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OPTIMAL CONFIGURATION SELECTION FOR A COOPERATING SYSTEM OF MOBILE MANIPULATORS

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ABSTRACT

In this paper, we examine the design and selection of optimal configuration of a system of multiple, wheeled mobile manipulators that can team up to cooperatively transport a large common object. Each individually autonomous mobile manipulator consists of a differentially driven wheeled mobile robot (WMR) with a passive, two degree-of-freedom (d.o.f), planar revolute-jointed arm mounted in the plane parallel to the base. The composite multi-degree-of-freedom vehicle, formed by placing a common object on the end-effector of two (or more) such mobile manipulator systems, possesses the ability to change its relative configuration as well as accommodate relative positioning errors of the mobile bases. However, the combination of the non-holonomic constraints of the wheeled mobile bases, the presence of passive joints and closed kinematic loops of the articulations require a careful treatment.

The suitable selection of the dimensions of the intermediate linkage and the overall configuration is critical to enhancing the overall performance. In this paper we will focus our attention on the formulation of a design-optimization problem to aid the selection of the configuration of the composite vehicle to attain near-isotropic manipulability of the common object.

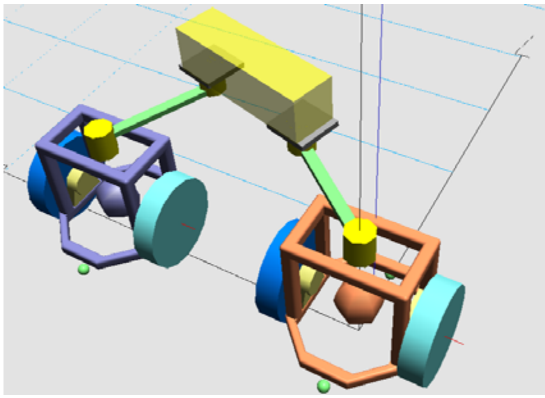
Keywords: Mobile manipulators, configuration optimization, isotropic manipulability.

INTRODUCTION

Our overall goal is the design, analysis and implementation of a flexible, scalable system of multiple wheeled mobile robots, that are individually autonomous but can team up to cooperatively transport large objects. Such frameworks for remotely controlled or remotely supervised cooperation of multiple autonomous mobile robots have applications for material handling tasks in many fields [1,2].

Figure 1: Sample individual mobile manipulator module.

Our individual autonomous robots take the form of differentially-driven Wheeled Mobile Robots (WMR), possessing a single rigid axle between two fixed disk wheels and the usual complement of nonholonomic kinematic constraints. Two or more wheeled mobile robots, with such rigid axles, cannot be arbitrarily coupled to each other due to the incompatibility of the velocities of the wheels. The potential degradation in the overall performance could include loss of mobility and manifestation of behaviors like rattle, shake, unintentional compliance, wear and tear of the tires and wheel slip. Hence, many approaches in the literature advocate the addition of further articulations (passive/active) between the various wheels/axles in order to accommodate the rigid body constraints. See Borenstein *et al.*, [3] for a description of some of the problems as well as a survey of some of the more common configurations of such systems.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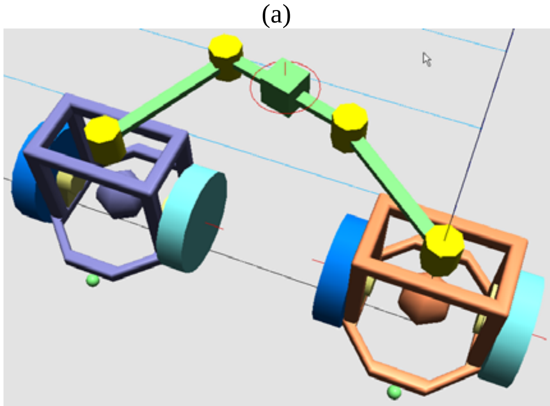
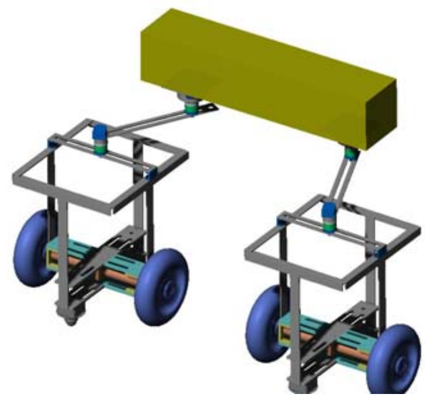

(b)

Figure 2: Collaboration of two mobile manipulators (a) Carrying common object (b) Forming an effective 4 bar linkage.

