IIE Transactions (2002) **35**, 000–000

Variable capacity sizing and selection of connections in a facility layout

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Received September 2001 and accepted May 2002

The variable capacity sizing and selection of connections in the facilities design context is discussed (to the best knowledge of the authors) for the first time in the open literature. A connection is defined as the connected part that links different sets of departments through which some interdepartmental material flows must go through. The goal of the problem is to select the location and capacity of the connections (and to assign the flows) so as to minimize the sum of the fixed connection installation costs and material movement cost in the material handing system. Mathematical programming formulations are presented for continuous and discrete capacity options. For the continuous unbounded capacity case, we prove that it can be reduced to the uncapacitated fixed charge facility location problem. For the discrete capacity case, a Lagrangian relaxation-based solution approach is developed. It provides a 'good' feasible solution as well as a lower bound for assessing the optimality gap. Computational results are reported. Our findings indicate that the discrete version of the problem can be effectively solved with the Lagrangian heuristic.

1. Introduction

Manufacturing facility design, in the most general case, involves the arrangement of a fixed number of departments so as to optimize a certain performance measure, such as travel time or manufacturing costs. This paper discusses the specific problem of variable capacity sizing and selection of connection parts within a facility layout. We use the term 'connection' in a generic sense. It is defined as the connected part that links different sets of departments. Some material flows have to pass through at least one of the connections, much like they have to pass through a material exchange point or aisle. The term may represent a variety of scenarios. Some examples are the following: (i) an Input/Output (I/O) station of a department that connects the department with its outside environment, also known as ingress/egress or pickup/dropoff for material; (ii) an aisle that connects different sets of departments; and (iii) a staging, inspection and distribution station. In all cases, the location and capacity sizing of the connections of a plant have a strong impact on the efficiency of the manufacturing Material Handling System (MHS). Suppose that we have a set of candidate connection sites. For each candidate connection, we have to decide its capacity size in terms of the volume of material that can flow through it. This would relate to the size of the door/gate or width of an aisle, for example. Different capacities have different construction costs associated with them. The goal of this paper is to develop an approach to select the location and the capacity of connections in order to minimize the sum of the fixed connection installation costs and proportional material movement costs in the MHS. We call it the variable capacity sizing and selection of connections problem. A special case of this system is illustrated in Fig. 1, where there are four departments and three candidate connections. We have to select the connections to open, and for each chosen connection, we have to determine its capacity. The lines that connect departments and connections are possible flow paths.

A large number of papers in the literature have considered capacity in the context of a material flow network design problem. Magnanti and Wong (1984) provide a comprehensive survey on the application of network design models and their resolution by mathematical programming techniques. Some capacitated models were discussed there. Other research works related to the capacity of a network include the papers by Khang and Fujiwara (1991), Herrmann *et al.* (1995) and Herrmann *et al.* (1996). All of those models assume that the capacity of the network links is known. On the other hand, some research focuses on the location of input/output stations, pickup stations, etc. Montreuil and Ratliff (1988) proposed a methodology for characterizing and locating

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Journal : IIET SPS Article No. : 010006 Dispatch : 8-7-2002 Pages : 11
PIPS No. : ☐ LE ☐ TYPESET
MS Code : IIET 010006 ☐ CP ☑ DISK

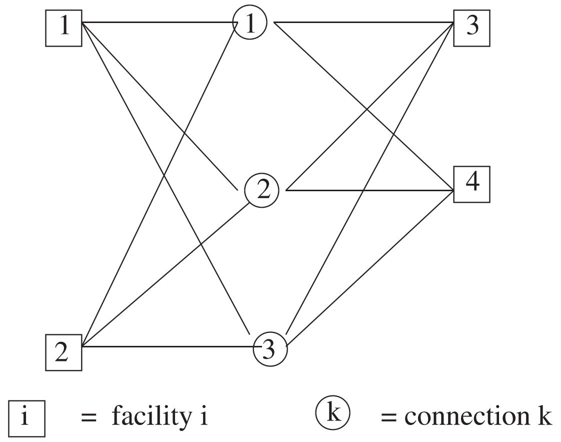

Fig. 1. An example.

input/output stations within a facilities layout. A linear programming model for optimizing the station locations under rectilinear distance and with rectangular boundary regions was presented. Kiran and Tansel (1989) developed a procedure to determine the best locations of pickup and delivery stations along a predefined flow path so as to minimize the system's operational cost. Some strongly polynomial solution methods were presented. Luxhoj (1991) presented a procedure, practical layout planning, to determine the location of facility ingress/egress points of departments in a manufacturing system. Benson and Foote (1997) provided a new distance metric, the shortest path distance between departments along aisles, to layout aisles and door locations.

Few researchers consider the capacity sizing of the connections. The fact remains, however, that connection capacity is a critical factor in many material handling systems. Traffic congestion in the system results in costly delays. Connection capacity will affect the selection of the connections and the assignment of material flows to the connections. The purpose of this paper is to introduce the variable capacity sizing and selection of connections problem, to analyze its properties, and to present efficient solution methods for solving practical versions of the problem.

The rest of this paper is organized as follows: Section 2 provides some preliminaries. Section 3 analyzes the case of continuous capacity, and develops properties of this problem that help reduce it to an uncapacitated fixed charge facility location problem. Section 4 details the discrete capacity case, which includes the development of a Lagrangian relaxation algorithm. Section 5 details our computational experience with these models. It aims at comparing the continuous and discrete cases, and also at detailing computational experience with the Lagrangian heuristic for the discrete case. Finally, Section 6 provides a summary and gives directions for future work. For ease of readability, the proofs of all properties and theorems have been placed in the Appendix.

2. Preliminaries

We assume that connections will be selected from a set of candidate sites K and let $|K| = n_K$, where $|\cdot|$ denotes a set's cardinality. Let $G = (N, A)$ be the material handling network, where N is the set of departments in the system, with $|N| = n_N$, and A is the arc set of material handling flow paths of the system. The term f_{ij} represents the flow from department i to j, for $(i, j) \in A$. The unit of flow could be truck trip, a pallet, or a unit part, etc., based on the particular situation. We assume that $f_{ij} > 0$ for all $(i, j) \in A$. This will simplify our presentation of the formulation, proofs and analysis. If some f_{ij} are equal to zero we can reformulate the problem by only defining decision variables for i, j combinations that have $f_{ij} > 0$. Flows have to go through one of the connections. We assume that the flow volumes are symmetric, i.e., $f_{ij} = f_{ji}$. The terms d_{ik} and d_{kj} represent the shortest distances from department i to connection k and from connection k to department j, respectively. We assume that the connections allow bi-directional flow. Other variations (e.g., unidirectional flows) can be analyzed in a similar manner. We will use the notation $Z^{\mathrm{P}}(\mathbf{x})$ to denote the objective function of a certain problem, where (P) indicates the problem and $\mathbf{x}$ indicates the vector of decision variables. The definitions of the problems considered in this paper can be found in Appendix A, Table A1.

