State University of New York at Buffalo
Mechanical and Aerospace Engineering Department

MAE 543: CONTINUOUS CONTROL

FINAL PROJECT

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Introduction.

In this project, we will covered the following theories in Part A:

A. Theories.
-Routh Hurwitz
-State Space
-Transfer Function
-Root Locus.
-Bode Diagram
-Proportional Control.
-Integral Control.
-Derivative Control.
-Observer.
-Lead Compensation.
-Lag Compensation.

In part B, we discussed the following topics and related them with the above theory.

B. Application.
-Stability
-Time Constant.
-Damping Ratio.
-Damping Coefficient.
-Critical damping or Over-Damping.
-Natural Frequency.
-Damping Natural Frequency.
-Accuracy in Steady State Response.
-Robustness with respect to Disturbance.
-Bandwidth.

A1. ROUTH-HURWITZ STABILITY CRITERION

Definition:

The *Routh-Hurwitz stability criterion* is a method for determining whether or not a system is stable based upon the coefficients in the system's ***characteristic equation***. It is particularly useful for higher-order systems because it does not require the polynomial expressions in the transfer function to be factored.

The procedure for using the Routh-Hurwitz criterion is as follows:

1. Write the characteristic equation (a polynomial in s) in the following form:

$$a_0 s^n + a_1 s^{n-1} + \ldots a_{n-1} s + a_n = 0$$

2. If any of the coefficients are zero or negative and at least one of the coefficients are positive, there is a root or roots that are imaginary or that have positive real parts.

3. Therefore, the system is unstable.

If all coefficients are positive, arrange the coefficients in rows and columns in the following pattern:

s^n	a_0	a_2	a_4	a_6	...
s^{n-1}	a_1	a_3	a_5	a_7	...
s^{n-2}	b_1	b_2	b_3	b_4	...
s^{n-3}	c_1	c_2	c_3	c_4	...
s^{n-4}	d_1	d_2	d_3	d_4	...
.	.	.	.		
.	.	.	.		
.	.	.	.		
s^2	e_1	e_2			
s^1	f_1				
s^0	g_1				

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

.
.
.

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

$$c_3 = \frac{b_1 a_7 - a_1 b_4}{b_1}$$

$$d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1}$$

$$d_2 = \frac{c_1 b_3 - b_1 c_3}{c_1}$$

.
.
.

Simplified model:

s^n	a_n	a_{n-2}	a_{n-4}
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}
s^{n-2}	$\dfrac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} = b_1$	$\dfrac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}} = b_2$	
$\vdots$	$\vdots$		
s^0	$\dfrac{b_1 a_{n-3} - a_{n-1} b_2}{b_1}$		

Note: the roots are in LHP is and only if all entries in 1st column have the same sign.

Example:

Given Characteristic Equation:

$$s^3 + (k_D - 2)s^2 + (5 + k_P)s + k_I = 0$$

Determine the stability conditions using Routh Table.

s^3	1	$(5 + k_P)$	0
s^2	$(k_D - 2)$	k_I	0
s^1	$\dfrac{(k_D - 2)(5 + k_P) - k_I}{(k_D - 2)}$	0	
s^0	k_I		

To stable, the first column of must have "+" sign.

$$k_D > 2$$
$$k_P > -5$$
$$k_I > 0$$
$$(k_D - 2)(5 + k_P) > k_I$$

The Routh-Hurwitz stability criterion states that the number of roots with positive real parts is equal to the number of changes in sign of the coefficients in the first column of the matrix. Note that the exact values are not required for the coefficients; only the sign matters.

If a system is stable (all of its poles are in the left half of the complex plane), then all the coefficients a_i must be positive and all terms in the first column of the matrix must be positive.

Example:

Given a system with characteristic equation

$$a_2 s^2 + a_1 s + a_0 = 0$$

Determine which values of a will make the system and which will make the system unstable.