One such configuration, called the composite Multi-Degree-of-Freedom (MDOF) vehicle [4], employs a compliant linkage with at least three degrees-of-freedom (d.o.f) to permit two rigid axles to be attached to a common platform**.** Two examples of such MDOF vehicles, the CLAPPER and the OMNIMATE, have been built. In both cases, the intermediate compliant linkage possesses two revolute (R) joints and one prismatic (P) joint. In the CLAPPER design, the two revolutes and one prismatic are arranged to form a passive RPR (Revolute, Prismatic, Revolute) linkage while in the OMNIMATE design, a passive PRR (Prismatic, Revolute, Revolute) linkage is formed between the two mobile bases. This compliant linkage enables the MDOF system to accommodate momentary controller errors without transferring any mutual forces/reactions between the platforms, thereby eliminating the excessive wheel slippage. However, the limitation with either such configuration is that the system remains forever assembled and cannot be reconfigured easily.

In contrast, in our design, each of the two differentially driven wheeled mobile bases is fitted with a passive planar 2 d.o.f arm mounted parallel to the ground plane. The resulting mobile manipulator module is shown in Fig. 1. A passive RRRR compliant linkage is formed between the two mobile bases when a common object is placed on the end-effectors of two adjacent modules as shown in Fig. 2(a). The bases of the individual articulated manipulators become the fixed pivots of the 4-bar linkage while the object takes on the role of the coupler link as shown in Fig. 2(b). This compliant linkage now plays a critical role in helping us to relax the rigid body constraints between the various axles. While the intermediate compliant linkage now possesses more than the minimum required d.o.f between the mobile bases, this does not necessarily pose a problem since the d.o.f may be reduced easily by holding one of the revolute joints fixed. Other advantages of such an approach include: (a) modularity/convenience of removing and/or adding further modules easily; and (b) the ability to change and optimize the configuration of the linkage while maintaining a fixed distance between the mobile bases. The CAD model and physical realization of the resulting overall system are shown in Fig. 3.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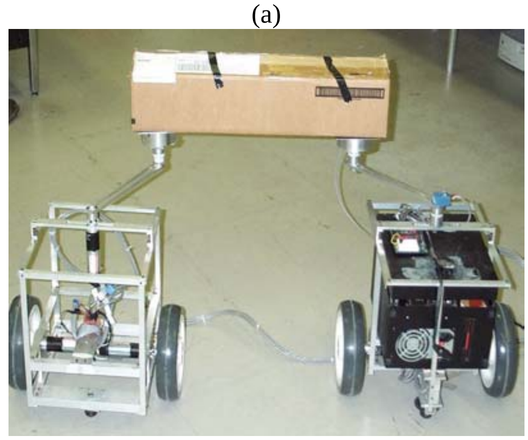
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Figure 3: Our MDOF system (ARNOLD) with compliant linkage; (a) CAD Model and (b) Physical Realization.

RESEARCH ISSUES

The resulting articulations, with suitable instrumentation and actuation, endow the composite MDOF vehicle with: (i) ability to accommodate changes in the relative configuration; (ii) redundant sensing for localizing the modules; and (iii) redundant actuation for moving the common object and compensate for environmental disturbances and errors. In particular, these articulations (with at least 3 d.o.f) permit the relaxation of the requirement for a common "Center of Rotation" between the multiple axles created by the non-holonomic constraints at the wheels. However, while the velocity-level kinematic constraints for the system are eliminated, other holonomic constraints (due to the closed kinematic loops) are introduced between the relative motions of the bases.

Closed kinematic loops present a number of subtleties that are not often seen in open kinematic chain manipulators. Notably, the kinematic configuration space of a closed loop manipulator is no longer a flat space but becomes a curved manifold embedded in a higher dimensional vector space. Further, as seen in Fig. 4, the existence of the holonomic (loop-closure) constraints, limit the d.o.f and hence not all articulations need to be actuated. Thus the selection of the location of the active joints also plays a vital role in determining the overall performance of the system.