Two basic cases will be considered. In the first one, we assume that the capacity of the connections is a continuous variable and the fixed connection installation costs are a linear function of location and capacity. In this case, we have two models based on whether or not the continuous capacity variable has an upper bound. The other case assumes that the capacity of the connections is selected from a set of discrete options.

3. The continuous capacity case

Suppose that the fixed connection installation costs include two components: The cost related to location, F_k, and the cost related to capacity, vc_k, where v is the cost of unit capacity and c_k is a continuous capacity variable. We assume that F_k depends on the location k because the potential connection sites may have different clearing costs of freeing up space (or important levels) in the existing layout. On the other hand, we assume that unit installation costs are the same throughout the layout. This is a reasonable assumption in the case of a shop floor setting (we note that this may not be true in urban location settings, where not only real estate but construction costs are site dependent). Thus the fixed installation cost of connection k is $F_k + vc_k$. The fixed installation costs are incurred at the beginning of the design horizon whereas the material handing costs are incurred on a uniform basis over the expected design horizon. To model this, we let α be the equal payment series present worth factor that translates, to net present worth, the cost per unit distance per unit flow evaluated over the design horizon. The parameter depends on the discount (interest) rate and its formula can be found in most engineering economics textbook, e.g., Park (2002).

Our decision variables are x_{ijk}, the fraction of flow *i-j* via connection $k \in K$; y_k which is an indicator variable where

$$y_k = \begin{cases} 1 & \text{if a connection is located at candidate site } k, \\ 0 & \text{otherwise,} \end{cases}$$

and c_k (defined earlier). For notational convenience we define $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{c}$ as vectors that denote the collection of decision variables x_{ijk}, y_k, and c_k, respectively.

Two models will be discussed in this section. In the first one, we assume that the continuous capacity variable has no upper bound. In the second one, there is an upper bound for the continuous capacity variable. This is done to model practical limits on connection size, e.g., width of an aisle or a door, or space available at a site.

3.1. *The unbounded case*

The formulation for this case is as follows:

$$(\mathrm{P}_1) \quad \min_{\mathbf{x},\mathbf{y},\mathbf{c}} Z^{\mathrm{P}_1}(\mathbf{x},\mathbf{y},\mathbf{c}) = \sum_{(i,j)\in A} \sum_{k \in K} \alpha f_{ij}(d_{ik} + d_{kj}) x_{ijk} + \sum_{k\in K} y_k(F_k + vc_k), \tag{1}$$

subject to

$$\sum_{k\in K} x_{ijk} = 1, \qquad \forall\,(i,j) \in A, \tag{2}$$

$$\sum_{(i,j)\in A} f_{ij}x_{ijk} \le c_k y_k, \qquad \forall\, k \in K, \tag{3}$$

$$x_{ijk} \ge 0,\ y_k \in \{0,1\},\ c_k \ge 0, \qquad \forall\,(i,j) \in A,\ k \in K. \tag{4}$$

The objective function (1) minimizes the total cost, which is the sum of the fixed connection costs and the total flow-weighted distance multiplied by the unit cost. It is a nonlinear function. Constraint (2) stipulates that the flows only travel through connections. Constraint (3) is a nonlinear capacity constraint. Constraint (4) represents the integrality and non-negativity constraints.

The problem (P_1) is a nonlinear mixed integer programming problem and is therefore difficult to solve directly (Nemhauser and Wolsey, 1988). We establish some properties of (P_1) with the hope of finding an effective solution procedure for this problem. See Appendix B for detailed proofs.

Property 1. *If* $(\mathbf{x},\mathbf{y},\mathbf{c})$ *is an optimal solution for the problem* (P_1), *then* $\forall k \in K$, $\sum_{(i,j)\in A} f_{ij}x_{ijk} = c_k y_k$.

It is easy to see that the property is true because the optimal solution could not have unused excess capacity, since we have to pay for that unused capacity. We can infer from this property that the optimal total capacity should be equal to the total flow, that is, $\sum_{k\in K} \sum_{(i,j)\in A} f_{ij}x_{ijk} = \sum_{(i,j)\in A} f_{ij} = \sum_{k\in K} c_k y_k$. Thus the term $v\sum_{k\in K} c_k y_k$ in the objective function is a constant if the solution is optimal.

Property 2. *There exists an optimal solution for the problem* (P_1) *in which* $x_{ijk} = 0$ *or* 1, $\forall (i,j) \in A, k \in K$, *if the capacities have no upper bound limit.*

According to Property 2, any flow will be fully assigned to the chosen open connection. Thus, the assignment variables, x_{ijk}, will naturally assume integer values.

To facilitate the development of an efficient solution strategy for (P_1), we consider a related uncapacitated fixed charge connection location problem, (P_1'). It is defined as follows:

$$(\mathrm{P}_1') \quad \min_{\mathbf{x},\mathbf{y}} Z^{\mathrm{P}_1'}(\mathbf{x},\mathbf{y}) = \sum_{(i,j)\in A} \sum_{k\in K} \alpha f_{ij}(d_{ik} + d_{kj})x_{ijk} + \sum_{k\in K} F_k y_k,$$

subject to

$$\sum_{k} x_{ijk} = 1, \qquad \forall\,(i,j) \in A,$$

$$x_{ijk} \le y_k, \qquad \forall\, k \in K,$$

$$x_{ijk} \ge 0,\ y_k \in \{0,1\} \qquad \forall\,(i,j) \in A,\ k \in K.$$

This can also be viewed as an uncapacitated fixed charge facility location problem if we treat each flow as a demand node and each connection as a facility.

We now claim that the problem (P_1) can be solved by the following two-step procedure:

Step 1. Solve the uncapacitated fixed charge connection location problem (P_1').
Step 2. Set $c_k = \sum_{(i,j)\in A} f_{ij}x_{ijk}, \forall k$.

