Outage Analysis and Optimal Power Allocation for Multinode Relay Networks

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***Abstract*—In this letter, a novel approach for outage probability analysis of the multinode amplify-and-forward relay network is provided. It is shown that the harmonic mean of two exponential random variables can be approximated, at high signal-to-noise ratio (SNR), to be an exponential random variable. The single relay case considered before is a special case of our analysis. Based on that approximation, an outage probability bound is derived which proves to be tight at high SNR. Based on the derived outage probability bound, optimal power allocation is studied. Simulation results show a performance improvement, in terms of symbol error rate, of the optimal power allocation compared to the equal power-allocation scheme.**

***Index Terms*—Amplify-and-forward protocol, outage probability, wireless relay networks.**

I. Introduction

DUE TO THE fading nature of the wireless channels, the transmitted signal undergoes severe degradation and this leads to the increased interest in diversity achieving techniques. Diversity means to deliver more than one copy of the transmitted symbols to the destination. Diversity can be achieved through different techniques such as frequency diversity, time diversity, and spatial diversity. Recently, there has been much interest in modulation techniques to achieve transmit diversity motivated by the increased capacity of multi-input multi-output (MIMO) channels [1].

Due to space constraints of the mobile units, the use of multiple antennas is limited. This gave rise to what is known as cooperative diversity, in which the nodes try to form a virtual multiple element transmit antenna. In [2], different protocols were proposed to achieve spatial diversity through node cooperation. Among those protocols are the decode-and-forward and amplify-and-forward protocols. The decode-and-forward protocol suffers from the error propagation problem. The amplify-and-forward protocol does not suffer from the error propagation problem but noise accumulates with the desired signal along the transmission. It was proved that the amplify-and-forward protocol with one relay node achieves full diversity of order two based on outage probability analysis. Note that the multinode decode-and-forward scenario has been considered in [3], in which power and resource allocations were analyzed.

In this letter, we provide an outage probability analysis for the multinode amplify-and-forward protocol with N relay nodes helping the source. In [2], the outage probability of the single relay amplify-and-forward network was obtained based on the limiting behavior of the cumulative distribution function (cdf) of certain combinations of exponential random variables. For the multinode amplify-and-forward protocol, the expression for the SNR will yield a summation of harmonic mean random variables. In [4], an expression for the outage probability for the multinode case was given without a proof. In this letter, a novel approach, based on approximating the harmonic mean of two exponential random variables by a single exponential random variable, is presented. The case of single relay amplify-and-forward protocol in [2] can be considered as a special case of our analysis with $N = 1$. Based on the derived outage probability bound, optimal power allocation is studied for the multinode amplify-and-forward protocol.

II. Outage Analysis of the Multinode Amplify-and-Forward Protocol

In this section, we describe the system model for the multinode amplify-and-forward protocol in which each relay only amplifies the source signal. We consider a cooperative strategy with two phases. In phase 1, the source transmits its information to the destination, and due to the broadcast nature of the wireless channel the neighbor nodes receive the information. In phase 2, N users help the source by amplifying the source signal. In both phases, we assume that the users transmit their information through orthogonal channels (through frequency division or time division multiplexing) and perfect synchronization between the cooperating nodes. We focus on one cooperation scenario, however, nodes can interchange their rules as source, relay, or destination.

In phase 1, the source broadcasts its information to the destination and N relay nodes. The received signals $y_{s,d}$ and y_{s,r_i} at the destination and the ith relay can be written, respectively, as

$$y_{s,d} = \sqrt{P_s} h_{s,d} x + \eta_{s,d} \tag{1}$$

$$y_{s,r_i} = \sqrt{P_s} h_{s,r_i} x + \eta_{s,r_i}, \quad \forall i = 1, 2, \ldots, N \tag{2}$$

where P_s is the transmitted source power, $\eta_{s,d}$ and η_{s,r_i} denote the additive white Gaussian noise (AWGN) at the destination and the ith relay, respectively, and $h_{s,d}$ and h_{s,r_i} are the channel coefficients from the source to destination and the ith relay, respectively. Each relay amplifies the received signal from the source and retransmits to the destination. The received signal at the destination in phase 2 due to the ith relay transmission is given by

