed, for the first time, a closed-form asymptotically tight (as SNR $\rightarrow \infty$) approximation of the outage probability for the DF cooperative ARQ relay scheme under fast fading (packet-rate fading or block fading) conditions, in which each relay forwards Alamouti-based retransmission signals. The closed-from expression provides significant insight into the merits of DF cooperative ARQ relaying relative to the direct ARQ scheme in fast fading scenarios and shows that the cooperative scheme achieves diversity order equal to $2L-1$ while the diversity order of the direct scheme is only L. Simulation and numerical studies illustrated that the closed-form approximation of the outage probability is tight at high SNR. Based on the asymptotically tight approximation of the outage probability, we were able to determine the optimum power allocation strategy for the DF cooperative ARQ relay scheme. It turns out that equal power allocation is not optimum in general and that the optimum power allocation strategy depends on the link quality of the channels related to the relay. It is also clear that we should allocate more power at the source and less at the relay. Further numerical and simulation studies illustrated the performance gains of the DF cooperative ARQ relay scheme with optimum power allocation relative to the equal power allocation scheme. We note that the outage probability developed in this paper may be used to analyze the delay and throughput performance of the DF cooperative ARQ relay scheme.

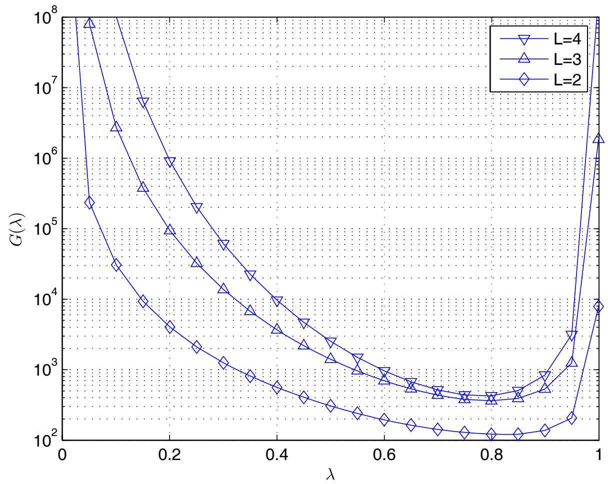

Fig. 5. Optimum power ratio λ for the DF cooperative ARQ scheme. When L=2, 3 and 4, the asymptotic power allocation is $\lambda = 0.8203$, $\lambda = 0.7969$ and $\lambda = 0.7838$, respectively.

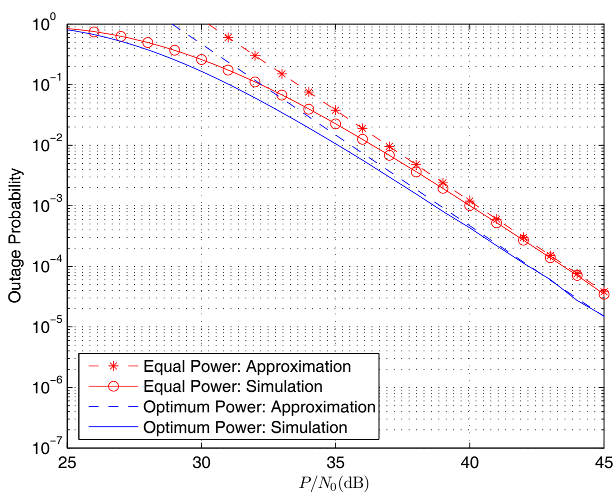

Fig. 6. Outage probability of the DF cooperative ARQ scheme with equal and optimum power allocations (L=2).