In the remainder of this paper, we will examine some of these key issues and challenges, which arise for the design and performance optimization of such systems. The important design parameters include the various link lengths and the configuration of the linkage at any instant of time and the suitable selection of the dimensions/parameters of the intermediate linkage thus becomes critical [5, 6]. In particular, configuration for the composite system must be chosen in order to minimize singular configurations of the articulations; and improve robustness to variations in the relative configuration due to local controller lapses and environmental disturbances.

From a design perspective, the overall task of design of the system can be considered to be composed of three overlapping stages. *First*, the designer has to select between competing component designs for the individual modules such as: selection of type, location and number of wheels, number of manipulators, type and number of articulations within the manipulator, among others – a process which we will loosely term “type synthesis”. *Second*, considerable freedom still exists from the viewpoint of selection of the characteristic dimensions of the wheeled base and manipulator links, locations of the manipulators with respect to the mobile base, among others – a process which we will term “dimensional synthesis”. *Finally*, since the system is highly redundant due to the presence of the multiple d.o.f within the system. In the absence of adequate number of task constraints, the system can possibly assume several infinities of postures/configurations permitting the selection of an “optimal” configuration – a process we will term “configuration synthesis”.

The evaluation of the suitability of a design requires the definition and use of either a local measure of performance for a particular task or some suitable global measure (by spatial or temporal integration of the local measure). In our work, we will examine the evaluation of a measure of manipulability of the overall system as our objective measure. With the development of suitable numerical measures of performance to serve as objective criteria, the above problem may be cast into the framework of a design optimization problem.

In this paper, we will focus our attention on the formulation of a design-optimization approach to the problem of selection of the “most suitable configuration” that maximizes the performance of the system in terms of carrying the common object while accommodating and correcting for relative positioning errors.

PREVIOUS LITERATURE

Performance measures play a critical role in design, optimization and control of robotic systems and a wide variety of local measures of performance, such as manipulability, isotropy index, condition number, dexterity, singularity have been developed and used in these contexts. In particular, we examine the literature related to manipulability for potential use as a quantitative measure of performance.

Yoshikawa [7] is credited with the first comprehensive treatment of manipulability for general open chains while Salisbury and Craig [8] are credited with the first use of manipulability in a design context. Since then various alternative formulations and various aspects of manipulability have been investigated by a number of authors. The singular value decomposition of the serial chain manipulator Jacobian and its geometric relationship with various manipulability measures to obtain further insights permitting interpretation of measures as the volume, proximity to singularity, and anisotropy/eccentricity of the manipulability ellipsoid. See Nakamura [9] for further details. Other researchers have examined the development of dynamics based measures by extending the notion of manipulability and isotropy to the dynamics domain [10].

Equivalent force performance measures for the local velocity based measures may be computed by virtue of the principle of virtual work and used for the synthesis and design of multi-d.o.f mechanisms. While many such measures have tended to be local, several authors have obtained global measures by spatial integration of local measures over a region [11] or by integration over time. The resulting performance measures have been employed as a means to evaluate the workspace quality [12], measure of kinematic accuracy [8], selection of operating regions [13], design of modular kinematic structures [14], and improvement of numerical properties for resolved velocity and acceleration control of manipulators [9].

However, as Park [15] and van den Doel and Pai [16] note, the sheer diversity of measures is partially due to the adoption of a local coordinate representation and hence advocate the formulation of coordinate independent measures using the unifying framework of differential geometry. They provide a geometric perspective to several of these measures that arise in the context of robotic manipulators and examine the use of the machinery of differential geometry to develop new scale and coordinate-invariant metrics.

Extensions to manipulability analysis have been studied both in the context of multiple cooperating robot arms [17, 18] as well as parallel manipulators [11, 19]. The distinction arises in that multiple cooperating robot arms often also feature redundancy of actuation, while parallel manipulators can feature many passive joints [20]. Park and Bobrow [21] examine the optimal base positioning of two cooperating robot manipulators in a geometric framework. Park and Kim [22, 23] note that while a number of coordinate-based formulations have been proposed for manipulability of systems with closed kinematic loops, particular case needs to be exercised due to some of the unique nonlinear characteristics of such systems. In their work, they present an elegant differential-geometric framework for analysis of manipulability of both redundantly-actuated multi-arm systems as well as exactly-actuated parallel manipulator systems in a unified fashion.

