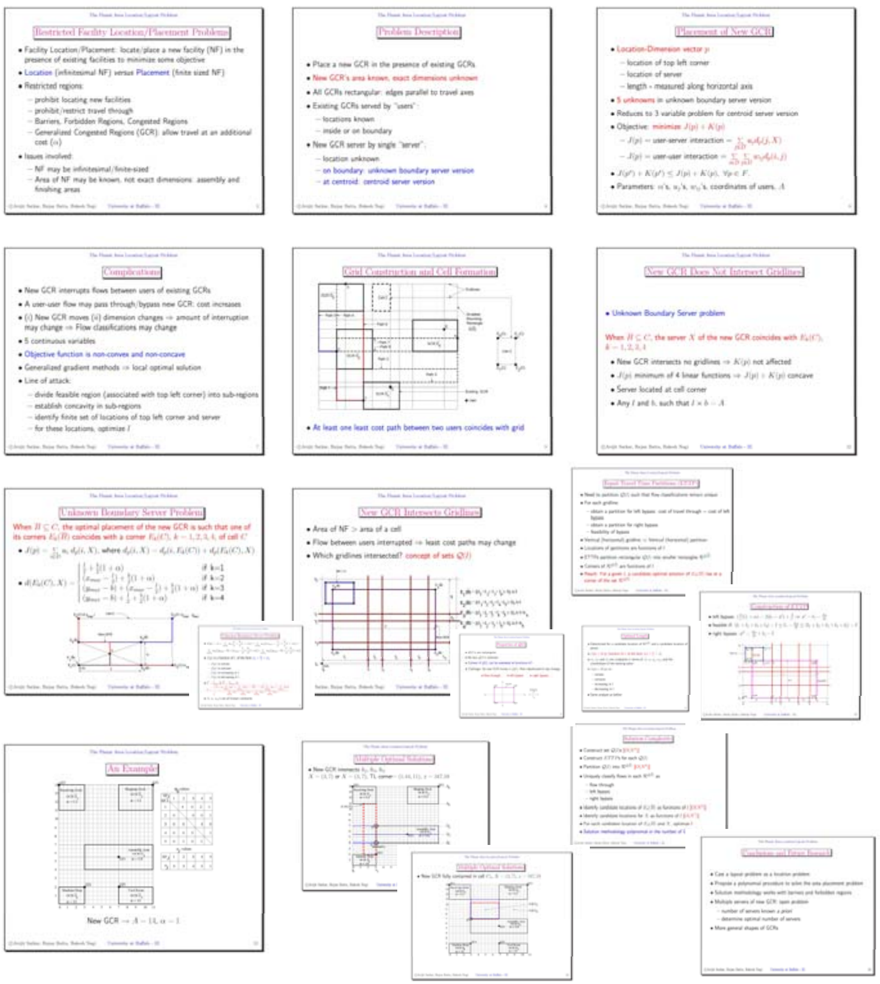
Congestion in Facilities Location and Layout: Deterministic and Stochastic Models

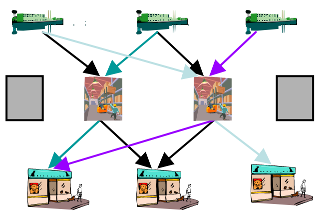
Grant #: DMI-0300370 (MES) PIs: Rakesh Nagi & Rajan Batta

Univ. at Buffalo (SUNY)

Task 1: Planar Facility Location/Layout with Generalized Congested Regions



Task 2: Connection Location Problem Simultaneous Sizing & Location of DCs



Research Motivation

- Traditionally, the location and sizing decision are treated separately
- Do not consider trade-off between Transportation cost, DC sizing and DC operating cost
- Capacity sizing based on congestion

General Model

$$\min \sum_i \sum_j \sum_k u_{ijk} f_{ij} x_{ijk} + \sum_k (F_k + t_k c_k) y_k$$

(Unit Transportation Cost: u_{ijk}; Sizing Cost: $F_k + t_k c_k$)

$$St \quad \sum_k x_{ijk} = 1, \quad \forall i, j,$$

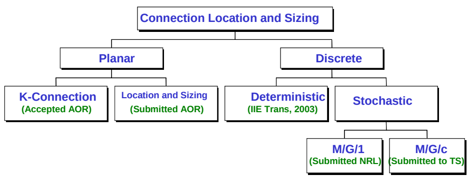
$$x_{ijk} \le y_k, \quad \forall i, j, k,$$

(Flow Assignment Constraints)

$$Pr\,(\text{All slots are occupied}) \le \beta_k, \ \forall k,$$

(Service Level Constraint)

$$x_{ijk}, c_k \ge 0, y_k \in \{0,1\}, \quad \forall i, j, k,$$



M/G/c Model

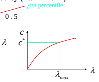
Model: An Approximate Formula

- For an *M/G/c* queueing system, the minimum number of servers for which the service level constraint is satisfied can be approximated by (Parikh 1977):

$$c \approx \frac{\lambda}{\mu} + q_\beta \sqrt{\frac{\lambda}{\mu}} + 0.5$$

(System load: $\frac{\lambda}{\mu}$; βth percentile: q_β)

- Relationship between c and λ given μ is a constant.



Deterministic Equivalent Form

$$\min \sum_i \sum_j \sum_k \alpha_{ijk} x_{ijk} + \sum_k F_k' y_k + \sum_k \gamma_k \sqrt{\sum_i \sum_j f_{ij} x_{ijk}}$$

(Concave Function)

$$St \quad \sum_k x_{ijk} = 1, \quad \forall i, j,$$

$$x_{ijk} \le y_k, \quad \forall i, j, k,$$

$$x_{ijk}, c_k \ge 0, y_k \in \{0,1\}, \quad \forall i, j, k,$$

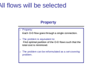
The Set-covering Problem

$$\min \sum_{W_s \in S} c_s z_s$$

$$St \quad \sum_{W_s \in S} z_s \le K, \forall k, \quad \text{(No more than potential DC sites are selected)}$$

$$\sum_{W_s \in S} b_{ijs} z_s \ge 1, \forall i, j, \quad \text{(All flows will be selected)}$$

$$z_s \in \{0,1\}, \forall W_s \in S,$$



Column Generation Approach

Master problem:

$$\min \sum_{W_s \in S'} c_s z_s$$

$$St \quad \sum_{W_s \in S'} z_s \le K, \forall k, \quad \text{Dual cost } \eta$$

$$\sum_{W_s \in S'} b_{ijs} z_s \ge 1, \forall i, j, \quad \text{Dual cost } \pi_{ij}$$

$$0 \le z_s \le 1, \forall W_s \in S',$$

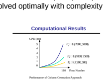
Column Generation Approach

- Pricing problem:

$$\min (F_k' - \eta) + \sum_i \sum_j (\alpha_{ijk} - \pi_{ij}) x_{ijk} + \gamma_k \sqrt{\sum_i \sum_j f_{ij} x_{ijk}}$$

$$St \quad x_{ijk} \in \{0,1\}, \forall i, j.$$

- From Shen, Coullard and Daskin (2003), the pricing problem can be solved optimally with complexity $O(mnk\log(mn))$



Computational Results

Benefit of the integrated model over the sequential Model:

- The benefit ranges from 0.8% to 14.9%
- The average benefit for all the problems is about 4%

M/G/1 Model