Arranged in matrix form, the coefficients are:

s^2	a_2	a_0
s	a_1	
1	$a_1 a_0 / a_2$	

The Routh-Hurwitz criterion states that all of the coefficients in the first column of coefficients must be positive, so for this case we must have $a_2 > 0$ and $a_1 > 0$. Since a_2 and a_1, a_0 must be greater than 0 as well.

As another example, consider the system with characteristic equation

$$s^3 + s^2 + 2s + 24 = 0$$

Arranged in matrix form, the coefficients are:

s^3	1	2
s^2	1	24
s	-22	0
1	24	

Since at least one of the coefficients (-22) is less than zero, this system is unstable. In fact, it has two roots in the right half-plane.

Example: Using Routh Table in determining the k value for controller.

Characteristic Equation:

$$s^3 + 3s^2 + ks + 5 = 0$$

Can we select k such that Tau < 1?
Can we select k such that Tau < 1/2?

Solution:
First substitute s=s-1 into the characteristic Equation,
Then use Routh Table to find the condition of k value for the system to be stable.

Substitute (s-1) with s,

$$(s-1)^3 + 3(s-1)^2 + k(s-1) + 5 = 0$$
$$\Rightarrow s^3 - 3s^2 + ks + 9 - 2k = 0$$

Not stable because there is a negative sign: $-3s^2$

Substitute (s-2) with s,

$$(s-2)^3 + 3(s-2)^2 + k(s-2) + 5 = 0$$
$$\Rightarrow s^3 + (k-3)s + (7-k)$$

Not stable because missing an s term.

So, find the condition where it will be stable using Routh Table.

s^3	1	k	0
s^2	3	5	0
s^1	3k-5	0	
s^0	5		

To stable, k>5/3.

Note: however, there is a limitation on using Routh Table to determine the stability of the system because it only determine one variable at each time, if we have a P.I controller or a P.D controller, or a P.I.D controller, we need to first determine their relation and then use the Routh Table to determine the conditions for stability.

A2. STATE SPACE

<u>State</u>: The state of a dynamic system is the smallest set of variable (called state variable) such that the knowledge of these variables at $t = t_0$, together with the knowledge of the input for $t \geq t_0$, completely determines the behavior of the system for any time $t \geq t_0$.

Thus, the state of a dynamic system at time t is uniquely determined by the state at time t_0 and the input for $t \geq t_0$, and it is independent of the state and input before t_0. Note that, in dealing with linear time-invariant systems, we usually choose the reference time t_0 to be zero.

<u>State Variable</u>: The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system.

<u>State Vector</u>: If n state variables are needed to completely describe the behavior of a given system. Then these n state variables can be considered the n components of a vector x.

<u>State space</u>: The n-dimensional space whose coordinate axes consist of the x_i axis, x_2 axis, ….., x_n axis is called a state space. Any state can be represented by a point in the state space.

Example:

$$\ddot{x} + 2\dot{x} + 5x = 4$$

$$State - space$$

$$z_1 = x, \qquad \dot{z}_1 = \dot{x} = z_2$$

$$z_2 = \dot{x}, \qquad \ddot{z}_2 = \ddot{x} = 4 - 2z_2 - 5z_1$$

$$y_1 = x, \qquad y_2 = \dot{x}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} 4$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + [0]$$

TF with State-space:

$$\dot{Z} = AZ + BU$$
$$Y = CZ + DU$$

Laplace transfer

$$sZ(s) - Z(0) = AZ(s) + BU(s)$$
$$(sI - A)Z(s) = BU(s)$$
$$Z(s) = (sI - A)^{-1}BU(s)$$

and

$$Y = C(sI - A)^{-1}BU(s) + DU(s)$$
$$\frac{Y}{U} = C(sI - A)^{-1}B + D$$

A3. TRANSFER FUNCTION:

A transfer function defines the relationship between the inputs to a system and its outputs. The transfer function is typically written in the frequency, or 's' domain, rather than the time domain. The LaPlace transform is used to map the time domain representation into the frequency domain representation.

