Fuzzy Set Theory Applications in Production Management Research: A Literature Survey

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Abstract

Fuzzy set theory has been used to model systems that are hard to define precisely. As a methodology, fuzzy set theory incorporates imprecision and subjectivity into the model formulation and solution process. Fuzzy set theory represents an attractive tool to aid research in production management when the dynamics of the production environment limit the specification of model objectives, constraints and the precise measurement of model parameters. This paper provides a survey of the application of fuzzy set theory in production management research. The literature review that we compiled consists of 73 journal articles and nine books. A classification scheme for fuzzy applications in production management research is defined. We also identify selected bibliographies on fuzzy sets and applications.

Keywords: Production Management, Fuzzy Set Theory, Fuzzy Mathematics.

1 Introduction

Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes (see for example Zadeh (1965)). Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. In an effort to gain a better understanding of the use of fuzzy set theory in production management research and to provide a basis for future research, a literature review of fuzzy set theory in production management has been conducted. While similar survey efforts have been undertaken for other topical areas, there is a need in production management for the same. Over the years there have been successful applications and implementations of fuzzy set theory in production management. Fuzzy set theory is being recognized as an important problem modeling and solution technique. A summary of the findings of fuzzy set theory in production management research may benefit researchers in the production management field.

Kaufmann and Gupta (1988) report that over 7,000 research papers, reports, monographs, and books on fuzzy set theory and applications have been published since 1965. Table 1 provides a summary of selected bibliographies on fuzzy set theory and applications. The objective of Table 1 is not to identify every bibliography and extended review of fuzzy set theory, rather it is intended to provide the reader with a starting point for investigating the literature on fuzzy set theory.

The bibliographies encompass journals, books, edited volumes, conference proceedings, monographs, and theses from 1965 to 1994. The bibliographies compiled by Gaines and Kohout (1977), Kandel and Yager (1979), Kandel (1986), and Kaufmann and Gupta (1988) address fuzzy set theory and applications in general. The bibliographies by Zimmerman (1983) and Lai and Hwang (1994) review the literature on fuzzy sets in operations research and fuzzy multiple objective decision making respectively. Maiers and Sherif (1985) review the literature on fuzzy industrial controllers and provide an index of applications of fuzzy set theory to twelve subject areas including decision making, economics, engineering and operations research.

As evidenced by the large number of citations found in Table 1, fuzzy set theory is an established and growing research discipline. The use of fuzzy set theory as a methodology for modeling and analyzing decision systems is of particular interest to researchers in production management due to fuzzy set theory's ability to quantitatively and qualitatively model problems which involve vagueness and imprecision. Karwowski and Evans (1986) identify the potential applications of fuzzy set theory to the following areas of production management: new product development, facilities location and layout, production scheduling and control, inventory management, quality and cost benefit analysis. Karwowski and Evans identify three key reasons why fuzzy set theory is relevant to production management research. First, imprecision and vagueness are inherent to the decision maker's mental model of the problem under study. Thus, the decision maker's experience and judgment may be used to complement established theories to foster a better understanding of the problem. Second, in the production management environment, the information required to formulate a model's objective, decision variables, constraints and parameters may be vague or not precisely measurable. Third, imprecision and vagueness as a result of personal bias and subjective opinion may further dampen the quality and quantity of available information. Hence, fuzzy set theory can be

Table 1: Selected Bibliographies of Fuzzy Set Theory

Reference Author(s)	Number of reference citations
Gaines and Kohout (1977)	763 (with 401 additional on topics closely related to fuzzy systems theory)
Kandel and Yager (1979)	1799
Zimmerman (1983)	54 (emphasis on fuzzy sets in operations research)
Maiers and Sherif (1985)	450 (emphasis on fuzzy sets and industrial controllers)
Kandel (1986)	952
Kaufmann and Gupta (1988)	220
Lai and Hwang (1994)	695 (emphasis on fuzzy multiple objective decision making)

used to bridge modeling gaps in descriptive and prescriptive decision models in production management research. In this paper, we review the literature and consolidate the main results on the application of fuzzy set theory to production management.

The purpose of this paper is to: (i) review the literature; (ii) classify the literature based on the application of fuzzy set theory to production management research; and, (iii) identify future research directions. This paper is organized as follows. Section 2 introduces a classification scheme for fuzzy research in production management research. Section 3 reviews previous research on fuzzy set theory and production management research. The conclusions to this study are given in Section 4.

2 Classification Scheme for Fuzzy Set Theory Application in Production Management Research

Table 2 illustrates a classification scheme for the literature on the application of fuzzy set theory in production management research. Seven major categories are defined and the frequency of citations in each category is identified. Quality management resulted in the largest number of citations (15), followed by project scheduling (14), and facility location and layout (14). This survey is restricted to research on the application of fuzzy sets to production management decision problems. Research on fuzzy optimization and expert systems are not generally included in this survey. Readers who are interested in fuzzy optimization and operations research should consult Negoita (1981), Zimmerman (1983) and Kaufmann (1986). A comprehensive review of fuzzy expert systems in industrial engineering, operations research, and management science may be found in Turksen (1992).

A total of 82 citations on the application of fuzzy set theory in production management research was found (see Table 3). The majority of the citations were found in journals (89%) while books and edited volumes also contributed (11%). Three journals, *Fuzzy Sets and Systems*, *International Journal of Production Research*, and *European Journal of Operational Research*, accounted for 55 percent of the citations. Table 4 provides a breakdown of the number of citations by topic and by year published. For example, three quality management articles where published in 1993. The three articles represent 20 percent of the research on fuzzy quality identified in this study, and 27 percent of the articles on fuzzy production management research that were found for 1993.

Table 2: Classification Scheme for Fuzzy Set Research in Production Management

	Research Topic	Number of Citations
1.	Job Shop Scheduling	9
2.	Quality Management	15
	a. Acceptance Sampling (6)	
	b. Statistical Process Control (5)	
	c. General Topics (4)	
3.	Project Scheduling	14
4.	Facility Location and Layout	14
	a. Facility Location (7)	
	b. Facility Layout (7)	
5.	Aggregate Planning	7
6.	Production and Inventory Planning	9
	a. Production Process Plan Selection Planning (5)	
	b. Inventory Lot Sizing Models (4)	
7.	Forecasting	14
	a. Simulation (1)	
	b. Delphi Method (3)	
	c. Time Series Analysis (8)	
	d. Regression Analysis (2)	
		Total = 82

Table 3: Summary of Journal and Book Citations on Fuzzy Set Theory in Production Management Research

Source	# Citations
Computers and Industrial Engineering	4
Computers and Mathematics with Applications	2
Decision Sciences	1
European Journal of Operational Research	6
Fuzzy Sets and Systems	24
Human Systems Management	1
IEEE Trans. on Engineering Management	1
IEEE Trans. on Systems, Man and Cybernetics	5
Inter. Journal of Operations and Production Management	1
Inter. Journal of Production Economics	3
Inter. Journal of Production Research	15
Inter. Journal of Quality and Reliability Management	1
Journal of the Operational Research Society	1
Journal of Risk and Insurance	1
Opsearch	3
Production Planning and Control	2
Project Management Journal	1
Quality and Reliability Engineering International	1
Above journals	73
Books and edited volumes	9
	Total = 82

Table 4: Citation Breakdown by Year and Research Classification

Examining Table 4, we observe that research on fuzzy project scheduling, facility location/layout and forecasting has been published over the last fifteen years. Research on job shop scheduling and quality management has increased in the last few years. Minimal research on fuzzy aggregate planning has been conducted over the past seven years.

3 Fuzzy Set Theory and Production Management Research

Extensive work has been done on applying fuzzy set theory to research problems in production management. Using the classification scheme developed in Section 2, research findings in each area of production management research will be reviewed.

3.1 Job Shop Scheduling

A number of papers on fuzzy job shop scheduling have been published. A summary of the direction of research on fuzzy job shop scheduling is found in Table 5. McCahon and Lee (1990) study the job sequencing problem when job processing times are represented with fuzzy numbers. The job sequencing algorithms of Johnson, and Ignall and Schrage are modified to accept triangular and trapezoidal fuzzy processing times. Makespan and mean flow time are used as the performance criteria in this work. The fuzzy sequencing algorithms are applied to job shop configurations involving n jobs and up to three workstations. McCahon and Lee (1992) modify the Campbell, Dudek, and Smith flow shop job sequencing heuristic to accept fuzzy processing times. Triangular fuzzy numbers are used to define job processing times in an n job and m workstation environment. Makespan and mean flow time are used to compare alternative sequences and to interpret the impact of the fuzzy processing times on job completion time, flow time and makespan. The article also provides a framework for interpreting and utilizing fuzzy makespan and mean flow time performance measures.

Ishii *et al.* (1992) investigate the scheduling of jobs under two shop configurations when job due dates are modeled with fuzzy numbers. Fuzzy due dates are defined by linear membership functions that reflect the level of satisfaction of job completion times. The first model addresses the n job and two machine open shop configuration. The aim of this problem is to determine the optimal speed of each machine and an optimal schedule with respect to an objective function consisting of the minimum degree of satisfaction among all jobs and costs of machine speed. The second model addresses an n job open shop with m identical machines. The objective in the second model is to develop a schedule that minimizes the maximum job lateness.

Tsujimura *et al.* (1993) study the three machine flowshop problem when job processing times are described by triangular fuzzy numbers. The optimal sequence is defined to be the sequence that minimizes the makespan. The solution methodology employed uses a modified version of Ignall and Schrage's branch and bound algorithm.