$$y_{r_i,d} = h_{r_i,d} \beta_i y_{s,r_i} + \eta_{r_i,d} \tag{3}$$

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and β_i satisfies the power constraint $\beta_i \leq \sqrt{P_i/(P_s|h_{s,r_i}|^2+N_0)}$ [2], where P_i is the ith relay power. The channel coefficients $h_{s,d}$, h_{s,r_i}, and $h_{r_i,d}$ are modeled as zero-mean complex Gaussian random variables with variances $\delta^2_{s,d}$, δ^2_{s,r_i}, and $\delta^2_{r_i,d}$, respectively. The channel coefficients are assumed to be available at the receiving nodes but not at the source node. The noise terms are modeled as zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension.

The outage probability for spectral efficiency R is defined as

$$P^{\text{out}}_{AF}(R) = \Pr\left\{\frac{1}{N+1} I_{AF} < R\right\} \tag{4}$$

where I_{AF} is the mutual information between the transmitted and received signals. The $1/(N+1)$ factor comes from the fact that the relays help the source through N uses of orthogonal channels. To calculate the mutual information between x and $\mathbf{y} = [y_{s,d}, y_{r_1,d}, \ldots, y_{r_N,d}]^T$ the system can be represented in a matrix form as [2]. However, it will become highly involved to get a closed-form expression for the mutual information using that approach. We apply a simple trick to get the mutual information between x and $\mathbf{y}$ by applying maximal ratio combiner (MRC) [5] to $\mathbf{y}$. The output of the MRC can be given by $r = \alpha_s y_{s,d} + \sum_{i=1}^N \alpha_i y_{r_i,d}$, where $\alpha_s = \sqrt{P_s} h^*_{s,d}/N_0$ and $\alpha_i = \sqrt{P_s}\beta_i h^*_{r_i,d} h^*_{s,r_i}/((\beta_i^2|h_{r_i,d}|^2+1)N_0)$. We can write r in terms of x as

$$\begin{aligned} r = &\left(\frac{P_s|h_{s,d}|^2}{N_0} + \sum_{i=1}^N \frac{P_s\beta_i^2 \left|h_{r_i,d}\right|^2 \left|h_{s,r_i}\right|^2}{\left(\beta_i^2\left|h_{r_i,d}\right|^2+1\right)N_0}\right) x \\ &+ \frac{\sqrt{P_s}h^*_{s,d}}{N_0}\eta_{s,d} + \sum_{i=1}^N \frac{\sqrt{P_s}\beta_i h^*_{r_i,d} h^*_{s,r_i}}{\left(\beta_i^2\left|h_{r_i,d}\right|^2+1\right)N_0} \\ &\times \left(\eta_{r_i,d} + h_{r_i,d}\beta_i \eta_{s,r_i}\right). \end{aligned} \tag{5}$$

Due to the orthogonality of the relays transmissions, the SNR at the MRC output is $\text{SNR}_{MRC} = \gamma_s + \sum_{i=1}^N \gamma_i$, where $\gamma_s = P_s|h_{s,d}|^2/N_0$ is the SNR due to the source transmission and $\gamma_i = P_s\beta_i^2|h_{r_i,d}|^2|h_{s,r_i}|^2/((\beta_i^2|h_{r_i,d}|^2+1)N_0)$ is the SNR due to the ith relay transmission. The probability density function (pdf) of $\mathbf{y}$ given x and the channel coefficients represents an exponential family of distributions [6]. Hence, it can be easily shown that r, given the channel coefficients, is a sufficient statistics for x, that is $p_{\mathbf{y}/x,r}(\mathbf{y}/x,r) = p_{\mathbf{y}/r}(\mathbf{y}/r)$, where $p_{\mathbf{y}/x,r}(\mathbf{y}/x,r)$ is the pdf of $\mathbf{y}$ given x and r and $p_{\mathbf{y}/r}(\mathbf{y}/r)$ is the pdf of $\mathbf{y}$ given r. Since r is a sufficient statistics for x, the mutual information between x and $\mathbf{y}$ equals the mutual information between x and r, that is $I(x;r) = I(x;\mathbf{y})$. Hence, the average mutual information satisfies

