# Two Tactical Models for Clustering Sensors in Wireless $Ad \ hoc$ Sensor Networks Operating in a Threat Environment

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#### Abstract

This paper builds upon recent work by Patel *et al.* [1] who presented a strategic model for clustering sensors in a threat environment. We consider two generalizations of this strategic model from a tactical perspective. The first generalization is in the modeling of the relocation cost when clusterhead locations are changed. The second generalization is in the modeling of the limited bandwidth capacity at clusterheads. Separate column generation heuristics are proposed for both generalizations. Computational studies are performed to explore the feasibility of the proposed heuristics and to study the effect of the problem parameters on the solutions obtained. Results of a case study are reported, based on a simulation testbed tool under development at the University at Buffalo (SUNY).

Keywords: Wireless *ad hoc* networks, sensor networks, maximal expected coverage, OR in military.

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## 1 Introduction

Patel, Batta and Nagi [1] present a model to deploy a given number of clusterheads over a specific time horizon, allowing relocation of clusterheads from one time period to the next and minimizing the weighted sum of expected demand covered and relocation cost. Their model is a strategic one in that it assumes a constant threat probability and relocation cost and also unlimited clusterhead capacity, and is useful for planning communication protocol for a military mission. When using this model in a tactical framework e.g. for adjustments in communication protocols during a mission, these assumptions are restrictive and unrealistic. The aim of this paper is to consider the relocation cost and clusterhead capacity constraints and show how these can be modeled and effectively dealt with.

Relocation cost is not a constant. When a clusterhead changes position, it must perform the following three activities: (i) notify sensors that it will not cover any more in the new position and disconnect the connections; (ii) establish new connections with the sensors that it starts covering in the new position; (iii) send updated information to other clusterheads. The first two activities depend on the number of sensors within a particular region, while activity three depends on the number of clusterheads and the size of the updated information. Clusterheads are also subject to bandwidth capacity constraints. Thus, for sensors which are covered by more than one clusterhead we need to decide which clusterhead it will be assigned.

To further motivate the need to consider relocation cost and clusterhead capacity in tactical modeling, we consider a problem instance with 10 sensors and 4 candidate clusterheads, in which sensor has unit demand, the number of clusterheads to be chosen is one, and the relocation cost is assumed to be 20 units for all time periods. Each clusterhead has unlimited bandwidth capacity and is capable of covering sensors within its coverage radius. Figure 1 represent the optimal solution obtained by solving the constant relocation cost/unlimited capacity model. The solution is to select candidate 1 as the clusterhead. Inclusion of a capacity constraints of 3 units will force the model to choose candidate 2 instead, due to capacity restriction. Now consider the effect of variable relocation cost. If the relocation cost reduces from 20 to 3, in a succeeding time period we may choose a different clusterhead.

In the first part of the paper, we present the variable relocation cost model, and in the second, the limited capacity model. Both models focus on the development of a network topology that maximizes the overall efficiency of the communication system in a tactical setting – which includes maximizing the expected coverage of sensors and minimizing the overhead cost of changing clusterhead locations.

# 2 Literature Review

Ad hoc network is a self organizing multi-hop wireless network that can be rapidly deployed [2]. All entities (sensors and clusterheads) in an *ad hoc* network can be mobile and communication between them is carried over a wireless medium which lacks a fixed



Figure 1: Motivating example

infrastructure. Sanchez, Evans and Minden [3] discuss some of the issues involved in *ad hoc* networks and broadly classify them as network topology, location management and routing management.

Network topology in *ad hoc* networks are either hierarchical or flat. Ramanathan and Steenstrup [4] present three key components, clustering, location management and virtual circuit management procedures, of Multihop Mobile Wireless Networks (MMWN) based on a hierarchical structure. Haas and Tabrizi [5] discuss some of the challenges and choices that need to be made in designing *ad hoc* network. In particular, they discuss the flat architecture and its advantages over the hierarchical architecture. On the other hand, Scachez, Evans and Minden [3] introduced the Highly Dynamic multi-hop wireless network (HDnet) and determined that the HDnet networking model, in terms of scalability and location management, offers a more practical and costeffective solution to mobile military networks than the classical *ad hoc* wireless model.

Location Management is the set of mechanisms used to determine where a sensor is with respect to the network infrastructure, which is a key issue in *ad hoc* communication. It provides a time-varying mapping between the sensor identifier and address. A number of location management strategies have been proposed. Sharony [6] partitioned a mobile network into logically independent subnetworks, where network nodes are members of physical and virtual subnets and their addresses are based on their current subnet affiliation. Pei and Gerla [7] considered mobility management in large, hierarchically organized multihop wireless networks, while using the notion of logical subnets to handle mobility. The problem with these strategies is that they do not exploit hierarchy at the network layer to reduce the frequency of location registration to distant location servers, that is, all events trigger a location update. Li *et al.* [8] proposed Grid Location Service (GLS), which relies on a grid-based geographic hierarchy overlaying the network area. Each node has a set of nodes functioning as location servers and maintains a table of immediate neighbors as well as second-degree neighbors.

In *ad hoc* networks routing has to be determined dynamically. The literature for routing protocols is divided into Proactive or Table Driven Routing Protocol, Reac-

tive or On-Demand Routing Protocol and Hybrid Protocol, the last being a combination of the first two. Some of the protocols cited in the literature are: Destination-Sequenced Distance-Vector Routing (DSDV) [9], Clusterhead Gateway Switch Routing (CGSR) [10], and Wireless Routing Protocol (WRP) [11]. In a reactive protocol, routes are determined on a needs basis; examples are Ad Hoc On-Demand Distance Vector Routing (AODV) [12], Dynamic Source Routing (DSR) [13], Associativity Based Routing (ABR) [14] and the Temporally Ordered Routing Algorithm (TORA) [15]. Hybrid protocols combine the advantages of both reactive and proactive protocols. Haas [16] proposed Zone Routing Protocol (ZRP), a hybrid protocol based on the notion of routing zones.

