Integrated Scheduling of Material Handling and Manufacturing Activities for Just-In-Time Production of Complex Assemblies

Muhammad F. Anwar, Rakesh Nagi*
Department of Industrial Engineering, 342 Bell Hall
State University of New York at Buffalo, Buffalo, NY 14260

Abstract

This paper considers the simultaneous scheduling of material handling transporters (such as Automatic Guided Vehicles or AGVs) and manufacturing equipment (such as machines and workcenters) in the production of complex assembled products. Given the shipping schedule for the end-items, the objective of the integrated problem is to minimize the cumulative lead time of the overall production schedule (i.e., total makespan) for on-time shipment, and to reduce material handling and inventory holding costs on the shop-floor. The problem of makespan minimization is formulated as a transportation integrated scheduling problem, which is NP-hard. For industrial sized problems, an effective heuristic is developed to simultaneously schedule manufacturing and material handling operations by exploiting the critical path of an integrated operations network. The performance of the proposed heuristic is evaluated via extensive numerical studies and compared with the traditional sequential scheduling approach. The superiority of the integrated heuristic is well demonstrated.

Keywords: Just-In-Time Production, Makespan, Material Handling Systems Scheduling.

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*To whom correspondence should be addressed. Email: nagi@eng.buffalo.edu

1 Introduction

Rapid advances in technology and changes in demand patterns to include customized features in manufactured products, and relatively shorter life cycle of manufactured goods has shifted the emphasis of manufacturing strategy from mass production to small batch manufacturing. When manufacturing small batches of similar or customized complex assemblies, besides manufacturing operations, a large number of material handling operations are also involved to transfer items between different workcenters. By some estimates, typically, a job spends on an average only 5% of its total flow time being processed on machines, and during the remaining time, either it is in a queue or being transported from one workcenter to another (Han and McGinnis, 1989). To survive in a highly competitive environment which demands short lead-time deliveries and low cost products, there is a need for better coordination and scheduling of production and logistic activities on the shop floor.

While significant research effort has been devoted to operation scheduling and vehicle scheduling in individual ways (see Section 2), the two problems are in fact tightly inter connected. It can be seen that the operations schedule provides the transportation requests, and in turn, the vehicle schedule provides the release time for operations on down stream machines. It is possible to conceive of a situation where a "good" operations schedule provides a very "poor" input to vehicle scheduling, and vice-versa, making the overall schedule unacceptable. Realizing this, recent interest (see Section 2.3) has been drawn towards the integrated problem in flow and job-shop settings. However, a similar attempt is not known to exist in an assembly environment, where the multi-level bill-of-materials impose precedence constraints among manufactured parts.

To address this void, the production environment addressed in this study is a manufacturing facility that manufactures complex assemblies in small batches and employs Automated Guided Vehicle (AGV) (or fork truck) based horizontal material handling transporters to transfer items between different machine cells. Such environments are common in the machine tool industry, heavy mechanical industry, and defense related high-technology products such as radar antennas, among other.

This study focuses on the simultaneous scheduling of processing and material handling operations for Just-In-Time (JIT) production of complex assemblies under multiple capacity constraints. These are critical issues for large and complex assembly manufacturing in which finished product cycle-times span over several months. The objective is to reduce the cumulative lead time of the production schedule (total makespan) and minimize WIP, inventory and material handling costs in order to "deliver these assemblies at the right time and with the right cost"(Schniederjan, 1993). To achieve these objectives, a JIT production strategy is employed by back-flushing all operations as late as possible, but without backlogging the end items. The problem has been formulated as a mixed integer linear program, and is NP-hard (see Section 3.1). Thus, for most practical sized problems heuristics are highly desirable. We propose a heuristic for the simultaneous scheduling of machines and material handling equipment to minimize cycle-time as well as material handling and inventory holding costs. The schedule of operations is generated by considering the critical path of the precedence network of operations obtained from bills-of-materials and routings of the manufactured items, the layout structure, and the information of transportation operations. To evaluate the performance of the proposed heuristic, the results of integrated scheduling approach are compared with the traditional sequential scheduling approach in an empirical study. We find that on the average, the integrated approach provides significant benefits in both lead-times and cost.

The paper is organized as follows. In Section 2, a brief literature review is presented. In Section 3, the problem description and the mathematical formulation of the problem are provided. In Section 4, the proposed heuristic is presented. In Section 5, the results of numerical tests are presented, which demonstrate the superiority of our proposed heuristic over sequential scheduling. In Section 6, practical applications and possible extensions of the proposed heuristic are discussed. Finally in Section 7, conclusions of this study are presented.

2 Background

Although there have been advances in production control techniques, for a multi-stage manufacturing/assembly environment, the Materials Requirements Planning (MRP) (or Manufacturing Resources Planning, MRP II) philosophy is still very popular in industry. MRP has been found as an effective way to incorporate relationships between end-items and the various component levels, and it attempts to complete end-items as close to their due-dates as possible while minimizing inventories. Production requirements of an end-item are translated into known production (or purchase) quantities and timing of components, based on available inventory, the Bill-Of-Materials (BOM) and item lead-times (Orlicky, 1975).

Despite these many advantages MRP offers, perhaps its major drawback is that it ignores detailed shop-floor capacity, and it assumes fixed production lead-times which are not realistic. Besides, the moving times between operations are either disregarded or largely approximated. The consideration of capacity constraints is essential to obtain realizable schedules on the shop-floor. And, in reality, not only the machines but also the material handling equipment are constrained resources. At times, machines are the bottlenecks, and at other times, non-availability of material handling equipment makes them the bottlenecks. This is especially true in expensive AGV-based material handling systems. Therefore, simultaneous scheduling of machines and material handling system needs to be considered in order to truly reduce the makespan and the cost of the final products. The following discussion of the literature focuses on job-shop scheduling of multi-level products, AGV scheduling, and material handling in a job-shop environment, as these are closely related to problem at hand.

2.1 Job-shop Scheduling of Multi-level Products

In the field of scheduling, the literature is vast, and it has been reviewed and classified by many authors. Panwalker and Iskander (1977) analyzed and summarized different priority rules presented in the literature. Rodammer and White (1988) surveyed different methods used in production scheduling. Blazewicz *et al.* (1991) have compiled a number of mathematical programming formulations for scheduling. McCarthy and Liu (1993) have listed the various types of scheduling problems and also the well known solution methods proposed in the literature. Most of these methods do not consider multi-level products.

Multi-stage, multi-item capacitated scheduling problem is addressed by various authors including Billington *et al.* (1983), Zahorik *et al.* (1984), Miltenburg and Sinnamon (1992, 1989), Agrawal *et al.* (1996), Anwar and Nagi (1997) in different problem environments.

It is noted that the above literature considers the scheduling of multi-level products with only manufacturing operations. Large and complex assemblies can seldom be manufactured in a single manufacturing cell or department, and the use of inter-cell material handling equipment such as AGVs or forklifts are essential. Incorporating material handling operations in an existing schedule is not trivial because in doing so, the entire schedule changes. Thus, the quality or effectiveness of the overall schedule cannot be guaranteed.

2.2 Scheduling of AGVs

Many authors have studied AGV-based material handling systems. The extensive literature has focused on the basic elements of AGV system design: (i) guide path network design (Egbelu and Tanchoco, 1986; Tanchoco and Sinriech, 1992), (ii) optimal number of AGVs (Maxwell and Muckstadt, 1982; Tanchoco *et al.*, 1987; Egbelu, 1987), (iii) vehicle dispatching (Egbelu and Tanchoco, 1984), (iv) vehicle routing (Broadbent *et al.*, 1985; Huang *et al.*, 1993; Kim and Tanchoco, 1991), and (v) traffic control (Wirth, 1983; Koff, 1987).

A static scheduling approach to AGVs is usually not possible because the manner in which AGVs process a given set of requests will define (or change) the future requests. Thus, the dispatching approach is usually adopted. However, as noted in the previous section, AGV dispatching will in-turn influence subsequent work-center scheduling, and there is potential loss of overall performance.

2.3 Integrated Material Handling in the Job-shop Environment

From the remarks in the aforementioned sub-sections, it is not hard to appreciate that the work-center and vehicle scheduling/dispatching problems are inextricably one. In the job-shop environment (single-level products), some efforts that consider the integrated problem have been reported in the literature. Ihsan and Hommertzheim (1992a) investigated various machine and AGV scheduling rules against certain scheduling criteria. Simultaneous scheduling of machines and AGV-based material handling equipment in a flow-shop is considered by Raman *et al.* (1986), Kise *et al.* (1991), and many other authors. In a flow-shop environment, the scheduling problem is comparatively simpler than that of a job-shop and assembly environment.

Pundit and Palekar (1990) proposed branch-and-bound as well as heuristic solution procedures for the simultaneous scheduling of machines and material handling vehicles in a job-shop environment. In the similar environment Ihsan and Hommertzheim (1992b) scheduled machine and material handling operations one at a time when they were needed, and assumed zero set-up time for all operations.