If x(t) is the input to the system and y(t) is the output from the system, and the LaPlace transform of the input is X(s) and the Laplace transform of the output is Y(s), then the transfer function between the input and the output is:

$$\frac{Y(s)}{X(s)}$$

In other words,

$$\text{Transfer function} \equiv \frac{L(output)}{L(input)} \text{,with initial conditions set to zero.}$$

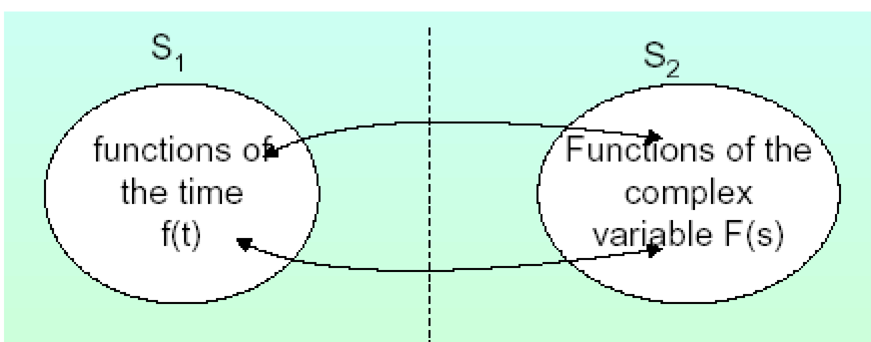
Laplace Transform:

Definition:

$$L(x(t)) = \int_{-\infty}^{\infty} e^{-st} x(t)dt \equiv X(s)$$

where s is the laplace variable.

The laplace transform thus map the “time domain” to a “laplace domain” as shown in the following figure:



The differentiation theorem:

1. $L(\dot{x}(t)) = sX(s) - X(0)$
2. $L(\ddot{x}(t)) = s^2 X(s) - sX(0) - \dot{x}(0)$
3. $L(\dddot{x}(t)) = s^3 X(s) - s^2 X(0) - sX(0) - \ddot{x}(0) - \dot{x}(0)$

Example:

System differential equation: $\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = F(x)$

$$L(\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x) = L(F(x))$$

$$[s^2X(s) - sX(0) - \dot{x}(0)] + 2\xi\omega_n[sX(s) - X(0)] + \omega_n^2 X(s) = F(s)$$
$$\Rightarrow [s^2 + 2\xi\omega_n s + \omega_n^2]X(s) = F(s) + [s + 2\xi\omega_n]X(0) + \dot{x}(0)$$

Where :
$[s + 2\xi\omega_n]X(0) + \dot{x}(0) =$ Initial Conditions.

$$\Rightarrow X(s) = \frac{F(s)}{[s^2 + 2\xi\omega_n s + \omega_n^2]} + \frac{[s + 2\xi\omega_n]X(0) + \dot{x}(0)}{[s^2 + 2\xi\omega_n s + \omega_n^2]}$$

Where:

Particular solution:
$$\Rightarrow L^{-1}\{X(s)\} = L^{-1}\{\frac{F(s)}{[s^2 + 2\xi\omega_n s + \omega_n^2]}\}$$

homogenous solution:
$$\Rightarrow L^{-1}\{X(s)\} = L^{-1}\left\{\frac{[s + 2\xi\omega_n]X(0) + \dot{x}(0)}{[s^2 + 2\xi\omega_n s + \omega_n^2]}\right\}$$

Transfer function(set all initial conditions equal to zero):

For output =x(t),

$$T.F = \frac{X(s)}{F(s)} = \frac{1}{[s^2 + 2\xi\omega_n s + \omega_n^2]}$$

For output $= \dot{x}(t)$,

$$T.F = \frac{sX(s)}{F(s)} = \frac{s}{[s^2 + 2\xi\omega_n s + \omega_n^2]}$$

The characteristic Equation is the denominator of the Transfer function:

Characteristic Equation: $=[s^2+2\xi\omega_n s+\omega_n^2]$
Classification of models:

1. Simple lag (inertia):

$$G(s)=\frac{1}{T(s+1)}$$

2. Oscillatory system:

$$G(s)=\frac{1}{s^2+2\xi\omega_n s+\omega_n^2}$$

3. Integration.

$$G(s)=\frac{1}{s}$$

A4. ROOT LOCUS

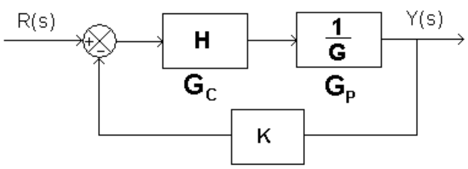
The locus of roots of the characteristic equation of the closed-loop system as the gain is varied from zero to infinity gives us the method its name. Such a plot clearly shows the contributions of each open open-loop pole or zero to the location of the closed-loop poles.

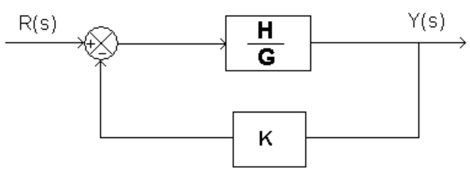
The root-locus enables us to find the closed-loop poles from the open-loop poles and zeros with the gain as parameter.

Since the method is a graphical one for finding the roots of the characteristic equation, it provides an effective graphical procedure for finding the roots of any polynomial equation arising in the study of physical systems.

Root-locus Method**:**

G(s) + kH(s) = 0





$$TF = \frac{\frac{H}{G}}{1 + K\frac{H}{G}} = \frac{H}{G + KH}$$

1) Symmetric about Real-axis

2) Number of loci. is equal to the order of characteristic equation
3) Start at k = 0 (roots of G(s))
4) End at k = ∞ (roots of H(s)) or at ∞

5) Asymptote at $\frac{k180^o}{n-m}$, k = ±1, ±3, …

6) Asymptote intersect real-axis at
(∑(Roots of G) - ∑(Roots of H))/(n-m)

7) Break-in and break-out points
Necessary condition : local extreme on real-axis for k

$$\frac{dk}{ds} = 0 = \frac{\frac{dG}{ds}H - G\frac{dh}{ds}}{H^2}$$

$G'H - H'G = 0$

8) Crossing of Im-axis

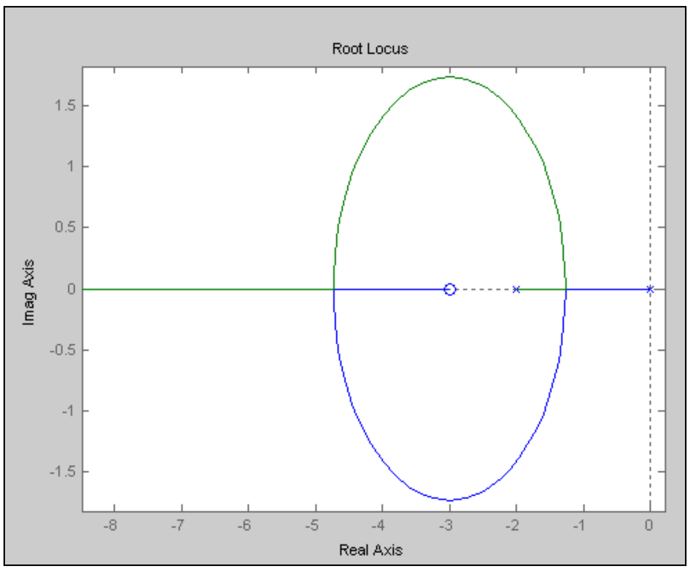
i) Substitute s = iw into G and H

ii) R-H

9) Behavior of Re-axis total number of real roots of G and H to the right to be odd.

