Placing a Finite Size Facility with a Center Objective on a Rectangular Plane with Barriers

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Abstract

This paper addresses the finite size 1-center placement problem on a rectangular plane in the presence of barriers. Barriers are regions in which both facility location and travel through are prohibited. The feasible region for facility placement is subdivided into cells along the lines of Larson and Sadiq (1983). To overcome complications induced by the center (minimax) objective, we analyze the resultant cells based on the cell corners. We study the problem when the facility orientation is known *a priori*. We obtain domination results when the facility is fully contained inside $1, 2$ and 3-cornered cells. For full containment in a 4-cornered cell, we formulate the problem as a linear program. However, when the facility intersects gridlines, analytical representation of the distance functions becomes challenging. We study the difficulties of this case and formulate our problem as a linear or nonlinear program, depending on whether the feasible region is convex or nonconvex. An analysis of the solution complexity is presented along with an illustrative numerical example.

Keywords: 1-center placement, finite size facility location, barrier, rectangular plane

1   Introduction

Location problems which impose restrictions on locating new facilities and/or travel through are typically referred to as *constrained* or *restricted* location problems. Such problems have the following two topographical properties. (1) The new facilities cannot be located within certain predescribed restricted areas in the plane. (2) It is not always necessary that any two points in the plane would be "simply communicating," i.e., the minimum travel distance between any two points in the plane may be made longer by the presence of the restricted regions.

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In real world location problems, situations are often encountered in which regions neither allow facility location nor travel through. Such regions are defined as **barriers** in the facility location literature. Examples of barriers would be impassable areas on the shop floor like machines, subassembly areas, input-output docks, etc. The available literature on location problems in the presence of barriers can be classified according to the following criteria:

- shape of barriers (arbitrary, polygonal, circular, rectangular, etc.)

- size of new facility (infinitesimal or finite size)

- travel metric used (Euclidean, rectangular, etc.)

- type of objective (median or center)

In one the earliest facility location papers dealing with barriers, Katz and Cooper [8] investigated a problem with a circular barrier considering the median objective and the Euclidean distance metric. Larson and Li [9] developed an efficient algorithm for determining the shortest feasible rectangular path between two points in the presence of polygonal barriers. Discretization results for the p-median problem in the presence of arbitrarily shaped barriers under the rectangular distance metric were obtained by Larson and Sadiq [10]. The authors introduced a grid construction procedure that splits the feasible region into cells. By converting the original problem into an equivalent p-median problem on a network, the authors proved that an optimal set of facility locations can be chosen based solely on the grid points, i.e., the nodes (barrier vertices and/or demand points) and the points of intersection of any two node traversal lines.

Batta, Ghose and Palekar [2] extended the results of [10] to include regions that prohibited facility location but allowed travel through. Aneja and Parlar [1] and Butt and Cavalier [3] developed heuristics for the 1-median problem in the presence of polygonal barriers under the l_p distance metric. Though the center problem in $\Re^2$ without barriers has been extensively studied in the literature (e.g., books of Drezner [5], Love *et al.* [11] and Francis *et al.* [6]), very few references can be obtained for the corresponding problem in the presence of barriers. Minimax locations problems on a network have been studied in the book by Handler and Mirchandani [7]. p-center problems on a network have been extensively studied in the book by Mirchandani and Francis [12].

Nandikonda, Batta and Nagi [14] address the 1-center problem with arbitrarily shaped barriers under the rectangular distance metric. The authors divide the feasible region into cells as outlined by [10]. To overcome complications due to the center objective, they introduce a new concept to classify cells based on their cell corners. A solution procedure has been developed for each class of cells. The overall complexity of the solution procedure is shown to be polynomially bounded.

Dearing, Hamacher and Klamroth [4] obtain dominating set results for the 1-center location problem in the presence of convex polygonal barriers under the rectangular distance metric. The feasible region is decomposed into cells. A bisector algorithm, that finds bisectors by adapting an algorithm of Mitchell [13], outputs the set of dominated points. Based on the domination results, the authors develop a polynomial (in the number of demand points and number of extreme points of the polygonal barriers) time algorithm to solve the resulting nonconvex optimization problem. Their work differs from the work of Nandikonda *et al.* [14] in its solution approach as well as the shape of barriers.

In all the above mentioned literature, the authors have assumed the new facility to be located as infinitesimal in size. However such an assumption is not always valid. Savas, Batta and Nagi [16] first considered the finite size "placement" problem of an arbitrarily shaped facility in the presence of arbitrary shaped barriers with the median objective and rectangular distance metric. By studying the behavior of the objective function, the authors identify the candidate(s) for optimal placement of a new facility with a fixed orientation and then for the same facility with a fixed server location. They also present a heuristic for the case when both facility orientation and server location are not known *a priori*. Our work differs from the work of Savas *et al.* [16] with respect to the nature of the objective. Also Nandikonda *et al.* [14] consider the same objective with an infinitesimal new facility whereas our work addresses the finite size facility placement problem.

In this paper, we consider the problem of "placing" a facility in the presence of barriers. This work can be practically applied to problems in layout analysis, where the objective, in many cases is to place a new department in the presence of existing departments (or to place a new machine in the presence of existing machines on the shop floor). An existing machine with its adjoining area comprising of space for associated equipment, input buffer, output buffer, scrap area can be considered to be a barrier.

The remainder of this paper is organized as follows. In §2, we describe and define our problem. In §3, we briefly revisit the grid construction procedure of [10] and the concept of "Equal Travel Time Lines" ($ETTLs$), as established in [2]. We then study and establish some new properties of $ETTLs$ in §4. §5 considers the facility placement problem with a given orientation; this section is split into several sub-sections, each dealing with a different case of the problem. We analyze the complexity of our solution procedure in §6 for the fixed orientation problem. We present an example problem and its detailed analysis in §7 to elucidate our solution methodology. Conclusions and directions for future research are outlined in §8.

2 Problem Description

Let there exist a finite number of **barriers** where travel through and facility placement are both prohibited. The existing **users** are distributed over a finite set of demand points located anywhere in the plane outside of or on the boundary of the barriers. A new facility is to be placed. The new facility communicates with the existing users through a **single server** located on its boundary. The 1-**center facility placement problem** is to determine the optimal placement of the facility such that the facility does not overlap with any of the existing barriers, and the maximum rectangular travel distance (circumventing barriers) from the server to any user is thereby minimized.

2.1 Definitions and Notations

We assume that each barrier is a closed and bounded area in $\Re^2$, with finite area. Let B_j (an open set) denote the set of points $(x, y) \in \Re^2$ contained strictly within barrier *j*. We also define $\overline{B_j} = B_j \cup \{boundary\ of\ barrier\ j\}$, to be a closed set. Each set $\overline{B_j}$ is called a barrier. Each barrier has a finite number of horizontal and vertical tangential lines. We let $B = \cup_j B_j$ and $\overline{B} = \cup_j \overline{B_j}$. Let H (an open set) denote the set of points contained strictly within the new facility that is to be placed and let $\overline{H} = H \cup \{boundary\ of\ the\ facility\}$. We note here that $B_i \cap B_j = \emptyset, i \neq j$ and $B_j \cap H = \emptyset$. The distinction between the inside and the boundary of a barrier/facility is necessary to allow travel on the boundary of the barrier/facility but not inside. Let $E(\overline{B})$ define the smallest rectangle (bounding rectangle) that encloses all barriers and users and whose sides are parallel to the x and y axes.

For a finite size facility, the coordinates of a single point cannot convey full information about the placement of the facility in $\Re^2$. Hence, we let $l = [X, \alpha]$ denote the *placement vector* for the facility. Here, $X = (x, y)$ represents the location, i.e., point coordinates for the server. The angle $0 \leq \alpha < 2\pi$ between the $+x$-axis and the line joining the server location and a predetermined point P on the facility (measured in the anti-clockwise direction) specifies the orientation of the facility. Figure 1 shows the placement vector. In summary, X is the location and α is the orientation of the facility. Together they determine the placement of the facility.

We will now define the feasible region for the finite-size facility placement problem. Let $\overline{H}(X, \alpha)$ be the set of points that correspond to the facility when the server is at X and has an orientation α. $\overline{H}(X, \alpha)$ is a closed set. The feasible region is defined as follows:

$$F = \{[X, \alpha] : H(X, \alpha) \cap B = \emptyset\}.$$

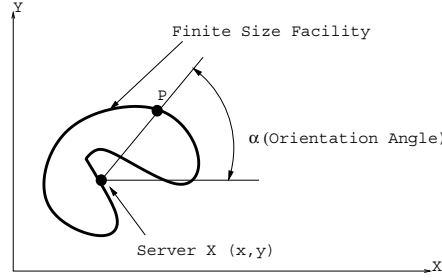We note here that F may be composed of mutually disjoint sets.