Although the approaches proposed by Pundit and Palekar (1990), and Ihsan and Hommertzheim (1992b) consider the simultaneous scheduling of machines and material handling vehicles in a job-shop environment, they do not consider the precedence relationship between different parts of the same end product. Complex assemblies require a large number of operations, and the consideration of precedence constraints is very important in scheduling of these assemblies. Therefore, precedence relationships and capacity constraints need to be considered in the integrated scheduling of machines and AGVs for producing multi-level assemblies in a JIT environment.

3 Problem Formulation

A general manufacturing facility that produces assemblies is considered. The facility consists of a set of work-centers $\{W_1, W_2, ...W_{w'}\}$ and a set of horizontal transporters $\{W_{w'+1}, W_{w'+2}, ...W_w\}$. Each work-center (transporter) W_Y may consist of f_Y functionally identical machines (equipment pieces). A general shop-floor layout is shown in Figure 1. Although, the shop shown is composed of manufacturing cells, this is not restrictive. We consider a set of final assemblies that must be produced in the facility. The set of manufactured (or make) parts includes all the final assemblies (P_f) and the semi-finished parts (P_s), and is represented by $P_m = P_f \cup P_s$. Each make part $p_k \in P_m$, is processed through a unique sequence of operations (or routing) $< O_{kl} >_{l=1}^{s_k}$, and each finished product p_i has a unique Bill-of-Material (BOM) which represents its assembly structure. The sample BOM of a finished product **A** along with the routing of the make items is shown in Figure 2. For a given shipping schedule of the end-items, the objective is to schedule the work-centers and transporters such that the cumulative production time (makespan) is minimized.

Figures 1 and 2 around here

Given the Bill-of-Materials (BOMs) of final products, the routings of the make parts, and the shipping schedule, an integrated operations network as proposed in Agrawal *et al.* (1996) can be developed (see Figure 3). We can enhance this network by inserting transportation operations between each pair of operations (predecessor - successor) that are performed on work-centers belonging to different cells/departments (see Figure 4). The processing times of these operations is the transport time required by the particular transporter to transport material from the upstream machine (of one cell) to the downstream machine (of another cell). For simplicity here, we assume that the transporter type is known for each part type (or operation). This can easily be extended to account for choices among transporter types.

Figures 3 and 4 around here

To formulate the problem, the following assumptions are made:

- Back-to-back production strategy is assumed.
- Machines/AGVs are assumed to be reliable.
- A machine can perform at most one operation at a time.
- An operation can be performed on one machine at a given time.
- Preemption of operations or AGV trips is not permitted.
- Processing times of all operations and due-dates of all final products are deterministic and known.
- Backlogging is not permitted.

- AGVs do not encounter congestions along the guide path (see Section 6).
- Sufficient input/output buffer space is available at each machine and each loading/unloading station.
- Entire batch of a (semi)finished product is transported on an AGV in a single trip.

3.1 Transportation Integrated Scheduling Problem (TISP)

For the production environment described above, the transportation integrated scheduling problem **(TISP)** has been formulated as a mathematical programming problem. In the integrated (and enhanced) network, let there be n operation types to be processed $\{O_1, O_2, ...O_n\}$. Let us define the set of indices of operations to be processed by workcenter Y as: $I_Y = \{j : O_j$ requires workcenter $Y\}$. We will loosely also refer to transporters as workcenters for convenience. An index set of the final operations of end-items is denoted by $E = \{j : O_j$ is a final operation of an end-item$\}$. The notations we use are as follows:

D_i = Due date of the i^{th} end-item.
S = Start time of the planning horizon.
$s(j)$ = Successor or immediate downstream operation of operation j.
S_j = Begin or start time of operation j.
F_j = Finish or completion time of operation j.
t_j = Processing time for the batch of operation j.
$K_{w(j)w(i)}$ = Time of an empty transporter travel from machine cell of operation j, $w(j)$, to machine cell of operation i, $w(i)$.

$$\psi_{ij} = \begin{cases} 1 & \text{if } O_j \text{ precedes } O_i \qquad (i, j \in I_Y,\ i \neq j,\ Y = 1, ..., w) \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_{jy} = \begin{cases} 1 & \text{if } O_j \text{ is performed on the } y^{th} \text{ functionally identical machine of work-center } W_Y \\ 0 & \text{otherwise} \qquad (\forall\ j \in I_Y,\ \ y = 1, 2, ..., f_Y) \end{cases}$$

A schedule may be defined by selecting a sequence of operations and assigning them to the required resources. Since this sequence defines the schedule, it also indicates the makespan. We can formally state the scheduling problem as follows **(TISP)**:

$$Minimize: \ \underset{i \in E}{Max}\ \{D_i\} - S$$

Subject to:

$$S_{s(j)} \geq F_j \qquad \text{for} \quad j = 1, 2, ..., n \tag{1}$$

$$F_j = S_j + t_j \qquad \text{for} \quad j = 1, 2, ..., n \tag{2}$$

$$\psi_{ij} + \psi_{ji} = 1 \qquad \text{for} \ \ i, j \in I_Y,\ i \neq j,\ Y = 1, ..., w \tag{3}$$

$$S_i - F_j \geq (\psi_{ij} + \phi_{iy} + \phi_{jy} - 3)M \quad \text{for } i, j \in I_Y,\ i \neq j,\ y = 1, ..., f_Y,\ Y = 1, ..., w' \tag{4}$$

$$F_j \leq S_i - K_{w(j)w(i)} + (3 - \psi_{ij} - \phi_{iy} - \phi_{jy})M + (\sum_{k \in I_Y, k \neq i} \psi_{ik} - \sum_{k \in I_Y, k \neq j} \psi_{jk} - 1)m$$

$$\text{for } i, j \in I_Y,\ i \neq j,\ y = 1, ..., f_Y,\ Y = w' + 1, ..., w \tag{5}$$

$$\psi_{ij} \geq \sum_{k=1}^{y-1} \phi_{jk} + \phi_{iy} - 1 \quad \text{for } i, j \in I_Y,\ i \neq j,\ y = 2, ..., f_Y,\ Y = w' + 1, ..., w \tag{6}$$

$$F_i \leq D_i \quad \text{for } i \in E \tag{7}$$

$$S \leq S_j \quad \text{for } j = 1, 2, ..., n \tag{8}$$

$$\sum_{y=1}^{f_Y} \phi_{jy} = 1 \quad \text{for } j \in I_Y,\ y = 1, ..., f_Y,\ Y = 1, 2, ..., w \tag{9}$$

$$\psi_{ij} \in \{0, 1\} \quad \text{for } i, j \in I_Y,\ i \neq j,\ Y = 1, 2, ..., w \tag{10}$$

$$\phi_{jy} \in \{0, 1\} \quad \text{for } j \in I_Y,\ y = 1, 2, ..., f_Y,\ Y = 1, 2, ..., w \tag{11}$$

The objective function minimizes the production makespan. Constraint (1) is due to the network structure, and it ensures that the starting time of the successor operation cannot be earlier than the completion time of its immediate predecessor operation j. Constraint (2) provides the relationship between the starting and the completion times for each operation j. Constraint (3) specifies the unidirectional nature of precedence relationship between two operations performed on the same workcenter in the scheduling sequence. Constraint (4) is the capacity constraint and provides the relationship between the starting and the completion times of any two operations on the same functionally identical machine of a processing workcenter W_Y; the constant M is a very large number. Constraint (5) is the capacity constraint for transporters and provides the relationship between the completion and the start times of any two transport operations on the same functionally identical vehicle/equipment W_Y, and also incorporates the empty travel/repositioning time if required; the constants m and M are very large numbers such that $m << M$. Constraint (6) sets the sequence of any two operations which are not scheduled on the same functionally identical vehicle/equipment (replicate) and ensures that the operations scheduled on lower replicate numbers precede in sequence (not necessarily in time) operations scheduled on higher replicate numbers. Constraint (7) indicates that the last operation of the i^{th} end product needs to be completed before its due date. Constraint (8) indicates that the start time of operation j is greater than or equal to the start time of the planning horizon. Constraint (9) indicates that if an operation O_j is performed on workcenter W_Y, then it must be performed on only one of its f_Y functionally identical machines. Finally, constraints (10) and (11) denote that ψ_{ij} and ϕ_{jy} are binary variables.

The vehicle routing problem (Orloff, 1976) and single stage machine scheduling problem (Lenstra and Rinnooy Kan, 1979) are accepted to be NP-hard, therefore, without proof, the integrated problem is assumed to be NP-hard. The proof can be made by a simple reduction of TISP to either of the aforementioned. Therefore, the existence of a polynomial time algorithm to solve this problem is highly unlikely.

Note also that many features of this formulation are similar to that of a general assembly job shop, and with the help of a dummy assembly, to that of a general job shop with AGVs. Thus, some common scheduling problems are subsumed in TISP.