PROBLEM FORMULATION

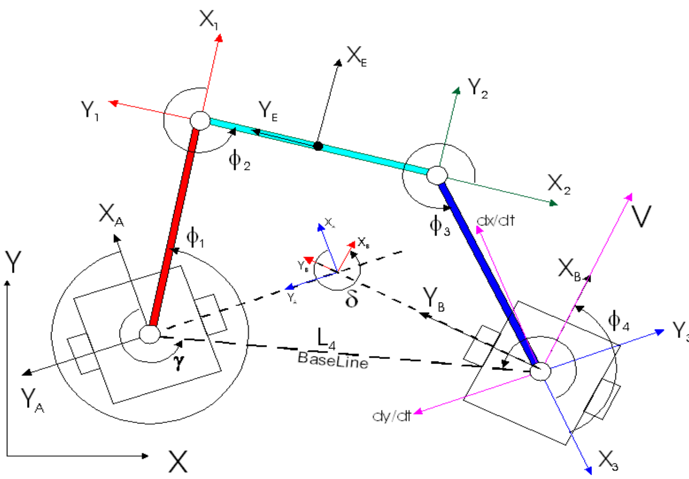

Figure 4: Schematic of the overall system depicting the critical model parameters.

An effective 4 bar linkage is formed when a common object is placed on the two passive 2R manipulators mounted on each of the two differentially driven mobile platforms. Figure 4 depicts the overall system along with the various relevant joint and link parameters where L_i's denote the various link lengths and the ϕ_i's denote the relative angles between the frame mounted on the i^{th} link and the previous frame using the Denavit-Hartenberg notation. L_4 is the length of virtual baseline between the centers of the two mobile platforms while γ is the angle made by this baseline with respect to the frame of reference mounted on Mobile Platform A (MP A). x and y are the position coordinates of the center of Mobile Platform B (MP B) with respect to MP A and δ represents the orientation of the frame of reference of MP B with respect to the frame of reference of MP A.

For the remainder of this paper, we will consider MP A to be fixed and consider only the relative motions of MP B. Further, we will also consider, the angle ϕ_4 can be controlled and maintained at a desired value by means of a servo-controlled motor. Our goal is to determine that particular configuration (or a set of configurations) of the system that will yield isotropic manipulability of the object. The midpoint of the commonly supported object (which forms the effective coupler link of the four-bar) is selected to be the end-effector of the overall system and a task frame ($O_EX_EY_E$) is attached. The vector of generalized coordinates for the system may be expressed as:

$$\mathbf{q} = [\phi_1 \quad \phi_2 \quad \phi_3 \quad L_4 \quad \gamma \quad x \quad y \quad \delta]^T \tag{1}$$

Several constraints can also be placed on the various generalized coordinates reducing the number required to define the configuration of the system. The configuration space manifold of the current system may be expressed in implicit form as:

$$\mathbf{\Phi}(\mathbf{q}) = \begin{pmatrix} L_1\cos\phi_1 + L_2\cos\phi_{12} + L_3\cos\phi_{123} - x \\ L_1\sin\phi_1 + L_2\sin\phi_{12} + L_3\sin\phi_{123} - y \\ \phi_1 + \phi_2 + \phi_3 - \delta \\ L_4\cos\gamma - x \\ L_4\sin\gamma - y \end{pmatrix} \tag{2}$$

where,

$$\cos\phi_{i,i+1,\ldots,n} = \cos(\phi_i + \phi_{i+1} + \ldots + \phi_n)$$
$$\sin\phi_{i,i+1,\ldots,n} = \sin(\phi_i + \phi_{i+1} + \ldots + \phi_n)$$

The first three constraints are obtained from the forward-kinematics equations for the articulated linkage, written in an inertial frame of reference instantaneously coincident with the reference frame of MP A. The last two equations express the position of MP B with respect to MP A, in terms of a virtual link along the baseline connecting the two centers. By differentiating the Eq. 2 we get:

$$\dot{\mathbf{\Phi}}(\mathbf{q},\dot{\mathbf{q}}) = \left[\frac{\partial \mathbf{\Phi}(\mathbf{q})}{\partial \mathbf{q}}\right]\dot{\mathbf{q}} = \mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = 0 \tag{3}$$

where

$$\mathbf{A}(\mathbf{q}) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & -1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & \cos\gamma & -L_4\sin\gamma & -1 & 0 & 0 \\ 0 & 0 & 0 & \sin\gamma & L_4\cos\gamma & 0 & -1 & 0 \end{bmatrix}$$