Clustering of nodes in *ad hoc* networks are performed to use the wireless resources efficiently by reducing congestion and for proper location and routing management. Clusterhead selection is an NP-hard problem [17] and hence most of the existing solution methodologies available are heuristic based approaches. Most of the clustering algorithms described in the literature assume that the links between nodes within a cluster are reliable and data can be communicated between them at all times. In this paper, a mathematical programming based approach based on a variation of a maximal expected covering location model due to Daskin [18] is adopted for dynamic clustering. We therefore review relevant literature on covering location problems. Toregas et al. [19] modeled the location of emergency service facilities as a Set Covering Location Problem (SCLP) where the objective is to cover all demand with the least number of facilities. Chapman and White [20] modeled the probabilistic version of SCLP ensuring that each node is served by a specified reliability level  $\alpha$ . Church and ReVelle [21] proposed the Maximal Covering Location Problem (MCLP) with the objective to cover maximum demand with a restricted number of facilities. Daskin [18] introduced a variant of the MCLP that considers the possibility that facilities may be unable to respond to demands at all times. The resultant model was labeled as MEXCLP (Maximum Expected Covering Location Problem).

Batta, Dolan and Krishnamurthy [22] attempted to relax three of the assumptions made in the MEXCLP model. Patel, Batta and Nagi [1] model the sensor network problem as a covering location problem with the objective of maximizing the expected demand covered by deploying a given number of clusterheads. In this paper, we generalize the Dynamic MEXCLP model due to [1] by considering variable relocation cost and bandwidth capacity constraints, which are pertinent at the tactical level.

# 3 Problem Formulation

Both Variable relocation cost and Capacity generalization models assume that the clusterheads are completely reliable while all links have (identical) steady-state probabilities of failure e.g. jamming in a military situation. Sensors are mobile with known velocity vectors. The clusterheads are also mobile and their potential locations constitute a set of discrete points. The time horizon is divided into equal time periods and relocation of clusterheads takes place at the beginning of each time period. Clusterheads are identical in all respects and have adequate bandwidth capacity to communicate with covered sensors. We define the parameters and the corresponding indices used in both generalizations as follows:

Parameters:

- $\Delta$  = set of potential clusterhead locations.
- $\Theta = \text{set of sensors.}$
- n = maximum number of clusterheads to be chosen.
- T = maximum number of time periods in the horizon under consideration.
- U = the distance beyond which a sensor is considered "uncovered".

 $D_{ikt}$  = distance between potential clusterhead location *i* and demand node *k* at time *t*.

 $d_k$  = demand per period of node k.

$$p =$$
 probability of a link failure per period (between any facility and demand node).  $(0$ 

 $r_{ikt} = \begin{cases} 1, \text{ if } D_{ikt} < U; \\ 0 \text{ otherwise.} \end{cases}$ 

### 3.1 Variable Relocation Cost Model

As stated earlier, in a two-level clustering sensor network, sensors communicate through the clusterheads and there is a single-hop peer-to-peer topology among clusterheads. Each clusterhead has information of the sensors covered by every other clusterhead and positions of the sensors are known at any time instant. Assignments are from a discrete set of potential clusterhead locations. When a clusterhead changes its position, it updates its own database and sends the updated information to other clusterheads. Sending and receiving of updated information incurs some cost, like battery consumption and bandwidth usage. Considering these as components of the relocation cost, we define:

- Constant Cost: Cost to initiate information transfer.
- **Registration Cost:** Cost to register newly covered sensors. It is a function of number of sensors concerned.
- **Deregistration Cost:** Cost to release sensors that will no longer be covered by the clusterhead in the new position. It is also a function of the number of sensors concerned.
- **Routing Table Updating Cost:** Cost incurred in sending the updated routing table to other clusterheads. It is a function of both number of clusterheads and the size of updated information (total number of sensors concerned).

### 3.1.1 Model Formulation

We define some additional terms for variable relocation cost generalization as follows:

For any sensor k, if a clusterhead moves from location i at time t - 1 to location s at any time t,  $w_{iskt}$  and  $\bar{w}_{iskt}$  are given by

Then, the relocation cost  $w'_{ist}$  at the beginning of time t is given by:

$$w'_{ist} = C + \sum_{k \in \Theta} [w_{iskt}(c + c_r) + \bar{w}_{iskt}(\bar{c} + c_r)]$$
(1)

From the definition of  $w'_{ist}$  we can see that the relocation cost is the summation of the cost incurred in registering, deregistering, updating routing table plus a constant cost. The decision variables of the problem are:

$$x_{it} = \begin{cases} 1, & \text{if clusterhead } i \text{ is chosen at time } t, \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{jkt} = \begin{cases} 1, & \text{if sensor } k \text{ is covered by at least } j \\ & \text{clusterheads at time } t, \end{cases}$$

$$0, & \text{otherwise.}$$

$$1, & \text{if a clusterhead is reassigned from } location i \text{ to location } s \text{ at the } location i \text{ to location }$$