Theorem 1. *The solution of the two-step algorithm is optimal for the problem* (P_1).

We conclude that the solution methods for the uncapacitated fixed charge facility location problem discussed in Mirchandani and Francis (1990) can be applied to solve (P_1'). In particular, we suggest the use of Erlenkotter's method (Erlenkotter, 1978). He used a dual-based procedure to obtain a near-optimal solution of the dual and the complementary slackness conditions to improve the bound.

A simple example is presented in the Appendix C (Example 1) to illustrate: (i) the procedure to obtain an optimal solution; and (ii) the optimal solution's properties as mentioned above.

3.2. *The bounded case*

In this subsection, we assume that there is an upper bound for the continuous capacity variable. Let M_k be the known upper bound of the capacity for connection k. The value of M_k might be determined, for example, from the space available at site k. This new problem (P_2) is the same as problem (P_1) except that we need to add a constraint set

$$c_k \le M_k, \qquad \forall\, k \in K.$$

Let (P'_2) be the capacitated fixed charge connection location problem related to (P_2). It is defined as follows:

$$(\mathrm{P}'_2) \quad \min_{\mathbf{x},\mathbf{y}} Z^{\mathrm{P}'_2}(\mathbf{x},\mathbf{y}) = \sum_{(i,j)\in A} \sum_{k\in K} \alpha f_{ij}(d_{ik}+d_{kj})x_{ijk} + \sum_{k\in K} F_k y_k,$$

subject to

$$\sum_{k} x_{ijk} = 1, \qquad \forall\,(i,j)\in A,$$

$$\sum_{(i,j)\in A} f_{ij}x_{ijk} \le M_k y_k, \qquad \forall\, k\in K,$$

$$x_{ijk} \ge 0,\ y_k \in \{0,1\} \quad \forall\,(i,j)\in A,\ k\in K.$$

This can also be viewed as a capacitated fixed charge facility location problem if we treat each flow as a demand node and each connection as a facility. Daskin (1995) provides some methods to solve problem (P'_2), including those that use Lagrangian relaxation. Note that Property 2 may not hold in this case. Using a similar argument to that presented in Section 3.1, we can establish that (P_2) can be solved by the following two-step procedure:

Step 1. Solve the capacitated fixed charge connection location problem (P'_2).
Step 2. Set $c_k = \sum_{(i,j)\in A} f_{ij}x_{ijk},\ \forall k$.

4. The discrete capacity case

4.1. *Formulation*

In many applications, the capacity variable is likely to have just a few discrete choices. For instance, doors/gates might come in standard size options. Suppose that L is the capacity option set and $|L| = n_L$. For simplicity in presentation, we assume that there are an equal number of capacity options for each connection (this can be achieved by using an infinite fixed cost when fewer options are provided). We let c_{kl} represent the lth capacity option of a connection at candidate site k if a connection is located there; F_{kl} denotes the fixed cost of locating a connection at candidate site k with capacity option l; x_{ijk} is the fraction of flow i-j via connection $k \in K$; and

$$y_{kl} = \begin{cases} 1 & \text{if a connection is located at candidate site } k \text{ with capacity option } l, \\ 0 & \text{otherwise.} \end{cases}$$

This problem can be formulated as follows:

$$(\mathrm{P}_3) \quad \min_{\mathbf{x},\mathbf{y}} Z^{\mathrm{P}_3}(\mathbf{x},\mathbf{y}) = \sum_{(i,j)\in A} \sum_{k\in K} \alpha f_{ij}(d_{ik}+d_{kj})x_{ijk} + \sum_{k\in K}\sum_{l\in L} y_{kl}F_{kl}, \tag{5}$$

subject to

$$\sum_{k\in K} x_{ijk} = 1, \quad \forall\,(i,j)\in A, \tag{6}$$

$$\sum_{(i,j)\in A} f_{ij}x_{ijk} \le \sum_{l\in L} c_{kl}y_{kl}, \quad \forall\, k\in K, \tag{7}$$

$$\sum_{l\in L} y_{kl} \le 1, \qquad \forall\, k\in K, \tag{8}$$

$$x_{ijk} \ge 0,\ \ y_{kl}\in\{0,1\}, \quad \forall\,(i,j)\in A,\ k\in K,\ l\in L. \tag{9}$$

The objective function (5) minimizes the total cost, which is the sum of the fixed connection costs and the total flow-weighted distance multiplied by the unit cost. Constraint (6) stipulates that the flows only travel through connections. Constraint (7) is a capacity constraint. Constraint (8) assures that only one capacity option is selected for each connection. Constraints (9) are the non-negativity and integrality constraints. In this case, we do not have the properties that are correspondent to the continuous capacity case, even for special cases like $F_{kl} = F_k + vc_{kl}$, where F_k is the installation cost related to location. The following example illustrates the different solution structure between the discrete case and the continuous case.

Consider again the layout shown in Fig. 1, with $\alpha = 1$. The travel distances and flow weights are the same as those in Appendix C (see Table A2). The capacity options and the related fixed costs are given in Table 1.

It is easy to check (by enumeration) that the unique optimal solution for this example is:

For the $\mathbf{x}$ variable, $x_{131} = 1$, $x_{141} = x_{142} = 0.5$, $x_{242} = 1$, and others equal to zero.

For the $\mathbf{y}$ variable, $y_{11} = y_{21} = 1$, and others equal to zero.

Substituting the solution into the objective function (5), we obtain an objective function value equal to 40.

The example shows that: (i) it is not necessary to have $\sum_{k\in K}\sum_{l\in L} c_{kl}y_{kl} \ne \sum_{(i,j)\in A} f_{ij}$ if $(\mathbf{x},\mathbf{y})$ is optimal; (ii) it is not necessary that the optimal solution for the (P_3) has the feature $x_{ijk} = 0$ or 1, $\forall i,j,k$, and (iii) the optimal objective value for the discrete case is greater than or equal

Table 1. Data for example (discrete case)

Connection	*Capacity option*			*Fixed cost*		
	1	*2*	*3*	*1*	*2*	*3*
1	5	10	15	6	11	16
2	5	10	15	6	11	16
3	6	12	16	7	13	17

to that for the continuous case given that the cost terms are correspondent. More precisely, the following observation can be easily verified.

