



Congestion in Facilities Location and Layout: Deterministic and Stochastic Models

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Task 1: Planar Facility Location/Layout with Generalized Congested Regions

Restricted Facility Location/Placement Problem

- Facility Location/Placement: locate a new facility (NF) in the presence of existing facilities to minimize some objective
- Location (multifacility NF) versus Placement (single user NF)
- Restricted regions:
 - prohibit locating new facilities
 - prohibit/restrict travel through
 - Barriers, Forbidden Regions, Congested Regions
 - Generalized Congested Regions (GCR) allow travel at an additional cost (α)
- Issues involved:
 - NF may be infinitesimal/finite-sized
 - Area of NF may be known, not exact dimensions: assembly and loading areas

Problem Description

- Place a new GCR in the presence of existing GCRs
- New GCRs may be known exact dimensions/unknown
- All GCRs rectangular; edges parallel to travel axes
- Existing GCRs served by "users"
- Locations known
 - inside or on boundary
- New GCR server by single "server":
 - location unknown
 - on boundary/unknown boundary server version
 - at centroid/centroid server version

Parameter of New GCR

- Location-Dimension vector p :
 - location of top-left corner
 - location of server
 - All GCRs rectangular; edges parallel to travel axes
 - length - measured along horizontal axis
- 3 unknowns in unknown boundary server version:
 - locations known
 - inside or on boundary
- Objective: minimize $f(p) + A(p)$
 - $f(p)$ = user-server interaction = $\sum_{i \in I} w_i d(p, X_i)$
 - $A(p)$ = user-user interaction = $\sum_{i, j \in I} w_i w_j d(X_i, X_j)$
 - $A(p) = A(p) + A(p) + A(p)$, $\forall p \in P$
 - Parameters: w_i, w_j, w_k, w_l , coordinates of users, α

Complexities

- New GCR interrupts flows between users of existing GCRs
- A user-user flow may pass through/bypass new GCR: cost increases
- (1) New GCR means (2) dimension changes in amount of interruption may change \Rightarrow Flow classifications may change
- 5 continuous variables
- Objective function is non-convex and non-concave
- Generalized gradient methods to local optimization
- Line of attack:
 - divide feasible region (associated with top-left corner) into sub-regions
 - establish convexity in sub-regions
 - identify finite set of locations of top-left corner and server
 - for these locations, optimize f

Grid Congestion and Cell Formation

- At least one least cost path between two users coincides with grid

New GCR Interacts With Unknown Boundary Server Problem

- Unknown Boundary Server problem
- When $\{i, j\} \subset C$, the server s of the new GCR coincides with $F_{ij}(C)$, $i, j = 1, 2, 3, 4$
- New GCR intersects no gridlines $\Rightarrow A(p)$ not affected
- $f(p)$ minimum of 4 linear functions $\Rightarrow f(p) + A(p)$ concave
- Server located at cell corner
- Any i and l , such that $i \neq l$

Unknown Boundary Server Problem

When $\{i, j\} \subset C$, the optimal placement of the new GCR is such that one of its corners $F_{ij}(C)$ coincides with a corner $F_{ij}(C)$, $i, j = 1, 2, 3, 4$ of cell C

$f(p) = \sum_{i \in I} w_i d(p, X_i) = \sum_{i \in I} w_i (d_{ij}(p, X_i) + \alpha d_{ij}(C, X_i))$

$d_{ij}(C, X_i) = \begin{cases} d_{ij}(C, X_i) & \text{if } i=1 \\ \max\{d_{ij}(C, X_i) - \alpha, 0\} & \text{if } i=2 \\ \max\{d_{ij}(C, X_i) - \alpha, 0\} & \text{if } i=3 \\ \max\{d_{ij}(C, X_i) - \alpha, 0\} & \text{if } i=4 \end{cases}$

New GCR Interacts With Unknown Boundary Server Problem

- Area of NF > area of a cell
- Flow between users interrupted \Rightarrow least cost paths may change
- Which gridlines intersect? Concept of using $Q(p)$

Least Cost Flow Problem (LCP)

Find to position (2) such that the objective function is a piecewise linear

Formulate LCP as an LP problem

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Formulate LCP as an LP problem

Formulate LCP as an LP problem

Ask Example

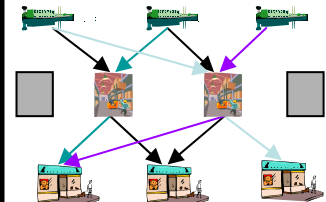
New GCR $\Rightarrow \alpha = 1, \lambda = 1$

Grid Congestion and Cell Formation

Column Generation Approach

- Use a least cost problem as a location problem
- Formulate a pricing problem to solve the one placement problem
- Iterative procedure until no more columns are generated
- Multiple copies of new GCR, not server
- Number of copies of new GCR, not server
- Number of copies of new GCR, not server
- Number of copies of new GCR, not server
- Number of copies of new GCR, not server