4 Transportation Integrated Scheduling Heuristic

Mathematical programming methods, as well as implicit enumeration methods, such as the branch-and-bound algorithm, can be applied to solve problem **(TISP)**, only for small dimensional cases. To address complex assembly structure problems, an effective heuristic has been developed. The proposed heuristic simultaneously schedules production and material handling operations in order to minimize the makespan by exploiting the critical path of the operations network.

4.1 Critical Path Concept

A network of operations, similar to that shown in Figure 4, is employed to prioritize operations and generate a production schedule in the proposed heuristic, which is similar to LETSA (Agrawal *et al.*, 1996) and SLIPSA (Anwar and Nagi, 1997) for the network shown in Figure 3.

The critical path of the operations network represents the time required to produce the final product assuming infinite resources. The infinite capacity critical path is fixed for a given product structure and determines the lower bound of the makespan. The critical path of the operations network is calculated using a FORWARD pass (or a push schedule), and early-start and early-finish times are recorded for each operation. Initially, the first and second operations from the head of the infinite capacity critical path are scheduled since they also belong to the capacitated critical path. Then considering precedence relationships, the next operation scheduled is the one that has the largest early finish-time (as it belongs to the new "infinite capacity critical path"), and the remaining operations are scheduled in the similar manner. In this way the deviation of the schedule from the lower bound of the makespan is attempted to be minimized. For example, the critical path of the product structure of Figure 2, computed from the operations network of Figure 3, consists of operations E.10-E.20-B.10-A.20 (or D.10-D.20-B.10-A.20), and the corresponding makespan is 21 time units. This, however, is the lower bound of the makespan. For a system that includes only one machine per workcenter, the optimal makespan for the example of Figure 2 is 35 time units, and the optimal schedule is given in Figure 5. Note that this schedule does not consider material handling operations. Therefore, it is not accurate and realistic in a situation which involves material handling time between different machine cells, and possibly material handling systems are the dynamic bottlenecks. Therefore, it usually underestimates the makespan.

Figure 5 around here

The operations network of Figure 4 consists of both production as well as material handling operations, and is employed for simultaneous scheduling of machines and AGVs. Now, the infinite capacity critical path consists of operations E.10-T.12-E.20-T.21-B.10-T.12-A.20 (or D.10-T.12-D.20-T.21-B.10-T.12-A.20), and its corresponding total time is 34 time units. Considering one machine (AGV) in each workcenter, the optimal makespan in this case is 50 time units, and the optimal schedule is presented in Figure 6. It can be seen that incorporating logistics in the shop-floor schedule provides a more realistic makespan, and the one without logistics is an underestimate.

Figure 6 around here

4.2 Transportation Integrated Problem Scheduling Algorithm (TIPSA)

The proposed algorithm addresses the problem **(TISP)**, and generates a "good" feasible schedule. It proceeds in a backward scheduling manner similar to MRP, in which the last operation is scheduled first, and the remaining operations are scheduled backwards in subsequent steps while respecting all precedence and capacity constraints.

The delivery schedule, product structures, routing data and transportation time between machine cells for loaded and empty vehicles are the inputs to the algorithm, and are used to construct an integrated operations network. Given this network, a set of feasible operations is defined to include all operations which do not have a successor operation, i.e., $F = \{O_j : s(j) = \emptyset, j = 1, 2, .., n\}$. The algorithm generates a schedule in four steps: (i) select an operation from the feasible set F, (ii) select a specific machine (AGV) from the required workcenter, (iii) schedule the selected operation; if an AGV requires empty travel to service that operation, include empty travel time, and (iv) update the set of feasible operations. The step-by-step algorithm is listed below.

The Algorithm

1. Input manufacturing system data including workcenters, their capacities and number of identical machines, and product data, i.e., part master records, BOMs, the delivery schedule, transportation time between machine cells for loaded and empty vehicles
2. Generate the operations network from the delivery schedule and BOM of end-items, routing data of their make items, and material handling operations
3. Using forward pass, compute Early Start (ES) and Early Finish (EF) times of all operations (for determining the dynamic infinite capacity critical paths)
4. Define the set of feasible operations as $F = \{O_j : s(j) = \emptyset, j = 1, 2, .., n\}$
5. While the feasible list of operations is non-empty, (i.e., $F \neq \emptyset$)
   - 5.1 Select the operation from the feasible list F that has the largest value of EF time (say O_j)
   - 5.2 Set its tentative finish time F_j equal to: (i) the earliest starting time of operation $s(j)$ from the partial schedule, if $s(j)$ exists, or (ii) the due date D_j if operation j is the last operation of a final assembly (i.e., $D_j \in E$)

5.3 For each identical machine (or AGV) y included in the required workcenter

  5.3.1 Set the tentative latest starting time for the operation O_j as $S_j = F_j - t_j$

  5.3.2 If the machine (AGV) is available during $[S_j,\ F_j]$ (and AGV backward empty travel, if any, is accomplished between $[F_j,\ F_j + K_{w(j)w(i)}]$ with i being the operation that will immediately follow j on the AGV y), S_j and F_j are the ideal starting and finish times respectively; go to step 5.4

  Else select MAX $\{S_j\}$, as the latest available starting time such that the machine (AGV) is available during $[S_j,\ S_j + t_j]$ (and AGV backward empty travel, if any, is accomplished between $[S_j + t_j,\ S_j + t_j + K_{w(j)w(i)}]$ with i being the operation that will immediately follow j on the AGV y), Set $F_j = S_j + t_j$

5.4 Schedule operation O_j at the latest available starting time S_j (over multiple machines or AGVs) on the corresponding machine (or vehicle)

5.5 Delete operation O_j from the operations network, and add all operations O_l to the feasible list such that $s(l) = O_j$

6. Compute the makespan of the resulting schedule
7. Compute the cost of production combining material, labor, work-in-process and material handling costs (see Appendix, and Agrawal *et al.*, 1996).

A final remark about step 5.3.2: Since the algorithm schedules operations on the critical path(s) first, it may happen that there are two operations on a specific machine (or vehicle) between which there is a gap where additional operation(s) could be scheduled at subsequent steps. The algorithm takes advantage of these gaps and inserts operations in them provided the precedence and capacity constraints are respected, and previously scheduled operations are not disturbed. For workcenters it is easy to check if the machine is available during $[S_j,\ F_j]$. For AGVs this insertion requires making sure that: (i) the gap is adequate to accommodate $[S_j,\ S_j + t_j + K_{w(j)w(i)}]$ where i is the operation that will immediately follow j on the AGV, and (ii) if the previous empty travel $[F_k,\ F_k + K_{w(k)w(i)}]$ can be replaced by $[F_k,\ F_k + K_{w(k)w(j)}]$ where k is the operation that was immediately before i on the AGV. These will verify that the AGVs routing remains connected during the schedule. To illustrate the algorithm, an example is presented in the following section.

4.3 A Detailed Example

Consider the finished product **A** shown in Figure 2 and its corresponding network of operations shown in Figure 4, which includes transportation operations for material transfer between cells. Let product **A** is to be delivered at 50 time units. Assuming one machine per workcenter and one workcenter per cell, the schedule of operations generated by **TIPSA** is presented in Figure 6. The steps and iterations involved in determining the production schedule are presented below and also in Table 1.

Using a forward pass, ES and EF times of all the operations in the network are determined. At the start of the iterations (step 4), the feasible list of operations ‘F’ contains only operation A.20.

Iteration 1: Since operation A.20 is the only operation in the feasible list (column 2 of Table 1) it is selected at step 5.1, and at step 5.2 the tentative finish time of operation A.20 is set to match the delivery time 50 (column 5 of Table 1). This finish time is tentative because constraint (4) of **TISP** has not yet been verified. At step 5.3.1 the tentative latest starting time is determined to be 43. This is verified at step 5.3.2, and the finish time is finally set to 50 (column 6 of Table 1). At step 5.4 operation A.20 is scheduled on workcenter 2 (column 7 of Table 1). At step 5.5 the list of feasible operations is updated and is given by F = $\{$T.12(for A.10), T.12(for B.10)$\}$ (column 8 of Table 1).

Iteration 2: At step 5.1 the operation from the feasible list ‘F’ with the largest EF time is selected, which in this case is the transportation operation T.12 (for B.10) with EF time of 27 (column 3 of Table 1). Step 5.2 sets the tentative finish time of operation T.12(for B.10) equal to 43 (column 5 of Table 1), which is the starting time of its immediate successor or down stream operation A.20. At step 5.3.1 the tentative latest starting time is determined to be 38. At step 5.3.2 the finish time is finalized to be 43, only after the verification of constraint (5) of **TISP** (column 6 of Table 1). At step 5.4, transportation operation T.12(for B.10) is scheduled on the AGV (column 7 of Table 1; see also Figure 6). Finally at step 5.5 the list of feasible operations is determined to be F = $\{$B.10, T.12 (A.10)$\}$ (column 8 of Table 1).