Observation. The optimal objective value for the discrete case is always greater than or equal to that for the continuous case, if their fixed costs of the connections has the same structure as $F_{kl} = F_k + vc_{kl}$.

If the fixed cost of locating a connection at a candidate site is not related to the capacity size options, then we always select the maximal capacity option and the problem can be reduced to the capacitated fixed charge facility location problem. Otherwise, an algorithm needs to be developed.

Before discussing solution algorithms, we note the following: For a selected set of connections such that their total capacity is greater than or equal to the total flows, the problem becomes a transportation problem. Specifically, if we are given y_{kl} and $\sum_{k\in K}\sum_{l\in L} y_{kl}c_{kl} \geq \sum_{(i,j)\in A} f_{ij}$, then the optimal assignment of flows to connections can be found by solving the following transportation problem:

$$\min_{\mathbf{x}} \sum_{(i,j)\in A}\sum_{k\in K} \alpha f_{ij}(d_{ik}+d_{kj})x_{ijk},$$

subject to

$$\sum_{(i,j)\in A} x_{ijk} = 1 \quad \forall\,(i,j)\in A,$$

$$\sum_{(i,j)\in A} f_{ij}x_{ijk} \leq c_{kl} \quad \forall\, k,l \text{ such that } y_{kl}=1,$$

$$x_{ijk} \geq 0.$$

We therefore assume for the rest of this section that the total capacity of the connections is greater than or equal to the total flows.

4.2. *Lagrangian relaxation algorithm*

In general, a greedy algorithm is one of the simplest heuristic ways to solve this kind of problem. Before we start to present a Lagrangian relaxation algorithm, we briefly discuss a greedy algorithm for this problem. The idea is as follows:

- Open all the connections and choose the maximal capacity option at each connection.
- Use the transportation problem to assign the flows to each connection.
- Close the connection that results to reduce the objective value the most.
- Repeat the procedure until the problem is infeasible or solution cannot be improved.

The greedy algorithm always does the best it can at each sequential decision. But this may not lead to the globally optimum solution. Example 2 in Appendix C shows that it exhibits poor performance at times. This motivates us to consider a Lagrangian relaxation heuristic algorithm. Another reason to use the Lagrangian relaxation algorithm is that it provides a lower bound for the original problem. This simultaneously allows us to evaluate how effective is the algorithm.

There are several choices related to which constraint is to be relaxed. In the capacitated fixed charge facility location problem, Klincewicz and Luss (1986) relax the constraints on the facility capacities. The corresponding subproblems become uncapacitated facility location problems. Other choices include relaxing assignment constraints or both; see Pirkul (1987) and Beasley (1993).

We choose to relax the capacity constraints (7). Let (P_3') be the Lagrangian relaxation problem:

$$(\mathrm{P}_3') \quad \min_{\mathbf{x},\mathbf{y}} Z^{\mathrm{P}_3'}(\boldsymbol{\mu}) = \sum_{(i,j)\in A}\sum_{k\in K} f_{ij}(\alpha(d_{ik}+d_{kj})+\mu_k)x_{ijk} + \sum_{k\in K}\sum_{l\in L} y_{kl}(F_{kl}-\mu_k c_{kl}), \tag{10}$$

subject to (6), (8), and (9).

The ideal choice of multipliers is such that they solve the Lagrangian dual problem, denoted by (DP):

$$(\mathrm{DP}) \quad \max_{\boldsymbol{\mu}} Z^{\mathrm{DP}}(\boldsymbol{\mu}). \tag{11}$$

The optimal value of the above problem (DP) provides the ‘best’ lower bound (using the Lagrangian method). For fixed values of the Lagrange multipliers μ_k, the optimal value of the $\mathbf{x}$ variables in the Lagrangian relaxation problem (P_3') can be established independently of the optimal values of the $\mathbf{y}$ variables. In fact, (P_3') can be solved by independently solving the following two sets of problems: a set of $|A|$ problems (SP_1) and a set of n_K problems (SP_2), where:

$$(\mathrm{SP}_1) \quad \min_{\mathbf{x}} Z^{\mathrm{SP}_1}(\boldsymbol{\mu}, i, j) = \sum_{k\in K} f_{ij}(\alpha(d_{ik}+d_{kj})+\mu_k)x_{ijk},$$

subject to

$$\sum_{k\in K} x_{ijk} = 1, \quad \forall\,(i,j)\in A,$$

$$x_{ijk} \geq 0, \qquad \forall\,(i,j)\in A,\ k\in K,$$

(which is decomposed on the indicies i and j), and

$$(\mathrm{SP}_2) \quad \min_{\mathbf{y}} Z^{\mathrm{SP}_2}(\boldsymbol{\mu}, k) = \sum_{l\in L} y_{kl}(F_{kl}-\mu_k c_{kl}),$$

subject to

$$\sum_{l\in L} y_{kl} \leq 1, \quad \forall\, k\in K,$$

$$y_{kl}\in\{0,1\}, \quad \forall\, k\in K,\ l\in L.$$

(which is decomposed on the index k).

We observe first that (SP_1) can be solved easily by setting $x_{ijs}=1$ for the s value that minimizes $f_{ij}\alpha(d_{ik}+d_{kj})+\mu_k)$ over all k. The optimal value for (SP_1) is

$$Z^{SP_1}(\boldsymbol{\mu}, i, j) = \min_{k \in K}\{f_{ij}\alpha(d_{ik} + d_{kj}) + \mu_k\}. \tag{12}$$

Next, we observe that (SP_2) is a 0–1 knapsack problem. It can be solved by setting $y_{kl} = 1$ if $F_{kl} - \mu_k c_{kl} \leq 0$ and is chosen such that $F_{kl} - \mu_k c_{kl}$ is minimized. The optimal value for (SP_2) is

$$Z^{SP_2}(\boldsymbol{\mu}, k) = \min\{\min_{l \in L}\{F_{kl} - \mu_k c_{kl}\}, 0\}. \tag{13}$$

Therefore, the optimal value for problem (P_2') is

$$Z^{SP_2'}(\boldsymbol{\mu}) = \sum_{(i,j)\in A} Z^{SP_1}(\boldsymbol{\mu}, i, j) + \sum_{k \in K} Z^{SP_2}(\boldsymbol{\mu}, k). \tag{14}$$

For a given $\boldsymbol{\mu}$, the optimal solution to the problem (P_3') is not likely to be feasible for the original problem (P_3). In particular, it may violate constraint (7), the capacity constraint that was relaxed. If, for some k, the constraint is violated, we will use a transportation algorithm to find a feasible solution, since for given $\mathbf{y}$ the problem is a transportation problem. This will provide a feasible solution to the original problem (P_3). Its objective value is an upper bound for (P_3), denoted by $UB(\boldsymbol{\mu})$.

