

Strategic capacity planning in supply chain design for a new market opportunity

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This paper addresses the problem of supply chain design at the strategic level when production/distribution of a new market opportunity has to be launched in an existing supply chain. The new market opportunity is characterized by a deterministic forecast expected to occur per period. The product (or service) is assumed to be produced (or provided) in a three-stage capacitated supply chain where the first stage concerns suppliers, the second stage producers and the final stage customers. There could be multiple alternatives at each stage which are defined as nodes. Nodes in each stage are connected to the next stage through capacitated transportation systems. Production capacity at the second stage (i.e. producers) are also limited since they may already be involved in other existing activities. The objective is to perform strategic capacity planning in the supply chain in order to meet the demand of the new opportunity at minimal cost. A linear running cost is associated with each node. If the decision is to increase the capacity of a node, then a fixed cost applies, followed by a cost that is proportional to the additional capacity.

The overall problem can be modelled as a large-scale mixed integer linear programming problem. A solution algorithm is developed to overcome difficulties associated with the size of the problem and is tested on empirical data sets. The overall contribution is an analytical tool that can be employed by managers responding to the new market opportunity at the strategic level for supply chain design.

1. Introduction

Supply chain networks are considered as solutions for effectively meeting customer requirements such as low costs, high product variety, quality and shorter lead times. The success of a supply chain lies in good strategic and tactical planning and monitoring at the operational level. Strategic planning is long-term planning and usually involves selecting providers and distributors, location and capacity planning of manufacturing/servicing units, among others. In the context of supply chain design we usually consider two aspects in the selection of partners: the qualitative aspect and the quantitative aspect. The qualitative aspects are the primary selection criteria, such as the financial position of the partner, quality policy, previous history, adaptability towards change of product type or market situations.

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In quantitative selection criteria we usually consider the cost effectiveness of the partners with respect to the supply chain. This consideration involves, for example, capacity planning, transportation network design, identification of distributors in the potential market. Plenty of models are discussed in the literature. In Erenguc *et al.* (1999) the authors proposed a model of a supply chain network composed of three stages: supplier network, producer network and distributor network, and defined the nature of the relationship between each stage. In Cohen and Lee (1988), a strategic model structure and a hierarchical decomposition approach for the supply chain are presented. Nagurney *et al.* (2002) presented a network equilibrium model and discussed qualitative properties of the model. For an extensive review of strategic production–distribution models in a global supply environment, the reader is referred to Vidal (1997) and Govil and Proth (2002).

In this paper we restrict ourselves to strategic capacity planning only. We consider the case of supply chain design for a new market opportunity when the demand in each period is known. All the potential partners have limited capacity (production and transportation) and the capacity can be increased by introducing new resources. The problem is to select providers and producers from among the available ones. The problem is modelled as a mixed integer programming problem, and the difficulty arises when the number of integer variable increases.

Integer linear programming problems constitute a subclass of combinatorial optimization problems (Hans 2001). Many problem-specific algorithms exist for finding the feasible solutions or even optimal solutions. Three basic methods, branch-and-bound (Mitten 1970), cutting plane algorithm, and dynamic programming, are widely used for solving integer linear programming problems (Winston 1993). Sometimes these methods are used in conjunction with others; for instance when the cutting plane method is used with branch-and-bound, the technique is known as the branch-and-cut algorithm (Hoffman and Padberg 1985) and when column generation (Dantzig and Wolfe 1960) is used in conjunction with branch-and-bound, it is referred to as branch-and-price (Vance *et al.* 1994). All the algorithms using branch-and-bound require a good starting bound which is usually obtained by LP relaxations. Lagrangian relaxation (Geoffrion 1974, Fisher 1985) is one of the available methods to obtain a good feasible bound.

In solving combinatorial optimization problems, there is always a trade-off between computational time and quality of the solution. In this paper we present a heuristic approach to find a good solution. Latterly, this solution can be used as an upper bound for the branch-and-bound algorithm developed to reach the optimal solution.

The remainder of the paper is organized as follows. Section 2 presents the problem description and formulation. The solution approach based on the key properties is presented in section 3. Numerical results of a computational study are presented in section 4. Finally, conclusions are presented in section 5.

