Optimal Partner Selection for Virtual Enterprises in Agile Manufacturing*

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Abstract

Selecting manufacturing partners in the new emerging world of agile manufacturing is an endeavor that is beset with the complexity and dynamics of the market, driven by customer needs, as well as the inherent subjectivity of the selection process. This problem consists of determining an optimal group of manufacturing companies that collectively respond to a specific business initiative as a virtual enterprise. Considering these requirements, traditional vendor selection methodologies do not lend themselves readily applicable to the agile environment. This paper develops a flexible optimization framework for optimal manufacturing partner selection in such an environment. The framework: (1) employs a Fuzzy-AHP approach to compute attribute preferences, (2) acquires linguistically, needs of the specific business initiative, and systematically modifies attribute preferences, (3) performs a search among alternative manufacturing companies to identify and rank a small number of efficient solutions, and (4) recommends an optimal solution while providing a decision aid for human decision making.

Keywords: Agile Manufacturing, Virtual Enterprise, Vendor selection, Analytic Hierarchy Process (AHP), Fuzzy Logic.

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1 Introduction

In face of fierce global competition and the brief moments of opportunity, manufacturing factories have become more focused, increasing the dependence on vendors in product realization. A multi-partnership manufacturing environment has thus emerged in which partner selection has become critical and is governed by market specific needs. 'Agile Manufacturing' is a new concept that subsumes characteristics and addresses problems of this new emerging manufacturing set up. Agility is the ability to thrive in a competitive environment of continuous and unanticipated change, to respond quickly to rapidly changing markets driven by customer-based valuing of products and services (Goldman, 1992).

Even though the problem of vendor selection has been well studied in the literature (see Section 2), the agile environment imposes additional dimensions to this already complex problem. A major such dimension stems from declining defense industrial base and the fact that contractors must increasingly rely on commercial vendors that may be physically distributed across the nation, or even the globe. In such a 'global market' scenario, contractors must consider manufacturability, cost, quality and cycle time concerns not just related to the factories in their vicinity but to a multiplicity of manufacturing facilities around the world. For example, several partner plants as well as several resources within each partner plant may be able to perform an operation. The process and partner selection become related and longer supply chains are encountered. Transport costs and times become significant in such multi-enterprise manufacturing. In addition, numerous other business type qualitative attributes like past-performance, service, organization structure become significant in selecting vendors in a customer and market driven agile environment. New strategic directions may require new criteria and/or a re-emphasis of existing criteria used in making the operational decisions necessary to implement them (Weber, *et al.,* 1991). The degree of importance associated with various evaluation criteria are therefore transient.

Coupled with these unique customer-driven requirements of agile manufacturing, the partner selection process, with its inherent subjectivity, becomes a highly dynamic and complex affair. Traditional methods offer little to aid the analyst in partner selection beset with these difficulties. Existing approaches do not provide an interactive environment that would accept user intent reflective of specific market needs, and systematically process them to modify the priorities on various attributes considered to yield quick and optimal solutions. This makes them inadequate in responding to the dynamics of vendor selection process in an agile environment.

This paper develops a flexible and interactive decision support system to aid in optimal selection of manufacturing partners for a business initiative in an agile manufacturing environment. This decision support system: (i) formally combines concrete quantitative information as well as users' *fuzzy* qualitative information, and (ii) employs state-space search algorithms to provide a quick and optimal selection of partners.

This work will supplement the decision support system for flexible synthesis of multi enterprise partnerships (Minis, *et al.,* 1993). The overall project is based on the vision of nation-wide factory network in which a large base of diversified suppliers and manufacturing partners will be linked electronically to facilitate concurrent engineering and product realization. Within this framework the product design team will be able to: (1) identify similar parts already under production in the network of manufacturing partners (Iyer and Nagi, 1994), (2) evaluate product manufacturability with respect to the capabilities of the potential partners, and (3) select the best partners for the realization of the design. Alternative rough-cut process routings and cost, quality and lead time ratings are generated by the manufacturability evaluation module which is one of the six building blocks of the overall decision support system (Minis, *et al.,* 1993).

The remainder of the paper is organized as follows. The literature surveys on vendor selection, Fuzzy Logic and Analytical Hierarchy Process are briefly discussed in Section 2. A description of the problem and its formulation are presented in Section 3. The overall framework for the selection of manufacturing partners is presented in Section 4. Section 5 presents our approach to partner selection in agile manufacturing. Section 6 discusses an example problem of a typical multi-partnership scenario in an agile manufacturing environment. System implementation issues are discussed in Section 7. Finally, conclusions and future work are discussed in Section 8.

2 Literature Review

Before discussing the flexible optimization framework, we present a brief review of previous work in vendor selection, highlighting the utility and significance of our approach. Further, the section also presents an overview of the theories and applications of Fuzzy Logic and Analytic Hierarchy Process (AHP) as they pertain to the problem of partner selection in agile manufacturing.

2.1 Vendor selection

Selecting the right manufacturing partners for a given product is one of the most fundamental decisions in a multi-product, multi-partner scenario. The partner selection problem is typical of managerial problems which involve tradeoff among multiple criteria that are largely qualitative. Dickson (1966) identified 23 different criteria commonly evaluated during the vendor selection process. The article ranks quality with extreme importance while delivery, performance history, technical capabilities, price, financial position are viewed as being of considerable importance. A recent survey paper (Weber, *et al.,* 1991) reviewed the past research in the vendor selection process with specific attention to criteria and analytical methods.

The entire research work in vendor selection can be classified under two broad categories: qualitative and quantitative. Several papers have analyzed the vendor selection problem qualitatively. However, given the economic importance and inherent complexity of the vendor selection process, research to develop quantitative techniques to address the problem has not been proportional to the need. The few proposed quantitative methodologies for vendor selection can be grouped into the following four categories:

- Matrix or Weight approach (Narsimhan, 1983; Lamberson, *et al.,* 1976; Gregory, 1986; Soukup, 1987),
- Linear Programming (Kingsman, 1986; Pan, 1989; Anthony and Buffa, 1977),
- Mixed Integer Programming (Narsimhan and Stoynoff, 1986; Chaudhary, *et al.,* 1991; Weber and Current, 1993), and
- Goal Programming (Buffa and Jackson, 1983).

Some of the well acknowledged quantitative methodologies suggested in these works include the *multiple objective approach* (Weber and Current, 1993), the *AHP approach* (Narsimhan, 1983) and the Lamberson's *quantitative approach for vendor selection* (Lamberson *et al.,* 1976). The multiple objective approach formulates the objective function Z as a function of several criteria $Z_n(x, y)$ that are to be optimized. The formulation provides a set of non-inferior solutions rather than a single optimal solution. The AHP approach decomposes the complex problem into a hierarchy of elements. The weights are quantified using a *pairwise comparison matrix.* The Lamberson's quantitative approach for supplier selection consists of three phases. The *Characteristic Rating* determines the utility or value of vendor attributes (u_i) for a particular industry. The *Vendor Survey* measures the degree to which a vendor possesses each attribute (v_i). The characteristic rating and the vendor survey are then used to establish a particular *Vendor Score* ($TSIR = \sum_{i=1}^{n} u_i.v_i$) called Total Source Indicated Rating.

These and other approaches to vendor selection require quantification of attributes' importance levels, that are largely qualitative, and assume these either to be provided by the user in numeric form or to be precisely deducible statistical results from surveys. However, decision makers often resist having to make what appears to them as precise numerical judgments in fuzzy situations (Arbel and Vargas, 1990). Further, in an agile manufacturing environment where the importance associated with different attributes is conditioned by the needs of the particular business initiative, repeating them for every specific customer-driven business initiative is a cumbersome task. Decision making in such an environment calls for a friendlier means of acceptance and methodical assimilation of decision makers' (DMs') subjective information for redefining attributes' priorities. Our approach to manufacturing partner selection accomplishes this by using Fuzzy-AHP, a fuzzy extension of Analytic Hierarchical Approach(AHP) (Saaty, 1986, 1990), to develop an interactive decision support system that would systematically address DMs' fuzzy concerns while computing priorities for selection criteria. Finally, unlike most vendor selection research which focuses at the determination of only one vendor for a specific task, our focus is on the determination of a sequence or a set of partners of a virtual enterprise that can collectively accomplish the deliverables of the business initiative in an optimal manner. Therefore a tree search for the optimal sequence of partners is introduced.

2.2 Analytic Hierarchy Process (AHP) and fuzzy logic

Analytic Hierarchy Process (Saaty, 1986, 1990) uses the principle of comparative judgment to construct pairwise comparisons of the relative importance of elements at some given level of a criteria hierarchy with respect to shared criteria or property at the level above, giving rise to a weight matrix. Priorities are synthesized from the second level down from multiplying local priorities by the priority of their corresponding criterion in the level above and adding for each element in a level according to the criterion it effects. This gives the global priority of that element. Field survey, databanks and other types of knowledge bases are the basis of such comparative judgments that crystallize into weights associated with various criteria. Such matrix-based approaches have been widely used in a gamut of areas including vendor selection process (Narsimhan, 1983).

A traditional AHP-based approach requires construction of comparison matrices in which the relative importance among attributes is expressed as precise numbers on a standard scale (usually from 1 to 10). The application of AHP method to partner selection in an agile environment however, has two limitations. Firstly, in the partner selection problem usually no single alternative is the optimal solution, but rather there may exist a set of alternatives that are often referred to as efficient, noninferior or nondominated solutions. AHP technique requires decision makers (DMs) to express their preferences for attributes using crisp numbers, and calculates a weight vector that quantifies the level of importance of attributes. Bryson and Mobolurin (1994) however, debate that other weight vectors may exist which are slight perturbations of AHP-derived weight vector, and could be viewed as approximately reflecting DMs' preferences. Given this, the inherent subjectivity in the DMs' level of importance for the attributes cannot be represented as a single weight vector. Such methodologies may therefore select only a single alternative as the optimal while overlooking others that may also fall within (or even better match) the DM's fuzzy preference. Conclusively, precise numbers fail to contain the subjectivity and vagueness in such decision making. Secondly, redefining attribute priorities to reflect specific market needs would require the tedious task of altering individual pairwise comparisons in the judgment matrix.