In order to find the best lower bound for the original problem (P_3), we need to find the optimal solution to the dual problem (DP). The dual problem can be solved by the subgradient approach (Fisher, 1981). Given an initial value $\boldsymbol{\mu}^0$, a sequence of values $\boldsymbol{\mu}^n$ is generated by the rule:

$$\mu_k^{n+1} = \max\left\{0, \mu_k^n + t^n\left(\sum_{(i,j)\in A} f_{ij}x_{ijk}^n - \sum_{l \in L} c_{kl}y_{kl}^n\right)\right\}, \tag{15}$$

where the values of $\mathbf{x}^n$ and $\mathbf{y}^n$ are the optimal solution to (P_3') for fixed $\boldsymbol{\mu}^n$. Here t^n is a positive scalar step size. It can be computed as follows:

$$t^n = \frac{\beta^n(UB - Z^{P_3'}(\boldsymbol{\mu}^n))}{\sum_{k\in K}(\sum_{(i,j)\in A} f_{ij}x_{ijk}^n - \sum_{l\in L} c_{kl}y_{kl}^n)^2}, \tag{16}$$

where β is a scalar satisfying $0 < \beta^n < 2$. We usually begin with $\beta^1 = 2$. The value of β^n is generally halved if the lower bound, $Z^{P_3'}(\boldsymbol{\mu}^n)$, has not increased in a given number of consecutive iterations. UB is the best upper bound so far on the original problem (P_3). $Z^{P_3'}(\boldsymbol{\mu}^n)$ is the objective function of the problem (P_3'). The algorithm terminates when one of the following conditions is true: (i) we have performed a pre-specified number of iterations; (ii) the upper bound equals the lower bound (in this case, the solution is optimal) or is close enough to the upper bound; or (iii) β^n becomes small.

The solution procedure can be described as follows. After relaxing constraint set (7), we initially fix the value of the Lagrange multipliers μ_k (see Theorem 2, later) and use the greedy algorithm to find a feasible solution for the original problem. It is an upper bound for the original problem. As indicated above, the resulting problem can be solved by solving two sets of simple problems, (SP_1) and (SP_2). The optimal solutions for (SP_1) and (SP_2) will be an optimal solution for the relaxed problem (P_3') for the given μ_k. The objective value of the problem (P_3') provides a lower bound for the original problem (P_3). If the solution for the relaxed problem is feasible to the original problem, we compute and update the upper bound of the original problem. If it is not, use the transportation problem to find a feasible solution to update the upper bound. If the termination conditions are satisfied, we stop. Otherwise, we decide if we need to update the Lagrange multipliers μ_k. If we decide to terminate the Lagrangian procedure, the best upper bound gives us a heuristic solution. The following is a step-by-step description of the procedure:

Step 1. Relax constraint set (7), fix the initial μ_k, and find an initial upper bound by the greedy algorithm.
Step 2. Solve (SP_1) and (SP_2), compute and update the lower bound.
Step 3. If the solution for the relaxed problem is feasible to (P_3), update the upper bound; if not, use the transportation problem to find a feasible solution, then update the upper bound.
Step 4. If the termination conditions are satisfied, stop; otherwise, update μ_k, and go to Step 2.

The next theorem provides a good initial set of Lagrangian multipliers.

Theorem 2. *There exists an optimal dual solution in which* $\mu_k = F_{kl}/c_{kl}, \forall k \in K$, *if constraint* (7) *is satisfied.*

To help interpret the result of Theorem 2, we note that F_{kl}/c_{kl} is the ratio of fixed installation cost to capacity. Thus it is natural to set the initial Lagrangian multipliers as $\mu_k^0 = \min_l\{F_{kl}/c_{kl}\}$. By the theorem, it appears to be beneficial to set the Lagrangian multipliers such that $\mu_k = F_{kl}/c_{kl}$.

5. Computational experiments

The aim of this section is two-fold. First, we focus on the value of allowing more connection options, which entails a discussion of the continuous and discrete cases. Then, we test the performance of the heuristic algorithm for the discrete case.

5.1. *Comparison of continuous and discrete models*

First, we generated each department's location, which was decided by its x-coordinate and the y-coordinate. These coordinate values were randomly selected from $U(0, 1000)$, where U denotes a uniform distribution. The

Table 2. Comparison of optimal objective values

Data set	*Number of capacity options in discrete case*			*Continuous case*
	3	*5*	*9*	
1	1224 200	1219 160	1218 920	1218 230
2	1859 440	1854 250	1852 065	1850 490

candidate sites of connections were also decided by its x-coordinate and the y-coordinate, where the x-coordinate values and y-coordinate values were randomly selected from the minimum value of the x-coordinate and the y-coordinate values of department's location to maximum value of x-coordinate and the y-coordinate values of department's location. For each pair of distinct departments, the amount of flow was randomly drawn from $U(5, 30)$. We assume $\alpha = 1$. About the fixed connection installation costs, let $F_k = 1000$ and $v = 50$ for the continuous case and $F_{kl} = 1000 + 50c_{kl}$ for the discrete case. Two small sets of test problems are considered. Set 1 has 15 departments and 20 candidate connections. Set 2 has 20 departments and 30 candidate connections. In each set of data, we let the capacity option of each connection be three, five, and nine, respectively. We ensure that in each progressively higher option case the previous capacity options are included, where the minimum and maximum c_{kl} values are arbitrarily chosen. We use a solver (CPLEX 6.5) for these small size problems and obtain optimal objective values are given in Table 2. The results show that the optimal objective values of the discrete case decrease as the number of capacity options increase and approach to that of the continuous case for both data sets. They also suggest a decreasing marginal return in terms of improvement in objective function value as the number of capacity options increases.

5.2. *Computational experiments for Lagrangian algorithm*

In this subsection, we design experiments to test the performance of the Lagrangian algorithm for the discrete case. All of the experimental tests were carried out on HP-UX 11 servers. The algorithm was coded in C++. Specifically, we have two goals: (i) performance analysis of the algorithm for varying test data (size and parameter); and (ii) comparison of computation time with a Linear Mixed Integer Program solver (LMIP). The particular solver we used is CPLEX 6.5.