Figure 1: A Finite-Size Facility, its Location and Orientation

2.2 Problem Definition

There is interaction between users and the server. The extent of interaction between user i and the server is denoted by $w_i \geq 0$. The interaction between any user and server takes place through a shortest feasible rectangular distance path that penetrates no barriers or facilities. Let $d_l(i, X)$ represent the length of such a shortest rectangular path between the user i and server at X when the facility placement is l. The subscript l signifies that the distance is a function of the facility placement.

Let D denote the set of all users. For a given facility placement $l = [X, \alpha]$, we define the function $J(l)$ as follows:

$$J(l) = \max_{i \in D} \ \{w_i d_l(i, X) + \gamma_i\},$$

where $w_i d_l(i, X)$ is the weighted travel distance between user i and the server X of the new facility and γ_i is the constant addendum associated with user i.

Then the weighted 1-center problem with arbitrarily shaped barriers can be denoted as follows:

$$\min_{l \in F} J(l).$$

3 Background

3.1 Grid Construction and Cell Formation

A grid construction procedure in the presence of barriers that divides the feasible region F into cells has been established in the work of Larson and Sadiq [10]. The same procedure is followed here. Let $Q(\overline{B_j})$ denote the points of tangency of barrier $\overline{B_j}$, i.e., points on the boundary of $\overline{B_j}$ through which a horizontal and/or a vertical line can be passed and for which all points on the boundary sufficiently close to this point lie in or either on one side of the line or the other. The grid construction procedure is outlined as follows:

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(i) Draw lines parallel to the x and y axes through all barrier vertices and the users, with each line terminated at the first barrier interior encountered (i.e., point in B), else at $E(\overline{B})$, as illustrated in Figure 2. (ii) Exclude from the set of lines in (i) any line extending from a barrier vertex that is not a user and where the vertex is the endpoint of the line. An example of the grid construction procedure is illustrated in Figure 2. For further details of the grid construction procedure, we refer the interested reader to [10].
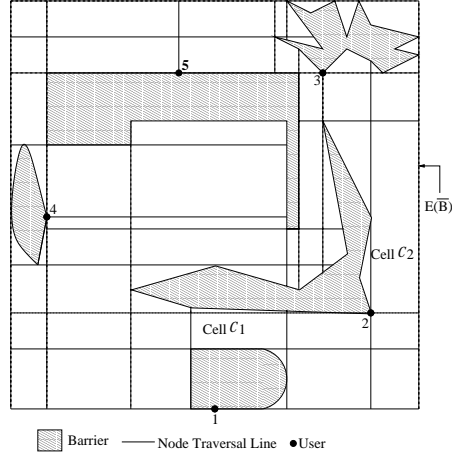

Figure 2: Example of the Grid Construction Procedure

The resulting set of lines are called "node traversal lines" in [10]. For simplicity in presentation, let us refer to these lines as gridlines. We let L_h denote the set of horizontal gridlines and L_v denote the set of vertical gridlines. We define $L = L_h \cup L_v$ as the set of all gridlines. The barriers and L divide F into a number of **cells**, as illustrated in Figure 2. Each cell boundary is composed of solely gridlines or segments of barrier boundaries and gridlines. For a given cell $\mathcal{C}$, let us consider the points (x_{min}, y_{min}), (x_{max}, y_{min}), (x_{max}, y_{max}), (x_{min}, y_{max}), where $x_{min}, y_{min}, x_{max}, y_{max}$ are the respective bounds on x and y in the cell. Clearly, at least one of the four points (x_{min}, y_{min}), (x_{max}, y_{min}), (x_{max}, y_{max}), or (x_{min}, y_{max}) is contained in $\mathcal{C}$. All such points contained in $\mathcal{C}$, up to a maximum of four, are henceforth referred to as *cell corners* of $\mathcal{C}$. We will later utilize the following results, as proved in [10] and [16]:

Result 1 [10]: *A shortest feasible rectangular path from a user to an infinitesimal point located in a cell $\mathcal{C}$ passes through a cell corner of $\mathcal{C}$.*

Result 2 [10]: *A shortest path in $\Re^2$ between any (x_a, y_a) and (x_b, y_b) can be found by restricting travel to nodal paths, i.e., paths containing a sequence of nodes $a - n_1 - n_2 - \cdots - n_k - b$ where $(a, n_1), (n_1, n_2), \ldots, (n_k, b)$ are pairs of simply communicating nodes (two nodes communicate simply if the presence of barriers causes no net increase in the travel distance between them).*

Result 3 [16]: *A nodal path between two points (x_a, y_a) and (x_b, y_b) can be represented as a*

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"traversal line path" $P(a, b)$, i.e., a path that comes in contact with a sequence of horizontal and vertical gridlines, $h_1, h_2, \ldots, h_m$ and $v_1, v_2, \ldots, v_n$.

Cells that do not share a common boundary with a barrier $\overline{B}$ are rectangular and have four cell corners. Cells that share their boundaries with a barrier may not be rectangular and hence may have less than four cell corners. Let the subset ζ^p denote all cells with p cell corners. Based on the previous discussion, we denote by ζ a set of all cells, then

$$\zeta = \zeta^1 \cup \zeta^2 \cup \zeta^3 \cup \zeta^4.$$

3.2 Equal Travel Time Lines and Subcells

For a fixed location $X \in \mathcal{C}$, we can meaningfully talk of the assignment of users to cell corners of $\mathcal{C}$. If the assignment of users to cell corners does not change upon moving the location X in $\mathcal{C}$, the distance functions are linear and monotonous over the cell $\mathcal{C}$. However, any change in the assignment of users to cell corners necessitates the construction of "Equal Travel Time Lines" ($ETTL$). The concept of $ETTL$s was introduced in the work of Batta *et al.* [2] and subsequently utilized in the work of Nandikonda *et al.* [14]. Details regarding construction of $ETTL$s can be found in [2] and [14].
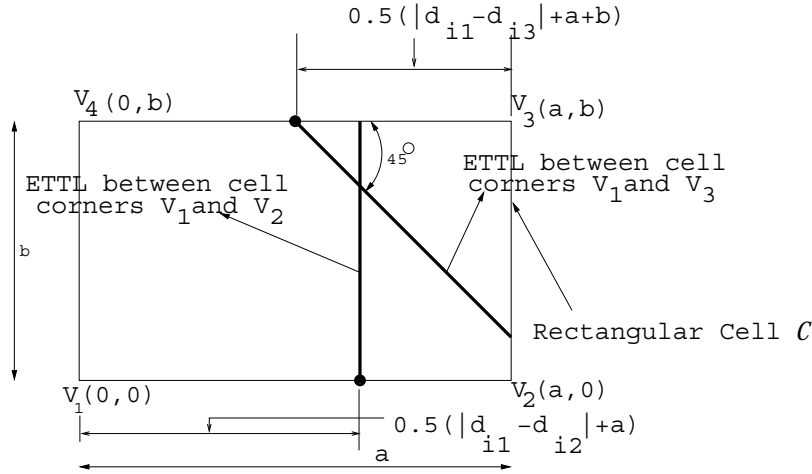

Figure 3: Example of an ETTLs in a Rectangular Cell

Let us consider a cell $\mathcal{C}$ with corners V_k, $k = 1, 2, 3, 4$ as shown in Figure 3. For the sake of simplicity and without loss of generality, let the corners V_1, V_2, V_3, V_4 have coordinates $(0, 0)$, $(a, 0)$, (a, b), $(0, b)$ respectively. Let the shortest rectangular distance from user i to cell corner V_k be denoted by d_{ik} for $k = 1, 2, 3, 4$. If $|d_{i1} - d_{i2}| < a$, where a is the length of the edge $\overline{V_1 V_2}$, then an $ETTL$ with respect to i between corners V_1 and V_2 is generated such that it is perpendicular to the edge $\overline{V_1 V_2}$ at a distance $\frac{1}{2}\{|d_{i1} - d_{i2}| + a\}$ from the cell corner that is closest to user i. The cases