Thus, dynamic MEXCLP with variable relocation cost is formulated as follows: (P1) Maximize

$$\sum_{t=1}^{T} \sum_{k \in \Theta} \sum_{j=1}^{n} (1-p) p^{j-1} d_k y_{jkt} - \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{s \in \Delta, s \neq i} z_{ist} w'_{ist},$$

subject to:

$$\sum_{j=1}^{n} y_{jkt} - \sum_{i \in \Delta} r_{ikt} x_{it} \le 0 \qquad \forall \ k \in \Theta, \ t = 1, \dots, T,$$
(2)

$$\sum_{i \in \Lambda} x_{it} \le n \qquad \forall \ t = 1, \dots, T, \tag{3}$$

$$\sum_{s \in \Delta, s \neq i} z_{ist} \ge x_{i,t-1} - x_{it} \qquad \forall \ i \in \Delta, \ t = 1, \dots, T,$$

$$\tag{4}$$

$$\sum_{i \in \Delta, i \neq s} z_{ist} \ge x_{s,t} - x_{s,t-1} \qquad \forall \ s \in \Delta, \ t = 1, \dots, T,$$
(5)

$$\sum_{s \in \Delta} z_{ist} \le 1 \qquad \forall \ i \in \Delta, \ t = 1, \dots, T,$$
(6)

$$z_{ist} \in \{0,1\} \qquad \forall \ i, s \in \Delta, \ s \neq i, \ t = 1, \dots, T,$$

$$(7)$$

 $x_{it} \in \{0,1\} \qquad \forall \ i \in \Delta, \ t = 1, \dots, T,$ (8)

$$y_{jkt} \leq 1 \qquad \forall \ j = 1, \dots, n, \ k \in \Theta, \ t = 1, \dots, T.$$

$$(9)$$

Here, the objective function maximizes the expected demand covered while considering relocation, registration and routing table updating cost. If node k is covered by m clusterheads at time t, Constraint (2) assigns each of the variables  $y_{1kt}, y_{2kt}, \ldots, y_{mkt}$  a value of 1 since the objective function is a maximization function containing the term  $y_{jkt}$ . Constraint (3) restricts the maximum number of clusterheads to be chosen to n for any time t. Constraint (4) ensures that at least one clusterhead at location i moves to another location if the number of clusterheads at time t - 1 is greater than the number of clusterheads at time t at location i. Constraint (5) is same as (4), but in a reverse sense. Constraint (6) ensures that there is only one clusterhead at each location after the relocation. Constraints (7) and (8) are binary constraints. Constraint (9) is a non-negativity constraint.

#### 3.1.2 Column Generation Heuristic

Column generation (CG) is a widely used technique to solve large-scale combinatorial optimization problems in the context of scheduling, set-partitioning, and vehicle routing. It is typically used in a multi-period model when the number of solutions available for each period is very large. The reader is referred to the following papers for a description of the technique in a variety of different applications: [23] solve the fleet routing and scheduling problem using a CG based approach. [24] solve set-partitioning problem encountered in the context of traffic assignment in satellite communication using a CG

heuristic. [25] adopt a CG heuristic to develop conflict-free routes on a bi-directional network for automated guided vehicles. [26] solve a linear multi-commodity flow problem using an iterative partial pricing scheme that is motivated by the CG approach of Dantzig-Wolfe decomposition.

CG is an iterative scheme where a sub-problem generates feasible solutions and a master problem evaluates and selects these feasible solutions. The sub-problems in our scheme are time separable and generate the optimal clusterhead assignment for each time period (based on the current dual multipliers). The master problem picks the best solution for each time period. We define some additional parameters and variables used in the master problem and sub-problem for the CG approach:

> $F_t$  = set of feasible solutions for time t.  $x_{qit}$  = value of  $x_{it}$  in solution q for time t.  $y_{qjkt}$  = value of  $y_{jkt}$  in solution q for time t.

The decision variables of the problem are:

$$F_{qt} = \begin{cases} 1, & \text{if solution } q \text{ is selected at time } t, \\ 0, & \text{otherwise.} \end{cases}$$

#### 3.1.3 Master Problem

The master problem evaluates the set of solutions available for each time period and selects one for each period such that the objective value is maximized. It is stated as follows:

Maximize

$$\sum_{t=1}^{T} \sum_{k \in \Theta} \sum_{j=1}^{n} \sum_{s \in F_t} (1-p) p^{j-1} d_k F_{qt} y_{qjkt} - \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{s \in \Delta, s \neq i} z_{ist} w'_{ist}$$

subject to:

$$\sum_{s \in \Delta, s \neq i} z_{ist} \geq \sum_{q \in F_{t-1}} F_{q,t-1} x_{qi,t-1} - \sum_{q \in F_t} F_{qt} x_{qit}$$
  
$$\forall t = 1, \dots, T, \quad i \in \Delta$$
(10)

$$\sum_{i \in \Delta, i \neq s} z_{ist} \geq \sum_{q \in F_t} F_{qt} x_{qst} - \sum_{q \in F_{t-1}} F_{q,t-1} x_{qs,t-1}$$
$$\forall t = 1, \dots, T, s \in \Delta$$
(11)

$$\sum_{q \in F_t} F_{qt} = 1 \quad \forall \quad t = 1, \dots, T,$$

$$(12)$$

$$\sum_{s \in \Delta} z_{ist} \leq 1 \quad \forall \ i \in \Delta, t = 1, \dots, T,$$
(13)

$$F_{qt} \in \{0,1\} \quad \forall q, t = 1, \dots, T,$$
 (14)

$$z_{ist} \geq 0 \qquad \forall \ t = 1, \dots, T, \ \forall \ i, s \in \Delta, s \neq i.$$
(15)

$$z_{ist} \leq 1 \quad \forall \ t = 1, \dots, T, \ \forall \ i, s \in \Delta, s \neq i.$$

$$(16)$$

The master problem is solved as a relaxed linear program with no binary constraint on the  $F_{qt}$  variables. This is referred to as the relaxed master problem (RMP). The problem starts with an initial basic feasible solution. Constraints (10) and (11) are similar to the Constraints (4) and (5) except that the decision variables are  $F_{q,t}$  and  $F_{q,t-1}$  instead of  $x_{it}$  or  $x_{st}$ . Constraint (12) ensure that only one solution can be selected for each time period. Let  $\beta, \gamma, \delta$  be the dual multipliers generated after solving the relaxed master problem for Constraints (10), (11) and (12) respectively. The subproblem generates the feasible solutions using these dual multipliers.