2. Problem description and formulation

2.1. Problem description

We consider a three-echelon system in which the first echelon concerns providers, the second echelon concerns producers and the last echelon concerns distributors. We assume that all the partners under consideration already meet the prerequisite requirements. All distributors have a definite demand in each period on a given

horizon. Providers provide the raw material/semi-finished products to selected producers, and these producers fulfil the demand of each distributor. Each provider and producer has its production cost and transportation costs to the next stage. These costs are invariant in time. The transportation costs vary from one pair (provider–producer and producer–distributor) to another. Each provider and each producer has limited production and transportation capacities. The transportation and production capacities can be extended by investing in resources. We assume that investment is possible only at the beginning of the first period. The transportation costs and production costs are linear functions of quantities. Investment cost is also a linear function of quantity, but with an additional fixed cost which does not depend upon quantities but only on the entities it is related to. The problem is to select the most economic combination of providers and producers such that they satisfy the demand imposed by all individual distributors.

2.2. Mixed integer programming formulation

Let $i \in \{1, 2, \dots, P\}$ be the providers, $j \in \{1, 2, \dots, M\}$ be the producers and $k \in \{1, 2, \dots, D\}$ be the distributors. The demand is deterministic and is known for each distributor on time horizon T . We consider four investments in this model: investment to enhance the capacity of providers, investment to enhance the capacity of producers and investment to enhance the transportation capacity between provider–producer and producer–distributor. The notations used are as follows:

- P_i : raw material cost per unit at provider i .
- m_j : production cost per unit at producer j .
- $u_{i,j}$: transportation cost per unit from provider i to producer j .
- $v_{j,k}$: transportation cost per unit from producer j to distributor k .
- $R_{i,j}$: available transportation capacity from provider i to producer j .
- $r_{i,j}$: added transportation capacity from provider i to producer j .
- $S_{j,k}$: available transportation capacity from producer j to distributor k .
- $s_{j,k}$: added transportation capacity from producer j to distributor k .
- G_i : available production capacity for provider i .
- g_i : added production capacity for provider i .
- H_j : available production capacity for producer j .
- h_j : added production capacity for producer j .
- α : discount rate.
- β : depreciation factor.
- $x_{i,j}^t$: raw material shipped from provider i to producer j in period t .
- $y_{j,k}^t$: product shipped from producer j to distributor k in period t .

In this model, we have the following four investment costs:

Investment cost for new transportation capacity $r_{i,j}$ from provider i to producer j

$$\text{Cost} = \begin{cases} A_{i,j} + a_{i,j} r_{i,j} & \text{if } r_{i,j} > 0 \\ 0 & \text{if } r_{i,j} = 0 \end{cases}.$$

Investment cost for new transportation capacity $s_{j,k}$ from producer j to distributor k

$$\text{Cost} = \begin{cases} B_{j,k} + b_{j,k} s_{j,k} & \text{if } s_{j,k} > 0 \\ 0 & \text{if } s_{j,k} = 0 \end{cases}.$$

Investment for provider i to enhance its capacity by g_i

$$\text{Cost} = \begin{cases} E_i + e_i g_i & \text{if } g_i > 0 \\ 0 & \text{if } g_i = 0 \end{cases}.$$

Investment for producer j to enhance its capacity by h_j

$$\text{Cost} = \begin{cases} F_j + f_j h_j & \text{if } h_j > 0 \\ 0 & \text{if } h_j = 0 \end{cases}.$$

The above investment costs shows that the investment cost is zero if the capacity addition is zero and consists of a fixed cost and quantity proportional to the added capacity if the capacity addition is positive.

Now, the problem which we denote by P_1 can be formulated as follows:

$$\begin{aligned} P_1 \text{MIN} & \sum_{t=1}^T \sum_{i=1}^P \sum_{j=1}^M C_i (p_i + u_{i,j}) x_{i,j}^t + \sum_{t=1}^T \sum_{j=1}^M \sum_{k=1}^D Ct (m_j + v_{j,k}) y_{j,k}^t \\ & + \sum_{i=1}^P \sum_{j=1}^M (C_1 - D_1)(a_{i,j} r_{i,j} + A_{i,j} U_{i,j}) + \sum_{j=1}^M \sum_{k=1}^D (C_1 - D_1)(b_{j,k} s_{j,k} + B_{j,k} V_{j,k}) \\ & + \sum_{i=1}^P (C_1 - D_1)(e_i g_i + E_i W_i) + \sum_{j=1}^M (C_1 - D_1)(f_j h_j + F_j X_j). \end{aligned}$$