$$\begin{aligned} I_{AF} =& I(x;r) \\ \leq& \log\left(1+\frac{P_s|h_{s,d}|^2}{N_0}+\sum_{i=1}^N \frac{P_s\beta_i^2\left|h_{r_i,d}\right|^2\left|h_{s,r_i}\right|^2}{\left(\beta_i^2\left|h_{r_i,d}\right|^2+1\right)N_0}\right) \end{aligned} \tag{6}$$

with equality for x zero-mean, circularly symmetric complex Gaussian random variable [1]. It is clear that (6) is increasing in β_i's. Hence, to maximize the mutual information, the power constraint should be satisfied with equality, yielding

$$\begin{aligned} I_{AF} = \log\Big(1 + &|h_{s,d}|^2 \text{SNR}_{s,d} \\ &+ \sum_{i=1}^N f\left(\left|h_{s,r_i}\right|^2 \text{SNR}_{s,r_i}, \left|h_{r_i,d}\right|^2 \text{SNR}_{r_i,d}\right)\Big) \end{aligned} \tag{7}$$

where $\text{SNR}_{s,d} = \text{SNR}_{s,r_i} = P_s/N_0$, $\forall i = 1,\ldots,N$ and $\text{SNR}_{r_i,d} = P_i/N_0$, $\forall i \in [1,N]$ and $f(v,u) = uv/(u+v+1)$.

Let the vector $\mathbf{p} = [P_s, P_1, P_2, \ldots, P_N]^T$. At high SNR, we can neglect the 1 term in the denominator of the $f(\cdot,\cdot)$ function [7]. Equation (4) can be rewritten as

$$\begin{aligned} &P^{\text{out}}_{AF}(\mathbf{p}, R) \\ &\simeq \Pr\left\{\left(\frac{P_s}{N_0}|h_{s,d}|^2 + \sum_{i=1}^N \frac{\frac{P_s}{N_0}\left|h_{s,r_i}\right|^2 \frac{P_i}{N_0}\left|h_{r_i,d}\right|^2}{\frac{P_s}{N_0}\left|h_{s,r_i}\right|^2 + \frac{P_i}{N_0}\left|h_{r_i,d}\right|^2}\right) < \left(2^{(N+1)R}-1\right)\right\}. \end{aligned} \tag{8}$$

Define the random variables $w_1 = (P_s/N_0)|h_{s,d}|^2$ and $w_{i+1} = ((P_s/N_0)|h_{s,r_i}|^2(P_i/N_0)|h_{r_i,d}|^2)/((P_s/N_0)|h_{s,r_i}|^2 + (P_i/N_0)|h_{r_i,d}|^2)$, $\forall i \in [1,N]$. The random variable w_1 is an exponential r.v. with rate $\lambda_1 = N_0/(P_s\delta^2_{s,d})$. To calculate the outage probability, we consider an approach based on approximating the harmonic mean of two exponential random variables by an exponential random variable.

The w_j's for $j \in [2, N+1]$ are the harmonic mean of two exponential random variables. The cdf for w_j, $j = 2,\ldots,N+1$ is given by [7]

$$P_{w_j}(w) = 1 - 2w\sqrt{\zeta_{j1}\zeta_{j2}}e^{-w(\zeta_{j1}+\zeta_{j2})}K_1(2w\sqrt{\zeta_{j1}\zeta_{j2}}) \tag{9}$$

where $\zeta_{j1} = N_0/P_s\delta^2_{s,r_{j-1}}$, $\zeta_{j2} = N_0/P_{j-1}\delta^2_{s,r_{j-1}}$, and $K_1(.)$ is the first order modified Bessel function of the second kind [8]. The function $K_1(.)$ can be approximated as $K_1(x) \simeq (1/x)$ for small x [8] from which we can approximate the cdf of w_j at high SNR as $P_{w_j}(w) = \Pr\{w_j < w\} \simeq 1 - e^{-w(\zeta_{j1}+\zeta_{j2})}$, which is the CDF of an exponential random variable of rate $\lambda_j = (N_0/P_s\delta^2_{s,r_{j-1}}) + (N_0/P_{j-1}\delta^2_{r_{j-1},d})$. Defining the random variable $W = \sum_{j=1}^{N+1} w_j$, the CDF of W, assuming the λ_i's to be distinct, can be obtained to be

$$\Pr\{W < w\} \simeq \sum_{k=1}^{N+1}\left(\prod_{m=1,m\neq k}^{N+1}\frac{\lambda_m}{\lambda_m-\lambda_k}\right)(1-e^{-\lambda_k w}). \tag{10}$$