Iteration 3: At step 5.1, operation B.10 is selected (column 4 of Table 1) from the feasible list ‘F’ $\{$B.10, T.12 (A.10)$\}$ because it has the largest EF time of 22 (column 3 of Table 1). Step 5.2 sets the tentative finish time of operation B.10 equal to 38 (column 5 of Table 1), which is the starting time of its immediate successor operation T.12 (for B.10). At step 5.3 the latest starting time is determined to be 38-6 = 32, and the finish time is finalized to be 38 (column 6 of Table 1). Now constraint (4) of **TISP** has been verified. At step 5.4, operation B.10 is scheduled on workcenter 1 (column 7 of Table 1). Finally, at step 5.5 the list of feasible operations is determined to be F = $\{$T.12(A.10), T.21(E.20), T.21(D.20)$\}$ (column 8 of Table 1).

Iteration 4: At step 5.1 operation T.12(A.10) is selected (column 4 of Table 1) from the feasible list ‘F’ due to its largest EF time of 18 (column 3 of Table 1). Step 5.2 sets its tentative finish time equal to the starting time of its immediate successor operation A.20 which is 43 (column 5 of Table 1). At step 5.3.1 the tentative latest starting time is determined to be 43-5 = 38. However, the AGV is busy during the tentative start and finish times. At step 5.3.2 the latest start and finish times are finalized to 30 and 35 respectively (column 6 of Table 1). Note that an empty travel of the AGV is accomplished between operations T.12(A.10) and T.12(B.10). Now constraints (5) and (6) of **TISP** have been verified. At step 5.4, transportation operation T.12(A.10) is scheduled on the AGV (column 7 of Table 1). Finally, at step 5.5 the list of feasible operations is determined to be F = $\{$T.21(E.20), T.21(D.20), A.10$\}$ (column 8 of Table 1).

Note that if the largest value of EF time is the same for more than one operations in the feasible list ‘F’, then in step 5.1 ties are broken arbitrarily (see iterations 5, 7, 8, and 12). The process is repeated till the end of 16^{th} iteration when all the operations are scheduled and the list of feasible operations ‘F’ becomes empty. Now the schedule is complete (see Table 1 for complete steps). The Gantt chart of the resulting schedule is presented in Figure 6, which corresponds to the makespan of 50 time units.

Table 1 around here

On the other hand, sequential scheduling follows a similar iterative procedure for scheduling the operations network of Figure 3, which excludes transport operations. The resulting schedule, which is also the optimal schedule for machines only, is presented in Figure 5. According to this schedule, the "reverse" first-come-first-served (FCFS) ordering of transport operations is T.12(B.10)-T.12(A.10)-T.21(E.20)-T.21(D.20)-T.12(D.10)-T.12(E.10)-T.21(I.10). Further, empty travel operations will be inserted in this sequence whenever the ending cell of an operation is not the same as the starting cell of the consecutive operation. The insertion procedure for this sequence of transport operations is performed in the existing machine schedule. The resulting schedule is presented in Figure 7. The total makespan of this schedule is 57 time units, and is 14% higher than the makespan of the TIPSA schedule.

Figure 7 around here

5 Results and Discussion

The performance of TIPSA was evaluated considering a large number of assembly production examples. Since the problem at hand is NP-hard, optimal results are possible only for small examples which are of limited interest. For examples of industrial dimension, we have considered performance over sequential scheduling. In sequential scheduling, initially the schedule of machines was generated by LETSA (Agrawal *et al.*, 1996), which has shown better performance than many other well-known dispatching/heuristic methods applied to multi-level assembly scheduling. This is followed by scheduling of material handling system using a "reverse" FCFS scheduling rule, i.e., we begin from the end of the schedule of machines and work our way backward, each time inserting the transport (and empty travel) operations in the order in which they are required. The choice of FCFS for scheduling AGVs is two-fold: (i) FCFS is a way we can maintain validity of the initial schedule of machines, and (ii) FCFS is known to provide good schedules (usually second to and at times better than LETSA) for only machine scheduling for multi-level assemblies (Agrawal *et al.*, 1996).

TIPSA treats manufacturing and transport operations alike, and in an attempt to delay operations as late as possible, it implicitly tries to minimize empty travel by selecting the appropriate AGV in step 5.4. This emulates a "reverse" closest vehicle rule when the ideal times are not being achieved. For the sequential scheduling model, two methods of vehicle dispatching were employed. The first method assigned a vehicle to the FCFS transport operation at random. The second method employed a modified nearest vehicle rule (similar to TIPSA) to assign the FCFS transport operation to the AGV which minimizes empty travel. Note that the assignment of manufacturing operations to specific machines as well as their sequence relationships are retained during both these insertion procedures.

Three sets of examples were studied, each with two different ranges for number of cells in the facility structure. The example sets included large BOMs corresponding to assemblies of three different types. Typical parameters for the three types of complex assembly BOMs used are presented in Table 2. Two values for the number of cells are chosen for each BOM type to study the effect of varying the proportions of manufacturing and material handling operations.

Table 2 around here

For all the examples, the part, BOM, routing, cell, and workcenter structure data were generated at random from uniform distributions with specified ranges. For each case of the BOM structure (defined in Table 2), the facility consisted of five to nine cells for the first set of examples, and two to five cells for the second set of examples. In each cell, there were one to three workcenters, each with one to two identical machines. The workcenter data included set-up and run costs per unit time (both chosen randomly between \$20/hour and \$80/hour). AGV operation cost per unit time was also chosen similarly. Purchased parts were assigned a cost between \$160 and \$800. Routings for manufactured parts were made up of five to ten operations, each including set-up and run times. The run times were chosen randomly between 5 and 10 minutes, and set-up times were chosen randomly between 10 and 30 minutes. For each problem instance generated, three values of transport times were chosen by multiplying the range of random run times by a multiplier (also called TR/RU ratio) equal to: 1, 5, and 10. The following two performance measures were employed:

1. Mean percentage improvement in makespan, where

   percentage improvement in makespan = $\frac{(Makespan)_{SEQUENTIAL} - (Makespan)_{TIPSA}}{(Makespan)_{SEQUENTIAL}}$ x 100

2. Mean percentage improvement in cost, where

   percentage improvement in cost = $\frac{(Cost\ of\ Schedule)_{SEQUENTIAL} - (Cost\ of\ Schedule)_{TIPSA}}{(Cost\ of\ Schedule)_{SEQUENTIAL}}$ x 100

It is also possible that for some examples, the makespan (cost) of the sequential schedule is smaller than the makespan (cost) obtained by TIPSA. In this case, the performance measure can take up a negative value. The performance of each model was evaluated at three different transport time to run-time (TR/RU) ratios, and by varying the number of AGVs from 1 to 3 in the MHS, and considering two different ranges for number of cells in the facility structure. Thus, each instance of a BOM model was considered with 18 variations. Varying the (TR/RU) ratio is essential in capturing different manufacturing environments and the effects of their facility sizes (which impact transportation and empty travel times). Considering different number of AGVs is helpful in changing the capacity of the MHS. This variation can also be very helpful in cost/benefit analysis of including additional AGVs in the MHS. If there is a significant reduction in the makespan (or average product cycle-time) by including additional AGVs, and the cost of additional AGVs could be offset by the reduction in makespan (or cycle-time) and WIP cost (over some reasonable planning horizon), then the fleet of AGVs in the MHS could be increased (see also Section 6). Finally, as mentioned earlier, varying the distribution of machines and the number of cells can help change the ratio of manufacturing to material handling operations. The influence of these factors on the performance measures can also be studied using ANOVA (ANalysis Of VAriance) of empirical results.

5.1 Makespan comparisons

To study the performance of integrated scheduling on the makespan of the schedule, the following numerical examples were attempted: (i) 75 examples of WIDE BOMs, (ii) 60 examples of LONG BOMs, and (iii) 69 examples of LARGE BOMs. As mentioned earlier, 18 variations of each example were further explored for the two cases of vehicle dispatching rules.

5.1.1 Random vehicle assignment

In this section the results for makespan comparison are presented when the random vehicle rule was employed for dispatching AGVs in sequential scheduling. The mean percentage improvement in the makespan for the three models are presented in Tables 3 through 5. In majority of the cases, the integrated schedules obtained by TIPSA resulted in shorter makespans than those generated by sequential scheduling for all the models regardless of the (TR/RU) ratio, the number of cells and the number of AGVs. The results indicate that the mean percentage improvement in makespan increases as the (TR/RU) ratio increases for all models. The same improvement is true for the increase in the number of cells. These factors increase the material handling workload, and TIPSA has more opportunity for improvement over sequential scheduling (see top three graphs of Figure 8). Makespan improvements also increase with the increase in the number of AGVs in the system, especially for higher (TR/RU) ratios. The reason for this, even though increasing the number of AGVs reduces material handling workload per vehicle, is the high empty travel in sequential FCFS-based vehicle scheduling. FCFS scheduling is constrained to the sequence of transport operations generated by the machine schedule, and therefore may require excessive empty travel to satisfy AGV routing connectivity in case of random assignment of vehicles. On the other hand, as mentioned earlier, TIPSA employs a "reverse" closest vehicle rule when the ideal times are not being achieved and selects the AGV that requires minimum empty travel. TIPSA also takes more effective advantage of gaps in the critical path scheduling procedure than the sequential procedure (see Section 4.2). However, in models LONG and LARGE at (TR/RU) ratio of 1, the improvement in makespan was not very significant, and did not change much with the increase in the number of AGVs. In fact, in some of these cases, the improvement in makespan deteriorated slightly with the increase in the number of AGVs. This can be explained by the reduction in material handling workload per vehicle and the reduced opportunities to improve over a sequential schedule.