Subjected to:

$$x_{i,j}^t \leq R_{i,j} + r_{i,j}; \quad t \in \{1, \dots, T\}, i \in \{1, 2, \dots, P\}, j \in \{1, 2, \dots, M\} \quad (1)$$

$$y_{j,k}^t \leq S_{j,k} + s_{j,k}; \quad t \in \{1, \dots, T\}, i \in \{1, 2, \dots, P\}, j \in \{1, 2, \dots, M\} \quad (2)$$

$$\sum_{j=1}^M x_{i,j}^t \leq G_i + g_i; \quad i \in \{1, 2, \dots, P\}, t \in \{1, 2, \dots, T\} \quad (3)$$

$$\sum_{k=1}^D y_{j,k}^t \leq H_j + h_j; \quad j \in \{1, 2, \dots, M\}, t \in \{1, 2, \dots, T\} \quad (4)$$

$$\sum_{i=1}^P x_{i,j}^t = \sum_{k=1}^d y_{j,k}^t; \quad t \in \{1, \dots, T\}, j \in \{1, 2, \dots, M\} \quad (5)$$

$$\sum_{j=1}^M y_{j,k}^t = d_k^t; \quad t \in \{1, \dots, T\}, k \in \{1, 2, \dots, D\} \quad (6)$$

$$r_{i,j} \leq Z_{i,j}^r \cdot U_{i,j}; \quad i \in \{1, 2, \dots, P\}, j \in \{1, 2, \dots, M\} \quad (7)$$

$$s_{j,k} \leq Z_{j,k}^s \cdot V_{j,k}; \quad j \in \{1, 2, \dots, M\}, k \in \{1, 2, \dots, D\} \quad (8)$$

$$g_i \leq Z_i^g \cdot W_i; \quad i \in \{1, 2, \dots, P\} \quad (9)$$

$$h_j \leq Z_j^h \cdot X_j; \quad j \in \{1, 2, \dots, M\} \quad (10)$$

$$U_{i,j} \in \{0, 1\}, V_{j,k} \in \{0, 1\}, W_i \in \{0, 1\}, X_j \in \{0, 1\} \quad (11)$$

$$C_t = (1 + \alpha)^{T-t+1}, D_t = (1 - \beta)^{T-t+1} \quad t \in \{1, 2, \dots, T\}, \quad (12)$$

In the above formulation, the constraints (1) and (2) guarantee that the shipment must be less than or equal to the available transportation capacity $R_{i,j} + r_{i,j}$ and $S_{j,k} + s_{j,k}$, respectively. Similarly, constraints (3) and (4) guarantee that the delivery in each period must not exceed the capacity. Constraint (5) guarantees that the total shipment, in each period, from all providers to all producers must be equal to the total shipment from all producers to all distributors. Constraint (6) guarantees that the total shipment from all producers to any distributor in each period must be equal to the demand of the distributor in that period. $Z_{j,k}^s$, $Z_{i,j}^r$, Z_i^g , and Z_j^h are big numbers greater than the total demand and $U_{i,j}$, $V_{i,j}$, W_i and X_j are binary integer variables. These binary variables are associated with the fixed cost in the objective function and take positive values only if capacity enhancements are made.

The problem would be simple and can be solved using the Minimum cost flow algorithm efficiently if we knew precisely the investment locations. The problem arises when we introduce binary variables and the size of the problem is large. For instance, the number of binary variables required for this problem is $(P + M + P M + M D)$. In the above model we assume that investment takes place in the first period only just to reduce the size of the problem. In the latter case the number of binary variables will be equal to $(P + M + P M + M D)T$. But we can see that the formulation can easily be generalized for multi-period investment and a multi-product chain. The above formulation is a mixed integer programming formulation.

We denote by P_2 the problem P_1 in which the variables $U_{i,j}$, $V_{i,j}$, W_i and X_j can take any value in $[0, 1]$. Thus, P_2 is obtained by relaxing constraints (11) in P_1 .

3. Properties and solution approach

3.1. Result 1

There exists at least one optimal solution to the problem P_2 in which constraints (7)–(10) are saturated.

Proof

Assume for instance, that $g_i < Z_i^g W_i$ for an optimal solution S_1 . Then consider a solution S_2 that is the same as S_1 , except that $W_i = g_i/Z_i^g$. S_2 is still feasible.

Furthermore,

$$C(S_1) - C(S_2) = E_i(W_i - g_i/Z_i^g) \geq 0, \quad (13)$$

where $C(\cdot)$ denotes the cost related to solution (\cdot) . Since $E_i \geq 0$, $C(S_1) \geq C(S_2)$. Two cases are possible: either $C(S_1) > C(S_2)$ and S_1 is not optimal, which contradicts the hypothesis, or $C(S_1) = C(S_2)$, and S_2 and S_1 are both optimal. This completes the proof.