#### 3.1.4 Sub-Problem

The sub-problem is formulated to generate a new column that has a favorable reduced cost to enter the basis of the master problem. It is solved for each time period t separately. The sub-problem for time periods t = 1 to t = T - 1 is stated as below:

$$\sum_{k \in \Theta} \sum_{j=1}^{n} (1-p) p^{j-1} d_k y_{jkt} + \sum_{i \in \Delta} [\beta_{it} - \beta_{it+1} + \gamma_{it+1} - \gamma_{it}] x_{it} - \delta_t$$

subject to:

$$\sum_{j=1}^{n} y_{jkt} - \sum_{i \in \Delta} r_{ikt} x_{it} \leq 0 \quad \forall k \in \Theta,$$
(17)

$$\sum_{i \in \Lambda} x_{it} \leq n \tag{18}$$

$$x_{it} \in \{0,1\} \quad \forall i \in \Delta, \tag{19}$$

$$y_{jkt} \leq 1 \quad \forall \quad j = 1, \dots, n, \ k \in \Theta.$$
 (20)

Since the dual multipliers  $\alpha$  are available only for t = 1 to t = T - 1, the sub-problem for t = T has different objective function. The objective function for t=T is: Maximize

$$\sum_{k \in \Theta} \sum_{j=1}^{n} (1-p) p^{j-1} d_k y_{jkt} + \sum_{i \in \Delta} x_{it} (\alpha_{it} - \beta_{it}) - \gamma_t$$

The solution for the sub-problem is a new variable  $F_{st}$  for the master problem. Thus for each time period an MILP is solved to get the new solution for that time period.

### 3.2 Limited Clusterhead Capacity

In order to incorporate the capacity constraint, each sensor needs to be assigned to one of the clusterheads that is "covering" it. However, we still have link failure probabilities and correspondingly, need multiple coverage of sensors. Thus multiple coverage still remains while preferential assignment, representing value of assigning a primary clusterhead to a sensor, is incorporated. Due to the mobility of sensors, the optimal selection of clusterheads for one time period may not be optimal for the entire horizon. Thus the tradeoff now is between multiple coverage and preferential assignment on one side and relocation cost on the other. Model parameters are defined as follows:

Parameters:

 $c_{ik}$  = value of preference of assignment of sensor k to clusterhead i

 $Q_{it}$  = capacity of clusterhead *i* during time period *t* 

C = cost per unit change in the number of clusterheads at any location i (one-half of relocation cost).

The decision variables are:

$$x_{it} = \begin{cases} 1, \text{ if clusterhead i is chosen at time } t, \\ 0 \text{ otherwise.} \end{cases}$$

$$z_{ikt} = \begin{cases} 1, \text{ if sensor } k \text{ is assigned to clusterhead } i \\ \text{ during time period } t, \\ 0 \text{ otherwise.} \end{cases}$$

$$y_{jkt} = \begin{cases} 1, \text{ if sensor } k \text{ is covered by at least } j \\ \text{ clusterheads at time } t, \\ 0 \text{ otherwise.} \end{cases}$$

 $w_{it} =$ positive difference in the number of clusterheads at

location i between time t - 1 and time t.

The capacitated dynamic MEXCLP can be formulated as follows:

#### Formulation:

(P2) Maximize

$$\sum_{t=1}^{T} \sum_{k \in \Theta} \sum_{j=1}^{n} (1-p) p^{j-1} d_k y_{jkt} + \sum_{i \in \Delta} \sum_{k \in \Theta} c_{ik} z_{ikt} - \sum_{t=1}^{T} \sum_{i \in \Delta} C w_{it},$$

subject to

$$\sum_{j=1}^{n} y_{jkt} - \sum_{i \in \Delta} r_{ikt} x_{it} \le 0 \qquad \forall k \in \Theta, \ t = 1, \dots, T,$$
(21)

$$\sum_{i \in \Delta} x_{it} \le n \qquad \forall t = 1, \dots, T,$$
(22)

$$w_{it} \ge x_{it-1} - x_{it} \qquad \forall i \in \Delta, \ t = 1, \dots, T,$$
(23)

$$w_{it} \ge x_{it} - x_{it-1} \qquad \forall i \in \Delta, \ t = 1, \dots, T,$$
(24)

$$z_{ikt} \le r_{ikt} x_{it} \qquad \forall i \in \Delta, k \in \Theta, \ t = 1, \dots, T,$$

$$(25)$$

$$\sum_{i \in \Delta} z_{ikt} \le 1 \qquad \forall k \in \Theta, \ t = 1, \dots, T,$$
(26)

$$\sum_{k \in \Theta} d_k z_{ikt} \le Q_{it} \qquad \forall i \in \Delta, \ t = 1, \dots, T,$$
(27)

$$x_{it} \in \{0, 1\} \qquad \forall i \in \Delta, \ t = 1, \dots, T,$$

$$(28)$$

$$w_{it} \ge 0 \quad \forall i \in \Delta, \ t = 1, \dots, T,$$

$$(29)$$

$$y_{jkt} \le 1 \quad \forall j = 1, \dots, n, \ k \in \Theta, \ t = 1, \dots, T,$$

$$(30)$$

$$z_{ikt} \in \{0, 1\}, \quad \forall i \in \Delta, k \in \Theta, \ t = 1, \dots, T.$$

$$(31)$$

The objective function is similar to the dynamic MEXCLP [1] with an additional term which represents preferred assignment of sensors to clusterheads. Thus the objective function maximizes demand covered and preferential assignment while allowing for relocation of clusterheads over a time horizon. Constraints (21) to (24) are same as in variable relocation generalization. Constraint (25) ensures that sensor k is assigned to clusterhead location i only if location i is occupied by a clusterhead and sensor k can be covered from location i. Constraint (26) ensures that a sensor is assigned to only one clusterhead during a time period. Constraint (27) is the capacity constraint for each clusterhead for each time period. Constraints (28), (29), (30) and (31) are binary constraints.