We generate network data as in Section 5.1. We let the number of capacity options be three or six for each connection now. The capacity options, c_{kl}, were randomly decided in the following manner: (i) for three capacity options, $c_{k1} \sim U(0.7\rho, 1.0\rho)$, $c_{k2} \sim U(1.1\rho, 1.4\rho)$, $c_{k3} \sim U(1.5\rho, 1.80\rho)$; (ii) for six capacity options, $c_{k1} \sim U(0.7\rho, 0.85\rho)$, $c_{k2} \sim U(0.90\rho, 1.05\rho)$, $c_{k3} \sim U\ (1.1\rho, 1.25\rho)$; $c_{k4} \sim U(1.30\rho, 1.45\rho)$, $c_{k5} \sim U(1.50\rho,\ 1.65\rho)$,

Table 3. Parameter values for test problems

	Parameter	*Small*	*Medium*	*Large*
1	Number of departments	10–30	55–65	70–80
2	Number of connections	15–45	110–160	180–200
3	Number of capacity options	3 or 6	3 or 6	3 or 6
4	Number of non-zero flows	45–435	1485–2080	2415–3160

$c_{k6} \sim U(1.65\rho, 1.80\rho)$; where $\rho = 2\sum_{(i,j)\in A} f_{ij}/|K|$, or $\rho = 3\sum_{(i,j)\in A} f_{ij}/|K|$, and $|K|$ is the total number of candidate connection sites. The term $\sum_{(i,j)\in A} f_{ij}/|K|$ is the average flow for the total number of candidate connection sites. We use different ρ values in order to study how the capacity option affects the algorithm's performance. Therefore, two sets of data were generated according to different ρ values. The parameter values for test problems are summarized in Table 3. We use the term "heuristic gap" to evaluate the efficiency of our algorithm. Heuristic gap is defined as (*best upper bound* $-$ *best lower bound*)/ *bestlower bound* $\times$ 100. According to the "weak duality" theorem (Shapiro, 1979), any objective value of the relaxed problem is a lower bound of the original problem. The objective value of any feasible solution for the original problem provides a upper bound of the original problem. Therefore, as illustrated in Fig. 2, we can guarantee that the solution for the original problem is near-optimal if the heuristic gap is very small.

Tables 4–6 provide the computational results for three (different size) test data sets. The large capacity refers to $\rho = 3\sum_{(i,j)\in A} f_{ij}/|K|$, while for the small capacity $\rho = 2\sum_{(i,j)\in A} f_{ij}/|K|$. Heuristic time is the CPU time for the LR algorithm. For small size problems, LMIP time is the CPU time to get the optimal solution when using LMIP. For medium size problems, the LMIP time records the CPU time to get the first feasible solution using default parameter settings, since we can not get the optimal solution within 1 hour for this size of problem. In some cases, LMIP could not provide any feasible solution within 1 hour. We denote this situation as '>3600'. For

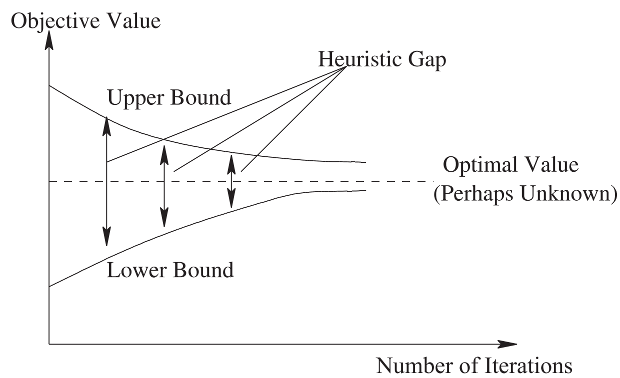

Fig. 2. Heuristic gap as a function of the number of iterations.

Table 4. Computational result for small sized problems

	Dept number	*Option number*	*Heuristic gap (%)*	*Heuristic time (s)*	*LMIP time (s)*
Large capacity	10	3	3.74	0.07	0.24
	10	6	1.88	0.07	0.31
	20	3	3.40	0.38	1.48
	20	6	2.52	0.65	15.72
	30	3	2.63	7.86	219.23
	30	6	1.81	8.37	74.57
Small capacity	10	3	1.39	0.07	0.16
	10	6	2.33	0.07	0.37
	20	3	1.87	0.83	3.58
	20	6	1.15	1.03	39.30
	30	3	1.58	5.86	77.05
	30	6	1.33	6.87	449.83

Table 5. Computational result for medium sized problems

	Dept number	*Option number*	*Heuristic gap (%)*	*Heuristic time (s)*	*LMIP time (s)*
Large capacity	55	3	1.96	255.60	545
	55	6	1.27	305.27	790
	60	3	2.45	698.42	3482
	60	6	1.44	694.49	3390
	65	3	1.86	779.64	2460
	65	6	1.36	1066.45	>3600
Small capacity	55	3	1.70	265.20	306
	55	6	1.15	295.43	324
	60	3	1.27	643.21	3572
	60	6	1.24	650.32	>3600
	65	3	1.23	767.24	>3600
	65	6	1.12	891.29	>3600

Table 6. Computational result for large sized problems

	Dept number	*Option number*	*Heuristic gap (%)*	*Heuristic time (s)*
Large capacity	70	3	2.10	2183.88
	70	6	1.67	1570.23
	75	3	2.38	2259.78
	75	6	1.35	2423.09
	80	3	1.82	4169.62
	80	6	1.51	2966.22
Small capacity	70	3	1.28	1899.93
	70	6	1.92	643.88
	75	3	1.66	2101.88
	75	6	0.99	1996.69
	80	3	1.84	2403.31
	80	6	1.61	1405.79

Table 7. Average gap for different problem sizes and parameter values

Size	*Capacity size*		*Capacity options*		*Average*
	Large capacity	*Small capacity*	*3*	*6*	
Small	2.66	1.61	2.44	1.84	2.14
Medium	1.77	1.28	1.75	1.30	1.50
Large	1.81	1.55	1.85	1.51	1.68

large size problems, we only applied the LR heuristic algorithm.