Tables 3, 4 and 5 around here

The percentage of examples which resulted in varied degrees of improvement in makespan over sequential scheduling for all the models and their 18 variations over (TR/RU) ratios, number of AGVs, and number of cells are presented in Table 6.

Table 6 around here

For the random vehicle assignment case, TIPSA resulted in an improvement of the makespan in 96 percent of all the examples tested, in 1 percent of the examples the makespan was same, and in only 3 percent of the examples sequential scheduling generated better schedules than TIPSA. In the few examples that sequential scheduling did better than TIPSA, the makespans were only 3.6% shorter on the average. The slightly better performance of sequential scheduling over TIPSA in these examples is due to the heuristic nature of both the procedures. Finally, even with an average of only three cells in the facility, improvement in makespan was significant at higher (TR/RU) ratios, and is very evident from numerical results of a large number of examples.

These above remarks were further validated using ANOVA studies which revealed that the main effect of each of the parameters (# cells, # AGVs, TR/RU ratio) on the makespan improvement was highly significant. The only significant effect of two-way interaction was for # AGVs and TR/RU ratio, and the three-way interaction was not significant.

5.1.2 Modified nearest vehicle assignment

In this section the results for makespan comparison are presented when the modified nearest vehicle rule was employed for dispatching AGVs in sequential scheduling. The mean percentage improvement in the makespan for the three models are presented in Tables 7 through 9. For WIDE BOMs TIPSA resulted in more than 10 percent improvement in the makespan in 58 percent of all the examples tested. At (TR/RU) ratio of 10, more than 15 percent improvement was achieved in 46 percent of the examples and more than 20 percent improvement in 13 percent of the examples, regardless of number of AGVs and the number of cells in the system.

Tables 7, 8 and 9 around here

In case of LARGE (respectively LONG) BOMs, at (TR/RU) ratio of 10, an improvement of more than 10 percent in the makespan was achieved in 48 (resp. 40) percent of the examples, more than 15 percent in 29 (resp. 24) percent of the examples, and more than 20 percent in 18 (resp. 14) percent of the examples, regardless of number of AGVs and the number of cells in the system.

The results indicate that the mean percentage improvement in the makespan increases with the increase in the (TR/RU) ratio and the number of cells (except in case of model WIDE). As noted in the previous case, these two factors increase the material handling workload, and TIPSA has more opportunity for improvement over sequential scheduling. Makespan improvements decrease with the increase in the number of AGVs in the system because increasing the number of AGVs reduces material handling workload per vehicle. The middle three graphs of Figure 8 also indicate the effect of above variables and show that TIPSA results in better schedules when there is increased workload due to larger number of cells in the facility structure, higher (TR/RU) ratio, and less number of AGVs in the system. The varying relationship between percentage improvement in makespan and (TR/RU) ratio in case of model WIDE (see Figure 8) could be attributed to several factors including the product structure that provides large number of scheduling choices at each step of the algorithms, and the number of cells. For the model WIDE, ANOVA studies revealed that the main effect of # AGVs was not significant, while the two-way interactions of: (i) # Cells and TR/RU ratio, and (ii) # AGVs and TR/RU ratio were significant. For the other models, the main effect of each of the parameters on the makespan improvement was highly significant, and all two-way interactions were not significant. The three-way interaction was not significant in any of the models.

In summary, TIPSA provided improved makespans in 93 percent of all the examples tested, in 2 percent of the examples the makespan was same, and in only 5 percent of the examples sequential scheduling generated better schedules than TIPSA. In the few examples that sequential scheduling did better than TIPSA, the makespans were only 3.88% shorter on the average.

As expected, increasing the number of AGVs resulted in reduced makespan as well as cost (see Section 5.2). We observed that in few examples for integrated scheduling, there was no reduction in makespan as we increased the number of AGVs. These were usually cases where the AGVs were not the bottleneck. It was also noted that in few cases, with the increase in the number of AGVs, while the makespan was reduced, the cost of the schedule increased. This was due to the excessive empty travel time of AGVs between different cells. This happens because the attempt to minimize the makespan by using multiple AGVs might cause an increase in the total empty travel with respect to the single AGV case. It is further noted that no explicit attempt is made in the scheduling procedure to minimize empty travel.

5.2 Cost comparisons

The costs of TIPSA schedules were compared with those of the modified nearest vehicle rule for dispatching AGVs in sequential scheduling. Both TIPSA and the sequential procedure in this case try to minimize to an extent empty travel of vehicles (for the sake of reducing the makespan), and accordingly somewhat minimize the cost of empty travel. The mean percentage savings in cost for the three models are presented in Tables 10 through 12. The results indicate that for all models, the mean percentage saving in the cost increases with an increase in the (TR/RU) ratio.

Tables 10, 11 and 12 around here

TIPSA resulted in more than 10 percent savings in the cost in 88 percent of the examples for WIDE BOMs at higher (TR/RU) ratios. TIPSA resulted in more than 15 percent savings in the cost in 87 percent of the examples, more than 20 percent savings in 72 percent of the examples, more than 25 percent savings in 63 percent of the examples, and more than 30 percent savings in the cost in 30 percent of the examples at the (TR/RU) ratio of 10, regardless of the number of AGVs and the number of cells in the facility structure for model WIDE. At (TR/RU) ratio of 10, the savings in the cost was more than 10 percent in 65 (resp. 36) percent of the examples, more than 15 percent in 40 (resp. 13) percent of the examples, and more than 20 percent in 24 (resp. 5) percent of the examples for LARGE (resp. LONG) BOMs, regardless of the number of AGVs and the number of cells in the facility structure.

In summary, TIPSA resulted in cost savings in 97 percent of all the examples tested, and there was on the average a cost increase of only 1.13 percent in 3 percent of the examples. The savings in cost are more significant at larger (TR/RU) ratios with larger number of cells in the facility structure, and less number of AGVs in the system (see bottom three graphs of Figure 8). This is because with the increase in the number of transport operations and material handling workload (i.e., with MHS system as the potential bottleneck)

TIPSA has a better opportunity to reduce WIP costs as compared to the sequential scheduling procedure. These observations were also validated in the ANOVA study.

Figure 8 around here

6 Practical Applications and Extensions of TIPSA

TIPSA has been developed to support the integrated scheduling of material handling system and machining workcenters for the production of large assemblies. Apart from providing dispatch lists for workcenters and AGVs, it can be used as a production planning and system design support tool. In this section, we outline some such applications, and we also present seminal ideas on the possible extensions of TIPSA.

1. **Scheduling and Planning**: The schedule of operations generated by **TIPSA** can be directly implemented on the shop floor. Due to the low computational time, the algorithm can be used on a rolling horizon basis to account for changes in the product mix as well the changes in the environment. The data generated by **TIPSA**, which are based on the actual finite capacity schedule of manufacturing and transport operations, can be used to provide realistic input for production planning, e.g., changing number of shifts, ordering raw-materials, etc. It can be also be employed to validate shipping schedules before accepting customer orders.
2. **Integrating Lot-sizing**: In a similar framework, Anwar and Nagi (1997) have incorporated lot-sizing in the network scheduling for only manufacturing operations of large assemblies. The procedure attempts to merge a pair of common nodes and reschedules the modified network, and at each iteration accepts the merge that provides the most improvement in the makespan. The process is repeated until there are no common nodes in the network or the makespan cannot be improved further. This work demonstrated significant improvements in the makespan as well as the cost over lot-for-lot schedules (even for small set-ups). This result and the current work demonstrate promising benefits of a three-way integration between lot-sizing, logistics and workcenter scheduling in assembly environments. It is not an exaggeration to expect that the combined benefits of judicious lot-sizing and integrated transportation in shop-floor scheduling could provide surprisingly high (e.g., exceeding 50%) reductions in product cycle times over the methods commonly used in the industry.
3. **AGV Fleet Sizing**: Despite the analytical work in fleet sizing, it is well recognized that the results are usually a lower bound on the number of AGVs, and should be validated by subsequent simulation studies (see e.g., Maxwell and Muckstadt, 1982). The low computational complexity of TIPSA makes it very suitable to be employed in simulation studies to study the effect of varying the number of AGVs in the system. As indicated in Section 5, increasing the number of vehicles in the MHS generally reduces product cycle times. It was also recognized that as the number of vehicles are increased, the processing workcenters become the usual bottlenecks, and further reductions in cycle times are not significant. TIPSA can be employed as an efficient scheduling engine to solve the integrated problem over a large simulation horizon, and this can be repeated for different fleet sizes. The reduction in

average product cycle time can then be plotted as a function of the fleet size. A similar plot can be generated for average production costs, which includes material, labor, WIP, and loaded and empty transportation costs. These results can be helpful in providing input to an engineering economic analysis for fleet sizing. Of course, similar studies can be performed for determining the optimal number of machines in manufacturing workcenters.