Appendix A
Proof of Lemma 1

For any partition $\boldsymbol{U}$ of the interval $[0, t]$, denoted as $\boldsymbol{U} = \{u_0, u_1, ..., u_J\}$ with $u_0 = 0$ and $u_J = t$, we can obtain upper and lower bounds of the event $\{\mathsf{u}_{s_1,...,s_M} + \mathsf{v}_{s_1,...,s_M} < t\}$ as follows:

$$\{\mathsf{u}_{s_1,...,s_M} + \mathsf{v}_{s_1,...,s_M} < t\} \subseteq \bigcup_{j=1}^{J} \{u_{j-1} \leq \mathsf{u}_{s_1,...,s_M} < u_j\} \cap \{\mathsf{v}_{s_1,...,s_M} < t - u_{j-1}\}, \tag{50}$$

$$\{\mathsf{u}_{s_1,...,s_M} + \mathsf{v}_{s_1,...,s_M} < t\} \supseteq \bigcup_{j=1}^{J} \{u_{j-1} \leq \mathsf{u}_{s_1,...,s_M} < u_j\} \cap \{\mathsf{v}_{s_1,...,s_M} < t - u_j\}. \tag{51}$$

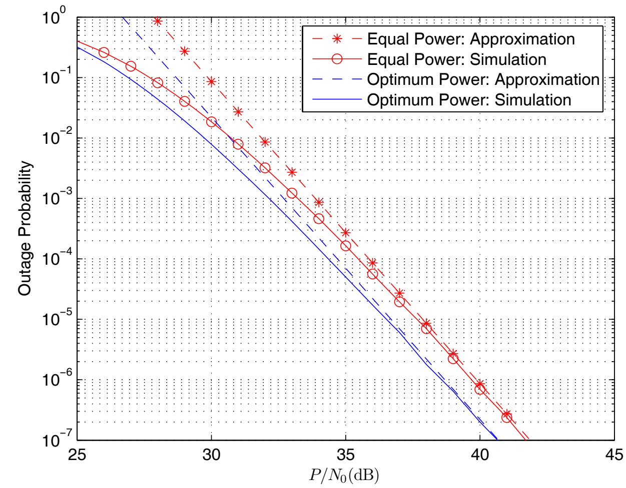

Fig. 7. Outage probability of the DF cooperative ARQ scheme with equal and optimum power allocations (L=3).

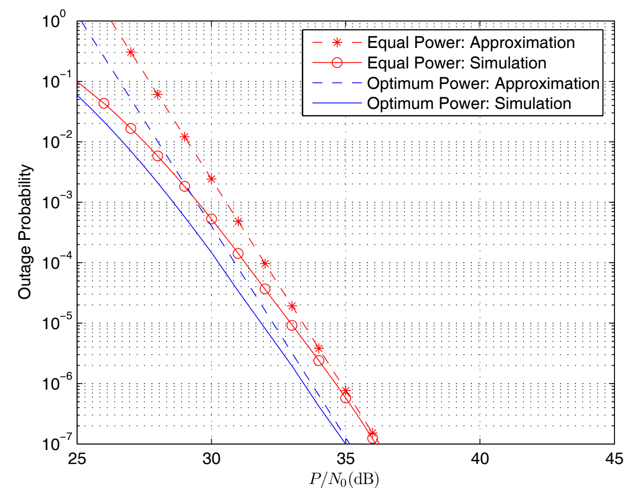

Fig. 8. Outage probability of the DF cooperative ARQ scheme with equal and optimum power allocations (L=4).

The upper and lower bounds in (50) and (51), respectively, are considered as a union of rectangles between u_{j-1} and u_j for $0 \leq j \leq J$. First, let us focus on the upper bound. The probability of the subset $\{u_{j-1} \leq \mathsf{u}_{s_1,...,s_M} < u_j\} \cap \{\mathsf{v}_{s_1,...,s_M} < t - u_{j-1}\}$ can be calculated as

$$\begin{aligned} &\Pr\left[u_{j-1} \leq \mathsf{u}_{s_1,...,s_M} < u_i, \mathsf{v}_{s_1,...,s_M} < t - u_{j-1}\right] \\ &= \{\Pr\left[\mathsf{u}_{s_1,...,s_M} < u_j\right] - \Pr\left[\mathsf{u}_{s_1,...,s_M} < u_{j-1}\right]\} \\ &\quad \times \Pr\left[\mathsf{v}_{s_1,...,s_M} < t - u_{j-1}\right]. \end{aligned} \tag{52}$$