$$a_{11} = -L_1\sin\phi_1 - L_2\sin\phi_{12} - L_3\sin\phi_{123}$$
$$a_{12} = -L_2\sin\phi_{12} - L_3\sin\phi_{123}$$
$$a_{13} = -L_3\sin\phi_{123}$$
$$a_{21} = L_1\cos\phi_1 + L_2\cos\phi_{12} + L_3\cos\phi_{123}$$
$$a_{22} = L_2\cos\phi_{12} + L_3\cos\phi_{123}$$
$$a_{23} = L_3\cos\phi_{123}$$

By partitioning the vector of generalized velocities into $\dot{\mathbf{q}}_1 = [\dot\phi_1 \quad \dot\phi_2 \quad \dot\phi_3 \quad \dot L_4 \quad \dot\gamma]^T$ and $\dot{\mathbf{q}}_2 = [\dot x \quad \dot y \quad \dot\delta]^T$ and the matrix $\mathbf{A}(\mathbf{q})$ into two submatrices, $\mathbf{A}_1, \mathbf{A}_2$, Eq. (3) may be rewritten as:

$$\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = [\mathbf{A}_1(\mathbf{q})]_{5\times5}\dot{\mathbf{q}}_1 + [\mathbf{A}_2(\mathbf{q})]_{5\times3}\dot{\mathbf{q}}_2 = \mathbf{0} \quad (4)$$

Given that the constraints are independent and the matrix $\mathbf{A}(\mathbf{q})$ is full rank, the vector of dependent velocities $\dot{\mathbf{q}}_1$ can be written in terms of the vector of velocities $\dot{\mathbf{q}}_2$ as:

$$\dot{\mathbf{q}}_1 = -[\mathbf{A}_1]^{-1}[\mathbf{A}_2]\dot{\mathbf{q}}_2 \quad (5)$$

However, the velocities $\dot{\mathbf{q}}_2 = [\dot{x} \quad \dot{y} \quad \dot{\delta}]^T$ are themselves not independent since they pertain to the velocities of the frame mounted on MP B, which are constrained by the nonholonomic constraints of the platform. With respect to an inertial frame, instantaneously fixed at the location of mobile platform A, these constraints may be expressed as:

$$\dot{x}\cos(\delta) - \dot{y}\sin(\delta) = 0 \quad (6)$$

Thus, the three velocities may also be expressed in terms of two independent velocities $\dot{\mathbf{u}} = [\dot{\theta}_L \quad \dot{\theta}_R]^T$ the velocities of left and the right wheels of the differentially driven mobile robot as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \cos & 0 \\ \sin & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{b} & \frac{-r}{b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix} = [\mathbf{K}]_{3\times2} \begin{bmatrix} \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix} \quad (7)$$

where r is the radius of the wheels of our mobile platform, b is the length of the wheel base. Thus, Eqn. 6 can be written in terms of the vector of independent wheel velocities, $\dot{\mathbf{u}} = [\dot{\theta}_L \quad \dot{\theta}_R]^T$, as:

$$\dot{\mathbf{q}}_1 = -[\mathbf{A}_1]^{-1}[\mathbf{A}_2][\mathbf{K}]\dot{\mathbf{u}} = [\mathbf{\Psi}]_{5\times2}\dot{\mathbf{u}} \quad (8)$$

The kinematic Jacobian that maps the joint rates $[\dot{\phi}_1 \quad \dot{\phi}_2]^T$ to the end effector velocity vector, $\dot{\mathbf{x}} = [\dot{x}_E \quad \dot{y}_E]^T$ may be obtained from the forward kinematics mapping as:

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \end{bmatrix} = \begin{bmatrix} -L_1\sin\phi_1 - (L_2/2)\sin\phi_{12} & -(L_2/2)\sin\phi_{12} \\ L_1\cos\phi_1 + (L_2/2)\cos\phi_{12} & (L_2/2)\cos\phi_{12} \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} \quad (9)$$

However, since $[\dot{\phi}_1 \quad \dot{\phi}_2]^T$ may be expressed in terms of the independent wheel velocities, $\dot{\mathbf{u}} = [\dot{\theta}_L \quad \dot{\theta}_R]^T$, as:

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix} \quad (10)$$

the effective System Jacobian may be obtained by combining Eq. (9) and (10) and written in terms of the vector of independent wheel velocities as:

$$\dot{\mathbf{x}} = \overline{\mathbf{J}}\,\dot{\mathbf{u}} \quad (11)$$

where

$$\overline{\mathbf{J}} = \begin{bmatrix} -L_1\sin\phi_1 - (L_2/2)\sin\phi_{12} & -(L_2/2)\sin\phi_{12} \\ L_1\cos\phi_1 + (L_2/2)\cos\phi_{12} & (L_2/2)\cos\phi_{12} \end{bmatrix} \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix}$$

and Ψ_{ij} is the entry on the i^{th} row and j^{th} column of the matrix, $\mathbf{\Psi}$, defined in Eq. (8).

The approach for determining this system Jacobian, $\overline{\mathbf{J}}$, is identical to the approach of Park and Kim (1998) when the system is not redundantly actuated, as discussed in Appendix A. However, it should be noted that the approach of Park and Kim (1998) lends itself easily to extensions to cases, where redundant actuation exists.

We now examine the singular value decomposition of this system Jacobian, $\overline{\mathbf{J}}$, in order to determine the nature of the manipulability ellipsoid and a suitable measure of manipulability for use in optimization to attain an isotropic configuration. Several authors [10,13,14] have designed manipulators to realize configurations where the condition number (or its reciprocal) of the Jacobian is close to (or equal to) 1. Under these conditions, the Jacobian is *isotropic* in that it preserves distances and angles permitting a unit sphere in the joint space to be mapped into a corresponding unit sphere in the task space. It is in this spirit that we will adopt a measure of manipulability, defined in terms of the singular values of this system Jacobian as:

$$f(\chi) = \frac{\sigma_{\max}}{\sigma_{\min}} - 1 \quad (12)$$

By using this measure of manipulability to serve as an objective function, an optimization based search for an isotropic configuration of the overall system is now possible. The vector of optimization parameters χ, containing all the relevant design parameters, can be chosen as:

$$\chi = [L_1 \quad L_2 \quad L_3 \quad L_4 \quad \phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4 \quad x \quad y \quad \gamma \quad \delta]^T \quad (13)$$

The system is subject to the same equality constraints introduced in Eq. (2) which result in a constrained optimization problem that may be stated as:

$$\underset{\chi}{Min}\; f(\chi) \quad (14)$$

$$s.t$$

$$\begin{bmatrix} L_1\cos\phi_1 + L_2\cos\phi_{12} + L_3\cos\phi_{123} - x \\ L_1\sin\phi_1 + L_2\sin\phi_{12} + L_3\sin\phi - y \\ \delta - \phi_4 - \phi_1 - \phi_2 - \phi_3 \\ L_4\cos\gamma - x \\ L_4\sin\gamma - y \end{bmatrix} = 0$$

OPTIMIZATION CASE STUDY

For the purposes of this case study we opt to fix several of the parameters of the optimization vector χ. In the initial prototype, shown in Fig. 4, we selected the three link lengths L_1, L_2, L_3 to be 27.94 cm (11 in.) each. The relative position and orientation between the frames of references on the two mobile platforms is also specified – x =0.00 cm (0 in), y = 60.96 cm (24 in.) and δ =0 – which eliminates them from the list of optimization variables. Further, the specification of the distance between the two mobile bases (x and y) in conjunction with the last two constraints of Eqn. 14, can be used to eliminate two further variables L_4 and γ. Thus, the *apriori* specification of various parameters results in a reduced set of optimization variables $\overline{\chi} = [\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4]^T$ and a reduced set of constraints permitting the constrained optimization problem to be restated as:

$$\underset{\overline{\chi}}{Min}\ f(\overline{\chi}) \tag{15}$$

$$s.t$$

$$\begin{bmatrix} L_1 \cos\phi_1 + L_2 \cos\phi_{12} + L_3 \cos\phi_{123} - x \\ L_1 \sin\phi_1 + L_2 \sin\phi_{12} + L_3 \sin\phi - y \\ \delta - \phi_4 - \phi_1 - \phi_2 - \phi_3 \end{bmatrix} = 0$$

The existence of these three constraints highlights the fact that the number of *independent* optimization variables is, in fact, *one*. Hence, in what follows, we examine the process of explicitly solving the three constraint equations in each iteration (to eliminate the dependent variables (ϕ_1, ϕ_2, ϕ_3) in favor of the independent variables (ϕ_4)). This effectively transforms the constrained optimization problem, Eq. (15), into an unconstrained optimization problem in a reduced (one-dimensional) design space. Further, since the reduced design space is one-dimensional, we also examine the nature of the obtained solution by performing a parameter sweep over the single independent variable, ϕ_4. The objective function is explicitly evaluated for a suitably discretized set of values of ϕ_4 in a desired range (0° to 180°) with the result depicted in Fig. 5.