Fuzzy logic is a new approach to the analysis of problems that are too complex or too ill-defined to be susceptible to analysis by conventional methods (Schmucker, 1984). It is an entirely non-numeric approach that allows the user to make verbal judgments through natural language expressions like 'very good', 'somewhat poor', etc. Such fuzzy judgments are then quantified using the extension principle applied to fuzzy set theory (Zadeh, 1975). Mathematical theory of fuzzy sets offers a promising approach to bridge this gap between technically necessary precision and the empirically more appropriate, but imprecise description of the problem (Zadeh, 1977a, 1977b). The idea of fuzzy logic is getting popular among researchers and has been extensively applied in areas such as risk analysis, computer security, and in numerous linguistic applications.

We develop a systematic approach to vendor selection that addresses both the issue of subjectivity and fuzziness in the selection process, and the uniqueness of the partner selection problem in an agile environment. Our approach utilizes linguistic expressions and fuzzy numbers for accepting and processing decision makers' judgments. The use of 'natural language expressions' for decision making in such illdefined problems exists in literature (Zadeh, 1972). Such natural language expressions provide a vehicle for the expression of imprecise and subjective evaluation by the decision maker in the absence of objective measures (Clements, 1977). Saaty's AHP approach (Saaty, 1986, 1990) has been extended to allow pairwise *verbal* comparison to synthesize priorities (Buckley, 1985). Van Laarhoven and Pedrycz (1983) constructed the AHP comparison matrix using verbal judgments that were mapped to triangular membership fuzzy functions.

This paper presents a methodology to select a set of manufacturing partners in an agile environment. The methodology overcomes the difficulty of containing subjectivity in decision makers' concerns by using fuzzy numbers to represent pairwise comparisons in the AHP judgment matrix. Fuzzy expressions are used for criteria emphases to capture specific market needs. The overall methodology: (1) blends fuzzy user information emphasizing market-specific needs with the information stored in a default judgment matrix, to quickly provide a redefined comparison matrix (without the need of repeated pairwise comparisons), (2) systematically synthesizes modified set of priorities for attributes, and (3) employs state-space search to explore the feasible combination of partners which are evaluated using the modified set of priorities or weights.

3 Problem Description

The problem consists of selecting manufacturing partners to perform the constituent responsibilities of a business initiative. These constituent responsibilities consist of accomplishing a set of product features. Processes required to produce and finish this set of features can be classified into primary, secondary and tertiary processes. Each class constitutes a set of alternative processes that can produce and/or finish one or more of the required set of features. Further, a particular process can be performed at alternative plant locations. Thus manufacturing a product involves selecting a sequence of process-plant pairs from a solution space represented as a tree (Figure 1). Each of the participating manufacturers (process-plant pair) are partners in the entire business initiative.

Insert Figure 1 Around Here

At a given level of processing, an alternative process performed at one of the feasible plant locations may complete one or more of the required product features. Note that, depending on the partner's resource capabilities, even the same process may accomplish a different set of features. The processing may leave some undesirable features which get added to the list of unfinished features to be accomplished subsequently. Unfinished features are completed using appropriate (secondary or tertiary) processes at feasible plant locations or partners. Branching in the tree consists of deciding alternative ways to accomplish the subsequent set of remaining product features. And, branching terminates when all the feasible combinations of partners capable of accomplishing the required set of features are exhausted.

Thus, selecting a set of partners for manufacturing a product involves an iterative process of selecting partners through the search tree until all the feature requirements are satisfied. The partner selection problem is typical of managerial problems which involve tradeoff among multiple criteria that are largely qualitative. Important criteria involved in partner selection are the cost, quality and lead time associated with the operations of a manufacturing partner. Note that when the subsequent processes are performed at different plant locations, transport cost becomes an added consideration. In addition, there are other business type qualitative information such as past performance, market position, service, etc. that influence selection of a particular partner in the virtual enterprise. A list of important attributes considered in partner selection is compiled as an attribute hierarchy in Figure 2. It is recognized that such attributes are industry-specific and may differ in structure and content. Our proposed methodology can incorporate redefined attribute hierarchies provided these characteristics are available in the databases of the partner plant models.

Insert Figure 2 Around Here

4 Optimal Partner Selection Framework

The objective is to develop a flexible optimization structure that: (i) will accept from the user the criteria of interest and their level of importance (reflecting the customer-specific needs), and (ii) will determine an optimal set of manufacturing partners. The approach uses Fuzzy-AHP, a *fuzzy* extension of Analytic Hierarchy Approach (AHP) (Saaty, 1986, 1990) for multicriteria decision making in fuzzy situations, to synthesize attribute weights. Manufacturing alternatives characterized by cost, quality, and leadtime, and organized in a tree structure are explored by a search algorithm. Evaluation of partner alternatives is accomplished by combining manufacturing characteristics and partner selection attributes with fuzzy combination weights.

The following fundamental issues constitute the building blocks of our partner selection framework.

1. **Manufacturability Evaluation [M0]** The cost, quality, and lead time rating of the various alternatives is obtained from the manufacturability assessment module [M0] developed as a part of the overall project (Minis, *et al.,* 1993). The module is supported by the information stored in partner plant models and has inputs from the design feasibility assessment module (Minis, *et al.,* 1993). The latter provides alternative process and plant options for each of the design features to be produced, in the form of a 'tree'.

2. **Selection Attribute Weights using Fuzzy-AHP [M1]** The module accepts user concerns in the form of *natural language expressions* through a computer interface, and combines it with existing data (from industrial surveys and statistical analysis) to re-emphasize attribute priorities using Fuzzy-AHP. These redefined attribute priorities reflect the specific needs of the business initiative. The user-interface provides a list of important attributes and their 'emphasis sheets' (see Section 5.1) at request, to assist the DM in expressing agile needs.

3. **Search algorithms for partner selection** [M2] Explicit enumeration and branch and bound search techniques are used to select the set of manufacturing partners that can collectively accomplish the entire set of product features. Feasible sequences of partners are evaluated with respect to cost, quality, leadtime, transport cost, and other selection attributes, the priorities of which are derived from the Fuzzy-AHP Weight Synthesizer [M1]. Further, Genetic Algorithms (GAs), a class of non-deterministic stochastic optimization techniques have also been considered to solve larger problems, but heuristically (Gupta and Nagi, 1995a).

Figure 3 is a schematic representation of the flexible optimal partner selection structure.

Insert Figure 3 Around Here

5 Flexible Optimization for Partner Selection

Selecting manufacturing partners in an agile environment involves three important tasks: (1) accepting the decision maker's concerns/judgments in a meaningful form and systematically combining them with the already available information to modify attribute weights, (2) efficiently searching for a group of manufacturing partners capable of satisfying the needs of the project, and (3) evaluating the alternatives, recommending the best, and presenting the other efficient ones. Our flexible optimization framework uses the following procedures to accomplish these tasks.

5.1 Attribute weights for a specific business initiative

As stated earlier, the uniqueness of the partner selection process in an agile environment together with the inherent subjectivity and fuzziness of the vendor selection process itself, summons for (i) a friendlier means of accepting decision makers judgments, and (ii) a systematic methodology for using this information for redefining priorities. This is accomplished by the following steps.

5.1.1 Fuzzy-AHP for computing attributes' weights

In Saaty's AHP, fuzziness is not directly represented by fuzzy notations but is indirectly modeled in a decision problem by forming a reciprocal judgment matrix using pairwise comparisons. However, as mentioned earlier, crisp numbers do not adequately contain the fuzziness in the pairwise comparisons. We overcome the difficulty of subjectivity and vagueness in the vendor selection problem by using fuzzy numbers as a superior means of representing pairwise comparisons in the AHP judgment matrix. A comparison matrix is constructed for the partner selection problem in which pairwise comparisons are fuzzy numbers. These fuzzy numbers are based on survey data and are representative of the static, longterm picture of vendor selection process in the specific manufacturing industry. Cheng and Mon (1994) have proposed a simple yet general algorithm for fuzzy AHP, using triangular fuzzy numbers, α-cuts, and interval arithmetic. The method uses fuzzy numbers to improve the scaling scheme proposed in Saaty (1990). Our approach uses this algorithm to calculate default *as well as modified attribute weights* for the critical selection criteria.

Insert Figure 4 Around Here

The algorithm utilizes triangular fuzzy numbers $\tilde{1}$ to $\tilde{9}$ to build the judgment matrix. The membership functions for these fuzzy numbers $\mu_{\tilde{A}}(x)$, are defined by the triplet (a_1, a_2, a_3) in Table 1 and are shown in Figure 4 (see Step 3 and Section 5.1.3 for more general fuzzy numbers). Subsequently, interval arithmetics is employed to solve for the fuzzy eigenvector. The steps in the algorithm are summarized below. For further details the reader can refer to Cheng and Mon (1994).

Insert Table 1 Around Here

- *Step 1:* The relative strengths of the criteria in the judgment matrix ($\tilde{A}$) are compared using triangular fuzzy numbers $\tilde{a_{ij}}$, such as

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \tilde{a}_{13} & \dots & \tilde{a}_{1(n-1)} & \tilde{a}_{1n} \\ \frac{1}{\tilde{a}_{12}} & 1 & \tilde{a}_{23} & \dots & \tilde{a}_{2(n-1)} & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \dots & \vdots & \vdots \\ \frac{1}{\tilde{a}_{1(n-1)}} & \frac{1}{\tilde{a}_{2(n-1)}} & \frac{1}{\tilde{a}_{3(n-1)}} & \dots & 1 & \tilde{a}_{n(n-1)} \\ \frac{1}{\tilde{a}_{1n}} & \frac{1}{\tilde{a}_{2n}} & \frac{1}{\tilde{a}_{3n}} & \dots & \frac{1}{\tilde{a}_{n(n-1)}} & 1 \end{bmatrix} \quad (1)$$

  where

  $\tilde{a}_{ij} \in \{\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}, \tilde{1}^{-1}, \tilde{3}^{-1}, \tilde{5}^{-1}, \tilde{7}^{-1}, \tilde{9}^{-1}\}$.