Then,

$$\begin{aligned} \lim_{\substack{s_i \to \infty \\ 1 \leq i \leq M}} \prod_{i=1}^{M} s_i^{d_1+d_2} \cdot \Pr\big[&u_{j-1} \leq \mathsf{u}_{s_1,...,s_M} < u_j, \\ &\mathsf{v}_{s_1,...,s_M} < t - u_{j-1}\big] \\ = ab \cdot \{f(u_j) - &f(u_{j-1})\} g(t - u_{j-1}), \end{aligned} \tag{53}$$

and for any monotonically increasing functions $f(t)$ and $g(t)$,

$$\sup_{\boldsymbol{U}} \lim_{\substack{s_i\to\infty\\1\le i\le M}} \prod_{i=1}^{M} s_i^{d_1+d_2} \cdot \Pr\left[\mathsf{u}_{s_1,\ldots,s_M} + \mathsf{v}_{s_1,\ldots,s_M} < t\right] \le ab \cdot \sum_{j=1}^{J} g(t-u_{j-1})\{f(u_j)-f(u_{j-1})\}. \quad (54)$$

Similarly,

$$\inf_{\boldsymbol{U}} \lim_{\substack{s_i\to\infty\\1\le i\le M}} \prod_{i=1}^{M} s_i^{d_1+d_2} \cdot \Pr\left[\mathsf{u}_{s_1,\ldots,s_M} + \mathsf{v}_{s_1,\ldots,s_M} < t\right] \ge ab \cdot \sum_{j=1}^{L} g(t-u_{j})\{f(u_j)-f(u_{j-1})\}. \quad (55)$$

The above upper and lower bounds are good for any partition $\boldsymbol{U} = \{u_0, u_1, ..., u_J\}$ over the interval $[0,\, t]$. Since $f'(t)$ is integrable, then, for $J \to \infty$, the sum terms in (54) and (55) converge to the same integral $\int_0^t g(x)f'(t-x)\mathrm{d}x$. Therefore we have the result in (4). □

APPENDIX B
PROOF OF LEMMA 2

We use induction to prove the result for any integer $n \ge 2$. When $n = 2$, it is easy to see that

$$F_2(\beta_1,\beta_2;t) = \int_0^t \int_0^{x_2} 2^{\beta_1 x_1+\beta_2 x_2} \mathrm{d}x_1 \mathrm{d}x_2$$
$$= \frac{(\ln 2)^{-1}}{\beta_1}\left\{\frac{(\ln 2)^{-1}}{\beta_1+\beta_2}\left(2^{(\beta_1+\beta_2)t}-1\right) - \frac{(\ln 2)^{-1}}{\beta_2}\left(2^{\beta_2 t}-1\right)\right\}$$
$$= \sum_{\delta_1\in\{0,1\}} \frac{(-1)^{2+\delta_1}(\ln 2)^{-2}}{\prod_{m=1}^{2}\left[\sum_{l=1}^{m} i_{m,l}(\boldsymbol{\delta})\beta_l\right]}\left(2^{\left[\sum_{l=1}^{2} i_{2,l}(\boldsymbol{\delta})\beta_l\right]t}-1\right), \quad (56)$$

i.e. the closed-form expression in (6) is valid for $n = 2$. Next, we assume that the result in (6) is good for $n = k$, where k is any fixed integer greater or equal to 2. Then, for $n = k + 1$,

$$F_{k+1}(\beta_1, ..., \beta_{k+1}; t)$$
$$= \int_0^t \int_0^{x_{k+1}} \cdots \int_0^{x_2} 2^{\beta_1 x_1+\beta_2 x_2+\cdots+\beta_{k+1}x_{k+1}} \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_{k+1}$$
$$= \int_0^t \underbrace{\int_0^{x_{k+1}} \cdots \int_0^{x_2} 2^{\beta_1 x_1+\beta_2 x_2+\cdots+\beta_k x_k} \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_k}_{F_k(\beta_1,\ldots,\beta_k;\, x_{k+1})} \times 2^{\beta_{k+1}x_{k+1}} \mathrm{d}x_{k+1}$$
$$= \int_0^t F_k(\beta_1, ..., \beta_k;\, x_{k+1})\, 2^{\beta_{k+1}x_{k+1}}\, \mathrm{d}x_{k+1}. \quad (57)$$