Ishibuchi *et al.* (1994) formulate an n job and m machine flowshop model with fuzzy job due dates. A nonlinear membership function is used to represent the grade of satisfaction with the completion time of a job. A scheduling objective of maximizing the minimum grade of satisfaction of a completion time is adopted. Two

Table 5: Fuzzy Job Shop Scheduling

Author(s)	# Machines	# Jobs	Fuzzification
Roy and Zhang (1996)	15	20	Fuzzy dispatch rules
Ishii and Tada (1995)	1	n	Fuzzy precedence relationships
Grabot and Geneste (1994)	3	6	Fuzzy dispatch rules
Han *et al.* (1994)	1	5	Fuzzy due dates
Ishibuchi *et al.* (1994)	10	20	Fuzzy due dates
Tsujimura *et al.* (1993)	3	4	Fuzzy processing times
Ishii *et al.* (1992)	2	n	Fuzzy due dates
	m	n	
McCahon and Lee (1992)	4	4	Fuzzy processing times and makespan
McCahon and Lee (1990)	1	4	Fuzzy processing times, makespan and flowtime
	2	6	
	3	4	

multi-start decent algorithms (first-move and best-move), a simulated annealing algorithm, and two taboo search algorithms (first-move and best-move) are applied in the solution methodology. The performance of the algorithms is compared using computer simulation based on a series of randomly generated test problems. The authors report that only the multi-start descent algorithms and the taboo search algorithms with a heuristic initial solution found satisfactory solutions with positive satisfaction grades for many test problems. As a result of the performances, a new approach is introduced by changing the objective function. The effectiveness of this approach is demonstrated using computer simulation.

Han *et al.* (1994) consider the n job, single machine maximum lateness scheduling problem with fuzzy due dates and controllable machine speeds. The objective is to find an optimal schedule and jobwise machine speeds which minimize the total sum of costs associated with dissatisfaction of all job completion times and jobwise machine speeds. A linear membership function is used to describe the degree of satisfaction with respect to job completion times. Incremental machine speed costs are defined as the cost associated with electrical power and/or labor. A polynomial time algorithm is employed to obtain solutions.

Grabot and Geneste (1994) use fuzzy logic to build aggregate dispatch rules in scheduling. The authors recommend that dispatch rules should be combined since individual dispatch rules are often dependent on the selected criterion of performance, the characteristics of the job shop, or the jobs themselves. For example, the combination of the shortest processing time and slack time rules can be expressed as: "if the operation duration is low (high) and the slack time is low (high) then the priority is high (low)". Linear membership functions are used to combine the dispatch rules. A six job, three machine job shop is studied using a simulator that evaluates the lateness, tardiness, flowtime, and average job lateness.

Ishii and Tada (1995) present an efficient algorithm for determining nondominated schedules for the n job single machine scheduling problem when a fuzzy precedence relationship exists between jobs. The bi-criteria objective of the algorithm is to minimize average job lateness while maximizing the minimal satisfaction level with respect to the fuzzy precedence relation. The complexity of the algorithm is studied and directions for future research on job shop scheduling with fuzzy precedence relations are identified.

Roy and Zhang (1996) develop a fuzzy dynamic scheduling algorithm (FDSA) for the n job m machine job shop scheduling problem. Fuzzy logic is used to combine conventional job shop scheduling rules to form aggregate heuristic rules. Membership functions for jobs, weighing schemes for priority rules employed in FDSA, and the fuzzy operators required in performing the fuzzy transformations are defined. Simulation experiments involving 20 jobs and up to 15 machines are conducted. Conventional priority rules (FCFS, SPT, EDD, and CR) are compared to three fuzzy heuristic rules under FDSA for the following performance measures: maximum and mean flow time, maximum and mean job lateness, and the number of tardy jobs. Results indicate that the fuzzy heuristic rules perform well in the job shop problems studied.

The job shop scheduling problem may be described as one in which a number of candidate jobs, each requiring processing time at various machines, are to be sequenced according to a dispatch rule so that a performance measure is optimized. Often, it is not possible to precisely define processing times (or even a probability distribution for processing times). Factors affecting the outcome of system performance such as the specification of job due dates, dispatch rules and precedence relationships among jobs and machines often are subjective. Fuzzy set theory, as demonstrated in the studies identified in this section, has contributed to job shop research by providing a means for capturing subjectivity in processing times, precedence relationships and performance objectives and incorporating them into the modeling and solution of job shop scheduling problems.

3.2 Quality Management

Research on fuzzy quality management is broken down into three areas, acceptance sampling, statistical process control, and general quality management topics. An overview of research on fuzzy quality management is found in Table 6.

3.2.1 Acceptance Sampling

Ohta and Ichihashi (1988) present a fuzzy design methodology for single stage, two-point attribute sampling plans. An algorithm is presented and example sampling plans are generated when producer's and consumer's risk are defined by triangular fuzzy numbers. The authors do not address how to derive the membership functions for consumer's and producer's risk.

Chakraborty (1988, 1994a) examines the problem of determining the sample size and critical value of a single sample attribute sampling plan when imprecision exists in the declaration of producer's and consumer's risk. In the 1988 paper, a fuzzy goal programming model and solution procedure are described. Several numerical examples are provided and the sensitivity of the strength of the resulting sampling plans is evaluated. The 1994a paper details how possibility theory and triangular fuzzy numbers are used in the single sample plan design problem.

Kanagawa and Ohta (1990) identify two limitations in the sample plan design procedure of Ohta and Ichi-

Table 6: Fuzzy Quality Management

Quality Area	Author(s)	Fuzzy Quality Application
Acceptance Sampling	Otha and Ichihashi (1988)	Single-stage, two-point attribute sampling plan
	Chakraborty (1988, 1994a)	Single sample, attribute sampling plan
	Kanagawa and Ohta (1990)	Extend work of Otha and Ichihashi (1988) to include nonlinear membership function
	Chakraborty (1992, 1994a)	Single-stage Dodge-Romig LTPD sampling plans
Statistical Process Control	Bradshaw (1983)	Introduces fuzzy control chart concept
	Wang and Raz (1990)	X-bar chart
	Raz and Wang (1990)	
	Kanagawa *et al.* (1993)	Fuzzy control charts for process average and process variability
	Wang and Chen (1995)	Economic statistical design of attribute np-chart
General Quality Management	Khoo and Ho (1996)	Quality function deployment
	Glushkovsky and Florescu (1996)	Quality improvement tools
	Gutierrez and Carmona (1995)	Multiple criteria quality decision model
	Yongting (1996)	Process capability analysis

hashi. First, Ohta and Ichihashi's design procedure does not explicitly minimize the sample size of the sampling plan. Second, the membership functions used, unrealistically model the consumer's and producer's risk. These deficiencies are corrected through the use of a nonlinear membership function and explicit incorporation of the sample size in the fuzzy mathematical programming solution methodology.

Chakraborty (1992, 1994b) addresses the problem of designing single stage, Dodge-Romig lot tolerance percent defective (LTPD) sampling plans when the lot tolerance percent defective, consumer's risk and incoming quality level are modeled using triangular fuzzy numbers. In the Dodge-Romig scheme, the design of an optimal LTPD sample plan involves solution to a nonlinear integer programming problem. The objective is to minimize average total inspection subject to a constraint based on the lot tolerance percent defective and the level of consumer's risk. When fuzzy parameters are introduced, the procedure becomes a possibilistic (fuzzy) programming problem. A solution algorithm employing alpha-cuts is used to design a compromise LTPD plan, and a sensitivity analysis is conducted on the fuzzy parameters used.

3.2.2 Statistical Process Control

Bradshaw (1983) uses fuzzy set theory as a basis for interpreting the representation of a graded degree of product conformance with a quality standard. When the costs resulting from substandard quality are related to the extent of nonconformance, a compatibility function exists which describes the grade of nonconformance associated with any given value of that quality characteristic. This compatibility function can then be used to construct fuzzy economic control charts on an acceptance control chart. The author stresses that fuzzy economic control chart limits are advantageous over traditional acceptance charts in that fuzzy economic control charts provide information on the severity as well as the frequency of product nonconformance.

Wang and Raz (1990) illustrate two approaches for constructing variable control charts based on linguistic data. When product quality can be classified using terms such as ‘perfect’, ‘good’, ‘poor’, etc., membership functions can be used to quantify the linguistic quality descriptions. Representative (scalar) values for the fuzzy measures may be found using any one of four commonly used methods: (i) by using the fuzzy mode; (ii) the alpha-level fuzzy midrange; (iii) the fuzzy median; or (iv) the fuzzy average. The representative values that result from any of these methods are then used to construct the control limits of the control chart. Wang and Raz illustrate the construction of an x-bar chart using the ‘probabilistic’ control limits based on the estimate of the process mean, plus or minus three standard errors (in a fuzzy format), and by control limits expressed as membership functions. Raz and Wang (1990) present a continuation of their 1990 work on the construction of control charts for linguistic data. Results based on simulated data suggest that, on the basis of sensitivity to process shifts, control charts for linguistic data outperform conventional percentage defective charts. The number of linguistic terms used to represent the observation was found to influence the sensitivity of the control chart.

Kanagawa *et al.* (1993) develop control charts for linguistic variables based on probability density functions which exist behind the linguistic data in order to control process average and process variability. This approach differs from the procedure of Wang and Raz in that the control charts are targeted at directly controlling the underlying probability distributions of the linguistic data.