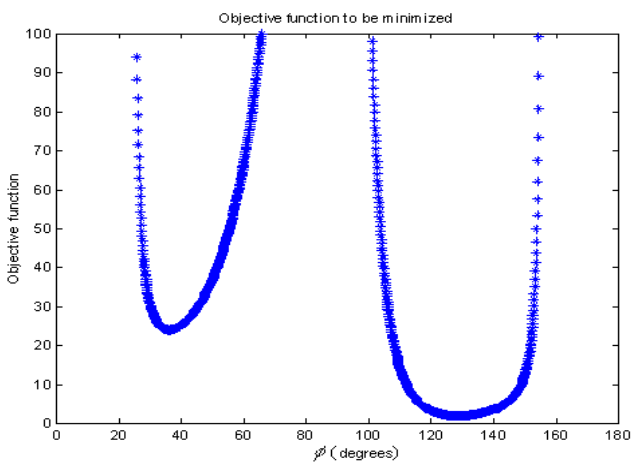

Figure 5: Objective function $f(\overline{\chi})$ for a range of ϕ_4.

We would like to note that there exist regions within this range of values of ϕ_4 where the objective function becomes ill-defined or equality constraints cannot be satisfied (and the corresponding objective function values are not plotted). The plot shows two distinct minima occurring with the lowest value for the range of ϕ_4 between 120° and 145°. By selecting our initial guess in this region, the application of a gradient based numerical optimization search yields the optimal value of ϕ_4=128.5915° where the objective function value attains a minimum of 1.9922. While a minimum is attained we see that the isotropic configuration where the objective function takes on a value of 0 can never attained for this set of values for the parameters. We also note that the value of the objective function does not change significantly around the minimum value. This may be interpreted to be indicative of the robustness of the resulting solution to fluctuations in attaining and maintaining the optimal configuration. Based on this optimal ϕ_4, the optimal values of the other 3 dependent parameters are easily obtained. The resulting "best configuration" for the articulated composite system with near-isotropic manipulability for the center of the object (modeled by the second link) is shown in Fig. 6 below.

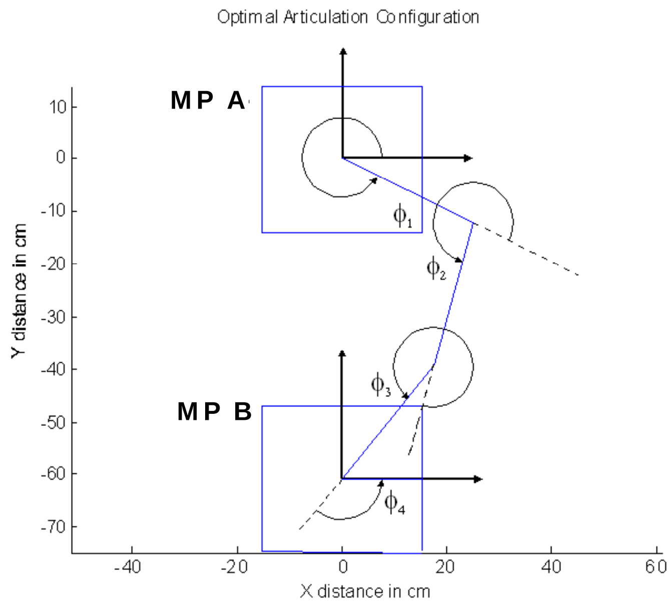

Figure 6: Optimal configuration of the dual robot system.