The outage probability can be expressed in terms of the cdf of W as $P^{\text{out}}_{AF}(\mathbf{p},R) \simeq \Pr\{W < (2^{(N+1)R}-1)\}$.

Expanding the expression in (10) and rearranging the terms, the cdf of W can be written as

$$\begin{aligned} \Pr\{W < w\} \simeq \sum_{n=1}^{N+1}&\left(\sum_{k=1}^{N+1}\left(\prod_{m=1,m\neq k}^{N+1}\frac{\lambda_m}{\lambda_m-\lambda_k}\right)\lambda_k^n\right) \\ &\times(-1)^{n+1}\frac{w^n}{n!} + \text{H.O.T.} \end{aligned} \tag{11}$$

where H.O.T. stands for the higher order terms. To prove that the system achieves a diversity of order $(N+1)$ we need to have the coefficients of w^n to be zero for $n \in [1,N]$ (since if we assume equal power allocation and define $\text{SNR} = P_s/N_0$, substituting for the λ_i's we get the factor $1/\mathrm{SNR}^n$ multiplying w^n). This requirement can be reformulated in a matrix form as

$$\underbrace{\begin{bmatrix} \lambda_1 & \dots & \lambda_{N+1} \\ \lambda_1^2 & \dots & \lambda_{N+1}^2 \\ \vdots & \vdots & \vdots \\ \lambda_1^{N+1} & \dots & \lambda_{N+1}^{N+1} \end{bmatrix}}_{\mathbf{V}} \underbrace{\begin{bmatrix} \prod_{m=2}^{N+1} \frac{\lambda_m}{\lambda_m - \lambda_1} \\ \prod_{m=1, m\neq 2}^{N+1} \frac{\lambda_m}{\lambda_m - \lambda_2} \\ \vdots \\ \prod_{m=1}^{N} \frac{\lambda_m}{\lambda_m - \lambda_{N+1}} \end{bmatrix}}_{\mathbf{q}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ c_1 \end{bmatrix}. \tag{12}$$

To prove (12), consider the following system of equations $\mathbf{Va} = \mathbf{c}$, where $\mathbf{c} = [0, 0, \dots, 1]^T$ and prove that $\mathbf{q} = c_1\mathbf{a}$ for some nonzero constant c_1. Noting that the columns of the $\mathbf{V}$ matrix are scaled versions of the columns of a Vandermonde matrix, i.e., it is a nonsingular matrix, the solution for this system of equations can be found as $a = \mathbf{V}^{-1}\mathbf{c} = (1/\det(\mathbf{V}))\mathbf{adj}(\mathbf{V})\mathbf{c}$, where $\mathbf{adj}(\mathbf{V})$ is the adjoint matrix of $\mathbf{V}$. The determinant of the $\mathbf{V}$ matrix can be expressed as $\det(\mathbf{V}) = (\prod_{j=1}^{N+1} \lambda_j) \prod_{k=1}^{N+1} \prod_{m>k}^{N+1} (\lambda_m - \lambda_k)$ [9]. Due to the structure of the $\mathbf{c}$ vector, we are only interested in the last column of the $\mathbf{adj}(\mathbf{V})$ matrix. The ith element of the $\mathbf{a}$ vector can be obtained as $a_i = ((-1)^N/\lambda_i) \prod_{j=1, j\neq i}^{N+1} (1/(\lambda_j - \lambda_i))$. It is clear that $\mathbf{q} = c_1\mathbf{a}$, where $c_1 = (-1)^N \prod_{i=1}^{N+1} \lambda_i$. The outage probability, substituting for the λ_i's, can now be expressed as

$$P_{AF}^{\mathrm{out}}(\mathbf{p}, R) \sim \frac{1}{(N+1)!} \cdot \frac{1}{P_s \delta_{s,d}^2} \cdot \prod_{i=1}^{N} \frac{P_s \delta_{s,r_i}^2 + P_i \delta_{r_i,d}^2}{P_s P_i \delta_{s,r_i}^2 \delta_{r_i,d}^2} \left(2^{(N+1)R} - 1\right)^{N+1} N_0^{N+1}. \tag{13}$$