Wang and Chen (1995) present a fuzzy mathematical programming model and solution heuristic for the economic design of statistical control charts. The economic statistical design of an attribute np-chart is studied under the objective of minimizing the expected lost cost per hour of operation subject to satisfying constraints on the Type I and Type II errors. The authors argue that under the assumptions of the economic statistical model, the fuzzy set theory procedure presented improves the economic design of control charts by allowing more flexibility in the modeling of the imprecisions that exist when satisfying Type I and Type II error constraints.

3.2.3 General Topics in Quality Management

Khoo and Ho (1996) present a framework for a fuzzy quality function deployment (FQFD) system in which the ‘voice of the customer’ can be expressed as both linguistic and crisp variables. The FQFD system is used to facilitate the documentation process and consists of four modules (planning, deployment, quality control, and operation) and five supporting databases linked via a coordinating control mechanism. The FQFD system is demonstrated for determining the basic design requirements of a flexible manufacturing system.

Glushkovsky and Florescu (1996) describe how fuzzy set theory can be applied to quality improvement tools when linguistic data is available. The authors identify three general steps for formalizing linguistic quality characteristics: (i) universal set choosing; (ii) definition and adequate formalization of terms; and (iii) relevant linguistic description of the observation. Examples of the application of fuzzy set theory using linguistic characteristics to Pareto analysis, cause-and-effect diagrams, design of experiments, statistical control charts, and process capability studies are demonstrated.

Gutierrez and Carmona (1995) note that decisions regarding quality are inherently ambiguous and must be resolved based on multiple criteria. Hence, fuzzy multicriteria decision theory provides a suitable framework for modeling quality decisions. The authors demonstrate the fuzzy multiple criteria framework in an automobile manufacturing example consisting of five decision alternatives (purchasing new machinery, workforce training, preventative maintenance, supplier quality, and inspection) and four evaluation criteria (reduction of total cost, flexibility, leadtime, and cost of quality).

Yongting (1996) identifies that failure to deal with quality as a fuzzy concept is a fundamental shortcoming of traditional quality management. Ambiguity in customers' understanding of standards, the need for multicriteria appraisal, and the psychological aspects of quality in the mind of the customer, support the modeling of quality using fuzzy set theory. A procedure for fuzzy process capability analysis is defined and is illustrated using an example.

The application of fuzzy set theory in acceptance sampling, statistical process control and quality topics such as quality improvement and QFD has been reviewed in this section. Each of these areas requires a measure of quality. Quality, by its very nature, is inherently subjective and may lead to a multiplicity of meanings since it is highly dependent on human cognition. Thus, it may be appropriate to consider quality in terms of grades of conformance as opposed to absolute conformance or nonconformance. Fuzzy set theory supports subjective natural language descriptors of quality and provides a methodology for allowing them to enter into the modeling process. This capability may prove to be extremely beneficial in the further development of quality function deployment, process improvement tools and statistical process control.

3.3 Project Scheduling

A summary of research on fuzzy project scheduling is found in Table 7. Examining Table 7, we note that the majority of the research on this topic has been devoted to fuzzy PERT. Prade (1979) applies fuzzy set theory to the development of an academic quarter schedule at a French school. When data are not precisely known, fuzzy set theory is shown to be relevant to the exact nature of the problem rather than probabilistic PERT or CPM. The aim of this work is to show how and when it is possible to use fuzzy concepts in a real world scheduling problem. An overview of a fuzzy modification to the classic Ford solution algorithm is presented along with a 17 node network representation for the academic scheduling problem. Calculations are demonstrated for a small portion

Table 7: Fuzzy Project Scheduling

Author(s)	Model Classification	Model Attributes
Shipley *et al.* (1996)	Fuzzy PERT	8 activity network for selling and producing a television commercial
Chang *et al.* (1995)	Solution procedure for fuzzy projects	uses fuzzy Delphi method, and combines composite and comparison methods
Lorterapong (1994)	Fuzzy CPM	fuzzy resource constrained project scheduling
Hapke *et al.* (1994)	Fuzzy project scheduling support system	53 activity network for resource allocation in software development
Nasuation (1994)	Fuzzy CPM	studies fuzzy slack
McCahon (1993)	Fuzzy PERT	compares fuzzy network and PERT over four basic network configurations of 4 to 8 activities
DePorter and Ellis (1990)	Fuzzy CPM	project crashing formulation
Buckley (1989)	Fuzzy PERT	discrete and continuous possibility distributions
Lootsma (1989)	Fuzzy PERT	compares stochastic PERT and fuzzy PERT
McCahon and Lee (1988)	Fuzzy PERT	triangular activity times
Kaufmann and Gupta (1988)	Fuzzy CPM	tutorial on fuzzy CPM
Dubois and Prade (1985)	Fuzzy PERT	tutorial on fuzzy PERT
Chanas and Kamburowski (1981)	Fuzzy PERT	11 activity, 9 node network
Prade (1979)	Fuzzy PERT	17 node network model for scheduling academic programs

of the overall scheduling problem.

Chanas and Kamburowski (1981) argue the need for an improved version of PERT due to three circumstances: (i) the subjectivities of activity time estimates; (ii) the lack of repeatability in activity duration times; and (iii) calculation difficulties associated with using probabilistic methods. A fuzzy version of PERT (FPERT) is presented in which activity times are represented by triangular fuzzy numbers.

Kaufmann and Gupta (1988) devote a chapter of their book to the critical path method in which activity times are represented by triangular fuzzy numbers. A six step procedure is summarized for developing activity estimates, determining activity float times, and identifying the critical path. A similar tutorial on fuzzy PERT involving trapezoidal fuzzy numbers may be found in Dubois and Prade (1985).

McCahon and Lee (1988) note that PERT is best suited for project network applications when past experience exists to allow the adoption of the beta distribution for activity duration times and when the network contains approximately 30 or more activities. When activity times are vague, the project network should be modeled with fuzzy components. A detailed example demonstrates modeling and solving an eight activity project network when activity durations are represented as triangular fuzzy numbers.

Lootsma (1989) identifies that human judgment plays a dominant role in PERT due to the estimation of activity durations and the requirement that the resulting plan be tight. This aspect of PERT exposes the conflict between normative and descriptive modeling approaches. Lootsma argues that vagueness is not properly modeled by probability theory, and rejects the use of stochastic models in PERT planning when activity durations are estimated by human experts. Despite some limitations inherent in the theory of fuzzy sets, fuzzy PERT, in many respects, is closer to reality and more workable than stochastic PERT.

Buckley (1989) provides detailed definitions of the possibility distributions and solution algorithm required for using fuzzy PERT. A ten activity project network example in which activity durations are described by triangular fuzzy numbers, is used to demonstrate the development of the possibility distribution for the project duration. Possibility distributions for float, earliest start, and latest start times are defined, but not determined, due to their complexity.

DePorter and Ellis (1990) present a project crashing model using fuzzy linear programming. Minimizing project completion time and project cost are highly sought yet conflicting project objectives. Linear programming allows the optimization of one objective (cost or time). Goal programming allows consideration of both time and cost objectives in the optimization scheme. When environmental factors present additional vagueness, fuzzy linear programming should be used. Linear programming, goal programming and fuzzy linear programming are applied to a ten activity project network. Project crashing costs and project durations are determined under each solution technique.

McCahon (1993) compares the performance of fuzzy project network analysis (FPNA) and PERT. Four basic network configurations were used. The size of the networks ranged from four to eight activities. Based on these networks, a total of thirty-two path completion times were calculated using FPNA and PERT. The performance of FPNA and PERT was compared using: the expected project completion time, the identification of critical activities, the amount of activity slack, and the possibility of meeting a specified project completion time. The results of this study conclude that PERT estimates FPNA adequately. When estimating expected project completion time however, a generalization concerning compared performance with respect to the set of critical activities, slack times and possibility of project completion times cannot be made. When activity times are poorly defined, the performance of FPNA outweighs its cumbersomeness and should be used instead of PERT.

Nasution (1994) argues that for a given alpha-cut level of the slack, the availability of the fuzzy slack in critical path models provides sufficient information to determine the critical path. A fuzzy procedure utilizing interactive fuzzy subtraction is used to compute the latest allowable time and slack for activities. The procedure is demonstrated for a ten event network where activity times are represented by trapezoidal fuzzy numbers.

Hapke *et al.* (1994) present a fuzzy project scheduling (FPS) decision support system. The FPS system is used to allocate resources among dependent activities in a software project scheduling environment. The FPS system uses L-R type flat fuzzy numbers to model uncertain activity durations. Expected project completion time and maximum lateness are identified as the project performance measures and a sample problem is demonstrated for a software engineering project involving 53 activities. The FPS system presented allows the estimation of project completion times and the ability to analyze the risk associated with overstepping the required project completion time.

Lorterapong (1994) introduces a resource-constrained project scheduling method that addresses three performance objectives: (i) expected project completion time; (ii) resource utilization; and (iii) resource interruption. Fuzzy set theory is used to model the vagueness that is inherent with linguistic descriptions often used by people when describing activity durations. The analysis presented provides a framework for allocating resources in an uncertain project environment.

Chang *et al.* (1995) combine the composite and comparison methods of analyzing fuzzy numbers into an efficient procedure for solving project scheduling problems. The comparison method first eliminates activities that are not on highly critical paths. The composite method then determines the path with the highest degree of criticality. The fuzzy Delphi method (see Kaufmann and Gupta (1988)) is used to determine the activity time estimates. The solution procedure is demonstrated in a 9 node, 14 activity project scheduling problem with activity times represented by triangular fuzzy numbers.