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between other adjacent cell corners can be dealt with similarly. If $|d_{i1} - d_{i3}| < a + b$, where b is the length of the edge $\overline{V_2 V_3}$, then an $ETTL$ with respect to i between corners V_1 and V_3 is generated that touches the cell boundary at a distance $\frac{1}{2}\{|d_{i1} - d_{i3}| + a + b\}$ from the cell corner that is closest to user i. It is pertinent to mention here that $ETTL$s generated due to diagonally opposite cell corners are inclined at $45°$ to the edges of a 4-cornered cell, e.g., the $ETTL$ generated due to V_1 and V_3 makes an angle of $45°$ with the edges $\overline{V_2 V_3}$ and $\overline{V_3 V_4}$. After the construction of $ETTL$s, which are induced on a cell $\mathcal{C}$ by user i, has been completed, the same procedure is repeated for all other users $k, k \neq i$. $ETTL$s partition a cell $\mathcal{C}$ into subcells SC. All $ETTL$s generated by the users in Figure 2 are illustrated in Figure 4. For example, consider $ETTL$ AB in cell $\mathcal{C}$ in Figure 4. AB is an $ETTL$ in the following sense: if X is located at a point on AB, a shortest rectangular path between X and user 1 goes through either cell corner V_1 or V_2 (in other words, one has to cover equal distance (hence the name $ETTL$) to travel from user 1 to X, either through V_1 or V_2). Clearly $ETTL$ AB helps to uniquely assign user 1 to cell corner V_1 or V_2 depending on the location of $X \in \mathcal{C}$.
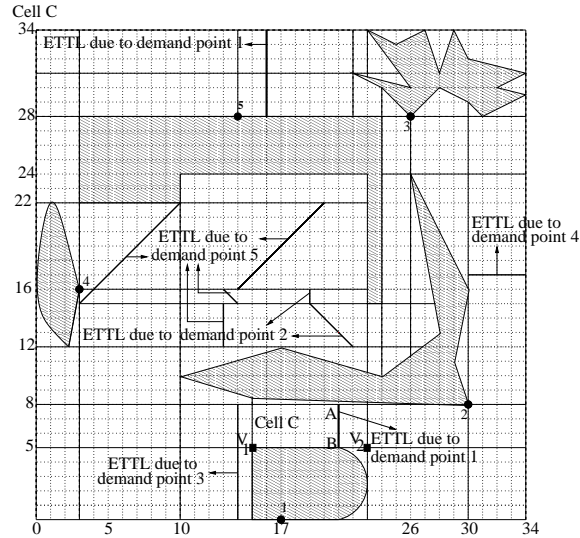
Figure 4: Cells and subcells

It is pertinent to mention here the following result proved in [14].

Result 4: *A demand point generates at most one $ETTL$ in a four-cornered cell.*

4 Additional Properties of $ETTL$s

4.1 Existing $ETTL$s may move

As discussed in §3.2, an $ETTL$ may be generated between any pair of cell corners of a 4-cornered cell. Due to the finite size of the new facility, an existing $ETTL$ may move. In fact, the following are now possible.

1. Due to its finite size and shape, the length of the shortest rectangular path between a user i and the new facility's server X, $d(i, X)$, may increase. Consider the $ETTL$ generated in the cell $\mathcal{C}$ between adjacent cell corners V_1 and V_2, as illustrated in Figure 5a. Due to placement of the new facility as illustrated in Figure 5b, $d(i, X)$ increases by $2p$ (note that user i is uniquely assigned to cell corner V_1). Hence the $ETTL$ moves closer to V_1 by p. We conclude that the position of an existing $ETTL$ may thus move when a finite sized facility is fully contained in a cell. Similar analysis can be performed for $ETTL$s between other pairs of adjacent as well as opposing cell corners.
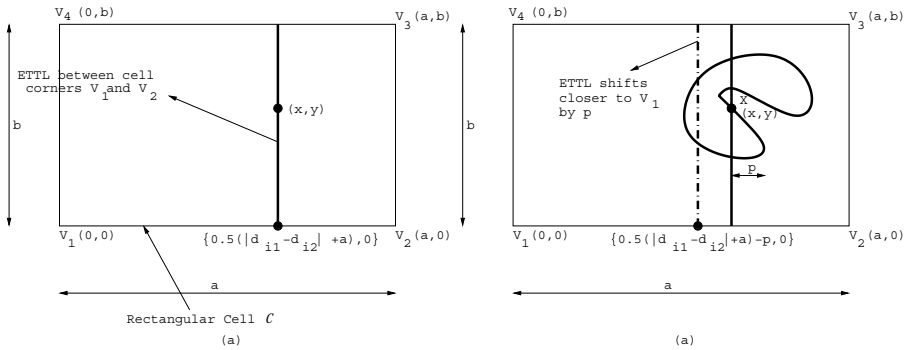

Figure 5: Movement of $ETTL$s

2. The finite sized new facility may block cell corners of cell $\mathcal{C}$ through which a user i communicates with $X \in \mathcal{C}$. Consequently $d(i, X)$ may increase and an existing $ETTL$ in $\mathcal{C}$ may move. However unlike the previous case 1, the new facility now intersects gridlines. The amount of movement of the $ETTL$ is a function of the increment of $d(i, X)$. We refer the interested reader for more details to [15].

Our solution methodology proposed later is based on a cell decomposition approach. When the new finite size facility is not fully contained in a cell, i.e., it intersects gridlines (§5.2.2), the shortest rectangular path between users and the server of the new facility is affected. Consequently the unique assignment of users to cell corners may change. Accurate distance measurements between users and the server is central to our solution methodology. Hence it is crucial to consider the movement of $ETTL$s. Note that this complexity is not encountered in [14] due to the infinitesimal size of the new facility.

4.2 $ETTL$ **intersections in a cell**

We conclude this section with the following lemma, which helps to simplify our analysis.

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Lemma 4.2.1. *ETTLs between both pairs of opposing cell corners in a four-cornered cell cannot exist.*

Proof : Consider a 4-cornered cell $\mathcal{C}$ with cell corners V_1, V_2, V_3, V_4. Assume that an $ETTL$, labeled $ETTL1$ exists between V_1 and V_3 due to user 1 associated with barrier B_1, as illustrated in Figure 6a. This happens when V_2 is not rectangularly visible to 1. (Otherwise an $ETTL$ would not exist between V_1 and V_3 and an $ETTL$ could exist between V_2 and V_3 or V_1 and V_2.) However V_1 and V_3 may be rectangularly visible to 1 if the cell is formed due to barrier traversal lines associated with B_1, as illustrated in Figure 6b.
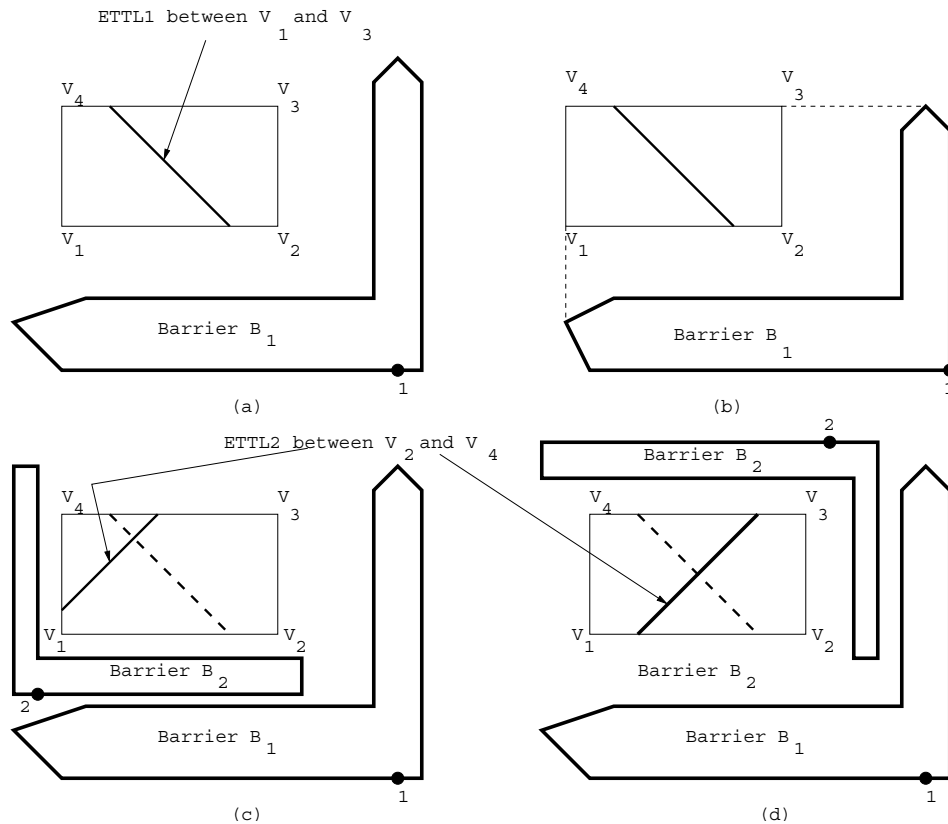

Figure 6: Figure for Lemma $4.2.1$

Let us also assume that user 2, associated with barrier B_2 generates an $ETTL$, labeled $ETTL2$ in C between V_2 and V_4, as shown in Figure 6c and Figure 6d. Depending on the placement of B_2, V_1 and V_3 (as in Figures 6c and 6d) will not be rectangularly visible to 2. In either case, a shortest rectangular path from server location 1 to V_1 (in Figure 6c) and to V_3 (in Figure 6d) must pass through cell corner V_2. Hence an $ETTL$ would exist between (adjacent cell corners) V_2 and V_3 in the case depicted in Figure 6c and between (adjacent cell corners) V_1 and V_2 in the case depicted Figure 6d, not between (opposite) cell corners V_1 and V_3. This contradicts our first assumption. The lemma follows. ■

Lemma 4.2.1 helps to restrict the number of subcells generated, as evident later in solution complexity analysis in §6. Based on the previous background, we now move on to the actual solution methodology of our problem.