4. **Integrating Dynamic Routing**: As discussed in Section 2, extensive research has been accomplished on dynamic routing and collision avoidance in AGV systems. This vast body of research has been motivated by a desire to maximize the benefits derived from expensive AGV systems, while automating complex decisions of routing, collision avoidance and traffic management. This is especially important when there are multiple vehicles serving the transportation requirements of the manufacturing/assembly shop. This research demonstrates, albeit via empirical studies, the significant benefits of integrating logistics and scheduling on the shop-floor. It is hoped that this demonstration can be enriched by incorporating these powerful strategies of dynamic routing (path planning) and collision avoidance for AGV control.

7 Conclusions

The problem of integrated material handling system and machine (processor) scheduling in a general job-shop that produces large assemblies, where both capacity and precedence constraints are critical, was addressed. The transport integrated scheduling problem was formulated as an integrated mathematical programming problem. An effective heuristic was developed for simultaneous scheduling of machines and material handling system. The proposed heuristic employs a critical-path-based scheduling approach and a Just-In-Time methodology in which operations are scheduled as late as possible (i.e., only when they are required), but without violating due dates, in order to minimize the production makespan of large and complex assemblies as well as WIP costs. To evaluate the results of the proposed heuristic, the improvement in makespan and cost were considered over the sequential scheduling of machines followed by the scheduling of AGVs to process machine requests. The initial schedule was obtained by LETSA (which performs better than many other commonly used scheduling heuristics), then AGVs were scheduled to process machine requests on a FCFS basis.

A large number of numerical examples showed that the schedules obtained by TIPSA result in considerable improvement in makespan and cost over the sequential scheduling of machines followed by scheduling of AGVs. In a situation, where excessive material handling operations are involved and the material handling equipment/AGVs are also the dynamic bottlenecks, the schedule obtained by TIPSA would result in very significant improvement in makespan as well as cost over the sequential scheduling approach.

The proposed scheduling approach has multiple applications in practical production environments. Besides its application in production planning and control, the proposed heuristic can be very effectively used in cost/benefit analyses related to fleet-sizing, i.e., to determine the optimal number of AGVs in the MHS, while minimizing the makespan and the cost of the overall system schedule.

Some of the assumptions used in the development of this methodology can be generalized to represent more practical production situations. Further research could be directed to extend our approach of integrated scheduling of machines and material handling system to include lot-sizing. The other useful extension of this work would be the modification in integrated scheduling to account for conflict free routing of AGVs on the travel paths.

Appendix

Costing a Schedule

The production cost as well as work-in-process (WIP) costs are computed from the schedule of operations. The cost calculations are similar to that of Agrawal *et al.* (1996), and included here for the sake of completeness. For computing the total cost, we introduce:

L = Labor and machine rate per unit time; for simplicity we assume it to be equal for all operations.
M_j = Raw material cost at the beginning of a starting operation O_j. (Note that, starting operations of the network do not have predecessors.)
r = Interest rate compounded per hour (assumed to be 20% APR in our numerical tests).
$s(j)$ = Successor or immediate downstream operation of operation j.
q_j = Number of units of operation j required for one unit of its successor operation $s(j)$.

Let $C(x) = L[(1+r)^x - 1]/r$ be the function for compounding interest at a rate of L (equal payment series compound amount factor) for calculating the cost of transportation and processing operations, and let $I(x) = (1+r)^x$ be the function for compounding interest (single payment series compound amount factor) for calculating WIP cost. When (B) is the batch size of the end item, then processing time of each operation in the network is a function of batch size (B), and $x = F_j(B) - S_j(B) = t_j(B)$; and $t_j(B) = (\delta_j + B \times q_j \times r_j)$, where δ_j is the setup time for the batch at operation O_j and r_j is the run time per operation O_j.

The total cost of the (sub-)assembly till operation O_j can be computed from the schedule of operations using the following recursive expressions:

$$T_j(B) = \begin{cases} C(t_j(B)) + \sum_{l:\, s(l)=O_j} T_l(B) I(F_j(B) - F_l(B)) & \text{if } \exists l : s(l) = O_j \\ C(t_j(B)) + M_j I(t_j(B)) & \text{otherwise} \end{cases}$$

The cost function is also presented graphically in Figure 9 to show how the cost is accumulated at each step with the passage of time. For operation 10 shown in the figure, the total cost comprises three components, the initial raw material cost, the labor/operation cost (linear component), and the interest (non-linear component). During the idle time between operation 10 and loaded transportation operation, the value of the part is incremented by its WIP cost. During the transportation operation, the cost of the part is incremented by actual operation cost of AGV and the WIP cost. Whenever empty travel is required for repositioning of AGV, it will always be accompanied by a loaded transportation operation. Empty AGV travel incurs only the operating cost (linear component) and it has to be added to the total cost of the actual transportation operation to determine the exact cost of the part. Note that, during the AGV empty travel, WIP cost will continue to pile up for the components which are waiting to be transported to other machine cells. This concurrency has not been shown in the illustration.

Figure 9 around here

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Table 1: Steps of **TIPSA** for scheduling of product **A**

Iteration No.	**Operation from ‘F’**	**Early Finish Time**	**5.1**	**5.2**	**5.3**	**5.4**	**5.5**
			Operation Selected	**Tentative Finish**	S_j, F_j	**W.C. Selected**	**Feasible List ‘F’**
1	A.20	34	A.20	50	43, 50	2	{T.12(B.10), T.12(A.10)}
2	T.12(B.10) T.12(A.10)	27 18	T.12(B.10)	43	38, 43	AGV	{B.10, T.12(A.10)}
3	B.10 T.12(A.10)	22 18	B.10	38	32, 38	1	{T.12(A.10), T.21(D.20), T.21(E.20)}
4	T.12(A.10) T.21(E.20) T.21(D.20)	18 16 16	T.12(A.10)	43	30, 35	AGV	{A.10, T.21(D.20), T.21(E.20)}
5	T.21(E.20) T.21(D.20) A.10	16 16 13	T.21(E.20)*	32	27, 30	AGV	{A.10, T.21(D.20), E.20}
6	T.21(D.20) E.20 A.10	16 13 13	T.21(D.20)	32	19, 22	AGV	{A.10, D.20, E.20}
7	E.20 D.20 A.10	13 13 13	A.10*	30	24, 30	1	{C.10, D.20, E.20}
8	E.20 D.20 C.10	13 13 7	E.20*	27	24, 27	2	{C.10, D.20, T.12(E.10)}
9	D.20 C.10 T.12(E.10)	13 7 10	D.20	19	18, 19	2	{C.10, T.12(E.10), T.12(D.10)}
10	C.10 T.12(E.10) T.12(D.10)	7 10 12	T.12(D.10)	18	13, 18	AGV	{C.10, T.12(E.10), D.10}
11	C.10 T.12(E.10) D.10	7 10 7	T.12(E.10)	24	5, 9	AGV	{C.10, D.10, E.10}
12	C.10 D.10 E.10	7 7 5	D.10*	13	6, 13	1	{C.10, E.10}
13	C.10 E.10	7 5	C.10	24	21, 24	1	{E.10, T.21(I.10)}
14	E.10 T.21(I.10)	5 4	E.10	5	0, 5	1	{T.21(I.10)}
15	T.21(I.10)	4	T.21(I.10)	21	2, 5	AGV	{I.10}
16	I.10	1	I.10	2	1, 2	2	{ }

*Ties broken at random

Table 2: Parameters for numerical examples

Parameter	**BOM type**					
	WIDE		**LONG**		**LARGE**	
	MEAN	**RANGE**	**MEAN**	**RANGE**	**MEAN**	**RANGE**
Levels	5.12	4-7	8.35	5-11	10	6-14
Parts	90.8	22-239	32	12-60	43	15-115
Make Parts	40.6	12-126	13.5	6-26	29.3	9-82
No. of Cells (1)	7.2	5-9	7.1	5-9	7	5-9
No. of Cells (2)	3	2-5	3.1	2-5	3.1	2-5