Shipley *et al.* (1996) incorporate fuzzy logic, belief functions, extension principles and fuzzy probability distributions, and developed the fuzzy PERT algorithm, ‘Belief in Fuzzy Probabilities of Estimate Time’ (BIFPET). The algorithm is applied to a real world project consisting of eight activities involved in the selling and producing of a 30-second television commercial. Triangular fuzzy numbers are used to define activity durations. BIFPET is used to determine the project critical path and expected project completion time.

The specification of activity duration times is crucial to both CPM and PERT project management applications. In CPM, historical data on the duration of activities in exact or very similar projects exists, and it is used to specify activity durations for similar future projects. In new projects where no historical data on activity durations exists, PERT is often used. Probabilistic-based PERT requires the specification of probability distribution (frequently the beta distribution) to represent activity durations. Estimates of the first two moments of the beta distribution provide the mean and variance of individual activity durations. Fuzzy set theory allows the human judgment that is required when estimating the behavior of activity durations to be incorporated into the modeling effort. The versatility of the fuzzy-theoretic approach is further championed in resource constrained and project crashing scenarios where additional uncertainty is introduced when estimating resource availability and cost parameters. The studies cited in this section have demonstrated how fuzzy set theory can be used to assist researchers in realistically modeling project management problems when activity durations, resource availability and project related costs cannot be precisely identified.

3.4 Facility Location and Layout

The problems of facility location and layout have been studied extensively in the production management and engineering literature.

3.4.1 Facility Location

Narasimhan (1979) presents an application of fuzzy set theory to the problem of locating gas stations. Fuzzy ratings are used to describe the relative importance of eleven attributes for a set of three location alternatives. A Delphi-based procedure was applied, and the input of decision makers was used to construct membership functions for three importance weights for judging attributes. Computations are summarized for the selection decision. The author concludes that the procedure presented is congruent to the way people make decisions. The procedure provides a structure for organizing information, and a systematic approach to the evaluation of imprecise and unreliable information.

Darzentas (1987) formulates the facility location problem as a fuzzy set partitioning model using integer programming. This model is applicable when the potential facility points are not crisp and can best be described by fuzzy sets. Linear membership functions are employed in the objective function and constraints of the model. The model is illustrated with an example based on three location points and four covers.

Mital *et al.* (1987) and Mital and Karwowski (1989) apply fuzzy set theory in quantifying eight subjective factors in a case study involving the location of a manufacturing plant. Linguistic descriptors are used to describe qualitative factors in the location decision, such as community attitude, quality of schools, climate, union attitude, nearness to market, police protection, fire protection, and closeness to port.

Bhattacharya *et al.* (1992) present a fuzzy goal programming model for locating a single facility within a given convex region subject to the simultaneous consideration of three criteria: (i) maximizing the minimum distances from the facility to the demand points; (ii) minimizing the maximum distances from the facilities to the demand points; and (iii) minimizing the sum of all transportation costs. Rectilinear distances are used under the assumption that an urban scenario is under investigation. A numerical example consisting with three demand points is given to illustrate the solution procedure.

Chung and Tcha (1992) address the location of public supply-demand distribution systems such as a water supply facility or a waste disposal facility. Typically, the location decision in these environments is made subject to the conflicting goals minimization of expenditures and the preference at each demand site to maximizing the amount supplied. A fuzzy mixed 0-1 mathematical programming model is formulated to study both uncapacitated and capacitated modeling scenarios. The objective function includes the cost of transportation and the fixed cost for satisfying demand at each site. Each cost is represented by a linear membership function. Computational results for twelve sample problems are demonstrated for a solution heuristic based on Erlenkotter's dual-based procedure for the uncapacitated facility location problem. Extension to the capacitated case is limited by issues of computational complexity and computational results are not presented.

Bhattacharya *et al.* (1993) formulate a fuzzy goal programming model for locating a single facility within a given convex region subject to the simultaneous consideration of two criteria: (i) minimize the sum of all transportation costs; and (ii) minimize the maximum distances from the facilities to the demand points. Details and assumptions of the model are similar to Bhattacharya *et al.* (1992). A numerical example consisting of two facilities and three demand points is presented and solved using LINDO.

3.4.2 Facility Layout

Grobelny (1987a, 1987b) incorporates the use of ‘linguistic patterns’ in solving the facility layout problem. Linguistic patterns are statements, based on the fuzzy aggregated opinions of experts, which can be used as recommendations when solving a layout problem and as criteria for evaluating an existing algorithm. For example, if the flow of materials between departments is high, then the departments should be located close to each other. The linking between the departments and the distance between the departments represent linguistic (fuzzy) variables; the ‘high’ and ‘close’ qualifications represent values of the linguistic variables. The evaluation of a layout is measured as the grade of satisfaction as measured by the mean truth value, of each linguistic pattern by the final placement of departments. Both the 1987a and 1987b models are construction type algorithms based on a modification of Hillier and Conner’s HC-66 layout algorithm.

Evans *et al.* (1987) introduce a fuzzy set theory based construction heuristic for solving the block layout design problem. Qualitative layout design inputs of ‘closeness’ and ‘importance’ are modeled using linguistic variables. The solution algorithm selects the order of department placement which is manual. The algorithm is demonstrated by determining a layout for a six department metal fabrication shop. The authors identify the need for future research toward the development of a heuristic that address both the order and placement of departments, the selection of values for the linguistic variables, and the determination of membership functions.

Raoot and Rakshit (1991) present a fuzzy layout construction algorithm to solve the facility layout problem. Linguistic variables are used in the heuristic to describe qualitative and quantitative factors that affect the layout decision. Linguistic variables capture information collected from experts for the following factors: flow relationships, control relationships, process and service relationships, organizational and personnel relationships, and environmental relationships. Distance is also modeled as a fuzzy variable and is used by the heuristic as the basis for placement of departments. Three test problems are used to compare the fuzzy heuristic with ALDEP and CORELAP. The authors note that the differences achieved by each of the three methods is a function of the different levels of reality that they use.

Raoot and Rakshit (1993) formulate the problem of evaluating alternative facility layouts as a multiple criteria decision model (MCDM) employing fuzzy set theory. The formulation addresses the layout problem in which qualitative and quantitative factors are equally important. Linguistic variables are used to capture experts’ opinions regarding the primary relationships between departments. Membership functions are selected based on consultation with layout experts. The multiple objectives and constraints of the formulation are expressed as linguistic patterns. The fuzzy MCDM layout algorithm is demonstrated for the layout of an eight department facility.

Raoot and Rakshit (1994) present a fuzzy set theory-based heuristic for the multiple goal quadratic assignment problem (QAP). The objective function in this formulation utilizes ‘the mean truth value’, which indicates the level of satisfaction of a layout arrangement to the requirements of the layout as dictated by a quantitative or qualitative goal. The basic inputs to the model are expert’s opinions on the qualitative and quantitative relationships between pairs of facilities. The qualitative and quantitative relationships are captured by linguistic variables, membership functions are chosen arbitrarily. Three linguistic patterns (one quantitative; two qualitative) are employed by the heuristic to locate facilities. The performance of the heuristic is tested against a set of test problems taken from the open literature. The results of the comparison indicate that the proposed fuzzy heuristic performs well in terms of the quality of the solution.

Dweiri and Meier (1996) define a fuzzy decision making system (FDMS) consisting of four principal components: (i) fuzzification of input and output variables; (ii) the experts' knowledge base; (iii) fuzzy decision making; and (iv) defuzzification of fuzzy output into crisp values. The analytical hierarchy process is used to weight factors affecting closeness ratings between departments. A computer program based on FDMS then generates activity relationship charts which, in turn, are developed into layouts by FZYCRLP (Fuzzy Computer Relationship Layout Planning - a modified version of CORELAP). Simulation is used to compare layouts generated under FZYCRLP and CORELAP for a set of twelve test problems involving layouts ranging in size from seven to seventeen departments. Layouts generated under FZYCRLP performed well under fuzzy and non-fuzzy evaluation metrics.

Many of the factors affecting facility layout and location problems are difficult to precisely measure and therefore require considerable human judgment. Closeness measures are a key input in nearly all facility layout models and are often determined in the form of closeness ratings that are described by degree of importance in linguistics terms such as 'absolutely necessary', 'very important', and 'undesirable'. Subjective weights are often used in conjunction with closeness measures when utilizing a scoring criterion to determine the layout of departments in a facility. Fuzzy set theory effectively models the linguistic aspects of specifying closeness measures and the subjectivities involved when specifying closeness weights. Facility location models may also require the determination of subjective factor weights to measure the relative importance of various factors influencing the location decision. Single and multiple criteria optimization procedures are frequently used in modeling facility location problems. Fuzzy set theory allows subjectivity in the parameters of these models to be incorporated into the model formulation and solution.

3.5 Aggregate Planning

Rinks (1981) cites a gap between aggregate planning theory and practice. Managers prefer to use their own heuristic decision rules over mathematical aggregate planning models. Using fuzzy conditional "if-then" statements, Rinks develops algorithms for fuzzy aggregate planning. A set of linguistic terms relevant to aggregate planning are defined and used to construct manager protocols (decision rules). Exponential membership functions are adopted and used in the algorithms. The fuzzy algorithm framework is applied to the classic Holt, Modigliani, Muth, and Simon (HMMS) paint factory data set. The total cost solution generated by the fuzzy aggregate planning algorithm exceeds the linear decision rule solution of HMMS by 5.0 percent. The strengths of the fuzzy aggregate planning model over traditional mathematical aggregate planning models include its ability to capture the approximate reasoning capabilities of managers, and the ease of formulation and implementation. The robustness of the fuzzy aggregate planning model under varying cost structures is examined in Rinks (1982a). A detailed set of forty production rate and work force rules is found in Rinks (1982b).