5   Solution Procedure

5.1   Introduction

Before a new facility is placed, the gridlines that are drawn due to the existing barriers and users were adequate to provide the framework that is required to obtain the minimum feasible rectangular distance path between any two users. Placement of a new facility alters the existing framework. Specifically, two things happen. Firstly, the facility itself acts as a barrier to travel. Hence a new set of traversal lines parallel to the coordinate axes must be passed through the vertices of the facility and the server on the facility as outlined in [10]. [16] has referred to these lines as *facility traversal lines*. Secondly, some existing gridlines will be terminated as they will encounter the interior of the new facility. As a result of placing a new facility, a new grid structure will be obtained.

Savas *et al.* [16] consider the 1-median problem of placing a finite-sized facility. The authors take advantage of the concave property of the distance functions and the nature of the objective function which is a positive linear combination of concave functions. However the objective (*min-imax*) function of our problem does not help in deducing any such property. Hence it becomes imperative in our problem to represent the distance functions analytically, as shown in later sections. This becomes challenging especially because of the arbitrary shape of the barriers and the new facility to be placed. Some concepts of Nandikonda *et al.* [14] are useful here as their work deals with location of an infinitesimal facility under the center objective. However since our problem addresses the finite size facility placement problem, we cannot directly extend the results of [14]. Analogous to [14], we split up our analysis into different cases. Unlike [14] due to the finite size of the facility to be placed, we have to identify a feasible region of location in each case. Also, in our problem, if the new facility to be located moves, the assignment of users to cell corners may change (as described in §4).

To facilitate our analysis, we conclude this section by defining the smallest enclosing rectangle of the new facility as $E(\overline{H})$. Similar to what we have previously defined for a cell, let the corners of $E(\overline{H})$ be denoted as H_k, $k = 1, 2, 3, 4$. We note that as the facility orientation remains fixed, so does $E(\overline{H})$.

5.2 Optimal Placement Candidates for the Facility with Fixed Orientation

In this section, we deal with the placement of a facility with known orientation α_0. We split up our analysis into two cases:

- when the facility placement does not cut off any element of the set L

- when the facility placement cuts off at least one element of the set L

5.2.1 The facility placement does not cut off any element of L

We note at the outset that we are concerned with the location of the server such that the new facility does not cut off any element of the set L, i.e., the facility is fully contained within a cell $\mathcal{C}$. This means that the set of feasible locations for the server $\mathcal{F} \subseteq \mathcal{C}$. $\mathcal{F} \subseteq \mathcal{C}$ can be obtained by moving the facility inside the cell such that it does not cut off any element of L. Based on the previous background, we now consider the following cases:

Case 1: The facility is fully contained in a 1-cornered cell

A 1-cornered cell $\mathcal{C}$ can be generated in four ways by choosing any one of the four possible cell corners (x_{min}, y_{min}), (x_{max}, y_{min}), (x_{max}, y_{max}), (x_{min}, y_{max}). Let us consider the new facility to be contained fully inside $\mathcal{C}$ with cell corner V_1, as illustrated in Figure 7. The following lemma establishes the fact that in such a scenario, the facility can be moved inside the cell thereby bringing the server closer to V_1.

Lemma 5.2.1. *Consider the new facility to be fully contained in a 1-cornered cell. Its optimal location is either (i) a unique point lying inside the cell, or (ii) on the vertical or horizontal tangent passing through the only cell corner, or (iii) the cell corner itself.*
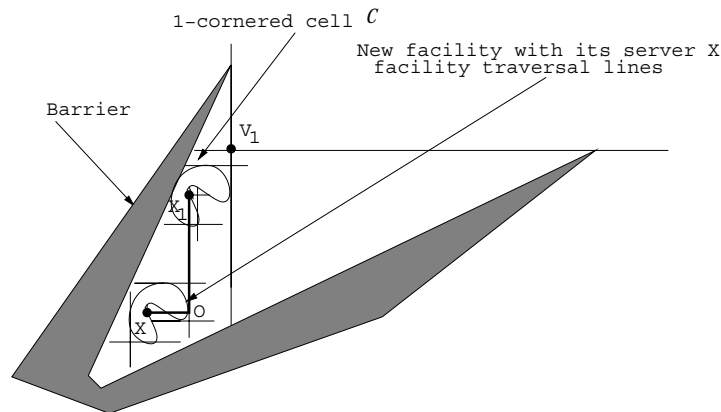

Figure 7: A 1-cornered cell (used in the proof of Lemma 5.1.1)

Proof: Let X be the optimal server location such that it is different from locations (i), (ii), and (iii) in the lemma statement. If the facility is moved from X to X_1 along the rectangular path XOX_1 (such that $\angle XOX_1 = 90°$ as illustrated in Figure 7) until at least one facility traversal line coincides with the tangent (horizontal or vertical depending on the shape of the 1-cornered cell) passing through V_1, the distance between the new server location X_1 and all users necessarily decreases. This follows from Result 1 [10] mentioned earlier and hence a contradiction is established. The optimal facility location (i) is thus obtained. The optimal location mentioned in (ii) may be obtained if X lies on $E(\overline{H})$. Location (iii) is a special case of locations (ii).

Since similar arguments can be made for any 1-cornered cell formed by choosing any one of the four points (x_{min}, y_{min}), (x_{max}, y_{min}), (x_{max}, y_{max}), (x_{min}, y_{max}), the lemma follows. ∎

Note that the optimal location of the new facility (for the fixed orientation problem) is given by the optimal location of X, its server. Clearly, the optimal server location for the case illustrated in Figure 7 is X_1. From Lemma 5.2.1, in a cell $\mathcal{C} \in \zeta^1$, the point X_1 dominates all other points. Hence it is sufficient to evaluate the 1-center objective function at X_1.

Case 2: The facility is fully contained in a 2-cornered cell

A 2-cornered cell $\mathcal{C}$ can be generated by choosing any two of the four possible cell corners (x_{min}, y_{min}), (x_{max}, y_{min}), (x_{max}, y_{max}), (x_{min}, y_{max}). This can be done in six ways. Figure 8a illustrates a 2-cornered cell with the cell corners $V_1(x_{min}, y_{min})$, $V_2(x_{max}, y_{min})$ while Figure 8b illustrates the case with $V_1(x_{max}, y_{min})$, $V_2(x_{min}, y_{max})$. We now state and prove the following lemma.

Lemma 5.2.2. *Consider the new facility to be fully contained in a 2-cornered cell. Its optimal location is such that at least one edge of the facility bounding rectangle $E(\overline{H})$ coincides fully or partially with one of the shortest rectangular paths (SRP) between the two cell corners.*

Proof: By definition, a cell corner must be either (x_{min}, y_{min}), (x_{max}, y_{min}), (x_{min}, y_{max}), or, (x_{max}, y_{max}). Hence six cases can be generated by choosing any two of the four aforementioned points as cell corners.

Let X be the optimal server location such that no edge of $E(\overline{H})$ coincides with the SRP between V_1 and V_2. However, as illustrated in Figure 8a (2-cornered cell with (x_{min}, y_{min}) and (x_{max}, y_{min}) as cell corners), we can move the facility along the perpendicular XX_1 until $E(\overline{H})$ coincides fully with the SRP between V_1 and V_2. Since the rectangular distance between X_1 and all users necessarily decreases, a contradiction is established.