- *Step 2:* A fuzzy eigen value $\tilde{\lambda}$ is a fuzzy number solution to

$$\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x}, \quad (2)$$

  where $\tilde{x}$ is a non zero $n \times 1$ fuzzy vector containing fuzzy numbers; (2) is equivalent to

$$(\tilde{a}_{i1} \otimes \tilde{x}_1) \oplus \dots \oplus (\tilde{a}_{in} \otimes \tilde{x}_n) = \tilde{\lambda} \otimes \tilde{x}_i; 1 \leq i \leq n, \quad (3)$$

  where $\otimes$ and $\oplus$ denote fuzzy multiplication and fuzzy addition, respectively (Kaufmann and Gupta, 1988; Cheng and Mon, 1993).

- *Step 3:* Fuzzy multiplication and addition is performed using interval arithmetic and α cuts. Triangular fuzzy numbers can be characterized alternatively, by defining the interval of confidence α (Cheng and Mon, 1993). For example, fuzzy number $\tilde{F}$ represented by the triplet (a_1, a_2, a_3) can be defined as:

$$\forall \alpha \in [0,1], \tilde{F}_\alpha = [a_1^\alpha, a_3^\alpha] = [a_1 + (a_2 - a_1)\alpha, a_3 - (a_3 - a_2)\alpha]. \quad (4)$$

  where $[a_1^\alpha, a_3^\alpha]$ are the α cuts for the triangular fuzzy number. From Kaufmann and Gupta (1991), $\tilde{P} = (1/a_3, 1/a_2, 1/a_1)$ approximates the inverse of the triangular fuzzy number $\tilde{F}^{-1}$ (where $\tilde{F} = (a_1, a_2, a_3)$). Thus, α-cuts for $\tilde{F}^{-1}$ are

$$\tilde{F}_\alpha^{-1} = \left(\frac{1}{a_3 - (a_3 - a_2)\alpha}, \frac{1}{a_1 + (a_2 - a_1)\alpha}\right) \quad (5)$$

In (1) and (2), for $0 < \alpha \leq 1$, and all i and j, let

$$\tilde{a}_{ij}^{\alpha} = [a_{ij1}^{\alpha}, a_{iju}^{\alpha}], \quad \tilde{x}_i^{\alpha} = [x_{i1}^{\alpha}, x_{iu}^{\alpha}], \quad \tilde{\lambda}^{\alpha} = [\lambda_1^{\alpha}, \lambda_u^{\alpha}]. \tag{6}$$

Substituting (6) in (3),

$$[a_{i11}^{\alpha}x_{11}^{\alpha}, a_{i1u}^{\alpha}x_{1u}^{\alpha}] \oplus \ldots \oplus [a_{in1}^{\alpha}x_{n1}^{\alpha}, a_{inu}^{\alpha}x_{nu}^{\alpha}] = [\lambda_1^{\alpha}x_{i1}^{\alpha}, \lambda_u^{\alpha}x_{iu}^{\alpha}] \tag{7}$$

or

$$a_{i11}^{\alpha}x_{11}^{\alpha} + \ldots + a_{in1}^{\alpha}x_{n1}^{\alpha} = \lambda_1^{\alpha}x_{i1}^{\alpha}, \quad a_{i1u}^{\alpha}x_{1u}^{\alpha} + \ldots + a_{inu}^{\alpha}x_{nu}^{\alpha} = \lambda_u^{\alpha}x_{iu}^{\alpha} \tag{8}$$

- *Step 4:* The degree of satisfaction of the decision maker $\hat{a}_{ij}^{\alpha}$, is defined as

$$\hat{a}_{ij}^{\alpha} = \lambda a_{iju}^{\alpha} + (1-\lambda)a_{ij1}^{\alpha} \quad \forall\lambda \in [0,1]. \tag{9}$$

where λ is index of optimism. Thus, the judgment matrix becomes:

$$\tilde{A} = \begin{bmatrix} 1 & \hat{a}_{12} = \lambda a_{12u}^{\alpha} + (1-\lambda)a_{121}^{\alpha} & \ldots & \hat{a}_{1n} = \lambda a_{1nu}^{\alpha} + (1-\lambda)a_{1n1}^{\alpha} \\ \frac{1}{\hat{a}_{12} = \lambda a_{12u}^{\alpha} + (1-\lambda)a_{121}^{\alpha}} & 1 & \ldots & \hat{a}_{2n} = \lambda a_{2nu}^{\alpha} + (1-\lambda)a_{2n1}^{\alpha} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\hat{a}_{1n} = \lambda a_{1nu}^{\alpha} + (1-\lambda)a_{1n1}^{\alpha}} & \frac{1}{\hat{a}_{2n} = \lambda a_{2nu}^{\alpha} + (1-\lambda)a_{2n1}^{\alpha}} & \ldots & 1 \end{bmatrix} \tag{10}$$

The eigenvectors corresponding to $\alpha = 0.1, 0.2, \ldots, 1$, and a fixed value of λ are calculated using computer code developed in C++. These eigenvectors represent the total arrangement order of attribute weights $[\mathbf{W}_j^{(\alpha,\lambda)} = \{w_j^{(\alpha,\lambda)}, \forall\alpha \in [0,1],\}$, for all attributes $j = 1, 2, \ldots, m]$, for all α-cuts with fixed λ values.

From the total arrangement order of attribute weights for all α-cuts with fixed λ values, we propose a *composite attribute weight vector* $\mathrm{W}_{composite\ j}^{\lambda}$, that quantifies the expected preference level of various selection attributes and is defined as

$$\mathrm{W}_{composite\ j}^{\lambda} = \frac{\sum_{\alpha} w_j^{(\alpha,\lambda)}\alpha}{\sum_{\alpha}\alpha}, \quad j = 1, \ldots, m; \tag{11}$$

5.1.2 Fuzzy expressions for criteria emphases

Our approach uses natural language expressions as a vehicle for accepting imprecise and subjective DMs' attribute preferences, reflecting specific market needs. The decision makers' level of importance for the selection criteria are accepted as *linguistic variables* such as 'extremely important' and 'fairly important'. The selected linguistic options re-emphasizing importance level for various attributes (specific to a business initiative) are modeled as operators on default fuzzy numbers.

Such linguistic options are defined for every member (criteria and their sub-components) in the selection criteria hierarchy. These form the contents of what is called an *emphasis sheet* of an attribute. The decision maker is provided with five level linguistic options. The user may invoke an emphasis sheet, such as the one illustrated in Figure 5, to re-emphasize its priority. For example, user's attractiveness to quality can be accepted as a natural language expression such as quality is *very important*. Here, the linguistic emphasis 'very important' for quality is modeled as operator on all the default fuzzy numbers representing pairwise comparisons of quality with other attributes.

Insert Figure 5 Around Here

5.1.3 Recomputation of attribute weights

The next critical task is to systematically combine the information gleaned from the user with the default fuzzy numbers collected through industrial surveys and other resources. We propose a scheme to modify the judgment matrix based on the information collected from the user, without having the user to alter pairwise comparisons repeatedly. According to this scheme, the user input pertaining to the severity or attractiveness of any attribute is superimposed on the default comparison matrix to give a modified matrix. The rule is that a linguistic emphasis chosen by the DM for an attribute will alter all the fuzzy numbers constituting the row and column vectors in the judgment matrix corresponding to the attribute emphasized. For example, the option 'very important' for quality systematically intensifies and weakens respectively, the fuzzy numbers in the row and column vectors corresponding to quality in the comparison matrix.

The membership functions of the affected triangular fuzzy numbers in the judgment matrix are intensified or weakened (by amount δ) to reflect the DMs' emphases. Figure 6 shows the membership functions at different levels of emphasis for fuzzy numbers $\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}$, and $\tilde{9}$. In terms of the triplets (a_1, a_2, a_3), the modified fuzzy number is represented as

$$\tilde{A} = (a_1, (a_2 + \delta), a_3)$$

where

$$\delta = \begin{cases} +2 & \textit{Extremely Important} \\ +1 & \textit{Very Important} \\ 0 & \textit{Important} \\ -1 & \textit{Moderately Important} \\ -2 & \textit{Not Important} \end{cases}$$

Insert Figure 6 Around Here

Thus every fuzzy number (except $\tilde{1}$) has five levels of intensity corresponding to the five linguistic options. As shown in Figure 6, fuzzy number $\tilde{1}$ has only three levels of modification (extremely important, very important, and important) and assumes the default (important) representation for 'moderately important' and 'not important' emphases. Unless altered, the default priority of an attribute corresponds to the median ('important') of the five level linguistic options and is equal to the weight calculated from the default comparison matrix.

While the fuzzy numbers $\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}$, and $\tilde{9}$ and their inverses are recommended for developing pairwise comparisons in the judgment matrix, more general triangular numbers defined by the triplets (a_1, a_2, a_3) can be employed (with $a_i \in \{1, 2, \ldots, 10\}$ and $a_1 \leq a_2 \leq a_3$). If larger deviations from the industry-specific or static definitions are desired to be expressed in the agile needs, the difference between a_3 and a_1 can be increased. This will allow, for instance, to have the relative strength of quality to supersede cost in the static definition and vice-versa in an agile initiative. Note that the five-level linguistic intensities for these (unsymmetric) fuzzy numbers can be expressed similarly using the appropriate (scaled) definitions of δ.

Using the algorithm and the composite weight vector definition presented in Section 5.1.1, the total arrangement order of modified priorities $\mathbf{W}_j^{(\alpha,\lambda)} = \{w_j^{(\alpha,\lambda)}\}$ (for all values of $\alpha = 0.1, 0.2, \ldots, 1.0$ and fixed values of λ), and the composite priorities $\mathrm{W}^{\lambda}_{composite\ j}$, of the selection criteria are computed. The alternative feasible sequences of partners are evaluated with respect to the total arrangement order of modified priorities as well as with respect to modified composite priorities of selection criteria.