Turksen (1988a, 1988b) advocates using interval-valued membership functions over the point-valued membership functions found in Rinks (1981, 1982a, 1982b) when defining linguistic production rules for aggregate planning. When applied to the HMMS data set, the interval-valued membership approach produced a total cost solution that exceeded the benchmark linear decision rule solution by 3 percent. Further analysis using a more parsimonious rule base consisting of 27 rules (as opposed to the original 40 rules defined by Rinks) produced robust results.

Ward *et al.* (1992) develop a C language program based on Rinks' fuzzy aggregate planning framework. The program contains Rinks' decision rules, membership functions, and the HMMS data. Using the program, the authors replicated Rinks' 1981 findings. The program was modified to include triangular and exponential membership functions and an expanded rule base. Cost increases of 2.0 to 4.5 percent resulted when membership functions were altered. The augmented rule base produced total cost solutions closely matching those of Rinks.

Gen *et al.* (1992) present a fuzzy multiple objective aggregate planning model. The model is formulated as a fuzzy multiple objective programming model with objective function coefficients, technological coefficients, and resource right-hand side values, represented by triangular fuzzy numbers. A transformation procedure is presented to transform the fuzzy multiple objective aggregate planning model into a crisp model. The transformation procedure and computational algorithm are demonstrated for a numerical example involving a six-period planning horizon. Multiple objectives of minimizing total production costs, inventory and backorder costs, and changes in the work force level we used.

Aggregate planning involves the simultaneous determination of a company's production, inventory, and workforce levels over a finite planning horizon such that total relevant cost is minimized. Many aspects of the aggregate planning problem and the solution procedures employed to solve aggregate planning problems lend themselves to the fuzzy set theory approach. Fuzzy aggregate planning allows the vagueness that exists in the determining forecasted demand and the parameters associated with carrying charges, backorder costs, and lost sales to be included in the problem formulation. Fuzzy linguistic "if-then" statements may be incorporated into the aggregate planning decision rules as means for introducing the judgment and past experience of the decision maker into the problem. In this fashion, fuzzy set theory increases the model realism and enhances the implementation of aggregate planning models in industry. The usefulness of fuzzy set theory also extends to multiple objective aggregate planning models where additional imprecision due to conflicting goals may enter into the problem.

3.6 Production and Inventory Planning

Fuzzy research findings in production and inventory planning are summarized in Tables 8a and 8b.

3.6.1 Production and Process Plan Selection

Kacprzyk and Staniewski (1982) address the problem of controlling inventory over an infinite planning horizon. An inventory system is represented as a fuzzy system, with the fuzzy inventory level as the output and fuzzy

Table 8: Fuzzy Production and Inventory Planning

a. Production/Process Plan Selection

Author(s)	Application	Method
Inuiguchi *et al.* (1994)	Process plan selection	Applies possibilistic programming to select best process plan
Zhang and Huang (1994)	Process plan selection	Formulates fuzzy integer programming model, selects best process plan subject to fuzzy objectives
Singh and Mohanty (1991)	Process plan selection	Dynamic programming used to select best process plan subject to fuzzy objectives
Lehtimaki (1987)	MPS selection	Selects MPS to maximize customer's fuzzy satisfaction level
Kacprzyk and Staniewski	Production planning	Develops model to control (1982) inventory over infinite planning horizon when demand is fuzzy

b. Inventory Management

Author(s)	Application	Method
Lee *et al.* (1991)	MRP lot-sizing	Develops fuzzy Silver- Meal, Wagner-Whitin, and part-period balancing algorithms
Lee *et al.* (1990)	MRP lot-sizing	Develops fuzzy part-period balancing algorithm
Park (1987)	Economic Order Quantity Model	Determines EOQ with fuzzy ordering cost and holding cost
Sommer (1981)	Withdraw from market	Satisfies fuzzy inventory and production capacity levels during withdrawal

replenishment as the input. Demand and system constraints on replenishment are also fuzzy. An algorithm is presented to find the optimal time-invariant strategy for determining the replenishment to current inventory levels that maximizes the membership function for the decision. The algorithm is demonstrated using a numerical example.

Lehtimaki (1987) studies MRP scheduling problems from the perspective of a multigoal decision problem. The decision problem at hand is to decide how to accommodate customer requests for order changes. To support this decision, the author suggests a more comprehensive service level concept. The features of this concept include: service level is measured by the degree of customer's satisfaction; satisfaction of the customer, who wishes a change depends on the accepted change, delivery date, and invoiced amount for the change; satisfaction of the other customers depends on the firmness of delivery dates; service level of competitors, business cycles, or seasonal variations have their effects as well; and effects on the company's reputation should also be taken into account.

The goal of maximizing customer satisfaction is vague and can be best modeled using fuzzy set theory. A set of candidate MPSs are generated to deal with approved customer order changes. Constraints governing MPS construction and goals are represented using membership functions. The selection of the best MPS is based on a maximizing decision, whereby the membership function of a fuzzy decision attains its maximum value. A numerical example involving four orders is used to demonstrate the decision model. The author acknowledges that theoretical and empirical research is needed to determine how to evaluate membership functions, aggregate goals and constraints, and rank alternatives.

Singh and Mohanty (1991) characterize the manufacturing process plan selection problem as a machine routing problem. The routing problem is formulated as a multiple objective network model. Each objective is defined by a fuzzy membership function as a means of capturing the imprecision that exists when defining objectives. A dynamic programming solution procedure identifies the network path representing the best process plan. A dual-objective example is demonstrated for a component part requiring three machining operations. Two of the machining operations can be performed at alternative machining centers, resulting in a network model consisting of six nodes and eight branches. Cost and processing time per component are each represented by triangular fuzzy numbers.

Zhang and Huang (1994) use fuzzy logic to model the process plan selection problem when objectives are imprecise and conflicting. Fuzzy membership functions are used to evaluate the contribution of competing process plans to shopfloor performance objectives. The optimal process plan for each part is determined by the solution of a fuzzy integer programming model. A consolidation procedure, which uses a dissimilarity criterion, then selects the process plan that best utilizes manufacturing resources. The algorithm is demonstrated for a problem consisting of three parts and eight process plans. The algorithm was also tested against non-fuzzy algorithms found in the literature. In some circumstances, more reasonable solutions where achieved as a result of the algorithm's ability to deal with the fuzziness inherent in manufacturing process planning.

Inuiguchi *et al.* (1994) compare possibilistic, flexible and goal programming approaches to solving a production planning problem. Unlike conventional methods, possibilistic programming allows ambiguous data and objectives to be included in the problem formulation. A production planning problem consisting of two manufacturing processes, two products and four structural constraints is considered. The problem is solved using possibilistic programming, flexible programming and goal programming. A comparison of the three solutions suggests that the possibilistic solution best reflects the decision maker's input, thereby emphasizing the importance of modeling ambiguity in production planning.

3.6.2 Inventory Management

Sommer (1981) uses fuzzy dynamic programming to solve a real-world inventory and production scheduling problem. Linguistic statements such as "the stock should be at best zero at the end of the planning horizon", and "diminish production capacity as continuously as possible", describe management's fuzzy aspirations for inventory and production capacity reduction in a planned withdrawal from a market. Fuzzy dynamic programming is used to determine the optimal inventory and production levels.

Park (1987) examines the economic order quantity model (EOQ) from the fuzzy set theoretic perspective. Trapezoidal fuzzy numbers are used to model ordering costs and inventory holding cost. The mode and median rules are suggested for transforming the fuzzy cost information into a scalar for input into the EOQ model. A numerical example using the median rule is presented to demonstrate a fuzzy EOQ model lot-sizing decision.

Lee *et al.* (1990) introduce the application of fuzzy set theory to lot-sizing in material requirements planning. A modified version of the part-period balancing algorithm is presented. Uncertainty in demand is modeled using triangular fuzzy numbers. A numerical example for an eight period horizon and four sets of demand data is used to demonstrate the algorithm. The authors identify two advantages in using fuzzy numbers and membership functions to model demand. First, fuzzy set theory allows both the uncertain demand and the subjective judgment of the decision maker to be incorporated into the lot-sizing decision. Second, fuzzy part-period balancing provides a richer source of data for the decision maker to use in terms of the membership values associated with the lot-sizes and costs. Lee *et al.* (1991) extend their 1990 treatment of the MRP lotsizing to include fuzzy modifications to the Silver-Meal, Wagner-Whitin, and part-period balancing algorithms. The authors argue that when demands of the master schedule are truly fuzzy, demand should be modeled using membership functions. The performance of the three fuzzy lot-sizing algorithms is compared based on nine sample problems.

Fuzzy set theory has been applied to problems in inventory management and production and process plan selection. The appeal of using fuzzy set theory in these production management problems echoes that of aggregate planning. Inventory management requires demand forecasts as well as parameters for inventory related costs such as carrying, replenishment, shortages and backorders. Precise estimates of each of these model attributes is often difficult. Similarly, in production and process plan selection problems imprecision exists in specifying demand forecasts, inventory and processing cost parameters, processing times, and routing preferences. Potential ambiguity is further increased when the problem is formulated with multiple objectives. The research studies reviewed in this section demonstrate the usefulness of fuzzy set theory in modeling and solving inventory, and production and process plan selection problems when data and objectives are subject to ambiguity.