Corollary

Since there exists at least one optimal solution such that constraints (7)–(10) are saturated, we can reduce the size of the problem P_2 by replacing $U_{i,j}$ by $r_{i,j}/Z_{i,j}^r$, $V_{j,k}$ by $s_{j,k}/Z_{j,k}^s$, W_i by g_i/Z_i^g and X_j by h_j/Z_j^h in the objective function and removing all the constraints (7)–(11).

3.2. Result 2

For $m = 1, 2, \dots$, we denote by P_2^m , derived from P_2 , obtained by:

- replacing $Z_{i,j}^r$ (resp. $Z_{j,k}^s, Z_i^g$ and Z_j^h) by $r_{i,j}^{m-1}$ (resp. $s_{j,k}^{m-1}, g_i^{m-1}$ and h_j^{m-1}) if $r_{i,j}^{m-1} > 0$ (resp. $s_{j,k}^{m-1} > 0, g_i^{m-1} > 0$ and $h_j^{m-1} > 0$);
- keeping the previous value of $Z_{i,j}^r$ (resp. $Z_{j,k}^s, Z_i^g$, and Z_j^h) otherwise, where $r_{i,j}^{m-1}, s_{j,k}^{m-1}, g_i^{m-1}$ and h_j^{m-1} belong to the optimal solution of P_2^{m-1} .

Then:

1. We can derive a feasible solution of P_1 from the optimal solution of P_2^m by setting $U_{i,j} = 1$ (resp. $V_{j,k} = 1, W_i = 1$ and $X_j = 1$) when $r_{i,j}^m > 0$ (resp. $s_{j,k}^m, g_i^m > 0$ and $h_j^m > 0$) and solving the new problem which we will call P_3^m .
2. Let S_2^m be the optimal solution of P_2^m . Then there exists m^* such that $S_2^{m^*}$ is a feasible solution of P_1 .

Proof

1. The solution S_2^m satisfies constraints (1)–(6) of problem P_2^m , which are also constraints of P_1 . Furthermore, in the solution S_2^m , constraints (7)–(10) are saturated and $U_{i,j}, V_{j,k}, W_i$, and $X_j \in [0, 1]$. They remain satisfied if these variables are fixed to 1 and the corresponding additional capacity is strictly positive. Then solving the problem P_3^m , we obtain a solution that is feasible for P_1 . This completes the proof of the first part of the result.
2. Consider the variables $r_{i,j}$ and $U_{i,j}$ that concern the transportation from providers to producers. According to result 1 and the hypothesis given in result 2, $r_{i,j}^m = a^{m-1} \cdot U_{i,j}^m$ replaces the constraint (7), where a^{m-1} is either
 - equal to $r_{i,j}^M, M < m$, where M , the greatest integer such that $r_{i,j}^M > 0$ belong to the optimal solution of P_2^M , if any;
 - or equal to the initial values of $Z_{i,j}^r$ if $r_{i,j}^k = 0$ for $k = 1, 2, \dots, m - 1$. Thus we have to consider two cases:

Case 1:

$U_{i,j}^k = r_{i,j}^k / r_{i,j}^{M_k}$, where $M_k < k$ is defined as M . This leads to:

$$\prod_{k=2}^m U_{i,j}^k = \prod_{k=2}^m \left(r_{i,j}^k / r_{i,j}^{M_k} \right) = r_{i,j}^m / r_{i,j}^{M_1},$$

and

$$r_{i,j}^m = r_{i,j}^{M_1} \prod_{k=2}^m U_{i,j}^k. \tag{14}$$

Since $U_{i,j}^k \in [0, 1]$, we see that $r_{i,j}^k$ decreases with k , i.e. with the number of iterations.

Case 2:

$r_{i,j}^k = 0$ for $k = 1, 2, \dots, m - 1$, and the next value of this variables, that is $r_{i,j}^m$, can only be obtained by solving P_2^m . Let E_k be the set of variables $r_{i,j}$ that are strictly positive in the optimal solution S_2^k of P_2^k , and F_k be the set of variables that are equal to zero in S_2^k .