Table 3: Percentage improvement in Makespan for Model WIDE*

Cells 5-9 WIDE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
WIDE-1	1	11.65	5.88	16.83	10.55	18.03	10.87
WIDE-2	5	8.69	7.33	42.63	9.49	55.71	6.24
WIDE-3	10	15.40	4.13	43.69	5.43	61.04	3.19
Cells 2-5 WIDE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
WIDE-1	1	9.45	6.15	10.05	6.32	10.14	6.42
WIDE-2	5	12.57	6.05	38.15	11.66	48.52	12.46
WIDE-3	10	10.11	7.24	43.49	6.34	58.12	7.12

Table 4: Percentage improvement in Makespan for Model LONG*

Cells 5-9 LONG	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LONG-1	1	2.89	3.45	2.71	3.65	2.78	3.66
LONG-2	5	6.77	8.87	16.77	11.85	20.50	15.95
LONG-3	10	13.15	8.04	26.57	11.91	31.67	15.86
Cells 2-5 LONG	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LONG-1	1	2.95	6.67	1.38	1.80	1.38	1.80
LONG-2	5	4.85	10.75	11.12	12.93	13.05	13.79
LONG-3	10	9.60	10.35	18.82	15.07	21.86	17.53

Table 5: Percentage improvement in Makespan for Model LARGE*

Cells 5-9 LARGE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LARGE-1	1	2.59	4.09	3.54	5.44	3.20	4.90
LARGE-2	5	12.22	12.25	19.51	13.43	21.60	16.86
LARGE-3	10	15.65	9.45	29.81	16.43	33.12	19.53
Cells 2-5 LARGE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LARGE-1	1	1.67	2.98	1.55	3.39	1.57	3.47
LARGE-2	5	7.87	9.14	12.21	10.55	13.10	12.68
LARGE-3	10	14.53	12.39	22.31	14.52	23.55	16.50

Table 6: Degree of Improvement in Makespan by Percentage of Examples

MODEL	**Cells 5-9**	**Percentage of Examples for Improvement in Makespan**						**Cells 2-5**	**Percentage of Examples for Improvement in Makespan**					
	AGV	[5, 10)%	[10, 20)%	[20, 30)%	[30, 40)%	[40, 50)%	> 50%	**AGV**	[5, 10)%	[10, 20)%	[20, 30)%	[30, 40)%	[40, 50) %	> 50%
WIDE-1	**1**	28	48	8				**1**	32	36	8			
	2	20	24	32	8	4		**2**	24	36	8			
	3	16	36	28	8	4		**3**	24	36	8			
WIDE-2	**1**	40	24	12				**1**	5	60	8			
	2				44	44	12	**2**	2		4	44	36	8
	3				4	8	88	**3**		4	4	12	32	48
WIDE-3	**1**	4	84	8				**1**	36	40	8			
	2			4	4	88	4	**2**			4	20	56	20
	3						100	**3**				4	8	88
LARGE-1	**1**	4	5					**1**	4		5			
	2	4	5					**2**	4					
	3	13	5					**3**	4					
LARGE-2	**1**	4	30	5				**1**	9	10	10			
	2	9	30	10	5	10		**2**	17	20	10	5	5	
	3	9	35	15	10	5	5	**3**	17	15	10	10	5	
LARGE-3	**1**	17	35	15	5			**1**	13	15	25			
	2	4	35	25	20	10	5	**2**	9	12	20	10	5	5
	3	4	25	35	15	10	15	**3**	9	25	15	5	10	10
LONG-1	**1**	15	9					**1**	10	9				
	2	15	13					**2**	5	9				
	3	15	9					**3**	5	9				
LONG-2	**1**	30	17	22	9			**1**	30	26	13			
	2	35	17	35	9	9		**2**	10	39	4	4	4	
	3	30	17	30	9		13	**3**	20	39	4	4	4	
LONG-3	**1**	25	35	22	13			**1**	20	17	22	13		
	2		13	9	22	35	4	**2**	30	13	22	30	4	
	3		13	4	22	26	18	**3**	25	13	22	20	9	4

Table 7: Percentage improvement in Makespan for Model WIDE

Cells 5-9 WIDE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
WIDE-1	1	11.65	5.88	7.85	7.08	8.31	5.84
WIDE-2	5	8.69	7.33	10.34	7.41	12.56	7.15
WIDE-3	10	15.40	4.13	11.29	7.16	16.07	7.28
Cells 2-5 WIDE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
WIDE-1	1	9.45	6.15	6.25	5.17	5.89	4.51
WIDE-2	5	12.57	6.05	15.21	5.18	15.46	9.30
WIDE-3	10	10.11	7.24	13.68	5.92	16.63	6.41

Table 8: Percentage improvement in Makespan for Model LONG

Cells 5-9 LONG	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LONG-1	1	2.89	3.45	0.97	1.25	0.79	1.27
LONG-2	5	6.77	8.87	5.36	6.46	5.51	6.06
LONG-3	10	13.15	8.04	10.45	7.51	9.43	6.82
Cells 2-5 LONG	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LONG-1	1	2.95	6.67	0.49	0.79	0.44	0.78
LONG-2	5	4.85	10.75	4.28	5.66	4.69	4.37
LONG-3	10	9.60	10.35	8.84	7.57	8.00	7.25

Table 9: Percentage improvement in Makespan for Model LARGE

Cells 5-9 LARGE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LARGE-1	1	2.59	4.09	1.83	3.12	1.05	3.01
LARGE-2	5	12.22	12.25	6.78	5.65	4.44	4.81
LARGE-3	10	15.65	9.45	12.14	11.14	8.41	6.89
Cells 2-5 LARGE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LARGE-1	1	1.67	2.98	0.85	3.35	0.75	3.46
LARGE-2	5	7.87	9.14	4.47	4.31	2.95	4.51
LARGE-3	10	14.53	12.39	8.88	7.10	5.49	5.29

Table 10: Percentage saving in Cost for Model WIDE

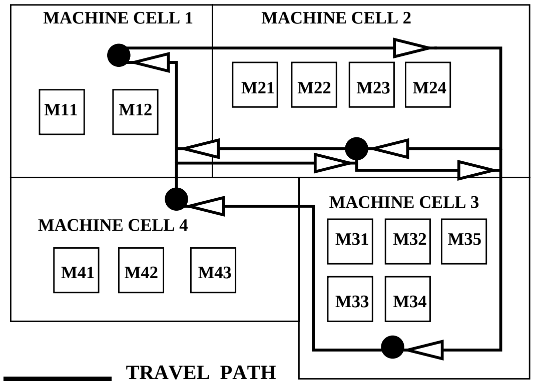
Cells 5-9 WIDE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
WIDE-1	1	3.42	4.54	2.20	1.35	1.46	0.77
WIDE-2	5	27.1	16.34	18.69	5.23	15.72	5.03
WIDE-3	10	38.88	8.90	29.26	6.02	26.03	6.17
Cells 2-5 WIDE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
WIDE-1	1	2.37	1.57	0.98	0.66	0.86	0.57
WIDE-2	5	16.46	5.46	12.99	5.36	10.29	4.78
WIDE-3	10	24.69	7.18	20.22	5.25	18.17	5.54

Table 11: Percentage saving in Cost for Model LONG

Cells 5-9 LONG	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LONG-1	1	0.58	0.64	0.46	0.50	0.49	0.55
LONG-2	5	8.23	5.31	5.24	3.80	4.53	3.05
LONG-3	10	16.67	7.47	10.59	5.73	9.08	4.96
Cells 2-5 LONG	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LONG-1	1	0.39	0.39	0.25	0.41	0.32	0.37
LONG-2	5	4.75	4.09	2.69	2.35	2.38	2.46
LONG-3	10	9.74	7.07	6.80	4.94	5.48	3.73

Table 12: Percentage saving in Cost for Model LARGE

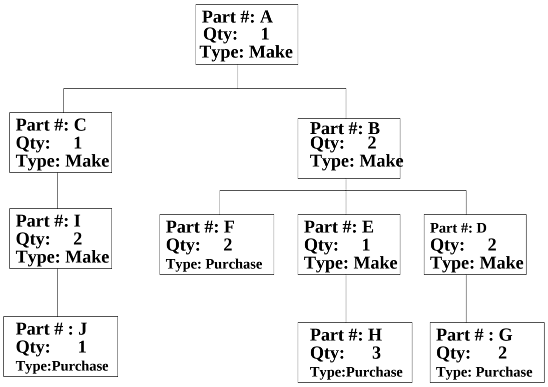
Cells 5-9 LARGE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LARGE-1	1	1.24	1.11	0.60	0.49	0.65	0.57
LARGE-2	5	14.05	8.16	7.58	4.42	5.30	3.93
LARGE-3	10	28.38	14.50	16.40	8.11	12.49	8.07
Cells 2-5 LARGE	**(TR/RU) ratio**	**No. of AGV=1**		**No. of AGV=2**		**No. of AGV=3**	
		MEAN	**STD**	**MEAN**	**STD**	**MEAN**	**STD**
LARGE-1	1	0.72	0.75	0.12	1.10	0.41	0.43
LARGE-2	5	7.88	4.56	4.30	2.79	3.04	2.53
LARGE-3	10	16.82	9.43	9.40	4.91	7.69	4.76