3.7 Forecasting

The first application of forecasting using fuzzy set theory to our knowledge appeared in Economakos (1979). A simulation based model was used to forecast the demand for electrical power when load components at various times of the day were described in linguistic terms. Interest in fuzzy forecasting has grown considerably since this initial article. Research on fuzzy forecasting is divided into three categories: qualitative forecasting employing the Delphi method, time series analysis and regression analysis. A summary of fuzzy forecasting applications is presented in Table 9.

Table 9: Fuzzy Forecasting

Author(s)	Forecasting Model Used	Application
Heshmaty and Kandel (1985)	Regression (4 independent variables)	Forecast sales of computers and peripheral equipment
Tanaka *et al.* (1982)	Regression (5 independent variables)	Predict prices of prefabricated houses
Chen (1996)	Time series	Forecast University of Alabama enrollment
Song *et al.* (1995)	Time series	Modification of earlier model
Sullivan and Woodall (1994)	Markov model and time series	Forecast University of Alabama enrollment
Song and Chissom (1994)	Time-variant time series	Forecast University of Alabama enrollment
Cummins and Derrig (1993)	Forecast selection decision model	Forecast insurance loss cost
Song and Chissom (1993b)	Time-invariant time series	Forecast University of Alabama enrollment
Song and Chissom (1993a)	Time-invariant and time variant time series	Outline procedure for conducting forecasts
Shnaider and Kandel (1989)	Time series	Forecast corporate tax revenues
Ishikawa *et al.* (1993)	Delphi method	Propose New Fuzzy Delphi Method (NFDM)
Kaufmann and Gupta (1988)	Delphi method	Tutorial on fuzzy Delphi
Murray *et al.* (1985)	Delphi method	Estimate student performance as measured by GPA

3.7.1 Delphi Method

Murray *et al.* (1985) identify fuzzy set theory as an attractive methodology for modeling ambiguity in the Delphi method. Ambiguity results due to differences in the meanings and interpretations that experts may attach to words. A pilot Delphi study using graduate business students (divided into control and test groups), as experts, was conducted to estimate the percentage of students attaining an “excellent” grade point average. Over four rounds of the Delphi, the control group received the typical feedback consisting of a summary of their responses. The test group received similar feedback but was also given feedback on a group average membership function. Methodological issues inherent in the study prevent a statistical analysis of the performance of the control and test groups with respect to the Delphi task. However, the authors demonstrate an appropriate approach for modeling ambiguity in the Delphi method, and provide insights on methodological issues for future research.

Kaufmann and Gupta (1988) present a detailed tutorial on the fuzzy Delphi method in forecasting and decision making. The authors outline a four-step procedure for performing fuzzy Delphi when estimates are represented by triangular fuzzy numbers. Fuzzy Delphi is demonstrated by example for a group of twelve experts engaged in forecasting the realization of a cognitive information processing computer.

Ishikawa *et al.* (1993) cite limitations in traditional and fuzzy Delphi methods and propose the New Fuzzy Delphi Method (NFDM). The NFDM has the following advantages: (i) fuzziness is inescapably incorporated in the Delphi findings; (ii) the number of rounds in the Delphi is reduced; (iii) the semantic structure of forecast items is refined; and (iv) the individual attributes of the expert are clarified. The NFDM consists of two methodologies: the Min-Max Delphi Method, and the Fuzzy Delphi Method via Fuzzy Integration. The Max-Min Delphi Method clarifies data of each forecaster by expertise, pursues the accuracy of the forecast from the standpoint of an interval representing possibility and impossibility, and identifies the cross point as the most attainable period. The Fuzzy Delphi Method via Fuzzy Integration employs the expertise of each expert as a fuzzy measure and identifies a point estimate as the most attainable period by the fuzzy integration of each membership function. The two methodologies are illustrated by way of a technological forecasting example using members of the Japan Society for Fuzzy Theory and Systems as experts.

3.7.2 Time Series

Shnaider and Kandel (1989) develop a computerized forecasting system to forecast corporate income tax revenue for the state of Florida. Historically, the conventional econometric models used to forecast Florida corporate income tax revenue failed to provide forecasts within acceptable error bounds. The root cause of the failure of the econometric models was their inability to deal with vague and imprecise information regarding the corporate strategies of taxpayers and the skewness in the distribution of the magnitude of tax payments. The fuzzy forecasting system consists of two parts. The first part uses moving averages technique to transform time series data on corporate tax revenue and the real per capita GNP in growth patterns described by fuzzy terms. Eighteen possible fuzzy growth patterns such as: 'moderate to rapid growth', 'slight positive nominal growth', 'very rapid growth', etc. are defined. The second part of the system utilizes the fuzzy forecast as its input and in turn generates the forecasted corporate income tax revenue. A control mechanism insures that the cumulative forecasting error stays within acceptable bounds.

Song and Chissom (1993a) provide a theoretic framework for fuzzy time series modeling. A fuzzy time series is applicable when a process is dynamic and has historical data that are fuzzy sets or linguistic values. Fuzzy relational equations are employed to develop fuzzy relations among observations occurring at different time periods. Two classes of fuzzy time series models are defined: time-variant and time-invariant. A seven-step procedure is outlined for conducting a forecast using the time-invariant fuzzy time series model. Song and Chissom (1993b) apply a first order, time-invariant time series model to forecast enrollments of the University of Alabama based on twenty years of historical data. The data was fuzzified and seven fuzzy sets were defined to describe "enrollments". The corresponding linguistic values ranged from "not many" to "too many". The seven-step procedure outlined in Song and Chissom (1993a) is used to fuzzify the data, develop the time series model, and calculate and interpret the output. The errors in the forecasted enrollments ranged from 0.1% to 11%, with an average error of 3.9%. The error resulting from the fuzzy time series model is claimed to be on par with error rates cited in the literature on enrollment forecasting.

Cummins and Derrig (1993) present a fuzzy-decision making procedure for selecting the best forecast subject to a set of vague or fuzzy criteria. Selecting the best forecast, as opposed to selecting the best forecasting model, most efficiently utilizes the historical information available to the forecaster. Fuzzy set theory is used to rank candidate forecasting methods in terms of their membership values in the fuzzy set of “good” forecasts. The membership values are then used to conclude the best forecast. The methodology is demonstrated in developing a forecast of a trend component for an insurance loss cost problem. A set of 72 benchmark forecasting methods are consolidated using fuzzy set theory. On the basis of three fuzzy objectives, a fuzzy set of “good” trend factors results. The authors conclude that fuzzy set theory provides an effective method for combining statistical and judgmental criteria in actuarial decision making.

Song and Chissom (1994) apply a first order, time-variant forecasting model to the Song and Chissom (1993b) university enrollment data set. The time variant-model relaxes the invariant-model assumption that at any time t, the possible values of the fuzzy time series are the same. A 3-layer backpropagation neural network was found to be the most effective method for defuzzifying the forecast fuzzy set. The neural network defuzzification method yielded the smallest average forecasting error over a range of model structures.

Sullivan and Woodall (1994) provide a detailed review of the time-invariant and time-variant time series models set forth by Song and Chissom (1993a, 1994). The authors present a Markov model for forecasting enrollments. The Markov approach utilizes linguistic labels and uses probability distributions, as opposed to membership functions, to reflect ambiguity. Using the enrollment data set of Song and Chissom, a Markov model is described and the parameter estimation procedure is compared with that of the fuzzy time series method. The Markov approach resulted in slightly more accurate forecasts than the fuzzy time series approach. The Markov and fuzzy time series models are compared to three conventional time-invariant time series models, a first-order auto-regressive model, and two second-order auto- regressive models. The conventional models which use the actual crisp enrollment data, outperformed the fuzzy time series or Markov model and are most effective when predicting values outside the range of the historical data.

Song *et al.* (1995) note that the fuzzy time series models defined in Song and Chissom (1993a), and those used to forecast university enrollments in Song and Chissom (1993b, 1994), require fuzzification of crisp historical data. The authors present a new model based on the premise that the historical data are fuzzy numbers. The new model is based on a theorem derived to relate the current value of a fuzzy time series with its past. The theorem expresses the value of a homogeneous fuzzy time series as a linguistic summation of previous values and the linguistic differences of different orders. A second form of the model relates the value of the fuzzy time series as the linguistic summation of various order linguistic backward differences. The authors note that the findings of this study are limited only to homogeneous fuzzy time series with fuzzy number observations.

Chen (1996) presents a modified version Song and Chissom’s time-invariant fuzzy time series model. The modified model is claimed to be “obviously” more efficient than Song and Chissom’s (1993b) model. The claim is based on the proposed model’s use of simplified arithmetic operations rather than the max-min composition operators found in Song and Chissom’s model. Forecasts based on the modified model and Song and Chissom’s model are compared to the historical enrollment data. The forecasted results of the proposed model deviate slightly from Song and Chissom. The statistical significance of the comparison is not addressed.

3.7.3 Regression Analysis

Tanaka *et al.* (1982) introduce fuzzy linear regression as a means to model casual relationships in systems when ambiguity or human judgment inhibits a crisp measure of the dependent variable. Unlike conventional regression analysis, where deviations between observed and predicted values reflect measurement error, deviations in fuzzy regression reflect the vagueness of the system structure expressed by the fuzzy parameters of the regression model. The fuzzy parameters of the model are considered to be possibility distributions which corresponds to the fuzziness of the system. The fuzzy parameters are determined by a linear programming procedure which minimizes the fuzzy deviations subject to constraints of the degree of membership fit. The fuzzy regression forecasting model is demonstrated by a multiple regression example in which the prices of prefabricated houses is determined based on quality of material, floor space, and number of rooms.