According to the induction assumption, we have

$$F_k(\beta_1, ..., \beta_k; x_{k+1})$$
$$= \int_0^{x_{k+1}} \int_0^{x_k} \cdots \int_0^{x_2} 2^{\beta_1 x_1+\beta_2 x_2+\cdots+\beta_k x_k} \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_k$$
$$= \sum_{\substack{\delta_1,\ldots,\delta_{k-1}\\ \in\{0,1\}}} \frac{(-1)^{k+\delta_1+\cdots+\delta_{k-1}}(\ln 2)^{-k}}{\prod_{m=1}^{k}\left[\sum_{l=1}^{m} i_{m,l}(\boldsymbol{\delta})\beta_l\right]}\left(2^{\left[\sum_{l=1}^{k} i_{k,l}(\boldsymbol{\delta})\beta_l\right]x_{k+1}}-1\right). \quad (58)$$

Substituting (58) into (57), we obtain

$$F_{k+1}(\beta_1, ..., \beta_{k+1}; t)$$
$$= \sum_{\substack{\delta_1,\ldots,\delta_{k-1}\\ \in\{0,1\}}} \frac{(-1)^{k+\delta_1+\cdots+\delta_{k-1}}(\ln 2)^{-k}}{\prod_{m=1}^{k}\left[\sum_{l=1}^{m} i_{m,l}(\boldsymbol{\delta})\beta_l\right]} \times \int_0^t \left(2^{\left[\sum_{l=1}^{k} i_{k,l}(\boldsymbol{\delta})\beta_l+\beta_{k+1}\right]x_{k+1}} - 2^{\beta_{k+1}x_{k+1}}\right)\mathrm{d}x_{k+1}$$
$$= \sum_{\substack{\delta_1,\ldots,\delta_{k-1}\\ \in\{0,1\}}} \frac{(-1)^{k+\delta_1+\cdots+\delta_{k-1}}(\ln 2)^{-(k+1)}}{\prod_{m=1}^{k}\left[\sum_{l=1}^{m} i_{m,l}(\boldsymbol{\delta})\beta_l\right]\cdot\left[\sum_{l=1}^{k} i_{k,l}(\boldsymbol{\delta})\beta_l+\beta_{k+1}\right]} \times \left(2^{\left[\sum_{l=1}^{k} i_{k,l}(\boldsymbol{\delta})\beta_l+\beta_{k+1}\right]t}-1\right)$$
$$- \sum_{\substack{\delta_1,\ldots,\delta_{k-1}\\ \in\{0,1\}}} \frac{(-1)^{k+\delta_1+\cdots+\delta_{k-1}}(\ln 2)^{-(k+1)}}{\prod_{m=1}^{k}\left[\sum_{l=1}^{m} i_{m,l}(\boldsymbol{\delta})\beta_l\right]\cdot\beta_{k+1}}\left(2^{\beta_{k+1}t}-1\right)$$
$$= \sum_{\substack{\delta_1,\ldots,\delta_{k}\\ \in\{0,1\}}} \frac{(-1)^{k+1+\delta_1+\cdots+\delta_{k}}(\ln 2)^{-(k+1)}}{\prod_{m=1}^{k+1}\left[\sum_{l=1}^{m} i_{m,l}(\boldsymbol{\delta})\beta_l\right]}\left(2^{\left[\sum_{l=1}^{k+1} i_{k+1,l}(\boldsymbol{\delta})\beta_l\right]t}-1\right), \quad (59)$$

where $\delta_k \in \{0, 1\}$, $i_{k+1,k+1}(\boldsymbol{\delta}) = 1$ and $i_{k+1,l}(\boldsymbol{\delta}) = \delta_k \cdot i_{k,l}(\boldsymbol{\delta}), l = 1, 2, ..., k$. Also, we have

$$\sum_{l=1}^{k+1} i_{k+1,l}(\boldsymbol{\delta})\beta_l = \begin{cases} \sum_{l=1}^{k} i_{k,l}(\boldsymbol{\delta})\beta_l + \beta_{k+1}, & \text{if } \delta_k = 1; \\ \beta_{k+1}, & \text{if } \delta_k = 0. \end{cases} \quad (60)$$

Therefore, the closed-form expression in (6) is valid for $n = k + 1$, which completes the proof. □

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