For the special case of single relay node $(N = 1)$ and letting $\mathrm{SNR} = P_s/N_0 = P_1/N_0$, we get

$$P_{AF}^{\mathrm{out}}(\mathrm{SNR}, R) \sim \frac{1}{2} \cdot \frac{1}{\delta_{s,d}^2} \cdot \frac{\delta_{s,r_1}^2 + \delta_{r_1,d}^2}{\delta_{s,r_1}^2 \delta_{r_1,d}^2} \left(\frac{2^{2R} - 1}{\mathrm{SNR}}\right)^2 \tag{14}$$

which is consistent with the result obtained in [2] for that simple case of single relay amplify-and-forward protocol.

From the expression in (13), let P be the total power and let $P_s = a_s P$ and $P_i = a_i P$ where $a_s + \sum_{i=1}^{N} a_i = 1$, $a_s > 0$, $a_i > 0$, $i = 1, \dots, N$. Define the SNR as $\mathrm{SNR} = P/N_0$, the diversity order of the system, based on the outage probability, is defined as $d_{AF}^{\mathrm{out}} = \lim_{\mathrm{SNR}\to\infty} -(\log P_{AF}^{\mathrm{out}}(\mathrm{SNR}, R))/(\log \mathrm{SNR}) = N + 1$. Hence, the system achieves a diversity of order $N + 1$, in terms of outage probability.

III. Multinode Amplify-and-Forward Optimal Power Allocation

The optimal power allocation is based on minimizing the outage probability bound in (13) under a total power constraint. Removing the fixed terms from the outage probability bound, our optimization problem can be written as

$$\mathbf{p}_{\mathrm{opt}} = \arg\min_{\mathbf{p}} \frac{1}{P_s^{N+1}} \prod_{i=1}^{N} \frac{P_s \delta_{s,r_i}^2 + P_i \delta_{r_i,d}^2}{P_i},$$
$$\textbf{subject to } P_s + \sum_{i=1}^{N} P_i \le P, \quad P_i \ge 0, \quad \forall i \tag{15}$$

where $\mathbf{p}$ is as defined in the previous section and P is the maximum allowable total power for one symbol transmission.

It can be easily shown that the cost function in (15) is convex in $\mathbf{p}$ over the convex feasible set defined by the linear power constraints. The Lagrangian of this optimization problem can be written as

$$L = \frac{1}{P_s^{N+1}} \prod_{i=1}^{N} \frac{P_s \delta_{s,r_i}^2 + P_i \delta_{r_i,d}^2}{P_i} + \tilde{\lambda}\left(P_s + \sum_{i=1}^{N} P_i - P\right) + \sum_{i=1}^{N} \mu_i (0 - P_i) \tag{16}$$

where the μ's serve as the slack variables. To minimize the outage bound, it is clear that we must have $P_i > 0\ \forall i$. The complementary slackness imply that since $P_i > 0$ then $\mu_i = 0\ \forall i$. Knowing that the log function is a monotone function and defining the $(N+1) \times 1$ vector $\mathbf{a} = [a_s, a_1, \dots, a_N]$, where $a_s = P_s/P$ and $a_i = P_i/P\ \forall i \in [1, N]$, the Lagrangian of the optimization problem in (15) can now be given as

$$f = -\log a_s + \sum_{i=1}^{N} \log\left(\frac{1}{a_i}\delta_{s,r_i}^2 + \frac{1}{a_s}\delta_{r_i,d}^2\right) + \lambda(\mathbf{a}^T \mathbf{1}_{N+1} - 1), \tag{17}$$