F \ T	1	2	3	4
1	0	5	7	10
2	3	0	4	6
3	5	5	0	3
4	2	2	6	0

DISTANCE TABLE

Figure 1: Layout



Part	Operation	Components Required	Processing Time	Workcenter	Cell_no
A	A. 10	C	6	WC # 1	1
	A. 20	B, C	7	WC # 2	2
B	B. 10	D, E, F	6	WC # 1	1
C	C. 10	I	3	WC # 1	1
D	D. 10	G	7	WC # 1	1
	D. 20	G	1	WC # 2	2
E	E. 10	H	5	WC # 1	1
	E. 20	H	3	WC # 2	2
I	I. 10	J	1	WC # 2	2

The processing time includes both the set-up time and the run time of the entire batch

Figure 2: Bill-of-Materials (BOMs) of final product A and routings of its make items

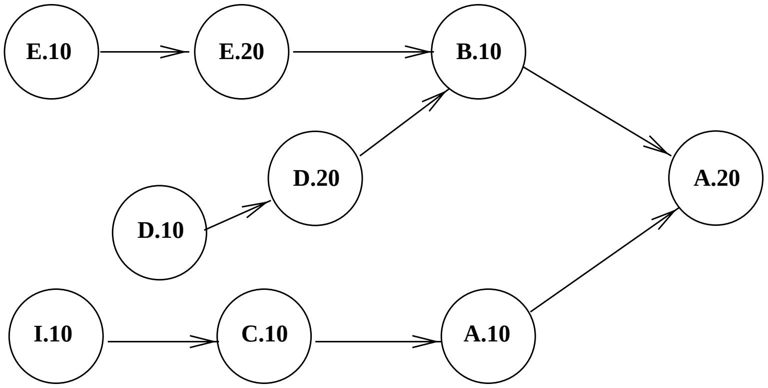


Figure 3: Operation Network for BOMs shown in Figure 2

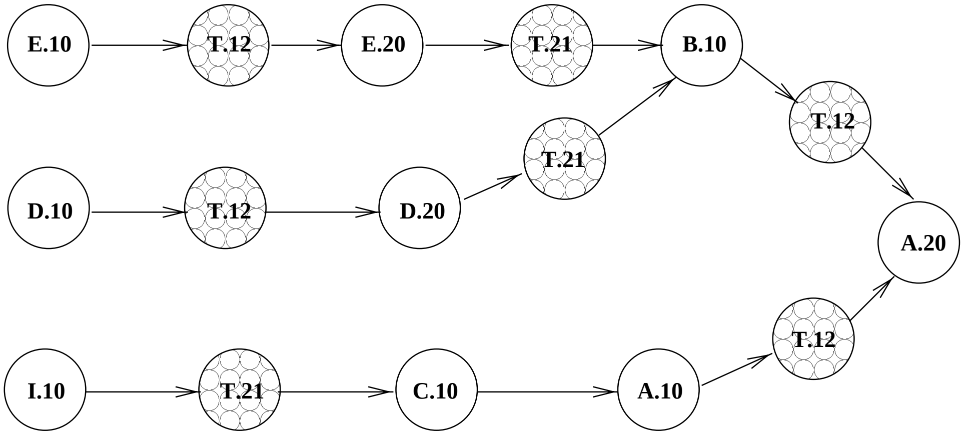


Figure 4: Operation Network for BOMs shown in Figure 2 including transportation operation

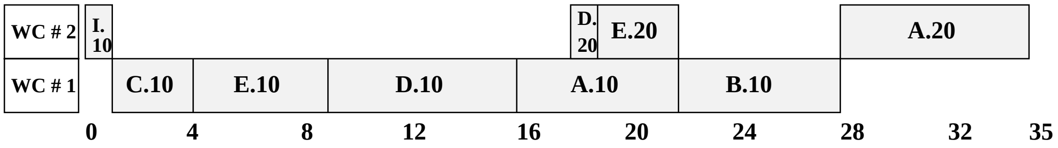


Figure 5: Gantt Chart of the Optimal Schedule for the example given in Figure 1 (without transportation operations)

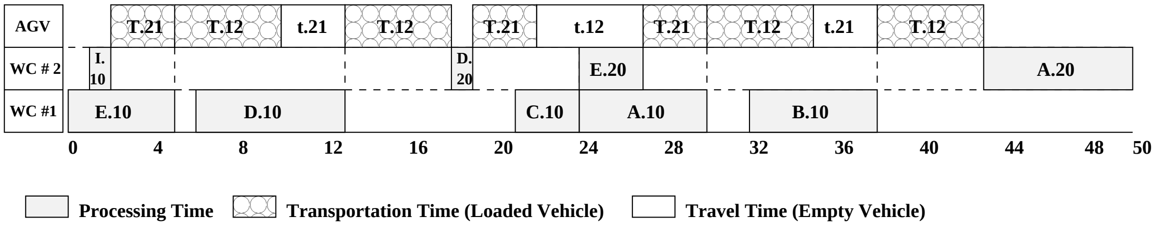


Figure 6: Gantt Chart of the Optimal Schedule generated by TIPSA for the example given in Figure 1 (including transportation operations)

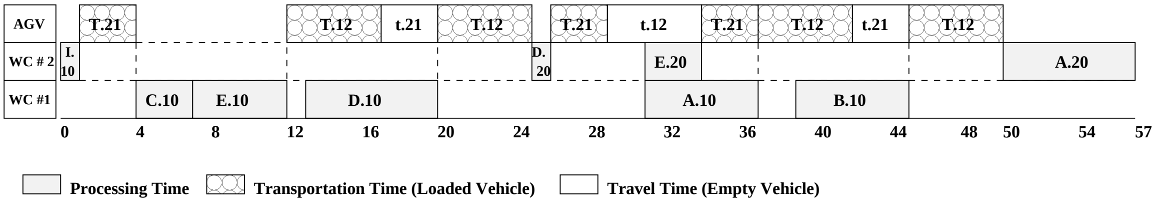


Figure 7: Gantt Chart of the Sequential Schedule for the example given in Figure 1 (including transportation operations)

Figure 8: Percentage Improvement in Makespan and Cost

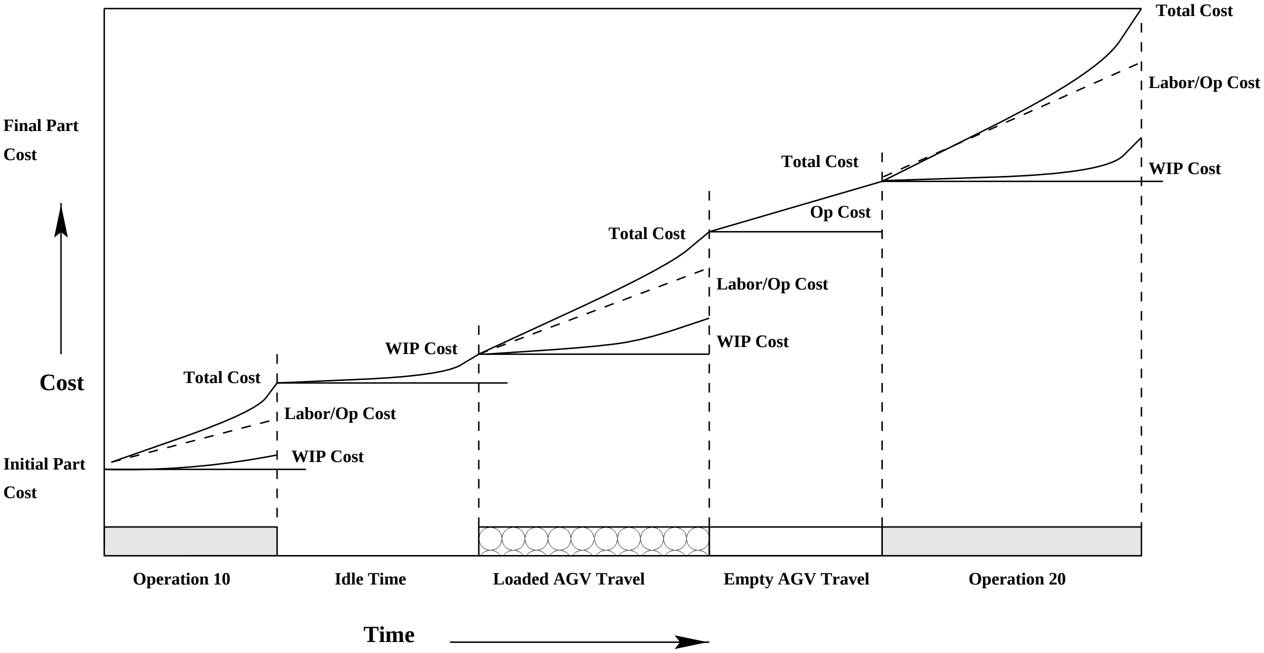


Figure 9: Example of the Cost Function