Heshmaty and Kandel (1985) utilize fuzzy regression analysis to build forecasting models for predicting sales of computers and peripheral equipment. The independent variables are: user population expansion, microcomputer sales, minicomputer sales, and the price of microcomputers. The forecasts for the sales of computers and peripheral equipment are given as fuzzy sets with triangular membership functions. The decision-maker then selects the forecast figure from within the interval of the fuzzy set.

Fuzzy set theory has been used to develop quantitative forecasting models such as time series analysis and regression analysis, and in qualitative models such as the Delphi method. In these applications, fuzzy set theory provides a language by which indefinite and imprecise demand factors can be captured. The structure of fuzzy forecasting models are often simpler yet more realistic than non-fuzzy models which tend to add layers of complexity when attempting to formulate an imprecise underlying demand structure. When demand is definable only in linguistic terms, fuzzy forecasting models must be used.

4 Conclusions

This paper has discussed an extensive literature review and survey of fuzzy set theory in production management research. Throughout the course of this study, it has been observed that (1) fuzzy set theory has been applied to most traditional areas of production management research, and (2) research on fuzzy set theory in production management research has grown in recent years. As illustrated in Table 3, 83 percent of the citations identified in this study were published in the last decade. Fifty-six percent of the research cited in this study were published within the last five years. Fuzzy research in quality management, forecasting, and job shop scheduling have experienced tremendous growth in recent years.

The appropriateness and contribution of fuzzy set theory to problem solving in production management research may be seen by parallelling its use in operations research. Zimmerman (1983) identifies that fuzzy set theory can be used in operations research as a language to model problems which contain fuzzy phenomena or relationships, as a tool to analyze such models in order to gain better insight into the problem and as an algorithmic tool to make solution procedures more stable or faster. These three notions clearly are applicable to research in the production management environment.

The research compiled in this review came from 18 journals and nine books and edited volumes. Our motivation was to identify where fuzzy set theory has been used in production research. Ideally, this foundation will assist researchers currently engaged in fuzzy set research in production management and may lead to the identification and stimulation of areas requiring additional research. This account should give production management researchers new tools and ideas on how to approach production management problems using fuzzy set theory. It also provides a basis for fuzzy set researchers to expand on the toolset for production management problems.

References

[1] Bhattacharya, U., Rao, J. R. and Tiwari, R. N. (1992) Fuzzy multi-criteria facility location problem, *Fuzzy Sets and Systems*, 51(3), 277-287.

[2] Bhattacharya, U., Rao, J. R. and Tiwari, R. N. (1993) Bi-criteria multi facility location problem in fuzzy environment, *Fuzzy Sets and Systems*, 56(2), 145-153.

[3] Bradshaw, C. W. (1983) A fuzzy set theoretic interpretation of economic control limits, *European Journal of Operational Research*, 13(4), 403-408.

[4] Buckley, J. J. (1989) *Fuzzy PERT, in Applications of Fuzzy Set Methodologies in Industrial Engineering*, Evans, G. W., Karwowski, W. and Wilhelm, M. R. (eds.), Elsevier Science Publishers B. V., Amsterdam, 103-114.

[5] Chanas, S. and Kamburowski, J. (1981) The use of fuzzy variables in PERT, *Fuzzy Sets and Systems*, 5(1), 11-19.

[6] Chakraborty, T. K. (1988) A single sampling attribute plan of given strength based on fuzzy goal programming, *Opsearch*, 25(4), 259-271.

[7] Chakraborty, T. K. (1992) A class of single sampling plans based on fuzzy optimization, *Opsearch*, 29(1), 11-20.

[8] Chakraborty, T. K. (1994a) Possibilistic parameter single sampling inspection plans, *Opsearch*, 31(2), 108-126.

[9] Chakraborty, T. K. (1994b) A class of single sampling inspection plans based on possibilistic programming problem, *Fuzzy Sets and Systems*, 63(1), 35-43.

[10] Chang, S., Tsujimura, Y., Gen, M. and Tozawa, T. (1995) An efficient approach for large scale project planning based on fuzzy Delphi method, *Fuzzy Sets and Systems*, 76(2), 277-288.

[11] Chen, S.-M. (1996) Forecasting enrollments based on fuzzy time series, *Fuzzy Sets and Systems*, 81(3), 311-319.

[12] Chung, K. and Tcha, D. (1992) A fuzzy set-theoretic method for public facility location, *European Journal of Operational Research*, 58(1), 90-98.

[13] Cummins, J. D. and Derrig, R. A. (1993) Fuzzy trends in property-liability insurance claim costs, *Journal of Risk and Insurance*, 60(3), 429-465.

[14] Darzentas, J. (1987) A discrete location model with fuzzy accessibility measures, *Fuzzy Sets and Systems*, 23(1), 149-154.

[15] DePorter, E. L. and Ellis, K. P. (1990) Optimization of project networks with goal programming and fuzzy linear programming, *Computers and Industrial Engineering*, 19(1-4), 500-504.

[16] Dubois, D. and Prade, H. (1985) *Possibility Theory: An Approach to Computerized Processing of Uncertainty*, Plenum Press: New York.

[17] Dweiri, F. and Meier, F. A. (1996) Application of fuzzy decision-making in facilities layout planning, *International Journal of Production Research*, 34(11), 3207-3225.

[18] Economakos, E. (1979) Application of fuzzy concepts to power demand forecasting, *IEEE Transactions on Systems, Man and Cybernetics*, 9(10), 651-657.

[19] Evans, G. W., Wilhelm, M. R. and Karwowski, W. (1987) A layout design heuristic employing the theory of fuzzy sets, *International Journal of Production Research*, 25(10), 1431-1450.

[20] Gaines, B. R. and Kohout, L. J. (1977) The fuzzy decade: a bibliography of fuzzy systems and closely related topics, *International Journal of Man-Machine Studies*, 9(1), 1-68.

[21] Gen, M., Tsujimura, Y. and Ida, K. (1992) Method for solving multiobjective aggregate production planning problem with fuzzy parameters, *Computers and Industrial Engineering*, 23(1-4), 117-120.

[22] Glushkovsky, E. A. and Florescu, R. A. (1996) Fuzzy sets approach to quality improvement, *Quality and Reliability Engineering International*, 12(1), 27-37.

[23] Grabot, B. and Geneste, L. (1994) Dispatching rules in scheduling: a fuzzy approach, *International Journal of Production Research*, 32(4), 903-915.

[24] Grobelny, J. (1987a) On one possible 'fuzzy' approach to facilities layout problems, *International Journal of Production Research*, 25(8), 1123-1141.

[25] Grobelny, J. (1987b) The fuzzy approach to facilities layout problems, *Fuzzy Sets and Systems*, 23(2), 175-190.

[26] Gutierrez, I. and Carmona, S. (1995) Ambiguity in multicriteria quality decisions, *International Journal of Production Economics*, 38(2/3), 215-224.

[27] Han, S., Ishii, H. and Fujii, S. (1994) One machine scheduling problem with fuzzy due dates, *European Journal of Operational Research*, 79(1), 1-12.

[28] Hapke, M., Jaszkiewicz, A. and Slowinski, R. (1994) Fuzzy project scheduling system for software development, *Fuzzy Sets and Systems*, 67(1), 101-117.

[29] Heshmaty, B. and Kandel, A. (1985) Fuzzy linear regression and its applications to forecasting in uncertain environment, *Fuzzy Sets and Systems*, 15, 159-191.

[30] Inuiguchi, M., Sakawa, M. and Kume, Y. (1994) The usefulness of possibilistic programming in production planning problems, *International Journal of Production Economics*, 33(1-3), 45-52.

[31] Ishibuchi, H., Yamamoto, N., Misaki, S. and Tanaka, H. (1994) Local search algorithms for flow shop scheduling with fuzzy due-dates, *International Journal of Production Economics*, 33, 53-66.

[32] Ishii, H. and Tada, M. (1995) Single machine scheduling problem with fuzzy precedence relation, *European Journal of Operational Research*, 87(2), 284-288.

[33] Ishii, H., Tada, M. and Masuda, T. (1992) Two scheduling problems with fuzzy due-dates, *Fuzzy Sets and Systems*, 6(3), 339-347.

[34] Ishikawa, A., Amagasa, M., Tomizawa, G., Tatsuta, R. and Mieno, H. (1993) The max-min Delphi method and fuzzy Delphi method via fuzzy integration, *Fuzzy Sets and Systems*, 55(3), 241-253.

[35] Kacprzyk, J. and Staniewski, P. (1982) Long-term inventory policy-making through fuzzy decision-making models, *Fuzzy Sets and Systems*, 8(2), 117-132.

[36] Kanagawa, A. and Ohta, H. (1990) A design for single sampling attribute plan based on fuzzy sets theory, *Fuzzy Sets and Systems*, 37(2), 173-181.

[37] Kanagawa, A., Tamaki, F. and Ohta, H. (1993) Control charts for process average and variability based on linguistic data, *International Journal of Production Research*, 31(4), 913-922.

[38] Kandel, A. (1986) *Fuzzy Mathematical Techniques with Applications*, Addison-Wesley: Reading, MA.

[39] Kandel, A. and Yager, R. (1979) A 1979 bibliography on fuzzy sets, their applications, and related topics, in *Advances in Fuzzy Set Theory and Applications*, Gupta, M. M., Ragade, R. K. and Yager, R. R. (eds.), North-Holland: Amsterdam, 621-744.

[40] Karwowski, W. and Evans, G. W. (1986)Fuzzy concepts in production management research: a review, *International Journal of Production Research*, 24(1), 129-147.

[41] Kaufmann, A. (1986) On the relevance of fuzzy sets for operations research, *European Journal of Operational Research*, 25, 330-335.