For the case depicted in Figure 8b, the size of the facility $\overline{H}$ is such that $E(\overline{H}) \cap B_j \neq \emptyset$. However as demonstrated in Figure 8b, a SRP can always be drawn between V_1 and V_2 that partially coincides with edges of $E(\overline{H})$. The conditions of the lemma are thus met. Similar arguments hold for other possible configurations of a 2-cornered cell. The lemma follows. ∎
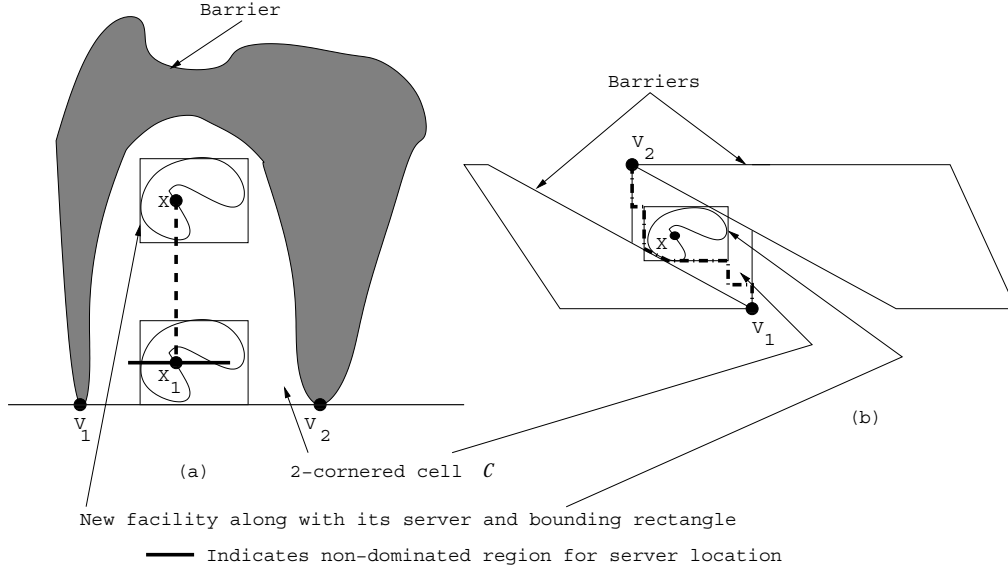
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Figure 8: A 2-cornered cell (used in the proof of Lemma 5.1.2)

The immediate implication of Lemma 5.2.2 is the following: for the case depicted in Figure 8a (i.e., when a 2-cornered cell is formed between adjacent cell corners), the "nondominated" region $\mathcal{D}$ is a line segment. Any point on this line $\mathcal{D}$ dominates any feasible $(x, y) \in \mathcal{C}$ such that $(x, y) \notin \mathcal{D}$. This line segment can be obtained by moving the facility inside the cell $\mathcal{C}$, such that $B_j \cap H = \emptyset$ and $E(\overline{H})$ coincides with the line segment joining V_1 and V_2. $\mathcal{D}$ is demonstrated in Figure 8 by a bold line.

For the case depicted in Figure 8b, (i.e., a 2-cornered cell with opposing cell corners), the server X can be moved along a line (joining X and some point on the facility's boundary) that makes an angle of $45°$ with the $+x$ axis without altering the minimum rectangular distance between a user i (constrained to pass through V_1 or V_2) and X. (The idea is similar to the case of 3-cornered cell and is discussed in detail in the following case.) The nondominated region $\mathcal{D}$ can be obtained by moving the facility inside $\mathcal{C}$ such that $B_j \cap H = \emptyset$ (in a similar manner as discussed next for 3-cornered cells).

Based on the previous background, we now proceed to determine the optimal server location. Clearly cell corners V_j and corners H_k of $E(\overline{H})$ are rectangularly visible for $j = k = 1, 2$. Hence the distance between user i and the facility server X can be expressed as follows:

$$d(i, X) = d(i, V_j) + d(V_j, H_j) + d(H_j, X),$$

where $d(i, V_j)$ and $d(H_j, X)$ are constants while $d(V_j, H_j)$ is a linear function of x and y.

The 1-center facility placement problem in a 2-cornered cell $\mathcal{C}$ is as follows:

$$\min_{X \in \mathcal{D}} \quad \max_{i \in D} \quad [w_i d(i, X) + \gamma_i].$$

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The 1-center facility placement problem can thus be solved by performing a line search over the nondominated segment $SP \in \mathcal{D}$.

Case 3: The facility is fully contained in a 3-cornered cell

Following the discussion in Case 2, four possible cases are generated when any three of the four cell corners (x_{min}, y_{min}), (x_{max}, y_{min}), (x_{max}, y_{max}), (x_{min}, y_{max}) are selected to represent a 3-cornered cell. Figure 9 shows a 3-cornered cell with the cell corners $V_1(x_{min}, y_{min})$, $V_2(x_{max}, y_{min})$ and $V_3(x_{min}, y_{max})$. We now state and prove the following lemma.

Lemma 5.2.3. *Consider the new facility to be fully contained in a 3-cornered cell. Its optimal location is such that at least one edge of the facility bounding rectangle $E(\overline{H})$ coincides with any edge of the cell that is not common to a barrier boundary.*
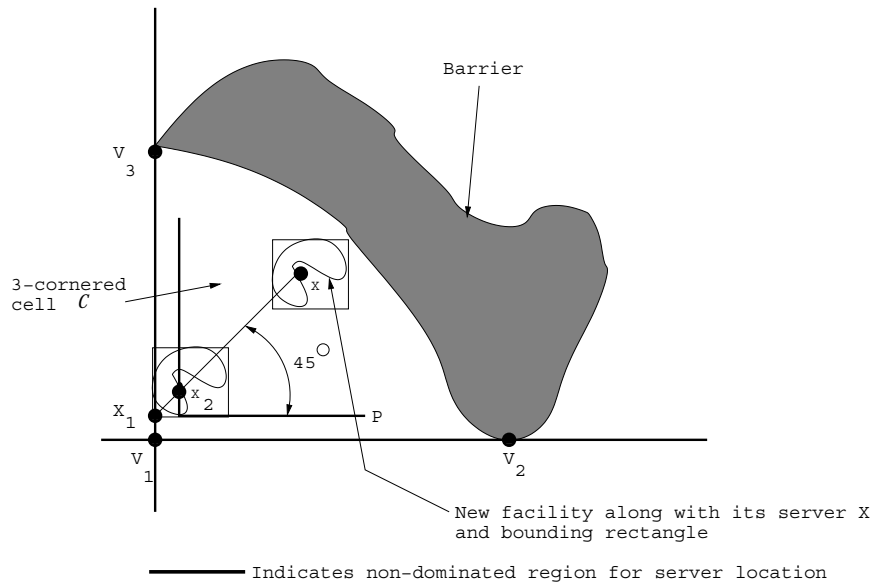

Figure 9: A 3-cornered cell (used in the proof of Lemma 5.1.3)

Proof: Let us consider the new facility to be contained fully inside a 3-cornered cell $\mathcal{C}$ with cell corners V_1, V_2, V_3, as illustrated in Figure 9. Let us consider the point X_1 on the edge V_1V_3 such that $\angle XX_1P = 45°$. By moving the server X of the new facility to X_2 along XX_1, the minimum rectangular distances from any user i (constrained to pass through V_2 or V_3) to the server X remains unaltered. Since we are considering the rectangular distance metric, any increase in y distance from X to V_2 or V_3 is compensated by a decrease in x-distance and vice versa. However for any user i constrained to pass through V_1, the minimum rectangular distance between i and X strictly decreases. Similar arguments apply if X_1 lies on V_1V_2. Since we assume that the facility is fully contained inside the cell, the lemma follows. ∎

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Other cases of 3-cornered cells can be dealt with in a similar manner. An immediate implication of Lemma 5.2.3 is the following: the nondominated region $\mathcal{D}$, when the new facility is fully contained in a 3-cornered cell, is composed of line segments that are parallel to the edges of the cell that are not common to a barrier boundary. $\mathcal{D}$ can be obtained by moving the facility inside $\mathcal{C}$ such that $B_j \cap H = \emptyset$. $\mathcal{D}$ is demonstrated in Figure 9 by bold lines.

Based on the previous background, the problem to determine the optimal server location over a segment $\mathcal{D}$ (when the new facility is fully contained in a 3-cornered cell), can be solved by performing a line search alike Case 2.

Before moving on to case 4, we observe that Lemmas 5.2.1, 5.2.2 and 5.2.3 extend the results obtained in Theorem 1 of Nandikonda *et al.* [14] to account for finite size of the new facility in our work.

Case 4: The facility is fully contained in a 4-cornered cell

A 4-cornered cell $\mathcal{C}$ may be subdivided into subcells SC by $ETTL$s between adjacent and/or opposing cell corners. Each subcell SC is defined by a set of linear line segments ($ETTL$s) and the boundary of a rectangular cell $\mathcal{C}$ to which SC belongs. It is straightforward to see that any SC is a convex polyhedron (for discussion, see Figure 10). Let $\mathcal{F}$ denote a feasible region within a SC where a server of the new facility can be located. Note, however, that the new facility itself may intersect $ETTL$s and may cross over into one or more neighboring subcells. One can show the following result Lemma 5.2.4.