An alternative formulation of (P2) can also be given. The variables and their indices for this formulation are same as (P2) except for  $Q_{it}$ , which is replaced by  $Q_i$ representing the capacity of clusterhead location *i* over the entire time horizon (note again that all the clusterheads are assumed to be identical). It can easily be shown that for an optimal solution of (P2) the  $w_{it}$  variables will take on 0-1 binary values. Furthermore, for an optimal solution of (P2) the  $y_{jkt}$  variables will also take on 0-1binary values. This is because the term  $\sum_{i\in\Delta} R_{ikt}x_{it}$  will be integral since  $R_{ikt}$  are 0-1binary constants and variables  $x_{it}$  assume only 0-1 binary values. The  $z_{ikt}$  variables need not take 0-1 values unless defined to do so. It might also be noted here that

the preferential assignment is an attempt to capture capacity restrictions and is not completely accurate. Because it only considers first order link failures and does not take subsequent assignments into consideration. In other words, if the link between a sensor and its most preferred clusterhead location breaks down, subsequent link assignments are not accounted for.

#### 3.2.1 Column Generation Formulation

Similar to dynamic MEXCLP with relocation cost model, we can apply column generation heuristic approach to capacitated dynamic MEXCLP. We define some additional parameters and variables used in the master problem and sub-problem for the CG approach:  $F_t$  = index set representing the available solutions for time t.

 $X_{sit}$  = value of  $x_{it}$  in solution  $s \in F_t$  for time t.

 $Y_{sjkt}$  = value of  $y_{jkt}$  in solution s for time t.

The decision variables of the problem are:

 $f_{st} = \begin{cases} 1 \text{ if solution } s \in F_t \text{ is selected at time } t, \\ 0 \text{ otherwise.} \end{cases}$ 

### 3.2.2 Master Problem

The master problem in this case is stated as follows: Maximize

$$\sum_{t=1}^{T} \sum_{k \in \Theta} \sum_{j=1}^{n} \sum_{s \in F_t} (1-p) p^{j-1} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{k \in \Theta} \sum_{s \in F_t} c_{ik} z_{sikt} f_{st} - \sum_{t=1}^{T} \sum_{i \in \Delta} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{k \in \Theta} \sum_{s \in F_t} c_{ik} z_{sikt} f_{st} - \sum_{t=1}^{T} \sum_{i \in \Delta} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{k \in \Theta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{k \in \Theta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{k \in \Theta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{k \in \Theta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{k \in \Theta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{k \in \Theta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{s \in F_t} \sum_{s \in F_t} C w_{it} d_k f_{st} y_{sjkt} + \sum_{t=1}^{T} \sum_{i \in \Delta} \sum_{s \in F_t} \sum_$$

subject to

$$-w_{it} + \sum_{s \in F_{t-1}} f_{st-1} x_{sit-1} - \sum_{s \in F_t} f_{st} x_{sit} \le 0,$$
  
$$\forall i \in \Delta, t = 1, \dots, T,$$
(32)

$$-w_{it} + \sum_{s \in F_t} f_{st} x_{sit} - \sum_{s \in F_{t-1}} f_{st-1} x_{sit-1} \le 0,$$

$$\forall i \in \Lambda, t = 1, T$$

$$(22)$$

$$\forall i \in \Delta, t = 1, \dots, T, \tag{33}$$

$$\sum_{s \in F_t} f_{st} = 1, \quad \forall \ t = 1, \dots, T,$$
(34)

$$f_{st} \in \{0, 1\}, \quad \forall s, t = 1, \dots, T,$$
(35)

$$w_{it} \ge 0, \quad \forall i \in \Delta, t = 1, \dots, T.$$
 (36)

The master problem is solved as relaxed linear program with no binary constraint on the variables. Constraints (32) and (33) are same as in (P2) except that we have variable  $f_{st}$  replacing  $x_{it}$  and  $y_{jkt}$ , whereas  $x_{it}$  and  $y_{jkt}$  are constants. Variable  $f_{st}$  represents a feasible solutions (column). Let  $\beta, \gamma, \delta$  be the dual multipliers corresponding to constraints (32), (33) and (34), respectively. The sub-problem generates the feasible solutions using these dual multipliers.

#### **3.2.3** Sub-problem for Period t

The objective of the sub-problem is precisely the reduced cost of an entering column to the master problem. Similar to the variable relocation cost model, the sub-problem is solved for each time period t separately. The sub-problem for time periods t = 1 to t = T - 1 is as follows:

Maximize

$$\sum_{k \in \Theta} \sum_{j=1}^{n} (1-p) p^{j-1} d_k y_{jkt} + \sum_{i \in \Delta} [\beta_{it} - \beta_{it+1} + \gamma_{it+1} - \gamma_{it}] x_{it} + \sum_{i \in \Delta} \sum_{k \in \theta} c_{ik} z_{ikt} - \delta_t$$

subject to

$$\sum_{j=1}^{n} y_{jkt} - \sum_{i \in \Delta} r_{ikt} x_{it} \le 0, \quad \forall k \in \Theta,$$
(37)

$$\sum_{i \in \Delta} x_{it} \le n, \quad \forall t = 1, \dots, T,$$
(38)

$$z_{ikt} \le r_{ikt} x_{it}, \quad \forall i \in \Delta, \ k \in \Theta, \ t = 1, \dots, T,$$

$$(39)$$

$$\sum_{i \in \Delta} z_{ikt} \le 1 \quad \forall k \in \Theta, \ t = 1, \dots, T,$$

$$\tag{40}$$

$$\sum_{k \in \Theta} d_k z_{ikt} \le Q_{it} \quad \forall i \in \Delta, \ t = 1, \dots, T,$$
(41)

$$z_{ikt} \in \{0,1\}, \quad \forall i \in \Delta, \ k \in \Theta, \ t = 1, \dots, T.$$

$$(42)$$

$$y_{jkt} \le 1, \quad \forall j = 1, \dots, n, \ k \in \Theta$$

$$\tag{43}$$

$$x_{it} \in \{0, 1\}, \quad \forall i \in \Delta.$$

$$\tag{44}$$

Due the non-availability of the dual multipliers  $\beta$  and  $\gamma$  for t = T, the objective function has a different form for t = T:

$$\sum_{k\in\Theta}\sum_{j=1}^{n}(1-p)p^{j-1}d_ky_{jkt} + \sum_{i\in\Delta}[\beta_{it}-\gamma_{it}]x_{it} + \sum_{i\in\Delta}\sum_{k\in\theta}c_{ik}z_{ikt} - \delta_t$$