Task 2: Connection Location Problem Simultaneous Sizing & Location of DCs



Plants Research Motivation

- Traditionally, the location and sizing decision are treated separately
- Do not consider trade-off between Transportation cost, DC sizing and DC operating cost
- Capacity sizing based on congestion

General Model

$$\min \sum_i \sum_j \sum_k \sum_l (u_{ijl} f_{ij} x_{ijk} + \sum_k (F_k + t_k c_k) y_k)$$

Unit Transportation Cost \rightarrow Sizing Cost

Flow Assignment Constraints:

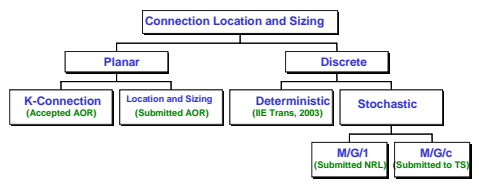
$$St \sum_k x_{ijk} = 1, \forall i, j, k$$

$$x_{ijk} \leq y_k, \forall i, j, k, k$$

Service Level Constraint:

$$Pr(\text{All slots are occupied}) \leq \beta_i, \forall k, i$$

$$x_{ijk}, c_k \geq 0, y_k \in \{0, 1\}, \forall i, j, k$$



M/G/c Model

Model: An Approximate Formula

For an M/G/c queueing system, the minimum number of servers for which the service level constraint is satisfied can be approximated by (Frank, 1977):

$$c \approx \frac{\lambda}{\mu} \left(1 + \frac{c-1}{c} \left(\frac{c}{\lambda} \right)^{\frac{1}{c}} \right)$$

Relationship between c and λ given μ is constant.

The Set-covering Problem

$$\min \sum_k c_k$$

$$St \sum_k z_k \leq K, \forall k, \text{--- No more than potential DC sites are selected}$$

$$\sum_{i,j} w_{ij} z_k \geq 1, \forall i, j, k, \text{--- All flows will be selected}$$

$$z_k \in \{0, 1\}, \forall k, k \in S$$

Column Generation Approach

Master problem:

$$\min \sum_k c_k z_k$$

$$St \sum_k z_k \leq K, \forall k, k \in S$$

$$\sum_{i,j} w_{ij} z_k \geq 1, \forall i, j, k, k \in S$$

$$0 \leq z_k \leq 1, \forall k, k \in S$$

Dual cost σ_k

Column Generation Approach

Pricing problem:

$$\min (F_k + \sigma_k) - \sum_{i,j} w_{ij} x_{ijk} + \gamma_k \left(\sum_{i,j} w_{ij} x_{ijk} \right)$$

$$St x_{ijk} \in \{0, 1\}, \forall i, j, k$$

From Shen, Couland and Dakin (2003), the pricing problem can be solved optimally with complexity $O(m \log(m))$

Computational Results

Benefit of the integrated model over the sequential Model:

- The benefit ranges from 0.8% to 14.9%
- The average benefit for all the problems is about 4%

M/G/1 Model

Formulation

Fixed Service Rate Case

$$\min \sum_k c_k$$

$$St \sum_k z_k \leq K, \forall k, k \in S$$

$$\sum_{i,j} w_{ij} z_k \geq 1, \forall i, j, k, k \in S$$

$$0 \leq z_k \leq 1, \forall k, k \in S$$

Variable Service Rate Case: Formulation

$$\min \sum_k c_k$$

$$St \sum_k z_k \leq K, \forall k, k \in S$$

$$\sum_{i,j} w_{ij} z_k \geq 1, \forall i, j, k, k \in S$$

$$0 \leq z_k \leq 1, \forall k, k \in S$$

Convexity Property: The objective function of the problem is convex

Solution Approach: Outer Approximation

Variable Service Rate Case: Sub-problem

$$\min \sum_k c_k$$

$$St \sum_k z_k \leq K, \forall k, k \in S$$

$$\sum_{i,j} w_{ij} z_k \geq 1, \forall i, j, k, k \in S$$

$$0 \leq z_k \leq 1, \forall k, k \in S$$

Variable Service Rate Case: Properties

- The Equal Utilization Rate Property: The utilization rates at the connections are identical for an optimal solution to the original problem
- $Z(p)$ is a convex function of p
- $\lambda_{ij}(p) = 0$, the rest of the equation is unique in $[0, 1]$
- Given T , we can find the optimal p by solving $Z'(p) = 0$

Conclusions and Further Work

- For approximate location optimization, the problem is reduced to sequential fixed charge location problem
- For approximate location optimization, the problem is solved iteratively by Lagrangian relaxation approach
- For multiple variable service rate case, Outer Approximation approach is required to solve the problem
- For multiple variable service rate case, Lagrangian relaxation approach is developed to solve the problem efficiently

Further Work

- Multi-server Connection Model
- Planar Connection Location Problem