DISCUSSION

In this paper, we examined the development of a framework for selection of the configuration of system of two collaborating wheeled mobile manipulators for transporting a common object while maximizing a suitable measure of manipulability. The developed optimization-based approach can be used to aid the designer to select both the *best set of dimensions* as well as *most suitable configuration* of the composite system. In the case study presented in this paper, we examined *only* the results of an optimization-based method to determine the "best configuration" for the entire system. This simplification was achieved by the apriori (and arbitrary) specification of the values of a set of parameters (L_1, L_2, L_3, x, y and δ). However, the framework developed here (in the form of the creation of a design-optimization problem) provides the designer with an invaluable tool to quickly evaluate various candidate sets of parameter values. Further, other optimization problems may also be easily created by limiting the restrictions placed on the set of optimization variables (e.g., by treating x and y as optimization variables). The resulting problems show a far greater richness (both in terms of feasible sets of optimal configurations as well as potential pitfalls) and are being examined as part of ongoing research.

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APPENDIX A: GEOMETRIC INTERPRETATION IN THE FRAMEWORK OF PARK AND KIM [23]:

Let K represents the k-dimensional manifold corresponding to the ambient space, and M denotes the m-dimensional $(m < k)$ curved manifold corresponding to the joint configuration space of the mobile manipulator system. Finally, N denotes the n-dimensional manifold corresponding to the end effector space of the mobile manipulator system. Let $\mathbf{x} = (x^1, \ldots, x^k)$ and $\mathbf{u} = (u^1, \ldots, u^m)$ and $\mathbf{f} = (f^1, \ldots, f^n)$ represent coordinates on K, M and N. The Riemannian metrics on each of these spaces may be expressed as positive definite matrices **E**, **G** and **H**, on K, M and N respectively.

Since M is embedded in K, **x** can be written explicitly as a function of **u**, i.e., $\mathbf{x} = \mathbf{x}(\mathbf{u})$. Similarly the forward kinematics may be written as:

$$\{\mathbf{f} : M \to N \mid \mathbf{u} \to \mathbf{f}(\mathbf{u})\} \quad (16)$$
$$\{\mathbf{x} : M \to K \mid \mathbf{u} \to \mathbf{x}(\mathbf{u})\}$$

The existence of these mappings creates a corresponding mapping of the Riemannian metrics through the tangent map. For example, the Riemannian metric of N is mapped onto the manifold M as $\mathbf{J}^T\mathbf{H}\mathbf{J}$, where the notation $\mathbf{J} = \nabla_{\mathbf{u}}\mathbf{f}$ is used to represent the derivative of forward kinematics map $\mathbf{f} : M \to N$ with respect to $\mathbf{u}$.

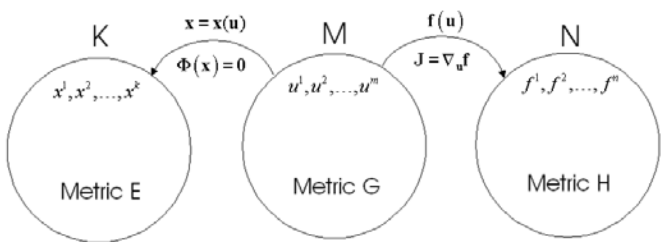

Figure 7: Mappings of the various manifolds.

In the current case, the ambient space K is parameterized by the vector of generalized coordinates, $\mathbf{x} = [\phi_1 \quad \phi_2 \quad \phi_3 \quad L_4 \quad \gamma \quad \theta_L \quad \theta_R]^T$ while the space of minimal (independent) coordinates, M, is parameterized by the vector, $\mathbf{u} = [\theta_L \quad \theta_R]^T$. Therefore the gradient of $\mathbf{x}$ with respect to $\mathbf{u}$ may be written as:

$$\nabla_{\mathbf{u}}\mathbf{x} = \begin{bmatrix} -\mathbf{A}_1^{-1}\mathbf{A}_2\mathbf{K} \\ 1 \quad 0 \\ 0 \quad 1 \end{bmatrix} \quad (17)$$

and the matrix **E** as:

$$\mathbf{E} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

Thus the projected metric **G** and **H** can be computed to be:

$$\mathbf{G} = (\nabla_{\mathbf{u}}\mathbf{x})^T \mathbf{E} (\nabla_{\mathbf{u}}\mathbf{x}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (19)$$

The eigen-decomposition of $\mathbf{J}\mathbf{G}^{-1}\mathbf{J}^T\mathbf{H}$ can now alternatively be used to determine the manipulability ellipsoid and formulate the objective function for our optimization, even in the presence of redundant actuation.