The solution of sub-problem  $(SP_t)$  is a set of  $x_{it}$ ,  $y_{jkt}$  and  $z_{ikt}$  values. These coefficient values are said to represent a solution  $f_{st}$ , which is added to the index set of solution  $F_t$  for period t in the master problem. If the sub-problem for every period t fails to produce a solution with strictly positive reduced cost, then the procedure terminates.

### 3.3 Initial Basic Feasible Solution

The Column Generation(CG) approach works in the feasible domain. The CG heuristic requires an initial basic feasible solution to start with. It starts with this solution and keeps on improving the objective function value by generating new solutions termed as "columns" and selecting the one that improves the objective function the most. The effectiveness of the CG approach is enhanced by the quality of the initial basic feasible solution. Hence it is important to develop a good heuristic for obtaining an initial basic feasible solution. [1] proposed Relocation and No-relocation heuristics for Dynamic MEXCLP. In our case, for both variable relocation cost and capacitated

model, we use the modified version of these two heuristics. We start CG with two basic feasible solutions for each time period generated by the Modified Relocation Heuristic (MRH) and Modified No-Relocation Heuristic (MNRH). The modified versions differ from [1]: Since, for variable relocation cost model, the relocation cost is different for every possible clusterhead relocation during each time interval, we use average relocation cost for each time interval while applying (RH). The average relocation cost is calculated by summing all relocation costs for a particular time interval and dividing them by the total number of possible relocations during that time interval. This might affect the solution quality of RH, but can still provide us a good initial feasible solution for CG heuristic. Similarly, for limited clusterhead capacity case, the demand covered by each potential is substituted by the sum of the demand covered and the preferred assignment for each potential location (refer [2] for details).

MRH is a greedy heuristic, which picks the best n locations, one at a time, for each time period. The heuristic takes into consideration the link failure probabilities and later adds relocation cost incurred in switching clusterheads locations in two successive time periods. MNRH is also a greedy heuristic in which the strategy is to place clusterheads at locations which cover maximum demand for all time periods assuming that relocation is not permitted. The heuristic also accounts for link failure probabilities in a similar manner to MRH, but does not reassign clusterheads.

Our results showed that in most randomly generated scenarios, the solution quality of the heuristics is good. However, particular examples can be contrived where each heuristic can perform very poorly. We therefore use solutions from both MRH and MNRH simultaneously as the initial columns (feasible solutions) of the CG procedure.

### 3.4 Solution Approach

We adopt the following solution approach for both the generalizations. At each iteration we solve RMP and then solve the sub-problems for time periods t=1 through t=T. The solutions with favorable reduced cost are simultaneously added to the master problem. Iterations are continued till the termination criterion is reached. We terminate CG iterations if none of the sub-problems give a solution of favorable reduced cost. In certain cases, the CG heuristic will keep on iterating with a very small increase in the objective function of the RMP. In such cases, other termination criteria can be utilized by a user: (i) a threshold time within which a solution is required, and (ii) a bound on the number of iterations.

# 4 Computational Results

Based on the solution methodologies proposed in Section 3, we present numerical results for the variable relocation cost model and limited capacity model. The Greedy and the CG heuristics for both models were implemented using the programming language C and the CPLEX 9.0 callable library. Solution times are reported in CPU seconds on an Intel Pentium IV processor, 3.2 GHz, 1 GB RAM workstation operating Red Hat Linux 9.0 platform. First we present the results for the variable relocation cost model and then the limited clusterhead capacity model. Note that, for both the models, we do not present test results comparing the performance of the CG heuristic with the CPLEX MIP solver since CPLEX could not find a feasible solution within the CG solution time even for the small problem instances.

### 4.1 Variable relocation cost case

In this, we compare our model with Patel et al. [1] model which considers constant relocation cost. Note that in [1] the relocation cost is defined as one half of the actual relocation cost because it is double counted: for the clusterhead assigned as well as the one unassigned, so in our case it is double of the actual value. First, we solve the variable relocation cost model using column generation (CG) heuristic. The CG procedure is terminated when the Relaxed Master Problem solution is within 2% of LP relaxation of (P1) and the objective function value, average variable relocation cost and the solution time are noted down. Then, we solve Patel et al. [1] model using their CG procedure with similar stopping criteria. While solving [1], the constant relocation cost(C) is set to one half of the average variable relocation cost in an attempt to obtain a fair comparison. After the [1] solution is obtained, it is evaluated using the tactical model criterion. Table 1 shows the results of the test run for the small and the medium size problems and Table 2 shows results for the large size problems. The results show that at the expense of significantly increased computation time (on average 83%) we can get a 1-3% improvement in objective function value. Whether or not this increased time is justifiable depends on the time available to deliver a tactical solution.

Next, we perform some test runs to determine the effect of various cost parameters on the solution quality of our model. We generate a random scenario for the test. First, we find the CG solution by setting constant relocation, registration, deregistration and routing table updating cost to one. Then, we increase these cost parameters in each run and note down the solution value. The results indicate that, as the constant cost increases, the objective function value decreases. It is also observed that the routing table updating cost has greater influence on the solution value than the registration and the deregistration cost.