5.2 Generation of alternatives for sequences of partners

Manufacturing in virtual enterprise is an effort of bringing together so called mini-factories, that may be located locally or globally, into a partnership deal. Numerous manufacturing facilities are therefore likely to be partners in any such business initiative. Searching for the best sequence of manufacturing partners involves selecting and evaluating feasible sequences of process-plant pairs from a large number of combinations. The following search techniques are used to explore good sequences of manufacturing partners:

1. **Explicit Enumeration:** This involves searching the entire search space, evaluating each possible process plan, and selecting the best alternatives. Explicit enumeration can be done using either depth-first or breadth-first techniques. In depth-first search, when a state is examined, all of its children and their decendents are examined before any of its siblings. For implementing the breadth-first search the search tree needs to be treated differently. Each layer would consists of process options and the set of feasible plant options for each process (see Figure 1). Breadth-first search would evaluate each process-plant pair at a given level and then proceed to the next layer. In case the various processes have large number of feasible plant options the depth-first search may be more efficient than the breadth-first search since with the latter, combinatorial explosion (due to high average number of descendants) may prevent the algorithm from finding a solution using the available computer memory.

2. **Branch and Bound method:** Branch and bound improves on the explicit enumeration by pruning off portions of the tree that have obviously inferior solutions. In an attempt to do so, the branch and bound algorithm starts with a 'good' feasible solution which is constantly improved on by the search process. Partial solutions are expanded in the search process, but only if the goodness measure so far along with a conservative estimate of the measure to goal

(lower bound of 'cost') is expected to be better than the 'good' feasible solution(s) at hand. Efficiency in search is due to the pruning of branches of partial solutions that even with a conservative estimate completion are worse than solution(s) explored already.

Finally, we have also considered Genetic Algorithms (GAs), a class of non-deterministic stochastic optimization techniques to solve larger problems heuristically (Gupta and Nagi, 1995a). Another advantage of using GAs is their ability to provide a population of 'good' solutions rather than a unique one.

Note that while the explicit enumeration only requires accumulation of attribute values for the sequence of partner, the other two approaches require some *a priori* evaluation or scoring of partial or complete sequences. In the GA, we have employed a single set of attribute weights (from the composite weights of the fuzzy-AHP, refer Section 4.2) and the hope is that the class of good solutions will be explored during the search, and can be evaluated more rigorously later. In the branch-and-bound procedure the accumulation of attributes in the partial sequence of partners is assigned scores using the total arrangement of weight vectors, and the cost-so-far is assumed to be the lowest among these scores. In addition, a simple cost-to-go or lower bound function is required. In our case we have assumed a null bounding function as the simplest approach (see Section 8 for proposed extensions).

5.3 Evaluation of a sequence of partners

Concurrent with (or after) the process of searching feasible sequences of partners, is the task of evaluating the alternatives. The search algorithm generates feasible sequences of partners $\langle(i^k, j^k, N_{ij}^k)\rangle_{k=1}^{n_P}$, where n_P is the total number of manufacturing partners in the sequence P. The $k-th$ partner in the feasible sequence P is represented by triplets (i^k, j^k, N_{ij}^k) where, i^k is the plant at which the $k-th$ manufacturing partner accomplishes a set of features N_{ij}^k using process j^k. The sequence of partners satisfies the following two conditions:

$$N_{ij}^k \cap N_{rs}^l = \emptyset, \ k \neq l; \tag{12}$$

$$card\left(\cup_{k=1}^{n_P} N_{ij}^k\right) = F. \tag{13}$$

where *card(.)* represents the cardinality of the set; and F is the total number of product features. Equation (12) ensures that each manufacturing partner accomplishes a unique set of features, and (13) ensures that all the features are accomplished by the selected manufacturers collectively.

Each manufacturing partner (k) in the feasible sequences of partners is characterized by the ratings (for accomplishing the set of features N_{ij}^k at plant i^k using process j^k) with respect to cost ($c_{ij}^k(N_{ij}^k)$), quality ($q_{ij}^k(N_{ij}^k)$), leadtime ($\tau_{ij}^k(N_{ij}^k)$), the transport costs ($T_{ij}^k(N_{ij}^k)$), and other selection criteria. It is very interesting to note that agile manufacturing philosophy aims at the efficient utilization of unused capacity among partners in a dual-use industrial infrastructure. This is made possible by leadtime ratings that reflect instantaneous capacity, and are derived from not only the actual resource set-up and processing times for the desired production quantity, but also instantaneous resource workload and queue sizes. In this paper however, we will assume that all these ratings are consistent with agile manufacturing principles, and have been provided by the manufacturability evaluation module [M0].

While ratings with respect to criteria such as cost and leadtime are additive over the partners in the sequence, the quality rating of each feature is dependent on the process selected. The simplest quality metric, *process capability index*, $\mathrm{c}_{\mathrm{P}_\mathrm{k}}$ defined in (14), is used to quantify quality. The tolerance limits are the ones that correspond to the part features that are accomplished by the process under consideration. If more than one set of tolerance limits correspond to this process, the one that provides the tightest tolerance zone is selected. The process variance is obtained from the plant model and may be process or equipment specific. The quality ratings of the each partner k in the sequence is aggregated using its 'yield' y_k, defined in (15).

$$\mathrm{c}_{\mathrm{P}_\mathrm{k}} = \frac{\text{(mean - lower tolerance limit)}}{3 \times \text{process variance}} = q_{ij}^k(N_{ij}^k) \tag{14}$$

$$\mathrm{y}_k = \mathrm{F}(3\mathrm{c}_{\mathrm{P}_\mathrm{k}}) = \mathrm{Prob}\left(-3\mathrm{c}_{\mathrm{P}_\mathrm{k}} \leq X \leq 3\mathrm{c}_{\mathrm{P}_\mathrm{k}}\right) \tag{15}$$

where X is a normally distributed random variable with mean 0 and variance 1. All the features in the set N_{ij}^k, accomplished by process j^k will have same yield, y_k. The effective yield Y_P of the sequence of partners is defined in (16).

$$\mathrm{Y}_P = \Pi_{k=0}^{n_P} \mathrm{y}_k, \tag{16}$$

The aggregate process capability index of the sequence of partners will be the value $\mathrm{C}_{\mathrm{P}_\mathrm{k}}$, such that condition in (17) is satisfied, and is used as the aggregate quality ratings of partners in the sequence.

$$\mathrm{Prob}\left(-3\mathrm{C}_{\mathrm{P}_\mathrm{k}} \leq X \leq 3\mathrm{C}_{\mathrm{P}_\mathrm{k}}\right) = \mathrm{Y}_P. \tag{17}$$

User preference for critical attributes and their relative importance, obtained from the Fuzzy-AHP Weight Synthesizer [M1], determine the dimension and parameters of the objective function. An objective function, which is expressed in terms of the ratings and the attribute weights is used to assign a score to the feasible group of manufacturing partners generated in the search. A general objective function formulation is:

Minimize:

$$S_P^{(\alpha,\lambda)} = \sum_{k=1}^{n_P} \left(\omega \times c_{ij}^k(N_{ij}^k) + \gamma \times \tau_{ij}^k(N_{ij}^k) + \delta \times T_{ij}^k(N_{ij}^k) + \ldots\ldots\right)$$
$$+ \beta \times \left(1 - \mathrm{Prob}\left(-3\mathrm{C}_{\mathrm{P}_\mathrm{k}} \leq X \leq 3\mathrm{C}_{\mathrm{P}_\mathrm{k}}\right)\right). \tag{18}$$

OR

$$S_P^{(\alpha,\lambda)} = \sum_{k=1}^{n_P} \left(\omega \times c_{ij}^k(N_{ij}^k) + \gamma \times \tau_{ij}^k(N_{ij}^k) + \delta \times T_{ij}^k(N_{ij}^k) + \ldots\ldots\right) + \beta \times (1 - \mathrm{Y}_P).$$

where ω, β, γ, and δ in (18) are the priorities associated with different attributes and assume values corresponding to the total arrangement order of modified attribute weight vectors $\mathbf{W}_j^{(\alpha,\lambda)} = \{w_j^{(\alpha,\lambda)}\}$ (for all values of $\alpha = 0.1, 0.2, \ldots, 1.0$ and fixed values of λ), and the composite weight vector $\mathrm{W}^{\lambda}_{composite\ j}$. The sequences of manufacturing partners are assigned scores $\mathbf{S_P}^{(\alpha,\lambda)} = \{S_P^{(\alpha,\lambda)}, \forall \alpha \in [0,1], \lambda \in [0,1]\}$, corresponding to the total arrangement order of the attribute weight vectors $\mathbf{W}_j^{(\alpha,\lambda)} = \{w_j^{(\alpha,\lambda)}, \forall \alpha \in [0,1], \lambda \in [0,1]\}$. The score vector $\mathbf{S_P}^{(\alpha,\lambda)}$ represents in a fuzzy sense, the membership function of the fuzzy score of alternative P. In addition, each feasible sequence of partners is assigned a composite score $\mathrm{S}^{\lambda}_{composite\ \ P}$, corresponding to the composite weight vector $\mathrm{W}^{\lambda}_{composite\ j}$.

The search algorithm generates and evaluates feasible sequences of manufacturing partners over the entire spectrum of modified weight vectors ($\mathbf{W}_j^{(\alpha,\lambda)}$) and the composite weight vector ($\mathrm{W}^{\lambda}_{composite\ j}$), while searching the solution space. Based on the composite score, if the best alternative f has a score $\mathrm{S}^{\lambda}_{composite\ \ f}$, less than the $\min_\alpha(\mathbf{S_P}^{(\alpha,\lambda)})$ of all the other alternative sequences, then the partner sequence f is the recommended best solution. A more conclusive test is if $\max_\alpha(\mathbf{S_f}^{(\alpha,\lambda)})$ is less than the $\min_\alpha(\mathbf{S_P}^{(\alpha,\lambda)})$ of all the other alternative sequences. Otherwise, all the alternative sequences of partners with $\min_\alpha(\mathbf{S_P}^{(\alpha,\lambda)})$ less than $\mathrm{S}^{\lambda}_{composite\ \ f}$, and alternative f constitute the set of non-inferior alternatives. In the context of this section, alternative f' is inferior to f if $S_{f'}^{(\alpha,\lambda)} \geq S_f^{(\alpha,\lambda)}, \forall \alpha \in [0,1], \lambda \in [0,1]$ and $S_{f'}^{(\alpha,\lambda)} > S_f^{(\alpha,\lambda)}$, for some $\alpha \in [0,1]$, and same $\lambda \in [0,1]$. Several linguistic decision making techniques such as the one suggested in Gupta and Nagi (1995b), can be used to help select a best alternative from a set of finite alternatives. However in this paper, the approach determines the ten best partner sequences together with their score distributions $\mathbf{S_P}^{(\alpha,\lambda)}$, providing the DM with a decision aid and the opportunity to select alternative partner sequences.