In general, the algorithm performs very well. The average heuristic gap for all combined data is about 1.77% and the computation time is less than 1 hour except for one case. Table 7 shows performance sensitivity of the algorithm to different parameters. A two-way ANOVA was performed. The p-value for the ρ value effect is 0.000 724, and the p-value for the number of options effect is 0.004 765. The results show that both the ρ value and the number of options have a significant effect on the accuracy of the algorithm.

6. Summary and future work

In this paper the variable capacity sizing and selection of connections problem has been studied. To our knowledge the determination of capacity size along with connection selection with fixed and variable capacity costs is new to the open literature. This work is motivated by the design of appropriate connections for a material handing system in the manufacturing facility. We note that application of this problem with revised assumption can be also found in other areas, such as urban planning, communication systems, and distribution systems. For the continuous capacity case without a bound, we prove that it can be reduced to the uncapacitated fixed charge facility location problem. For the discrete capacity case, a Lagrangian relaxation-based approach has been developed. This approach decomposes the original problem to two sets of simple mathematical programming problems. The computational results for three different size test data show that the algorithm is efficient in accuracy and the computational time is also affordable. An ANOVA test shows that the capacity size and the number of options effect are significant to the algorithm performance.

Capacities are important in many facility design problems. However, fixed capacities are less important than are 'practical capacities' in many cases. For example, the flows may not arrive at a connection uniformly over time. Thus, the practical capacity of a connection may be significantly different from the theoretical one. In order to get such a practical capacity we need to use knowledge of queuing theory in conjunction with an understanding of flow behavior. This is an area suggested for future work. The planar version of this problem could be another future work, in which the fixed connection installation cost is a function of the location (that can be chosen from some planar region).

Acknowledgements

The authors would like to acknowledge support from the National Science Foundation via Grant No. DMI-9800429. They would also like to thank two anonymous referees for their helpful comments that led to a much enhanced version of this paper.

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Appendices

Appendix *A*

The definitions of the problems considered in this paper are presented in Table A1.

Appendix *B*

Property 1. If $(\mathbf{x}, \mathbf{y}, \mathbf{c})$ *is an optimal solution for the problem* (P_1), *then* $\forall k \in K$, $\sum_{(i,j)\in A} f_{ij}x_{ijk} = c_k y_k$.

Proof. Suppose that the assertion is not true and let $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}^*)$ represent an optimal solution for (P_1). It follows that there exists a k such that $\sum_{(i,j)\in A} f_{ij}x^*_{ijk} \neq c^*_k y^*_k$. From constraints (3) and (4), it follows that $\sum_{(i,j)\in A} f_{ij}x^*_{ijk} < c^*_k y^*_k$ and that $y^*_k = 1$. Therefore, $\sum_{(i,j)\in A} f_{ij}x^*_{ijk} < c^*_k$. Let $c'_k = \sum_{(i,j)\in A} f_{ij}x^*_{ijk}$. We replace c^*_k by c'_k for all k where $\sum_{(i,j)\in A} f_{ij}x^*_{ijk} < c^*_k y^*_k$. If $\sum_{(i,j)\in A} f_{ij}x^*_{ijk} = c^*_k y^*_k$, we let c^*_k remain unchanged. Let the new $\mathbf{c}$ vector be labeled $\mathbf{c}'$. It is easy to see that the solution $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}')$ is feasible for (P_1). Moreover,

$$\begin{aligned} Z^{\mathrm{P}_1}(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}^*) &= \sum_{(i,j)\in A}\sum_{k\in K} \alpha f_{ij}(d_{ik} + d_{kj})x^*_{ijk} + \sum_{k\in K} y^*_k(F_k + vc^*_k) \\ &> \sum_{(i,j)\in A}\sum_{k\in K} \alpha f_{ij}(d_{ik} + d_{kj})x^*_{ijk} + \sum_{k\in K} y^*_k F_k + \sum_{k\in K} y^*_k v c'_k \\ &= Z^{\mathrm{P}_1}(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}'), \end{aligned}$$

which establishes a contradiction. Hence, the assertion of the property is correct. ■

Property 2. *There exists an optimal solution for the problem* (P_1) *in which* $x_{ijk} = 0$ *or* 1, $\forall (i,j) \in A, k \in K$, *if the capacities have no upper bound limit.*

Table A1. Problem definitions

Problem	*Definition*
(P_1)	Variable capacity sizing and selection of connections model without upper bound
(P'_1)	Uncapacitated fixed charge connection location problem related to (P_1)
(P_2)	Variable capacity sizing and selection of connections model with upper bound
(P'_2)	Capacitated fixed charge connection location problem related to (P_2)
(P_3)	Variable capacity sizing and selection of connections model with discrete options
(P'_3)	Lagrangian relaxation problem of problem (P_3)
(DP)	Lagrangian dual problem
(SP_1)	Subproblem of (DP)
(SP_2)	Subproblem of (DP)

Proof. Consider an optimal solution $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}^*)$ and suppose that there exists i, j, k such that $x_{ijk} \notin \{0,1\}$. Since $0 < x^*_{ijk} < 1$, it follows from (2) that there exists a $k' \neq k$ such that $x^*_{ijk'} > 0$ and that $x^*_{ijk} + x^*_{ijk'} \leq 1$. From the fact that $x^*_{ijk} > 0$, $x^*_{ijk'} > 0$ and $f_{ij} > 0$, it follows from (3) that $y^*_k = 1$ and $y^*_{k'} = 1$, respectively. Clearly, either $d_{ik} + d_{kj} \leq d_{ik'} + d_{k'j}$ or $d_{ik} + d_{kj} > d_{ik'} + d_{k'j}$. We consider the first case (the proof for the second situation is very similar). Let $x_{ijk} = x^*_{ijk} + x^*_{ijk'}$, $x_{ijk'} = 0$, $c_k = c^*_k + f_{ij}x^*_{ijk'}$, $c_{k'} = c^*_{k'} - f_{ij}x^*_{ijk'}$, $y_k = 1$ and $y_{k'} = 1$. The solution $(\mathbf{x}, \mathbf{y}, \mathbf{c})$ with these values for x_{ijk}, $x_{ijk'}$, c_k, $c_{k'}$, y_k and $y_{k'}$, and other values remaining unchanged is a feasible solution to (P_1). This can be verified by checking constraints (2) and (3). The objective function value for this new solution is less than or equal to the value for the $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}^*)$ solution. This can be verified by using Equation (1). By repeated application of this procedure we can arrive at a binary solution in which $x_{ijk} = 0$ or 1, $\forall (i,j) \in A, k \in K$. ■

Theorem 1. *The solution of the two-step algorithm is optimal for the problem* (P_1).