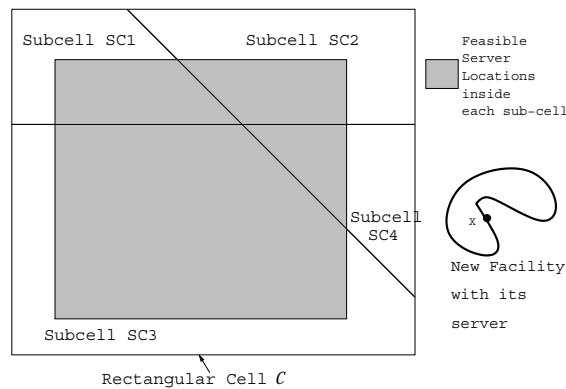**Lemma 5.2.4.** $\mathcal{F} \subseteq SC$ *are convex polyhedra.*

Figure 10: Figure for Lemma 5.1.4

Proof: Let us consider any subcell SC of a rectangular cell $\mathcal{C}$, as shown in Figure 10. $\mathcal{F} \subseteq SC$ is obtained by moving the new facility along the boundary of SC. Thus the server X moves parallel to the edges of SC. The lemma follows since each subcell SC is a convex polyhedron. ∎

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Hence $\mathcal{F} \subseteq SC$ can be represented by a set of inequalities

$$A^{\mathcal{F}} \begin{pmatrix} x \\ y \end{pmatrix} \leq b^{\mathcal{F}}. \tag{1}$$

The rectangular distance from a user i to the server X can be expressed as follows:

$$d(i, X) = d(i, V_j) + d(V_j, H_k) + d(H_k, X),$$

where $d(i, V_j)$ and $d(H_k, X)$ are constants while $d(V_j, H_k)$ is a linear function of the coordinates of the server, i.e., x and y. Note that a pair of V_j, H_k are rectangularly visible if $j = k$. Hence a user i assigned to cell corner V_j will communicate with the server X through corner H_j of the facility bounding rectangle. Since H_k, $k = 1, 2, 3, 4$ and X may not be rectangularly visible, Results 2 and 3 mentioned in §3.1 can be utilized to evaluate the distance function $d(H_k, X)$.

The 1-center facility placement problem over each $\mathcal{F} \subseteq SC \subseteq \mathcal{C}$ can be formulated as a linear program as follows:

$$\text{minimize } Z$$

subject to
$$w_i[d(i, V_j) + d(V_j, H_j) + d(H_j, X)] + \gamma_i \leq Z \qquad \text{for } i \in \mathcal{V}_k, \forall k \tag{2}$$

$$A^{\mathcal{F}} \begin{pmatrix} x \\ y \end{pmatrix} \leq b^{\mathcal{F}}. \tag{3}$$

$\mathcal{V}_k$ denotes the set of users uniquely assigned to cell corner V_k, such that $\cup_k \mathcal{V}_k = D$. Solution to the linear program gives the optimal 1-center location in $\mathcal{F}$.

5.2.2 The facility placement cuts off at least one element of L

When the new facility is placed over a number of gridlines, the facility may interfere with the shortest rectangular paths between the existing users. The consequence of the facility cutting off a number of gridlines is that the shortest feasible rectangular path from a user i to the server location X may have to travel around the facility, i.e., the facility itself may act as a barrier to travel between the user and its server. Hence $ETTL$s may move, as discussed earlier in Section 4.1. Consequently the assignment of users to cell corners may change. The placement of the new facility determines the cell corner through which a user i communicates with the server X of the new facility.

Let us consider a feasible initial placement l of the facility such that it cuts off at least one element of L. We assume here that no element of L coincides with any facility traversal line. Following the concept of Savas *et al.* [16], we identify the set of placements $\mathcal{Q}$ (of the server X) such that when $l \in \mathcal{Q}$, the facility will always interfere with the same element(s) of L. The idea is elucidated in Figure 11a, in which the new facility cuts off two gridlines, marked as Line A and Line B. The set $\mathcal{Q}$ obtained by moving the facility such that it interferes with no gridline other than Line A and Line B is also shown. It is pertinent to note the following here:

1. $\mathcal{Q}$ may be composed of mutually disjoint sets, as illustrated by Figure 11b.
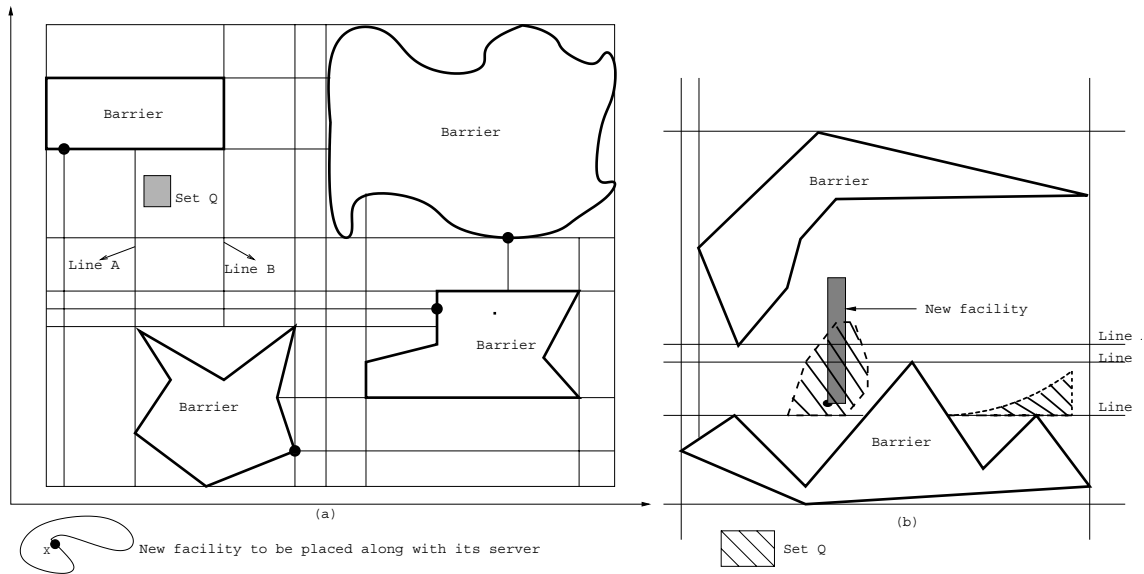
Figure 11: Illustration for the set $\mathcal{Q}$

2. $\mathcal{Q}$ may be nonconvex nonpolygonal, as illustrated by Figure 12a.

3. $\mathcal{Q}$ may be convex polygonal, as illustrated by Figure 11a.

4. $\mathcal{Q}$ may be nonconvex polygonal, as illustrated by Figure 12b.

To demonstrate the existence of the previous cases, we have considered a rectangular new facility in some cases for the sake of simplicity.
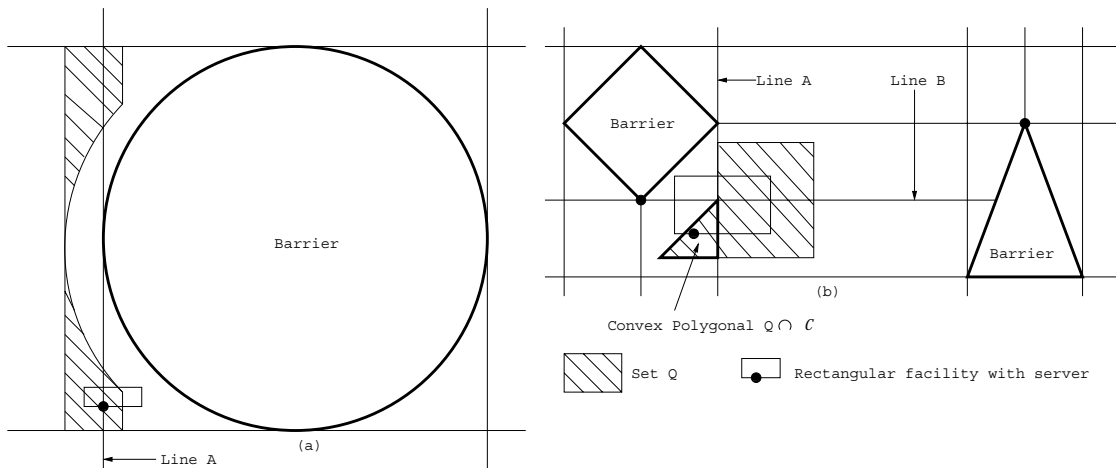
Figure 12: Illustration for different shapes of $\mathcal{Q}$

The 1-center problem is solved over each $\mathcal{Q} \cap \mathcal{C}$, since $\mathcal{Q}$ may span more than one cell $\mathcal{C}$. For the $\mathcal{Q} \cap \mathcal{C}$ under consideration, the shortest rectangular path from a user i to X is constrained to pass

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through the cell corners of $\mathcal{C}$. Also as illustrated in Figure 12, there is no guarantee that $\mathcal{Q}$ is convex. Since $\mathcal{Q}$ can be nonconvex, the optimal server location cannot be determined by formulating the problem as a linear program in general. However in some special cases, the 1-center problem can be formulated as a linear program. We present two such cases as follows:

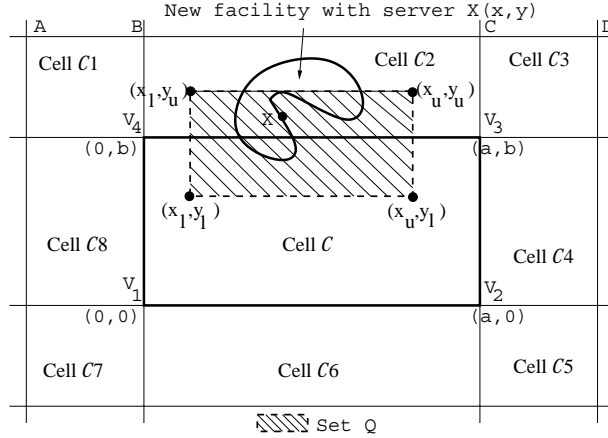**Case 1: $\mathcal{Q} \cap \mathcal{C}$ is rectangular**

Figure 13: $\mathcal{Q} \cap \mathcal{C}$ is rectangular

Let us consider a 4-cornered cell $\mathcal{C}$ as shown in Figure 13 with cell corners V_1, V_2, V_3, and V_4. Let $\mathcal{C}$ be bounded on all sides by gridlines (due to other barriers or users) parallel to the edges of $\mathcal{C}$. In other words, $\mathcal{C}$ is bounded on all sides by 8 rectangular cells labeled Cell $\mathcal{C}1$, Cell $\mathcal{C}2$, etc. Let us consider the gridline segments V_4V_3 and BC. The new facility is moved parallel to V_4V_3, BC, BV_1, and CV_2 such that it cuts only $V_4V_3 \in L$. $\mathcal{Q}$ obtained by cutting $V_4V_3 \in L$ is rectangular. Hence $\mathcal{Q} \cap$ Cell $\mathcal{C}$ and $\mathcal{Q} \cap$ Cell $\mathcal{C}2$ are rectangular.

The rectangular distance from a user i to the server X can be expressed as follows:
$$d(i, X) = d(i, V_j) + d(V_j, H_k) + d(H_k, X),$$
where $d(i, V_j)$ and $d(H_k, X)$ are constants while $d(V_j, H_k)$ is a linear function of the coordinates of the server, i.e., x and y. Since $H_k, k = 1, 2, 3, 4$ and X may not be rectangularly visible, Results 2 and 3 mentioned in §3.1 can be utilized to evaluate the distance function $d(H_k, X)$.

Hence the problem to determine the 1-center can be formulated as a linear program over the shaded region $\mathcal{Q} \cap \mathcal{C}$ as follows:

$$\text{minimize } Z$$

subject to
$$w_i[d(i, V_j) + d(V_j, H_k) + d(H_k, X)] + \gamma_i \leq Z \qquad \text{for } i \in \mathcal{V}_k, \forall k \tag{4}$$
$$x_l \leq x \leq x_u \tag{5}$$
$$y_l \leq y \leq y_u \tag{6}$$

where x_l, x_u, y_l, y_u represent the lower and upper bounds of the x-coordinate and y-coordinate

respectively of the $\mathcal{Q} \cap \mathcal{C}$ under consideration and $\mathcal{V}_k$ denotes the set of users uniquely assigned to cell corner V_k, such that $\cup_k \mathcal{V}_k = D$. A similar linear program can be formulated over $\mathcal{Q} \cap$ Cell $\mathcal{C}2$.

Case 2: $\mathcal{Q} \cap \mathcal{C}$ is convex polygonal

If $\mathcal{Q} \cap \mathcal{C}$ is convex polygonal, as illustrated in Figure 12b, $\mathcal{Q}$ can be represented by a set of inequalities

$$A^{\mathcal{Q}} \begin{pmatrix} x \\ y \end{pmatrix} \le b^{\mathcal{Q}}. \tag{7}$$

Hence the problem to determine the 1-center can again be formulated as a linear program, as in Case 1.

Case 3: $\mathcal{Q} \cap \mathcal{C}$ is nonconvex

However if $\mathcal{Q}$ is nonconvex, the problem to determine the 1-center involves finding the minimax location over a nonconvex set, which can be represented as an optimization problem with a linear objective function and a nonconvex feasible region. Hence the problem can be formulated as a nonlinear program and solved using standard nonlinear programming solvers.

The overall feasible region F is composed of a number of such regions $\mathcal{Q}$. The number and structure of $\mathcal{Q}$s depends on the number and shapes of barriers. However a number of $\mathcal{Q}$s will be rectangular or convex polygonal. As demonstrated earlier, the determination of the 1-center for such cases involves solving a linear program. However for nonconvex $\mathcal{Q}$s, the problem to determine the 1-center can be formulated as a nonlinear program and solved using standard nonlinear programming solvers. Some nonconvex regions might lend themselves to division into a finite collection of convex subsets. Here again, linear programming formulations are possible.

6 Solution Complexity

Our solution methodology for the fixed orientation problem can be split as two distinct cases: (i) when the facility is fully contained in a cell/subcell (ii) when the facility intersects gridlines. Hence the number of cells/subcells is an important factor that governs our solution complexity. The number of 1, 2, 3, 4-cornered cells generated depends on the number of barriers, their shapes and their placement. In fact, the number of cells generated is a direct function of the number of points of tangency of each barrier. Formally, N barriers generate at most CN horizontal and CN vertical gridlines where C is a constant that signifies the upper bound on the number of tangency lines of a barrier (in fact, this is our only restriction to the arbitrariness of barriers). Hence the maximal number of 4-cornered cells generated is $O(N^2)$. However these many 4-cornered cells are typically not generated as gridlines are terminated when they intersect barriers. The number of subcells into which 4-cornered cells can be split is a function of the number of $ETTL$s generated in each 4-cornered cell. Recall that a user generates at most 1 $ETTL$ in a 4-cornered cell, as proved in [14].

Also since $ETTL$s between opposing cell corners do not exist, as proved by Lemma 4.2.1, the number of subcells generated is also restricted. Hence N users, one associated with each barrier, can generate N $ETTL$s in a 4-cornered cell in the worst case, which can intersect in $O(N^2)$ ways. Hence the number of linear programs to solve is $O(N^4)$.

When the new facility to be placed intersects gridlines, the number of set $\mathcal{Q}$s formed depends on the number of gridlines the facility intersects (which in turn is dependent on the number, shapes and relative proximity of the barriers). It is pertinent to note here that the set $\mathcal{Q}$s are defined when the facility intersects successive horizontal and/or vertical gridlines. The new facility can intersect CN vertical gridlines in $O(N^2)$ ways because intersecting: (i) one at a time is CN ways, (ii) two at a time is $CN - 1$ ways because of the successive intersection requirement, and so on. Similarly, CN horizontal gridlines can be intersected in $O(N^2)$ ways, and the maximal number of set $\mathcal{Q}$s is $O(N^4)$. Note that if a facility cuts one vertical (or horizontal) gridline, $\mathcal{Q}$ can span at most two cells. Similarly if a facility cuts two vertical (or horizontal) gridlines, $\mathcal{Q}$ can span at most three cells. Hence considering only vertical (or horizontal) gridlines, the number of $\mathcal{Q} \cap \mathcal{C}$s is $O(N^5)$. Considering there exist both vertical and horizontal gridlines, the number of $\mathcal{Q} \cap \mathcal{C}$s is $O(N^6)$. Consequently there are $O(N^6)$ optimization problems to solve. The complexity of the entire solution procedure would depend on the complexity of the optimization problem in each such set $\mathcal{Q} \cap \mathcal{C}$.

7   Numerical Example

We now elucidate our solution methodology with a simple five-barrier example, as illustrated by Figure 4 in §3.2. We redraw Figure 4 as Figure 14 and Figure 15 for the convenience of the reader. Each barrier has one user. The coordinates of the barrier vertices and users are known. The weight (w_i) and addendum (γ_i) of user i are listed in Table 1.