where $\mathbf{1}_{N+1}$ is an all 1 $(N+1) \times 1$ vector. Applying first-order optimality conditions, $\mathbf{a}_{\mathrm{opt}}$ must satisfy

$$\frac{\partial f}{\partial a_s} = \frac{\partial f}{\partial a_i} = 0, \quad \forall i \in [1, N] \tag{18}$$

from which we get

$$\frac{1}{a_s}\left[1 + \sum_{j=1}^{N} \frac{\delta_{r_j,d}^2}{\delta_{r_j,d}^2 + \frac{a_s}{a_j}\delta_{s,r_j}^2}\right] = \frac{1}{a_i}\left[\frac{\delta_{s,r_i}^2}{\delta_{s,r_i}^2 + \frac{a_i}{a_s}\delta_{r_i,d}^2}\right]. \tag{19}$$

Since $a_s > 0$ and $a_j > 0$, then we can easily show that $a_s > a_i$ i.e., $P_s > P_i\ \forall i \in [1, N]$. This is due to the fact that the source power appears in all the SNR terms in (13) either through the source-destination direct link or through the harmonic mean of the source-relay and relay-destination links.

Using (18), we have

$$\frac{1}{a_j}\left[\frac{\delta_{s,r_j}^2}{\delta_{s,r_j}^2 + \frac{a_j}{a_s}\delta_{r_j,d}^2}\right] = \frac{1}{a_i}\left[\frac{\delta_{s,r_i}^2}{\delta_{s,r_i}^2 + \frac{a_i}{a_s}\delta_{r_i,d}^2}\right], \quad \forall i, j. \tag{20}$$

Define $c_i = (a_i/a_s) = (P_i/P_s), \forall i = 1, \cdots, N$ and using (19), we get

$$\frac{\delta_{r_i,d}^2}{\delta_{s,r_i}^2} c_i^2 + c_i - c = 0,\ \forall i \in [1, \cdots, N] \tag{21}$$

for some constant c. From (19), c should satisfy the following equation:

$$f(c) = c - \frac{1}{1 + \sum_{j=1}^{N} \frac{\delta_{r_j,d}^2}{\delta_{r_j,d}^2 + \frac{1}{c_j(c)}\delta_{s,r_j}^2}} = 0. \tag{22}$$

Since $P_i < P_s, \forall i$, then $c_i < 1, \forall i$. Hence, using (21), we have $c \in (0, 1 + \min_i(\delta_{r_i,d}^2/\delta_{s,r_i}^2))$. So we have reduced the $(N+1)$-dimensional problem to a single dimension search over the parameter c which can be done using a simple numerical search or any other standard method such as the Newton's method. Convexity of both the cost function and the feasible set in (15) imply global optimality of the solution of (22) over the desired feasible set. It is worth noting that minimizing the

TABLE I
OPTIMAL POWER ALLOCATION FOR ONE AND TWO RELAYS ($\delta^2_{s,d} = 1$ IN ALL CASES)

	$\delta^2_{s,r_i} = 10, \delta^2_{r_i,d} = 1$ relays close to the source	$\delta^2_{s,r_i} = 1, \delta^2_{r_i,d} = 10$ relays close to the destination
one relay	$P_s/P = 0.5393$ $P_1/P = 0.4607$	$P_s/P = 0.8333$ $P_1/P = 0.1667$
two relays	$P_s/P = 0.3830$ $P_1/P = P_2/P = 0.3085$	$P_s/P = 0.75$ $P_1/P = P_2/P = 0.125$

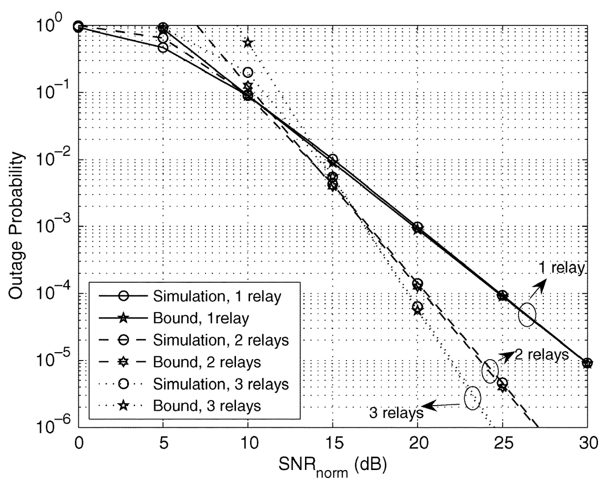

Fig. 1. Outage probability for one, two, and three nodes amplify-and-forward relay network.

outage probability bound derived in this paper will also result in minimizing the SER upper bound derived in [10].