[42] Kaufmann, A. and Gupta, M. M. (1988) *Fuzzy Mathematical Models in Engineering and Management Science*, North-Holland: Amsterdam.

[43] Khoo, L. P. and Ho, N. C. (1996) Framework of a fuzzy quality deployment system, *International Journal of Production Research*, 34(2), 299-311.

[44] Lai, Y.-J. and Hwang, C.-L. (1994) *Fuzzy Multiple Objective Decision Making Methods and Applications*, Springer-Verlag: Berlin.

[45] Lee, Y. Y., Kramer, B. A. and Hwang, C. L. (1990) Part-period balancing with uncertainty: a fuzzy sets theory approach, *International Journal of Production Research*, 28(10), 1771-1778.

[46] Lee, Y. Y., Kramer, B. A. and Hwang, C. L. (1991) A comparative study of three lot-sizing methods for the case of fuzzy demand, *International Journal of Operations and Production Management*, 11(7), 72-80.

[47] Lehtimaki, A. K. (1987) An approach for solving decision problems of master scheduling by utilizing theory of fuzzy sets, *International Journal of Production Research*, 25(12), 1781-1793.

[48] Lootsma, F. A. (1989) Stochastic and Fuzzy PERT, *European Journal of Operational Research*, 43(2), 174-183.

[49] Lorterapong, P. (1994) A fuzzy heuristic method for resource-constrained project scheduling, *Project Management Journal*, 25(4), 12-18.

[50] Maiers, J. and Sherif, Y. S. (1985) Applications of fuzzy set theory, *IEEE Transactions on Systems, Man and Cybernetics*, 15(1), 175-189.

[51] McCahon, C. S. (1993) Using PERT as an approximation of fuzzy project-network analysis, *IEEE Transactions on Engineering Management*, 40(2), 146-153.

[52] McCahon, C. S. and Lee, E. S. (1988) Project network analysis with fuzzy activity times, *Computers and Mathematics with Applications*, 15(10), 829-838.

[53] McCahon, C. S. and Lee, E. S. (1990) Job sequencing with fuzzy processing times, *Computers and Mathematics with Applications*, 19(7), 31-41.

[54] McCahon, C. S. and Lee, E. S. (1992) Fuzzy job sequencing for a flow shop, *European Journal of Operational Research*, 62(3), 294-301.

[55] Mital, A. and Karwowski, W. (1989) A framework of the fuzzy linguistic approach to facilities location problem, in *Applications of Fuzzy Set Methodologies in Industrial Engineering*, Evans, G. W., Karwowski, W. and Wilhelm, M. R. (eds.), Elsevier Science Publishers B. V., Amsterdam, 323-330.

[56] Mital, A., Kromodihardjo, S., Metha, M. and Karwowski, W. (1988) Facilities location: quantifying subjective criteria using fuzzy linguistic approach, in *Recent Developments in Production Research*, Mital, A. (ed.), Elsevier Science Publishers B. V., Amsterdam, 307-314.

[57] Murray, T. J., Pipino, L. L. and vanGigch, J. P. (1985) A pilot study of fuzzy set modification of Delphi, *Human Systems Management*, 5(1), 76-80.

[58] Narasimhan, R. (1979) A fuzzy subset characterization of a site-selection problem, *Decision Sciences*, 10(4), 618-628.

[59] Nasution, S. H. (1994) Fuzzy critical path, *IEEE Transactions on Systems, Man and Cybernetics*, 24(1), 48-57.

[60] Negoita, C. V. (1981) The current interest in fuzzy optimization, *Fuzzy Sets and Systems*, 6(3), 261-269.

[61] Ohta, H. and Ichihashi, H. (1988) Determination of single- sampling-attribute plans based on membership functions, *International Journal of Production Research*, 26(9), 1477-1485.

[62] Park, K. S. (1987) Fuzzy-set theoretic interpretation of economic order quantity, *IEEE Transactions on Systems, Man and Cybernetics*, 17(6), 1082-1084.

[63] Prade, H. (1979) Using fuzzy set theory in a scheduling problem: a case study, *Fuzzy Sets and Systems*, 2(2), 153-165.

[64] Raoot, A. and Rakshit, A. (1991) A ‘fuzzy’ approach to facilities lay-out planning, *International Journal of Production Research*, 29(4), 835-857.

[65] Raoot, A. and Rakshit, A. (1993) A ‘linguistic pattern’ approach for multiple criteria facility layout problems, *International Journal of Production Research*, 31(1), 203-222.

[66] Raoot, A. and Rakshit, A. (1994) A ‘fuzzy’ heuristic for the quadratic assignment formulation to the facility layout problem, *International Journal of Production Research*, 32(3), 563-581.

[67] Raz, T. and Wang, J. (1990) Probabilistic and membership approaches in the construction of control charts for linguistic data, *Production Planning and Control*, 1(3), 147-157.

[68] Rinks, D. B. (1981) A heuristic approach to aggregate production scheduling using linguistic variables, in *Applied Systems and Cybernetics* - Vol. VI, Lasker, G. E. (ed.), Pergamon Press: New York, 2877-2883.

[69] Rinks, D. B. (1982a) The performance of fuzzy algorithm models for aggregate planning under differing cost structures, in *Fuzzy Information and Decision Processes*, Gupta, M. M. and Sanchez, E. (eds.), North-Holland Publishing: Amsterdam, 267-278.

[70] Rinks, D. B. (1982b) A heuristic approach to aggregate planning production scheduling using linguistic variables: methodology and application, in *Fuzzy Set and Possibility Theory*, Yager, R. R. (ed.), Pergamon Press: New York, 562-581.

[71] Roy, U. and Zhang, X. (1996) A heuristic approach to n/m job shop fuzzy dynamic scheduling algorithms, *Production Planning and Control*, 7(3), 299-311.

[72] Shnaider, E. and Kandel, A. (1989) The use of fuzzy set theory for forecasting corporate tax revenues, *Fuzzy Sets and Systems*, 31(2), 187-204.

[73] Singh, N. and Mohanty, B. K. (1991) A fuzzy approach to multi-objective routing problem with applications to process planning in manufacturing systems, *International Journal of Production Research*, 29(6), 1161-1170.

[74] Shipley, M. F., De Korvin, A. and Omer, K. (1996) A fuzzy logic approach for determining expected values: a project management application, *Journal of the Operational Research Society*, 47(4), 562-569.

[75] Sommer, G. (1981) *Fuzzy inventory scheduling, in Applied Systems and Cybernetics* - Vol. VI, Lasker, G. E. (ed.), Pergamon Press: New York, 3052-3060.

[76] Song, Q. and Chissom, B. S. (1993a) Fuzzy time series and its models, *Fuzzy Sets and Systems*, 54(3), 269-277.

[77] Song, Q. and Chissom, B. S. (1993b) Forecasting enrollments with fuzzy time series - part I, *Fuzzy Sets and Systems*, 54(1), 1-9.

[78] Song, Q. and Chissom, B. S. (1994) Forecasting enrollments with fuzzy time series - part II, *Fuzzy Sets and Systems*, 62(1), 1-8.

[79] Song, Q., Leland, R. P. and Chissom, B. S. (1995) A new fuzzy time-series model of fuzzy number observations, *Fuzzy Sets and Systems*, 73(3), 341-348.

[80] Sullivan, J. and Woodall, W. H. (1994) A comparison of fuzzy forecasting and Markov modeling, *Fuzzy Sets and Systems*, 64(3), 279-293.

[81] Tanaka, H., Uejima, S. and Asai, K. (1982) Linear regression analysis with fuzzy model, *IEEE Transactions on Systems, Man and Cybernetics*, 12(6), 903-907.

[82] Tsujimura, Y., Park S. H., Change, I. S. and Gen, M. (1993) An effective method for solving flow shop problems with fuzzy processing times, *Computers and Industrial Engineering*, 25(1-4), 239-242.

[83] Turksen, I. B. (1988a) Approximate reasoning for production planning, *Fuzzy Sets and Systems*, 26(1), 23-37.

[84] Turksen, I. B. (1988b) An approximate reasoning framework for aggregate production planning, in *Computer Integrated Manufacturing*, NATO ASI Series, Vol. 49, Turksen, I. B. (ed.), Springer-Verlag: Berlin, 243-266.

[85] Turksen, I. B. (1992) Fuzzy expert systems for IE/OR/MS, *Fuzzy Sets and Systems*, 51(1), 1-27.

[86] Wang, J.-H. and Raz, T. (1990) On the construction of control charts using linguistic variables, *International Journal of Production Research*, 28(3), 477-487.

[87] Wang, R.-C. and Chen, C.-H. (1995) Economic statistical np-control chart designs based on fuzzy optimization, *International Journal of Quality and Reliability Management*, 12(1), 82-92.

[88] Ward, T. L., Ralston, P. A. S. and Davis, J. A. (1992) Fuzzy logic control of aggregate production planning, *Computers and Industrial Engineering*, 23(1-4), 137-140.

[89] Yongting, C. (1996) Fuzzy quality and analysis on fuzzy probability, *Fuzzy Sets and Systems*, 83(2), 283-290.

[90] Zadeh, L. A. (1965) Fuzzy Sets, *Information and Control*, 8, 338-353.

[91] Zhang, H.-C. and Huang, S. H. (1994) A fuzzy approach to process plan selection, *International Journal of Production Research*, 32(6), 1265-1279.

[92] Zimmerman, H.-J. (1983) Using fuzzy sets in operational research, *European Journal of Operational Research*, 13(3), 201-216.

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