### 4.2 Limited clusterhead capacity case

In this case too, we start of by comparing the Capacitated DMEXCLP with Uncapacitated DMEXCLP. First, we solve Patel *et al.* [1] model using their CG heuristic. The CG procedure is terminated when the Relaxed Master Problem solution is within 2% of LP relaxation. Then, using the elected clusterhead (fixing the  $x_{it}$  variables) values as input, we solve the Capacitated DMEXCLP (P2) model using the CG heuristic. Here, the CG procedure is terminated when the Relaxed Master Problem solution is within 5% of LP relaxation of (P2). Thus, the Uncapacitated model results are obtained by solving DMEXCLP model [1] and evaluating this solution using the tactical criterion. Next, we solve the capacitated DMEXCLP model independently. Table 3

Table 1: Model Comparison Results - Small and Medium Size Problems

Potential Ch/Sensor = 0-500, Displacement = 0-40, Demand = 10-20, Delta = 100-250, Theta = 100-250, Time Period = 5-15, Link Failure Probability = 0.2-0.5, Const Relocation Cost = 3-10,

Max No of Ch Chosen Range = $5-15$ , Coverage Radius = $40$ ,						
Routing Table Updating $Cost = 1$ , Registration $Cost = 1$ ,						
Deregistration $Cost = 1$						
	Constant	Relocation	Variable 1	Relocation	% Gain	
No					in Obj.	
	CG	CG	CG	CG	Value	
	Solution	Time	Solution	Time		
1	17145.1	8.71	17543.47	107.04	2.32	
2	9948.72	11.65	10213.6	56.44	2.66	
3	4814.32	7.17	4882.88	11.41	1.42	
4	10456.75	8.16	10664.36	110.84	1.99	
5	9883.76	8.11	10087.29	56.96	2.06	
6	6132.72	6.72	6295.48	59.15	2.65	
7	5729.36	3.08	5863.912	52.43	2.35	
8	7597.552	3.84	7828	55.93	3.03	
9	3605.76	2.43	3705.6	3.38	2.77	
10	6445.07	7.76	6611.09	16.36	2.58	

shows the results of the comparison for small and medium size problems. Table 4 shows the same information for large size problems. The increase in computational effort (average 87%) is about the same as that for the relocation cost case. However, the improvement in objective function value is smaller (in 1-2% range). Again, whether or not this increased computational effort is desirable depends upon the time available to deliver a tactical solution and importance of the associated benefit.

We also conducted tests to study the Capacitated DMEXCLP model behavior with respect to the change in parameters p and C. We considered a problem instance with 300 clusterheads, 200 sensors 11 time slots and 5 clusterheads to be chosen. It was observed that as p increases the objective function value decreases and the solution time increases. In case of the relocation cost model, the objective function value decreases with increase in C up to a threshold value. Further increase has no effect on the solution value. Similar behavior can be observed in solution time also, except that it

Table 2: Model Comparison Results - Large Size Problems

CH Location = $0-1000$ , Sensor Location = $0-500$ , Displacement = $0-30$ ,
Demand = 10-20, Delta = 200-500, Theta = 200-500,
Time Period = 5-15, Link Failure Probability = $0.2-0.5$ ,
Const Relocation Cost = 1-5, Max No of Ch Chosen = $5-15$ ,
Coverage Radius = 50, Routing Table Updating $Cost = 1$ ,
Registration Cost = 1, Deregistration Cost = $1$

	Constant Relocation		Variable Relocation		% Gain	
No					in Obj.	
	CG	CG	CG	CG	Value	
	Solution	$\operatorname{Time}$	Solution	Time		
1	55683.19	41.29	55953.42	275.59	0.49	
2	16770.43	28.01	16866.23	98.04	0.57	
3	35578.48	18.48	35792.9	117.71	0.60	
4	25202.56	12.7	25393.85	188.88	0.76	
5	23906.63	8.83	24153.14	126.18	1.03	
6	37584.21	53.79	38113.4	228.99	1.41	
7	31169.1	39.07	31682.32	212.53	1.65	
8	19284.65	24.72	19622.68	145.69	1.75	
9	16451.84	29.67	16711.71	139.09	1.58	
10	20105.18	23.11	20489.37	147.93	1.91	

increases with the increase in C (refer to Figures 2 and 3).

Finally, Analysis of Variance (ANOVA) test was carried out using the results obtained from CG heuristic to determine the factors affecting the solution quality (% expected coverage). We consider p, C, n, number of clusterheads, number of sensors, T, sensor demand, displacement, coverage radius and clusterhead capacity as the relevant factors governing the response variable. From the results of ANOVA, number of clusterheads chosen, number of sensors, time period, sensor demand, coverage radius and clusterhead capacity were found to have an effect on coverage either directly or through second level interaction.

Table 3: Model Comparison Results - Small and Medium Size Problems

$ m CH/Sensor \ Location = 0-500, \ Displacement = 0-30,$
Demand = 10-20, Delta = 100-250, Theta = 100-250,
Time Period = 5-15, Link Failure Probability = $0.2-0.5$ ,
Const Relocation Cost = $3-10$ , Max No of Ch Chosen = $5-15$ ,
Coverage Radius = $40$ , Bandwidth Capacity = $70$
Preferential Assignment $Cost = 20$

	Uncapaci	tated Model	Capacitat	ed Model	% Gain
No				in	
	CG	CG	CG	CG	Obj. Value
	Solution	$\mathbf{Time}$	Solution	$\mathbf{Time}$	
1	9239.04	60.72	9381.28	370.71	1.54
2	9193.32	47.38	9346.20	318.68	1.66
3	10840.36	58.72	11059.00	217.21	2.02
4	21513.92	43.95	22036.89	178.60	2.43
5	15313.44	36.14	15564.74	310.29	1.64
6	18364.96	19.88	18697.65	84.88	1.81
7	15227.04	16.97	15430.56	106.81	1.34
8	6508.96	18.33	6696.32	83.50	2.88
9	15313.44	36.14	15564.74	310.29	1.64
10	11227.46	33.61	11394.63	171.32	1.49