Proof. Suppose that $(\mathbf{x}^*, \mathbf{y}^*)$ is the optimal solution obtained in Step 1 and that Step 2 yields the vector $\mathbf{c}^*$. Therefore,

$$\sum_{k\in K} c^*_k y^*_k = \sum_{k\in K} y^*_k \left(\sum_{(i,j)\in A} f_{ij} x^*_{ijk} \right) = \sum_{(i,j)\in A} f_{ij} \left(\sum_{k\in K} y^*_k x^*_{ijk} \right)$$
$$= \sum_{(i,j)\in A} f_{ij} \left(\sum_{k\in K} x^*_{ijk} \right) = \sum_{(i,j)\in A} f_{ij}.$$

We now prove that $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}^*)$ is an optimal solution for problem (P_1).

First, feasibility is readily established by checking constraints (2), (3), and (4).

Now, we assume that $(\mathbf{x}', \mathbf{y}', \mathbf{c}')$ is an optimal solution for the problem (P_1). Then $(\mathbf{x}', \mathbf{y}')$ is a feasible solution for the problem (P'_1). Upon comparing the objective functions of (P_1) and (P'_1), we get $Z^{\mathrm{P}_1}(\mathbf{x}', \mathbf{y}', \mathbf{c}') = Z^{\mathrm{P}'_1}(\mathbf{x}', \mathbf{y}') + v\sum_{k\in K} c_k y'_k$. Since $(\mathbf{x}', \mathbf{y}', \mathbf{c}')$ is optimal to (P_1), from Property 1 we get $Z^{\mathrm{P}_1}(\mathbf{x}', \mathbf{y}', \mathbf{c}') = Z^{\mathrm{P}'_1}(\mathbf{x}', \mathbf{y}') + v\sum_{(i,j)\in A} f_{ij}$, where $v\sum_{(i,j)\in A} f_{ij}$ is a constant. Thus, $Z^{\mathrm{P}'_1}(\mathbf{x}', \mathbf{y}') = Z^{\mathrm{P}'_1}(\mathbf{x}^*, \mathbf{y}^*)$, and $Z^{\mathrm{P}_1}(\mathbf{x}', \mathbf{y}', \mathbf{c}') = Z^{\mathrm{P}'_1}(\mathbf{x}^*, \mathbf{y}^*) + \sum_{k\in K} c^*_k y^*_k = Z^{\mathrm{P}_1}(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}^*)$. This implies that $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{c}^*)$ is also an optimal solution for problem (P_1). ■

Theorem 2. *There exists an optimal dual solution in which* $\mu_k = F_{kl}/c_{kl}$, $\forall\, k \in K$, *if constraint (7) is satisfied.*

Proof. We assume that the above condition is not true for some optimal solution μ^*_k and derive another solution of greater or equal value satisfying the condition. Choose a connection k in the optimal solution violating the above condition. There are two cases:

If $\mu^*_k < F_{kl}/c_{kl}$, then $F_{kl} - \mu^*_k c_{kl} = d > 0$. According to the above discussion, in the optimal solution to (P'_3), $y^*_{kl} = 0$. Let $\mu'_k = \mu^*_k + d/c_{kl}$, which increases the Lagrangian multiplier since d is positive. It is easy to see that $F_{kl} - \mu'_k c_{kl} = 0$, and the second term of the objective function does not change. But the first term of the objective function may increase. Thus the objective function will not decrease.

If $\mu^*_k > F_{kl}/c_{kl}$, then $F_{kl} - \mu^*_k c_{kl} = d < 0$. According to the above discussion, in the optimal solution to (P'_3), $y^*_{kl} = 1$. Let $\mu'_k = \mu^*_k + d/c_{kl}$, which decreases the Lagrangian multiplier since d is negative. It is easy to see that $F_{kl} - \mu'_k c_{kl} = 0$. Now, the second term of the objective function will increase by $|d|$. The first term of the objective function will decrease by $\sum_{(i,j)\in A} f_{ij}x^*_{ijk}|d|/c_{kl}$. If constraint (7) is satisfied, then we have $\sum_{(i,j)\in A} f_{ij}x^*_{ijk}|d|/c_{kl} < |d|$. Thus the objective function will increase. The theorem follows. ■

Appendix C

Example 1. We use the layout shown earlier in Fig. 1. There are three flows: (1,3), (1,4), and (2,4). Suppose that $\alpha = v = 1$, and $F_1 = F_2 = F_3 = 1$. The travel distances and flow f_{ij} are as in Table A2.

From the first step, it is easy to get the following optimal solution:

For the $\mathbf{x}$ variable, $x_{131} = x_{141} = x_{242} = 1$, others equal to zero; For the $\mathbf{y}$ variable, $y_1 = y_2 = 1$, and $y_3 = 0$.

From the second step, set $c_1 = 6$, $c_2 = 4$, and $c_3 = 0$.

Substituting the solution into the objective function (1) we obtain the optimal objective value of 38. An enumeration check reveals that it is the optimal solution. It can be verified that this solution has the characteristics stated in Properties 1 and 2.

Example 2. There are two flows: (1,3) and (2,4). The total number of candidate connections are three and only one capacity option exists at each connection. Suppose that $\alpha = 1$, $F_1 = 12$, $F_2 = 20$, $F_3 = 12$, and $c_1 = c_3 = 1, c_2 = 2$. The travel distances and flow f_{ij} are as in Table A3.

Using the greedy algorithm, we find that the solution is to open connections 1 and 3 with an objective function value of 30. Using the Lagrangian relaxation heuristic,

Table A2. Data for Example 1 (continuous case)

Flow	f_{ij}	*Travel distance*		
		Connection 1	*Connection 2*	*Connection 3*
(1,3)	4	2	6	8
(1,4)	2	3	5	7
(2,4)	4	5	3	5

Table A3. Data for Example 2 (discrete case)

Flow	f_{ij}	*Travel distance*		
		Connection 1	*Connection 2*	*Connection 3*
(1,3)	1	4	2	4
(2,4)	1	2	3	2

the solution is to open just connection 2 and the resultant objective function value is 25. This also happens to be the optimal solution.

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Contributed by the Facilities Layout and Material Handling Department