Whatever $k = 1, 2, \dots$, we know that:

$$\sum_{i \in E_k} r_{i,j}^k = \text{constant} \tag{15}$$

since this quantity is the sum of the additional capacities, which is a constant (see problem P_1 and P_2^k). Furthermore, if the same variable $r_{i,j}$ belongs to E_{k_1} and E_{k_2} , $k_2 > k_1$, then according to (14):

$$r_{i,j}^{k_2} \leq r_{i,j}^{k_1}. \quad (16)$$

Since the number of variables $r_{i,j}$ is finite, the number of sets E_k , $k = 1, 2, \dots$, is finite. But this sequence of sets has the following property:

if $m_2 > m_1$, we cannot have $E_{m_2} \subset E_{m_1}$. The proof is straightforward:

If we had $E_{m_2} \subset E_{m_1}$, we would have

$$\sum_{r_{i,j} \in E_{m_2}} r_{i,j}^{m_2} \leq \sum_{r_{i,j} \in E_{m_2}} r_{i,j}^{m_1} \quad (\text{see 16})$$

and

$$\sum_{r_{i,j} \in E_{m_2}} r_{i,j}^{m_1} \leq \sum_{r_{i,j} \in E_{m_1}} r_{i,j}^{m_1} \quad \text{since } E_{m_2} \subset E_{m_1}$$

Thus

$$\sum_{r_{i,j} \in E_{m_2}} r_{i,j}^{m_2} < \sum_{r_{i,j} \in E_{m_1}} r_{i,j}^{m_1}.$$

But we know that (see 14) $\sum_{r_{i,j} \in E_{m_2}} r_{i,j}^{m_2} = \sum_{r_{i,j} \in E_{m_1}} r_{i,j}^{m_1}$. This contradiction proves the above claim.

As a consequence, there exists m_1 and m_2 such that $E_{m_1} \subseteq E_{m_2}$. If $E_{m_2} = E_{m_1}$, solutions $S_2^{m_1}$ and $S_2^{m_2}$ are identical and are a feasible solution of P_1 since the binary variables are 0 or 1. If $E_{m_1} \subset E_{m_2}$, we see that the size of E_m will converge since it is limited by the number of variables $r_{i,j}$ and we return to the previous case.

The same proof can be applied to variables $s_{j,k}$, g_i , h_i and the related binary variables. This completes the proof.

3.3. Solution method

We utilize result 1 and result 2 to develop the algorithm.

3.3.1. Initialization

To initialize the variables $Z_{i,j}^r$, we take the sum of the maximum demand of all distributors on periods $1, 2, \dots, T$. This value is an upper bound of the $Z_{i,j}^r$ values that represent the additional transportation capacities from provider to producers. We subtract the available transportation capacity $R_{i,j}$ from the obtained sum. If the difference is negative, we set it to zero. We apply a similar process for each of the sets $\{Z_{j,k}^s\}$, $\{Z_i^g\}$ and $\{Z_j^h\}$.

3.3.2. Algorithm

In this algorithm, we derive a feasible solution of P_1 from each S_2^m , the optimal solution of P_2^m as mentioned in the first part of result 2. We keep the best of the feasible solutions of P_1 obtained along the iterations. The detailed algorithm is presented in the appendix.

Remarks. We introduced a mechanism that forced the solutions S_2^m of problem P_2^m to converge to a feasible solution of P_1 , but we cannot claim that this feasible solution is optimal for P_1 since the mechanism introduces additional constraints to the set of feasible solutions of P_1 and thus reduces this set.

	Algorithm solution	Optimal solution	Percentage error
1	276518	275781	0.267
2	257359	257359	0.0
3	324934	322973	0.607
4	292007	292007	0.0
5	354365	354116	0.070
6	322126	325881	1.910
7	334013	333499	0.154
8	321299	319871	0.446
9	291253	290190	0.366
10	310869	310869	0.0

Table 1. Results

4. Numerical examples

In this section we present the solutions to 10 randomly generated examples and compare the solution with the corresponding optimal solutions. All the examples are generated for five providers, five producers, five distributors and for five time periods. The ranges of different parameters are as follows:

- Available production capacities of providers: 0–500;
- Available transportation capacities of providers: 0–500;
- Production cost of providers: 5–10;
- Available production capacities of producers: 0–500;
- Available transportation capacities of producers: 0–500;
- Production cost of producers: 20–35;
- Investment costs: 1000–5000;
- Proportional investment costs: 1–10;
- Demands of all distributors: 100–500;

The optimal solutions are obtained using a branch-and-bound algorithm. The results are presented in table 1.