6 Example Problem

This section illustrates the application of our flexible optimization framework for optimal partner selection. The task involves accomplishing a set of seven design features ($\mathbf{F} = \{f^r, r = 1, \ldots, 7\}$), for producing a new product. A design feature may be accomplished by multiple process options (primary, secondary, or tertiary). Furthermore, depending on the specific resource capabilities of a partner, a process option can accomplish one or more of the required set of product features (Table 2). These options, characterized by their attribute ratings are obtained from the Manufacturability Assessment Module [M0].

Insert Table 2 Around Here

There may be large number of feasible partner sequences in the solution space represented as a tree (Figure 7). The search process involves selecting a feasible sequence of manufacturing partners that will produce and finish the required set of features. Figure 7 illustrates the explosive nature of the solution space and delineates three of the several possible sequences of manufacturing partners. The evaluation of manufacturing partners is performed using a simple formulation of the objective function that considers four attributes: cost, quality and lead time ratings and the transport cost between plants, where the design features are processed.

Insert Figure 7 Around Here

As discussed in Section 5.1, the importance or severity associated with various attributes are synthesized using Fuzzy-AHP and are obtained from the Fuzzy-AHP Weight Synthesizer [M1]. Shown below is the default judgment matrix measuring the pairwise comparisons among the attributes (as fuzzy numbers $\tilde{1}$, $\tilde{3}$, $\tilde{5}$, $\tilde{7}$, and $\tilde{9}$) considered for computing the importance level of the four attributes. This data is directly obtained from the comparison matrix used by Narsimhan (1983) for supplier selection, except that the Saaty's scaling scheme is improved by using triangular fuzzy numbers.

$$\tilde{A} = \left[\begin{array}{l|cccc} & Cost\ (\omega) & Quality\ (\beta) & Leadtime\ (\gamma) & Transcost\ (\delta) \\ \hline Cost\ (\omega) & 1 & \tilde{3} & \tilde{1} & \tilde{3} \\ Quality\ (\beta) & \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{3}} & \tilde{1} \\ Leadtime\ (\gamma) & \frac{1}{\tilde{1}} & \tilde{3} & 1 & \frac{1}{\tilde{2}} \\ Transcost\ (\delta) & \frac{1}{\tilde{3}} & \tilde{1} & \tilde{2} & 1 \end{array}\right]$$

Let us consider that the business initiative under consideration requires a partner plan solution where 'cost' and 'leadtime' attributes are *extremely important* and *very important* respectively, and the DM emphasizes the same by selecting the corresponding linguistic options on the respective attribute emphasis sheets. The decision system alters the default matrix according to the scheme outlined in Section 5.1.3, and recomputes weights using α-cuts and interval arithmetic. The modified matrix is

$$\tilde{A}_{modified} = \left[\begin{array}{l|cccc} & Cost\ (\omega) & Quality\ (\beta) & Leadtime\ (\gamma) & Transcost\ (\delta) \\ \hline Cost\ (\omega) & 1 & \tilde{3}^{++} & \tilde{1}^{+} & \tilde{3}^{++} \\ Quality\ (\beta) & \frac{1}{\tilde{3}^{++}} & 1 & \frac{1}{\tilde{3}^{+}} & \tilde{1} \\ Leadtime\ (\gamma) & \frac{1}{\tilde{1}^{+}} & \tilde{3}^{+} & 1 & \frac{1}{\tilde{2}^{-}} \\ Transcost\ (\delta) & \frac{1}{\tilde{3}^{++}} & \tilde{1} & \tilde{2}^{-} & 1 \end{array}\right]$$

Using the Cheng and Mon (1994) algorithm on the modified matrix, the total arrangement of modified weight vectors $[\mathbf{W}_j^{(\alpha,0.5)} = \{w_j^{(\alpha,0.5)}, \forall\alpha \in [0,1],\}$, for $j = 1,2,3,4]$, are computed for λ

= 0.5 (moderate level of optimism). Composite weight vector $\mathrm{W}^{0.5}_{composite\ j}$ is computed using (11). Table 3 shows the modified weights and composite weights for cost (ω), quality (β), leadtime (γ), and transport cost (δ).

The search for the feasible plans is accomplished using an explicit enumeration (depth-first search technique). A depth-first search algorithm, written in C++ is used to generate feasible partner sequences $\langle (i^k, j^k, N^k_{ij}) \rangle^{n_p}_{k=1}$ satisfying constraints (12) and (13). The objective function in (19) is used to evaluate sequences of partners. The feasible combinations of manufacturing partners are assigned score $\mathbf{S_P}^{(\alpha,0.5)} = \{S_P^{(\alpha,0.5)}, \forall \alpha \in [0,1]\}$, and a composite score $\mathrm{S}^{0.5}_{composite\ \ P}$ corresponding to the total arrangement order of the attribute weight vectors $\mathbf{W}_j^{(\alpha,0.5)} = \{w_j^{(\alpha,0.5)}, \forall \alpha \in [0,1], \lambda = 0.5\}$ and composite weight vector $\mathrm{W}^{0.5}_{composite\ j}$, respectively from Table 3.

Insert Table 3 Around Here

$$\text{Minimize:} \qquad S_P^{(\alpha,0.5)} = \sum_{k=1}^{n_P} \left(\omega \times c^k_{ij}(N^k_{ij}) + \gamma \times \tau^k_{ij}(N^k_{ij})\right) + \delta \times T_{norm} \\ + \beta \times \left(1 - \mathrm{Prob}\left(-3\mathrm{C}_{\mathrm{P_k}} \le X \le 3\mathrm{C}_{\mathrm{P_k}}\right)\right). \tag{19}$$

$$T_{norm} = \left(\frac{\sum_{k=1}^{n_P - 1} T_{k,k+1}}{T_{max}}\right)$$

where,

N^k_{ij}	N^k_{ij} is the set of product feature accomplished by the $k-th$ partner using process j at facility i.
T_{norm}	normalized transport cost.
T_{max}	maximum transport cost for the routing
$T_{k,k+1}$	transport cost between i^k and $i^{(k+1)}$ $\forall k = 1, 2, ..., (n_P - 1)$
$c^k_{ij}(N^k_{ij})$	cost rating of $k-th$ manufacturing partner for producing set of feature N^k_{ij}
$\mathrm{C}_{\mathrm{P_k}}$	aggregate quality rating of manufacturing partner sequence P
$\tau^k_{ij}(N^k_{ij})$	lead time rating of $k-th$ manufacturing partner for producing set of feature N^k_{ij}
$\omega, \beta, \gamma, \delta$	attribute weights for cost, quality, leadtime and transport cost, respectively.

Insert Figure 8 Around Here

The best (or recommended) partner sequence for this application (Application 1), based on the composite score is presented in (20) that is a sequence of triplets, (plant_id, process_id, set of features accomplished).

$$\left(7,1,\left\{f^2\right\}\right)_1 - \left(8,1,\left\{f^1\right\}\right)_2 - \left(4,2,\left\{f^3,f^5\right\}\right)_3 - \left(7,3,\left\{f^4,f^6\right\}\right)_4 - \left(4,5,\left\{f^7\right\}\right)_5 \tag{20}$$

The recommended partner sequence has five partners satisfying the conditions in (12) and (13). Figure 8 shows the score distribution for the best ten partner sequences obtained after evaluation of 3069 noninferior, feasible solutions. The evaluation was performed using the total arrangement of weights and composite weights obtained from the fuzzy-AHP weight synthesizer. In the graph, alternatives are represented as a sequence of (plant_id, process_id) pairs, and the average score corresponds to the score using composite weights. The plot illustrates the utility and significance of our methodology in perspective. Even though the partner sequence in (20) is selected as the best solution based on the composite score, the sequence in (21) (the second best solution) dominates the best solution in the α-range 0.0 to 0.550. Conventional approaches would have selected solution in (21) as the best if the AHP-derived weight vector, computed using non-fuzzy representations of the DM inputs, had fallen in the spectrum of weights for α-range 0.0 to 0.550. Note that dominance relationship between the alternatives changes significantly over the range of alpha values. DMs can use other linguistic decision making techniques to select the best alternative among the non-dominated set (see e.g., Gupta and Nagi, 1995b).

Insert Figure 9 Around Here

If the business initiative under consideration had different requirements from the customer, and quick delivery was of primary importance, the selection process could be repeated with leadtime at an 'extremely important' emphasis. Without presenting the details of the new fuzzy-weights, we present the application results of our flexible optimization framework in Figure 9. This application (Application 2) identifies partner sequence in (21) as the best solution based on the composite score.

$$\left(7,1,\left\{f^2\right\}\right)_1 - \left(3,1,\left\{f^1\right\}\right)_2 - \left(8,3,\left\{f^4,f^5\right\}\right)_3 - \left(1,4,\left\{f^3,f^6\right\}\right)_4 - \left(6,5,\left\{f^7\right\}\right)_5 \tag{21}$$

Based on the results of the two applications, we can make the following two important observations: (i) the optimal solution recommended could change with different attribute emphases (which are assimilated effortlessly), and (ii) it is possible to have a conclusive choice of partners (supported through the decision aid of our framework), even while considering fuzzy-weights.

7 Systems Implementation

The software for the optimal selection of manufacturing partners for a business initiative in agile manufacturing has been implemented using C and C++ on the SUN/Unix platform. Routines for the Fuzzy-AHP module and the search module have been coded. An interactive user-interface has been designed for the X-window environment using SUIT (Simple User Interface Toolkit), an open domain subroutine library for graphics (Conway, 1992). The interface assists in collecting the information on DM's preferences for selection attributes that is critical to the selection process. A menu showing all the important selection attributes for partner selection is displayed and the user selects the criterion to be emphasized from this menu by clicking on the desired button. Another menu showing the five level linguistic options is displayed. A decision maker can express concerns in the form of *linguistic variables* such a 'very important', 'fairly important', thus relieving him from the burden of making judgments on a numeric scale. After the DM has expressed concerns specific to the business initiative, search for the best set of manufacturing partners is triggered by clicking on the 'SEARCH' button. Once the search is complete, the software displays the best sequence of manufacturing partners that can collectively accomplish the constituent responsibilities of the business initiative. Further, a graphical decision aid (such as Figure 8) assists the DM in viewing the fuzzy functions of the overall score of the ten best alternatives. This tool helps the analyst: (i) in building confidence towards the suggested optimal if it dominates all others, or (ii) breaking ties among competing ones considering other hard to capture qualitative concerns (e.g., patronage). Partner information such as the one presented in Figure 2 is stored in an object-oriented database system (ObjectStore, 1992).