# 5 Case Study

We present a case study to demonstrate how the capacitated DMEXCLP model proposed in Section 3.2 can be employed and used for tactical decision making in the military environment. For our case study, we consider a scenario with 15 clusterheads (10 air, 5 ground), 20 sensors (5 ground and 15 air) and 20 enemy targets (10 air and 10 ground). The scenario is generated using the simulation testbed tool developed at the University at Buffalo<sup>1</sup>. The testbed is a prototypical simulation tool built to provide "common operating picture (COP)" for friendly commanders and facilitate them to have a shared understanding in decision making. The tool allows the user to design

<sup>&</sup>lt;sup>1</sup>Refer to http:

www.eng.buffalo.edu/ $\sim\!$ nagi for more details about the project

Table 4: Model Comparison Results - Large Size Problems

$ m CH/Sensor \ Location = 0-500, \ Displacement = 0-30,$
Demand = 10-20, Delta = 200-400, Theta = 200-400,
Time Period = 5-15, Link Failure Probability = $0.2-0.5$ ,
Const Relocation Cost = 5-20, Max No of Ch Chosen = $10-25$ ,
Coverage Radius = 50, Bandwidth Capacity = $100$
Preferential Assignment $Cost = 20$

	Uncapacitated Model		Capacitat	ed Model	% Gain
No					in
	CG CG CG		CG	CG	Obj. Value
	Solution	Time	Solution	$\operatorname{Time}$	
1	59179.91	494.61	59685.17	11166.37	0.85
2	22395.95	165.76	22487.90	8368.80	0.41
3	38297.78	180.40	38519.20	981.02	0.58
4	33792.61	92.89	34133.28	2581.80	1.01
5	18951.84	158.76	19011.04	672.73	0.31
6	28820.72	702.16	29051.44	14893.66	0.80
7	28425.60	408.41	28604.32	12142.64	0.63
8	17413.44	398.19	17491.36	4541.48	0.45
9	18960.32	219.25	19213.84	4474.50	1.34
10	21564.56	241.74	21631.24	4013.10	0.31

range of scenarios (battlefield) and evaluate and test algorithms to perform network topology management, sensor simulation, target tracking and fusion in a battlefield environment. One of the major components of this testbed is network topology management scheme. It utilizes the capacitated DMEXCLP model described in Section 3.2 to determine the suitable network topology for distributed fusion. For the simulation case, we consider Buffalo east region map of 108  $Km^2$  area. The topography is generated using the terrain data (digital elevation model format) and vector map data (digital line graph format). The entities (sensors, clusterheads and targets) are randomly located in the theater and travel with the velocity range of 300-400 kmph. The other model parameters chosen for the study are as follows. The coverage radius (U) range is 15-25 km, link failure probability (p) is 0.2-0.5, constant relocation cost range is 5-10, clusterhead bandwidth capacity is 200 Mbps, maximum number of clusterhead to be chosen is 5, preferential assignment cost is 4 and sensor demand range is 10-15



Figure 2: Effect on varying parameters p and C on solution time



Figure 3: Effect on varying parameters p and C on solution value

Mbps. The simulation is allowed to run for 120 seconds. The waypoints for all the entities are generated for every  $1/10^{th}$  of a second. Therefore the total time period under consideration is 1200 time steps (i.e. T = 1200).

Due to the complexity of the problem, it is impractical to simulate for the entire time period under consideration. Therefore, we divide the entire time period into a sequence of smaller horizons and solve the network calculation problem over their periods. But this has its downside. If we solve the problem for every time interval separately the solution obtained, though optimal for that time interval, is sub-optimal at the global level. Moreover, it does not capture and utilize the events from the previous time periods. To overcome this problem we adopt a rolling horizon approach and overlap the intervals in the sequence. The length of the overlap and time interval (T) are specified by the user and in our case it is 30 time steps (3 seconds) and 15 time steps (1.5 seconds) respectively. However, this method requires the network calculation model to be invoked several times (80 invokes in this case). With these modifications to the model, we simulate the scenario for the stipulated simulation duration of 1200 seconds. Figures (4), (5), and (6) show the snapshots of the simulation results during different stages. For instance, Figure 4 shows the chosen clusterheads (large circles) and the sensors assigned to them during the first 3 seconds of the simulation. The dotted line represents the communication link between a clusterhead and a sensor. Apart from the clusterhead and sensor level information the model also provides the user with various performance measures (% of expected coverage during each time period and over the interval, number of single, double and triple coverage, etc.) at the global level. These measures to can be used to evaluate the effectiveness of various networking algorithms (Greedy, CG) and compare them with the other algorithms (MOBIC and Geographical based). This demonstrate the utility value of the tactical models proposed and their ease of use under a practical application.



Figure 4: Sensors communicating with the clusterheads during time period 1-30



Figure 5: Sensors communicating with ground clusterhead during time period 46-75

# 6 Conclusion

In this work we proposed two tactical models for managing communications between sensors in a threat environment, while using clusterheads. The first model incorporates



Figure 6: Sensors communicating with air clusterhead during time period 181-210

a variable relocation cost (includes constant relocation, registration, deregistration and routing table updating costs) and the second model considers limited bandwidth capacity of a clusterhead.

A CG heuristic is developed for each of the proposed models. In both cases, it was found that the complexity of the problem increased significantly compared to dynamic MEXCLP model [1]. This is evident from the significantly increased solution times for similar problem sizes for the dynamic MEXCLP model. However, the CG heuristic was found to perform better for large practical size problems. The following improvements can be done to the proposed models to address practical situations in a military context: The assumption that the sensor locations are known in advance for every time period can be relaxed to consider uncertainty of location with respect to time. Also, the uniform link failure probability assumption can be relaxed and time varying link failure probabilities can be used.

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