As we can see in these results, the solutions provided by the heuristic approach are close, if not identical, to the optimal solutions. None of these examples required more than ten iterations in order that P_2^m provide a feasible solution of P_1 .

5. Conclusion

In general, solving linear integer programming problems is difficult because of their combinatorial nature. In this paper the goal was to develop a good solution method for this specific problem without using linear integer programming methods. In our approach, because of the equality constraints in demand and supply, the heuristic algorithm which modifies the Z value at each iteration performed well. This heuristic solution can be used further as an upper bound for the branch-and-bound algorithm, if the optimal solution is of prime importance.

Appendix

Initialization

Define maxdemand = $\sum_{j=1}^D \text{Max}(d_j^t)_{t=1, \dots, T}$ and set

For $t = 1, \dots, T, j = 1, \dots, M, k = 1, \dots, D$

$Z_{i,j}^r = \text{Max}(\text{maxdemand} - R_{i,j}, 0)$, and similarly for the other $Z_s, Z_{j,k}^s, Z_i^g$ and Z_j^h .

We initialize all the Z s with a value which represents the maximal possible enhancement of the corresponding capacity.

Algorithm

In this algorithm, we derive a feasible solution of P_1 from each S_2^m , an optimal solution of P_2^m as mentioned in the first part of result 2. We keep the best of the feasible solution of P_1 obtained along the iterations.

1. Use initialization to set the Z values.
2. Set Best solution = A big positive number.
3. For count = 1, 2, ..., until the solution of P_2^{count} is identical to the solution of $P_2^{\text{count}-1}$
 - (a) Solve the problem P_2^{count} using the simplex algorithm.
 - (b) i. Define the new problem P_3^{count} derived from P_2^{count} by setting the relaxed binary variables ($U_{i,j}$, $V_{j,k}$, W_i , X_j) equal to 1 if they are greater than zero in the solution set of P_2^{count} .
 - ii. Compute the optimal solution of the problem P_3^{count} .
 - iii. Set Candidate solution = criterion value of P_3^{count} .
 - iv. If Candidate solution < Best solution replace the Best solution by the Candidate solution.
 - (c) Define the problem $P_2^{\text{count}+1}$ by setting $Z_{i,j}^r = r_{i,j}$ if $r_{i,j} > 0$ and similarly for other Z s in P_2^{count} .

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References

- COHEN, M. A. and LEE, H. L., 1988, Strategic analysis of integrated production-distribution systems: models and methods. *Operations Research*, **36**, 2, 216–218.
- DANTZIG, G. B. and WOLFE, P., 1960, Decomposition principles for linear programming. *Operations Research*, **8**, 101–111.
- ERENGUC, S. S., SIMPSON, N. C. and VAKHARIA, A. J., 1999, Integrated production/distribution planning in supply chains: An invited review. *European Journal of Operational Research*, **115**, 219–236.
- FISHER, M., 1985, An application orientated guide to langrangian relaxation. *Interfaces* **15**, **2**, 10–21.
- GEOFFRION, A., 1974, Langrangian relaxation for integer programming. *Mathematical Programming Study 2: Approaches to Integer Programming* (New York: North-Holland), pp. 82–114.
- GOVIL, M. and PROTH, J.-M., 2002, *Supply Chain Design and Management: Strategic and Tactical Perspectives* (London: Academic Press).
- HANS, E., 2001, *Resource loading by branch-and-price techniques*. Ph.D. thesis, Beta research school for operations management and logistics.
- HOFFMAN, K. L. and PADBERG, M., 1985, LP-based combinatorial problem solving. *Annals of Operations Research*, **4**, 145–194.
- MITTEN, L., 1970, Branch-and-bound methods: General formulation and properties. *Operations Research*, **5**, 268–279.
- NAGURNEY, A., DONG, J. and ZHANG, D., 2002, A supply chain network equilibrium model. *Transportation Research Part E*, **38**, 281–303.

- VANCE, P. H., BARNHARD, C., JOHNSON, E. L. and NEMHAUSER G. L., 1994, Solving binary cutting stock problems by column generation and branch-and-bound. *Computational Optimization and Applications*, **3**, 111–130.
- VIDAL, C. J. and GOETSCHALCKX, M., 1997, Strategic production-distribution models: A critical review with emphasis on global supply chain models. *European Journal of Operational Research*, **98**, 1–18.
- WINSTON, W. L., 1993, *Operations Research Application and Algorithms* (Belmont, CA: Duxbury Press).