The software system provides additional features of viewing all the feasible alternatives and subjecting them through different dominance tests. It allows the user to specify some important attributes with respect to which the dominated solutions can be eliminated. On the reduced set of "competing" alternatives the user can proceed with the evaluation using: (i) fuzzy-AHP weights, or (ii) user specified weights. The latter is helpful when the user would like to exercise more control on the choice of attributes to be selected for evaluation and would like to use other sets of combination weights. All these operations are made possible by a user-friendly interface.

8 Conclusion and Recommendations

Partner or vendor selection process involves strategic decision making and may have serious financial implications. While vendor selection has been an important problem in business and academic research committees, agile manufacturing adds new dimensions of complexity to it. Firstly, the dynamics of a market driven by customer needs affects the relative importance that the various criteria have in vendor selection process. Thus, the emphasis on attributes may need to be altered for every customer driven business initiative. Secondly, the inherent subjectivity of the associated information on attribute priorities needs to be captured in a simplistic (non-numeric) form, to reflect the specific needs realistically. Finally, the selection involves a group (not just one) partners that effectively accomplish the multiple product features in a virtual enterprise framework; process selection and transport cost become relevant in this environment.

In response to this reality, the paper presented a decision support system that used: (1) Fuzzy-AHP and a user-interface to encase the vagueness and subjectivity of the users' criteria of interest in a particular business initiative, and (2) Tree Search Algorithm for providing a set of alternative groups of manufacturing partners for the project. The proposed approach consists of constructing an AHP comparison matrix with default pairwise evaluation of attributes using fuzzy numbers, reflective of a long term, static picture of vendor selection process in the specific manufacturing industry. The use of fuzzy numbers enables to account for the inherent subjectivity in pairwise comparison. The user-interface and emphasis sheets help acquire agile initiative specific needs in linguistic choice form. The information thus obtained is combined with the data stored in default comparison matrix, and synthesized by Fuzzy-AHP to redefine relative priorities. Explicit enumeration and branch-and-bound techniques are used to search groups of manufacturing partners to best satisfy the requirements of the new venture. The feasible groups of manufacturing partners are evaluated based on their cost, quality, leadtime, and other attributes commonly considered in vendor selection. Additionally, the flexible optimization framework assists through a graphical decision aid, human decision making while considering competing alternatives.

The benefits that can be derived from the above work can be summarized as follows:

1. The software developed (in C/C++ language) enables the decision maker to determine potential manufacturers/suppliers that are competent to form a virtual-enterprise, without having to express concerns (preferences) related to attribute priorities in numeric form. The decision maker (DM) expresses the attribute preferences in linguistic form and the flexible optimization framework developed, translates and synthesizes redefined attribute preferences through a user-friendly interface.

2. The DM is further relieved from the cumbersome task of altering pairwise comparisons in the judgment matrix. The proposed methodology provides a simple procedure to alter the judgment matrix based on the linguistic preferences captured from the DM through the user-interface.

3. Since there may be more than a single solution to the partner selection problem. The proposed methodology systematically quantifies the performance ('goodness') of these alternatives over a spectrum of weights that encompass DM's fuzzy concerns/preferences.

4. The treatment selects the best alternative based on a *composite score*. In addition, the software system presents other non-dominated groups of partners, that may be competent to form a virtual enterprise in a more constrained environment. For example, the best sequence of partners recommended by the software system may become infeasible due to special management concerns (such as patronage, previous contracts). The selection of alternative sequences of manufacturing partners is further enhanced by a graphic decision aid, exhibiting the performance distribution of non-dominated alternatives.

5. Lastly, the software system offers flexibility to the decision maker to organize the selection procedure. Provisions are included for manual weight inputs, bypassing processing modules,

and extending the selection procedure.

There are several other aspects of vendor selection process that are open for exploration. The future research efforts may include expanding the attribute database. This will result in a comprehensive list of attributes that may become significant in certain scenarios. Examples of this could include length of contract, geographic zoning of vendors, quantity discounts, etc. The simplistic lower-bound procedure in our implementation of branch-and-bound can be improved, and other heuristic search techniques can be developed for large problems. We hope that this flexible optimization framework can also be applied to other combinatorial problems which involve subjective criteria that may change emphases repeatedly.

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Table 1: Triplets defining membership functions of triangular fuzzy numbers

Fuzzy number	Membership function $\mu_{\tilde{A}}(x)$
$\tilde{1}$	$(1, 1, 3)$
$\tilde{x}$	$(x-2, x, x+2)$, where x = 3, 5, 7, 9

Table 2: Process and plant options

Feature (f^k)	Process options (j^r)	Plant options (i^r)
f^1	Forging	9, 3, 8
	Injection Molding	4, 5
f^2	Injection Moulding	4, 3
	Forging	7, 3
f^3	Milling	1, 4
	Injection Molding	4
f^4	Turning	7, 8
f^5	Injection Molding	4, 5
	Turning	8, 1
	Milling	1, 5
f^6	Turning	1, 7
	Milling	1, 4
f^7	Grinding	6, 4

Table 3: Total arrangement of weight vectors

$W_j^{(\alpha,0.5)}$	ω	β	γ	δ
$W_j^{(0.1,0.5)}$	0.408	0.131	0.258	0.203
$W_j^{(0.2,0.5)}$	0.401	0.137	0.251	0.212
$W_j^{(0.3,0.5)}$	0.407	0.134	0.253	0.206
$W_j^{(0.4,0.5)}$	0.413	0.131	0.256	0.200
$W_j^{(0.5,0.5)}$	0.418	0.129	0.258	0.195
$W_j^{(0.6,0.5)}$	0.450	0.120	0.250	0.180
$W_j^{(0.7,0.5)}$	0.474	0.114	0.244	0.168
$W_j^{(0.8,0.5)}$	0.495	0.108	0.239	0.158
$W_j^{(0.9,0.5)}$	0.512	0.103	0.237	0.149
$W_j^{(1.0,0.5)}$	0.526	0.099	0.235	0.140
$\mathrm{W}^{0.5}_{composite\ j}$	0.473	0.114	0.244	0.169

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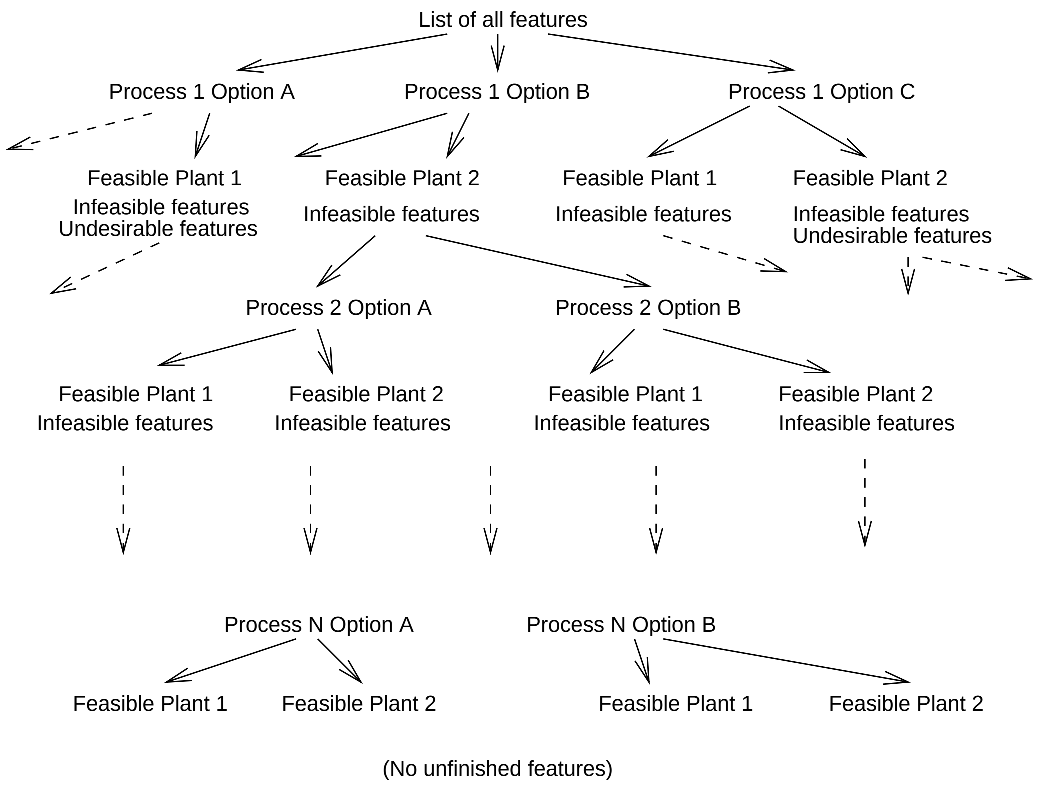