Table I gives numerical results for the optimal power allocation for one and two relays helping the source. From the results in Table I, it is clear that equal power allocation is not optimal. As the relays get closer to the source the equal power-allocation scheme tends to be optimal. If the relays are close to the destination, optimal power allocation can result in a significant performance improvement, in terms of SER, compared to the conventional equal power-allocation scheme, as will be seen in the simulation section.

IV. SIMULATION RESULTS

In this section, we present some simulations to prove the theoretical analysis presented in the previous sections. Fig. 1 shows the outage probability for one, two, and three relay nodes helping the source versus $\mathrm{SNR}_{\mathrm{norm}}$ defined as $\mathrm{SNR}_{norm} = (\mathrm{SNR}/2^R - 1)$, which is the SNR normalized by the minimum SNR required to achieve spectral efficiency R for complex additive white Gaussian noise (AWGN) channel. In the simulations, we used $R = 1$. For the single relay case, all the channel variances are taken to be 1. For the case of two relay nodes, all the channel variances are taken to be 1 except for the channel between the source and the second relay for which the channel variance is taken to be $\delta^2_{s,r_2} = 10$, which means that the second relay is close to the source. For the case of three relay nodes, all the channel variances are taken to be 1 except for the channel between the source and the second relay for which the channel variance is taken to be $\delta^2_{s,r_2} = 10$ and the channel between the source and the third relay for which the channel variance is taken to be $\delta^2_{s,r_3} = 5$. From Fig. 1 it is clear that the bound in (13) is tight at high SNR.

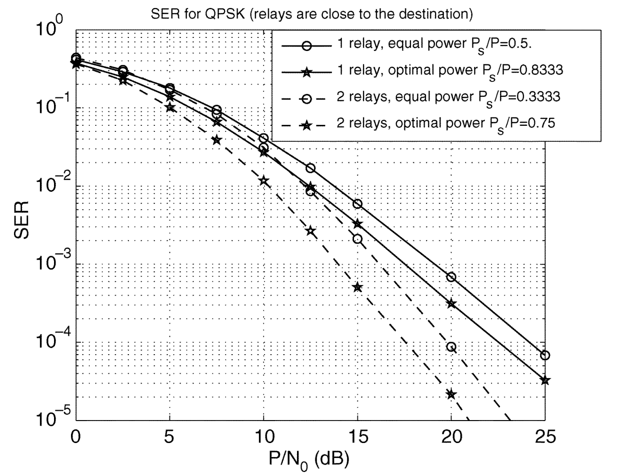

Fig. 2. Comparison of the SER for QPSK modulation using equal power allocation and the optimal power allocation for relays close to the destination.

Fig. 2 shows a comparison between the equal power and optimal power-allocation schemes for relays close to the destination ($\delta^2_{s,r_1} = \delta^2_{s,r_2} = 1$, $\delta^2_{r_1,d} = \delta^2_{r_2,d} = 10$ and $\delta^2_{s,d} = 1$). From Fig. 2 we can see that, using the optimal power-allocation scheme, we can get about 1-dB improvement for the single relay case and about 2-dB improvement for the two-relays case over the equal power assignment scheme.

V. CONCLUSION

In this letter, we provide outage probability analysis of the multinode amplify-and-forward protocol. We derive an outage probability bound for the multinode amplify-and-forward protocol which proves to be tight at high SNR. Based on the derived outage probability bound, we determine the optimal power allocation between the source and the relays that minimizes the outage probability bound, which is also the optimal power allocation based on minimizing the SER bound. We show that optimal power allocation can result in a significant performance improvement compared to the conventional equal power allocation scheme.

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