Table 1: Example data

User (i)	1	2	3	4	5
w_i	1	2	2	1	2
γ_i	3	5	2	1	4

The new facility (along with its server) is shown in Figure 14. The first step is to define the feasible region (hatched region as illustrated in Figure 14). Following the grid construction procedure of §3.1, we draw 11 horizontal (numbered 1 through 11 in boxes in Figure 15) and 13 vertical gridlines (numbered A through M in boxes in Figure 15) through the barrier vertices and users. As a result, sixty-four cells are generated. The number of 1, 2, 3 and 4-cornered cells is 0, 7, 22 and 35 respectively. In Figure 15, the cells are numbered in circles. $ETTL$s generated due to various users are also illustrated.
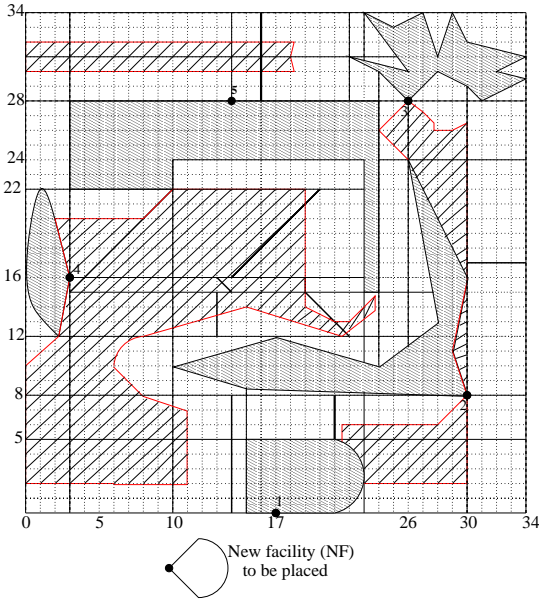
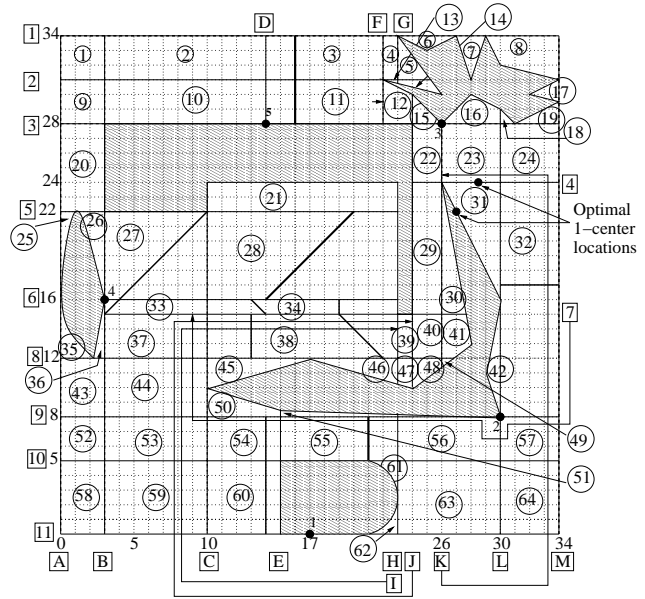Figure 14: Feasible region for placing new facility

Figure 15: Cells and gridlines numbered

The new facility (NF) can be fully contained in 7 cells (cells numbered 27, 28, 32, 59, 60, 63 and 64), all of which are 4-cornered. For all these cases, the weighted 1-center problem can be formulated as a linear program (LP). The (local) optimal 1-center location and the corresponding objective function values (z) for all these cases are listed in Table 2.

Table 2: Results when NF is fully contained in cells

NF fully contained in cell	# of sub-problems	Optimal 1-center (x^*, y^*)	z
27	1	$(3, 18)$	79
28	2	$(10, 18)$	65
		$(16, 18)$	77
32	2	$(30, 17)$	58
		$(30, 22)$	48
59	1	$(6, 3)$	92
60	1	$(10, 3)$	94
63	1	$(26, 3)$	74
64	1	$(30, 3)$	60

As an example, let us consider the case when the NF is fully contained in cell 63. The weighted 1-center (x, y) objective function for cell 63 is as follows:

min   max$\{[1(6 + (x - 23) + y) + 3], [2(3 + (30 - x) + (5 - y)) + 5], [2(27 + (30 - x) + (5 - y)) + 2], [1(31 + (x - 23) + (5 - y)) + 1], [2(39 + (30 - x) + (5 - y)) + 4]\}$,

with feasible server locations being bounded by $23 \le x \le 26$ and $2 \le y \le 3$. This 1-center problem can be formulated as an LP. The optimal 1-center for cell 63 is $x = 26$ and $y = 3$ with an objective function value of 74. Recall that when the NF is fully contained inside cells, $ETTL$s

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do not move. Also note that when a cell is split into subcells by an $ETTL$ (for example, cells 28 and 32), the weighted 1-center problem splits into two sub-problems (one for each subcell), each of which can be formulated as an LP.

When the NF intersects gridlines, set $\mathcal{Q}$s are formed. In this example, 38 such set $\mathcal{Q}$s are generated. Recall that $\mathcal{Q}$s may span more than one cell. In all such cases, the weighted 1-center problem has to be solved over $\mathcal{Q} \cap \mathcal{C}$s. Here, 60 $\mathcal{Q} \cap \mathcal{C}$ sets are found. Consequently, there are 60 optimization problems to solve, only one of which turned out to be a nonlinear program (NLP) (case 6 in Table 3). All other cases were formulated as LPs. The LPs and NLP were solved using the LP solver LINDO 6.1 and the NLP solver LINGO 8.0. The solution times in all cases were less than a second. We now present a few selected results, when the NF intersects gridlines.

For each of the following cases, Table 3 lists the gridline(s) intersected, shape of $\mathcal{Q}$s generated, shape of $\mathcal{Q} \cap \mathcal{C}$ (thereby indicating the number of resulting optimization problems), their type (LP or NLP) and the (local) optimal 1-center location along with its objective function value (z).

Table 3: Selected results when NF intersects gridlines

Case #	Lines intersected	Shape of $\mathcal{Q}$	Shape of $\mathcal{Q} \cap \mathcal{C}$	LP/NLP	Opt. 1-median (x^*, y^*)	z
1	L	mutually disjoint sets, convex polygonal(CP)	–	LP	$(30, 30)$	86
		rectangular(R)	–	LP	$(27, 22)$	42
2	L,4	CP	R	LP	$(28.67, 24)$	41.33
			CP	LP	$(26.89, 22.22)$	41.33
3	H,10	nonconvex non–polygonal(NC-NP)	R	LP	$(23, 6)$	94
			NC-NP	LP	$(23, 5)$	96
4	C	R	–	LP	$(8.5, 3)$	87
5	C,9	nonconvex, polygonal(NC-P)	CP	LP	$(8, 8)$	78
			CP	LP	$(8.25, 7)$	78.5
6	C,8	NC-NP	CP	LP	$(6, 13)$	72
			NC-NP	NLP	$(8, 12)$	70
7	C,6,7	R	R	LP	$(6, 15)$	68
			R	LP	$(6, 15)$	68
			R	LP	$(6, 16)$	70

The optimal 1-center locations (obtained by comparing all local minima) of the new facility are $(26.89, 22.22)$ and $(28.67, 24.00)$ (Case 2 in Table 3), both with objective function values of 41.33.

We make an interesting observation for Case 3. While one of the $\mathcal{Q} \cap \mathcal{C}$ related to cell 61 is nonconvex nonpolygonal, it is analogous to the 3-cornered cell for the fully contained case (see Lemma 5.1.3). Thus its boundary dominates the interior and a line search (through an LP) is adequate.

Note that since the new facility intersects gridlines, $ETTL$s may move. Case 7 in Table 3 is an

example in which the $ETTL$s in cells 26 and 32 move. However the assignment of users to cell corners does not change in any of the cases in this example.

Establishing domination results can significantly reduce the number of optimization problems to be solved. In this example, it can be verified that parts of the feasible region, for example, between (i) $y = 30$ and $y = 32$, (ii) $x = 0$ and $x = 3$, and (iii) all feasible locations contained strictly inside cell 42, are dominated. All such feasible locations are excluded from analysis. Moreover, it is worthwhile noting that we can attempt to prune some cells (in which the NF can be fully contained) based on a bound provided by the optimal 1-center location of an infinitesimal facility. The overall efficiency of the solution procedure can thus be further improved.

Finally, the number of optimization problems to be solved depends on the number of barriers, their shapes and relative proximity, as mentioned earlier. However, even for more complex problems, the computational time required to solve each of the mini optimization problems is expected to be negligible.

8 Conclusions

In summary, this paper addresses the optimal placement problem of a single finite size arbitrarily shaped facility in the presence of arbitrarily shaped barriers to travel with the center objective. The rectangular travel metric is employed. The server location and the facility orientation define a facility's placement. Due to the complexities associated with the finite size of the new facility, the placement problem with fixed facility orientation is studied in this paper. Based on the subdivision of the feasible region into cells, we obtain cell domination results for 1, 2, 3-cornered cells when the facility is fully contained inside such cells. When the new facility is fully contained in a 4-cornered cell (or a subcell generated by $ETTL$s), the 1-center placement problem can be formulated as a linear program. When the facility cuts gridlines, we analyze the problem and develop a solution methodology based on distinct but fixed sets of gridlines which the facility intersects.

Our work can be viewed as an extension of Savas *et al.* [16] who study a similar problem but with "median" objective and Nandikonda *et al.* [14] who consider the 1-center placement of an infinitesimal facility. [16] considers a median objective and proves the objective function to be concave. However since distances are being measured explicitly in our work, the procedure can be applied to a general class of problems irrespective of the objective considered. Unlike [16], we solely consider the user-server interaction. We note that since the new facility to be located itself acts as a barrier to rectangular travel, it can disrupt the level of interaction between two users. Study of the 1-center placement problem under a similar setting but subject to a constraint that the user-user interaction should not exceed a specified threshold is an interesting open problem. Proposing a solution methodology to the variable orientation problem (when the server location X_0 is known *a priori*) is also a future research direction.

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