Figure 1: Search Tree

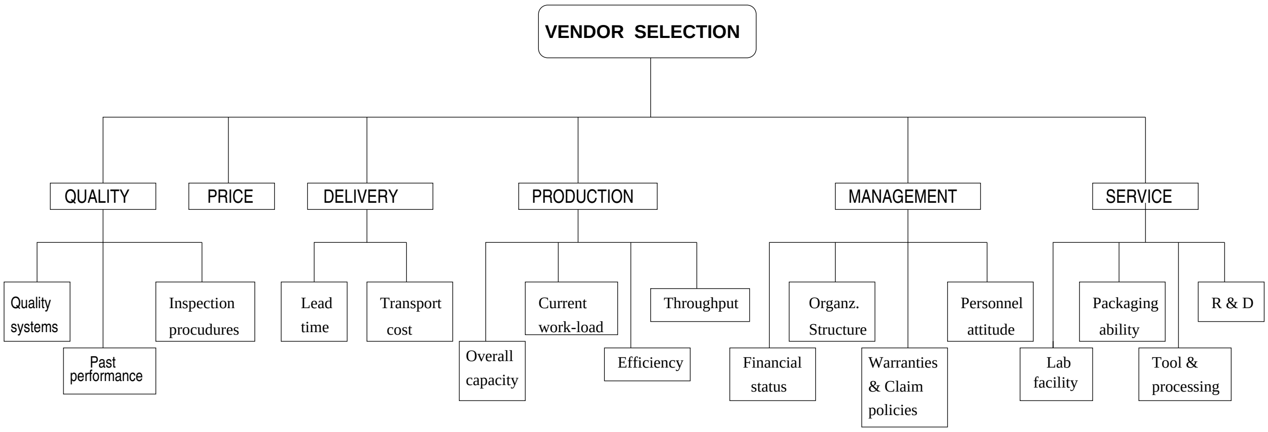


Figure 2: Vendor Selection Attribute Hierarchy

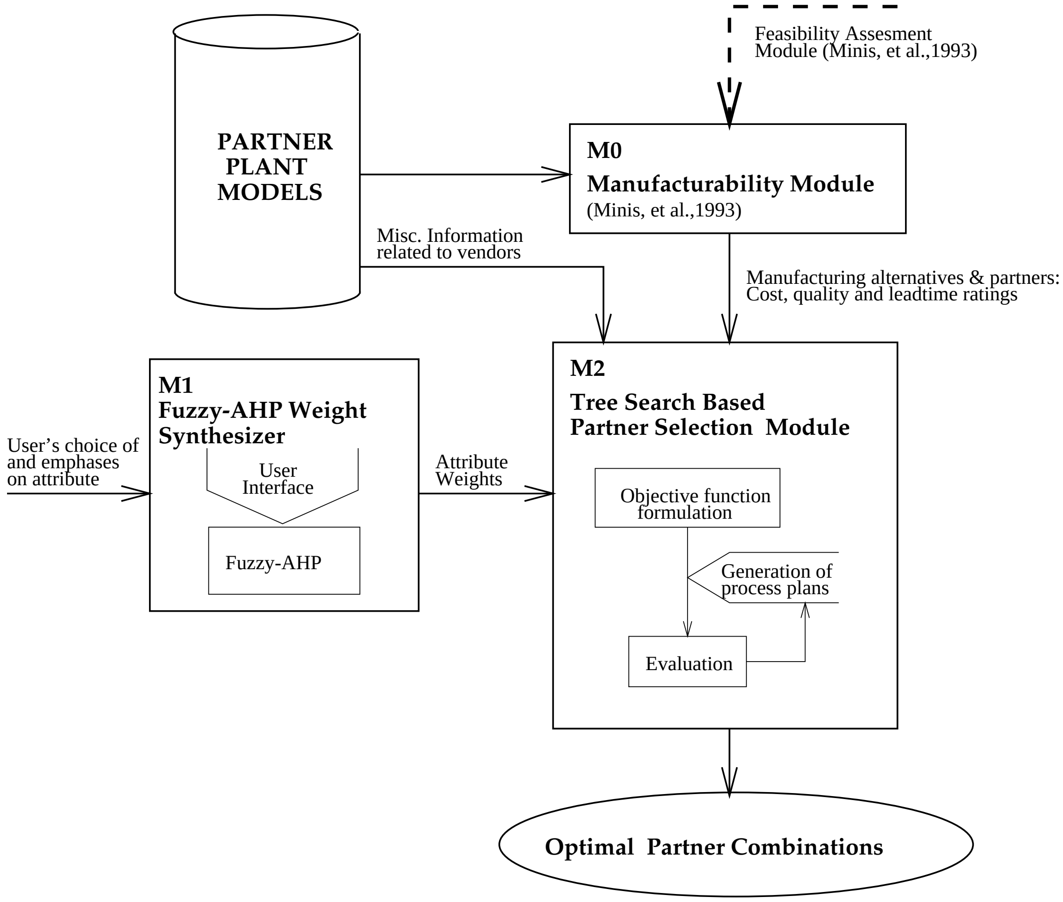


Figure 3: Flexible optimal partner selection framework

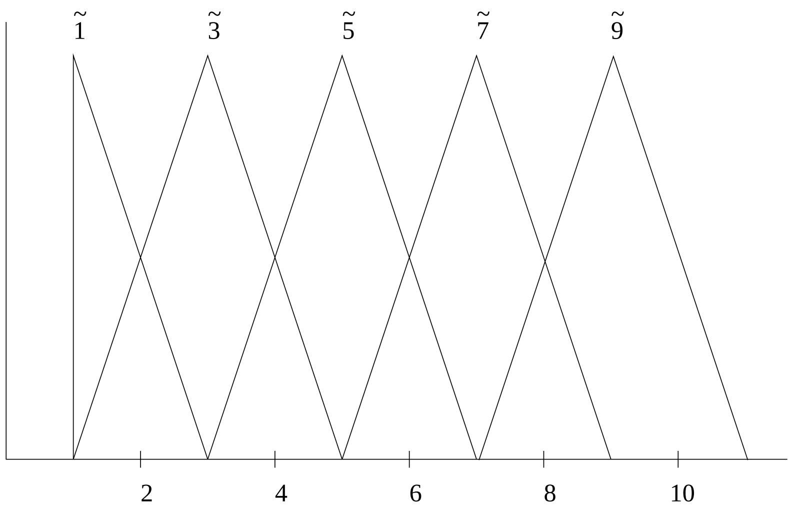


Figure 4: Triangular fuzzy numbers

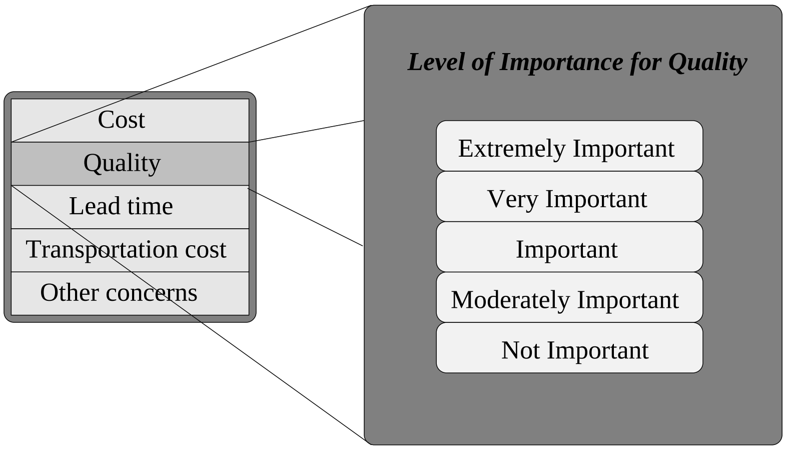


Figure 5: Emphasis sheet

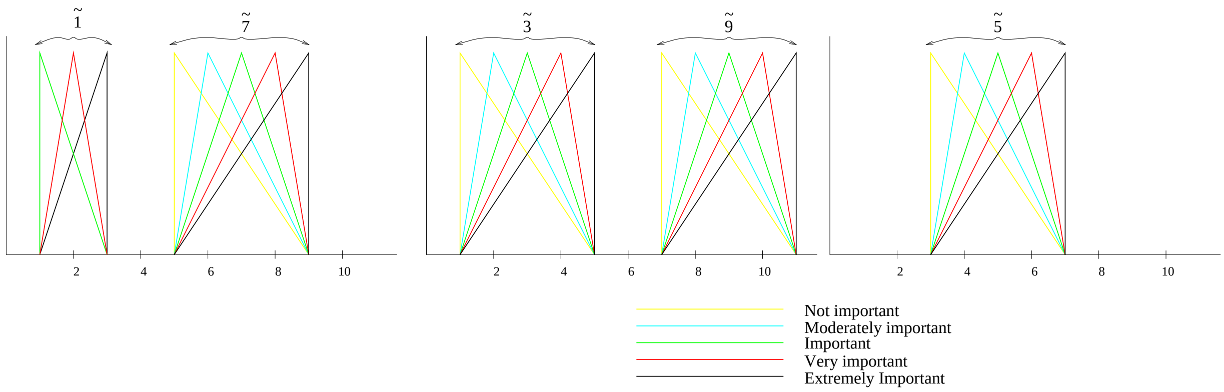
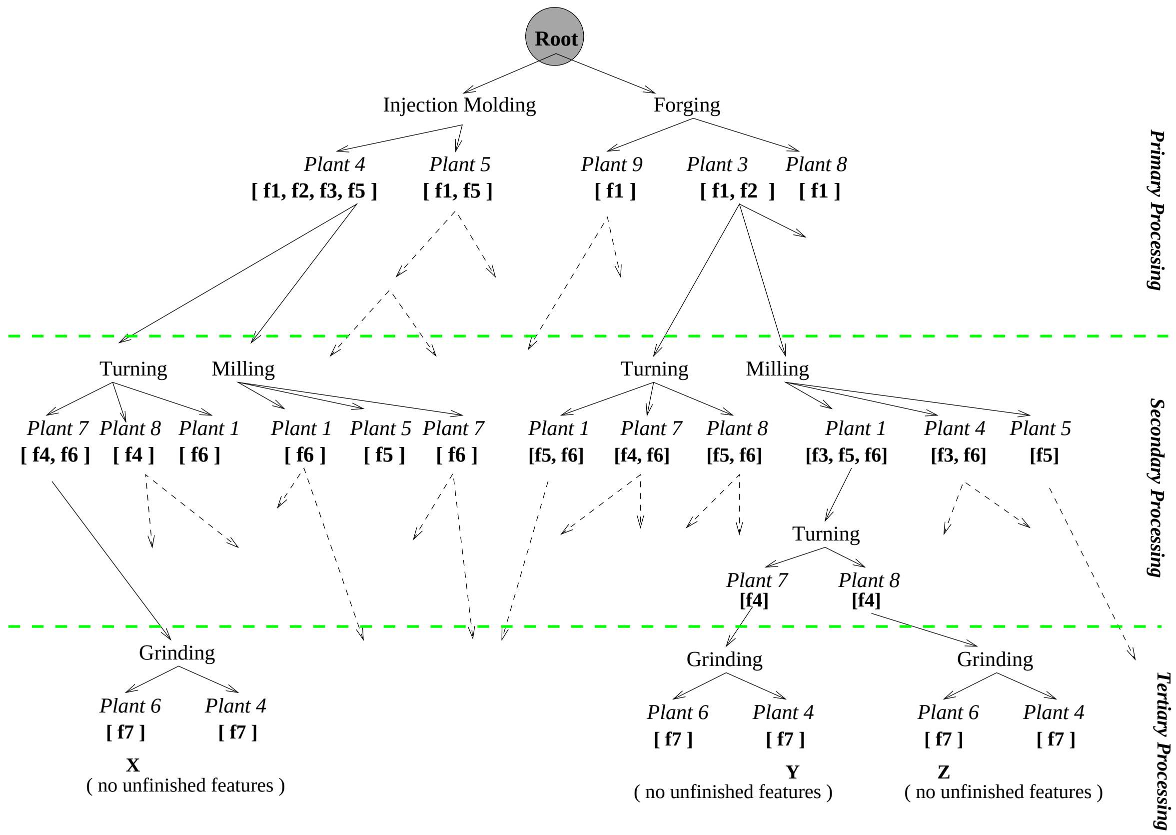


Figure 6: Modification scheme for fuzzy numbers



Feasible Sequences of Manufacturing Partners:

Sequence X P1(Injection Molding, Plant 4, [f1, f2, f3, f5]) - P2(Turning, Plant 7, [f4, f6]) - P3(Grinding, Plant 6, [f7])
Sequence Y P1(Forging, Plant 3, [f1, f2]) - P2(Milling, Plant 1, [f3, f5, f6]) - P3(Turning, Plant 7, [f4]) - P4(Grinding, Plant 4, [f7])
Sequence Z P1(Forging, Plant 3, [f1, f2]) - P2(Milling, Plant 1, [f3, f5, f6]) - P3(Turning, Plant 8, [f4]) - P4(Grinding, Plant 6, [f7])

Figure 7: Solution space

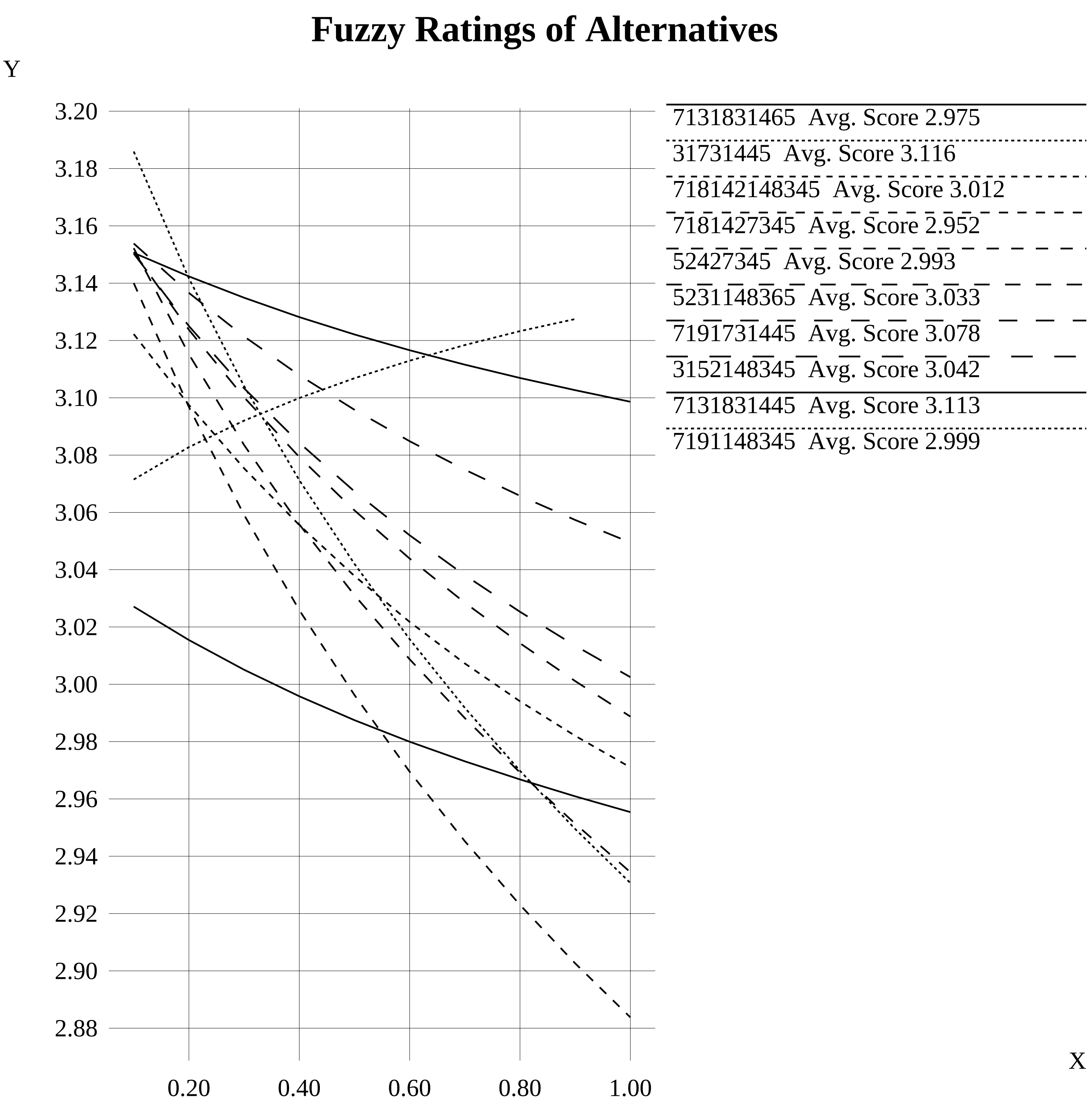


Figure 8: Fuzzy Ratings of Partner Sequences (Application 1)

Fuzzy Ratings of Alternatives

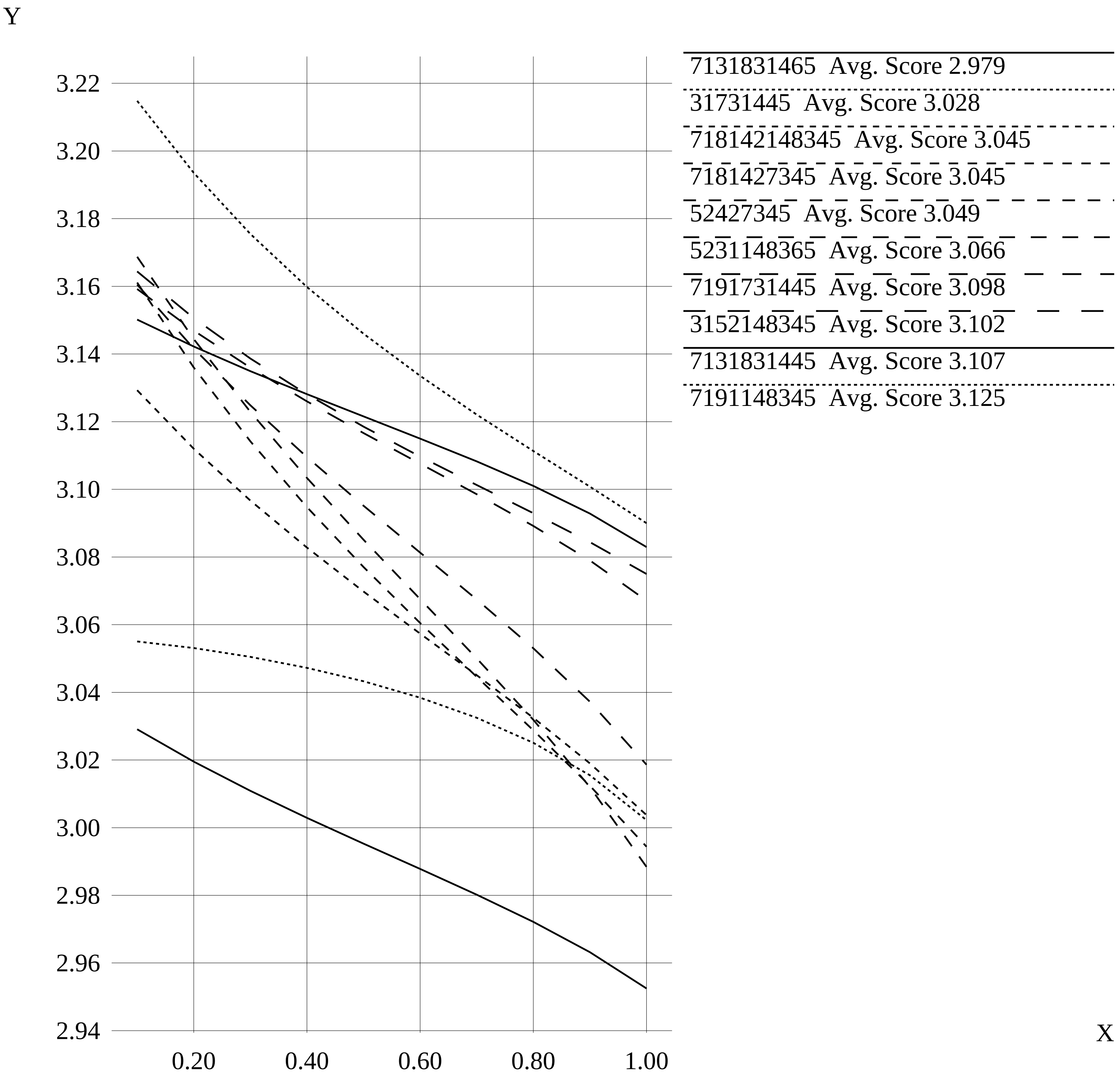


Figure 9: Fuzzy Ratings of Partner Sequences (Application 2)