Example:

S(S+2)+k(S+3) =0



i) Asymptote at

$$\frac{\pm 180}{n-m} = \frac{\pm 180}{2-1} = \pm 180^o$$

ii) Asymptote intersect real-axis at

$$\frac{-2+0-(-3)}{2-1} = 1$$

iii) Break-in and break-out points

$$(2S+2)(s+3)-(S^2+2S) = 0$$

$$S^2+6S+6=0$$

$$S = -3 \pm \sqrt{3} = -1.27, -4.73$$

iv) Crossing of Im-axis

$$wi(wi+2)+k(wi+3)=0$$
$$(2w+kw)i+3k-w^2=0$$
$$k=-2, w^2=3k$$
$$w=\pm\sqrt{6}i$$

Thus solution is k=0 and w=0

A5. BODE DIAGRAM.

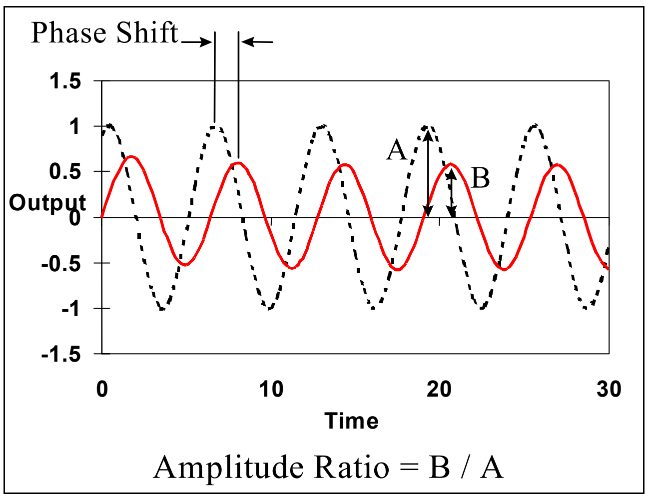
Introduction.

The frequency response of a system can be viewed in two different ways: via the Bode plot or via the Nyquist diagram. Both methods display the same information; the difference lies in the way the information is presented. The frequency response method has certain advantages, especially in real-life situations such as modeling transfer functions from physical data.

The frequency response is a representation of the system's response to sinusoidal inputs at varying frequencies. The output of a linear system to a sinusoidal input is a sinusoid of the same frequency but with a different magnitude and phase.

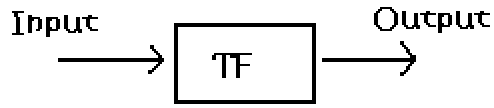
The frequency response is defined as the magnitude and phase differences between the input and output sinusoids. When a linear system is subjected to a sinusoidal input, its ultimate response is also a sustained sinusoidal wave, with the same frequency.

The figure below compares the output response of a system (solid line) with a sinusoidal input (dashed line) disturbing the system.



This particular graph shows the response of the system: $G(s) = \frac{3}{5s+1}$ to the sinusoidal input of: $u = \sin(t)$. The figure shows that the response of the system lags the input changes slightly, this lag is known as the phase shift. The ratio of the amplitudes of the input and output sinusoids is known as the amplitude ratio. Both the magnitude and the phase shift of a system will change with the frequency of the input into the system.

The frequency response is defined as: the transfer function with s replaced by $i\omega$ is called the "Frequency Response Function".



$$TF = \frac{Output}{Input}$$

We can substitute s with $i\omega$ and the response will be the Bode Diagram.

The Bode diagram can provide useful information about both:
How a given process will behave when a controller is provided,
How a particular process+controller combination will behave.

The Bode diagram provides information about the relationship between the input and output of any system whose input can be manipulated and whose output can be measured.

This clearly applies to a process where the input is an adjustment, e.g. a control valve, and the output is a measurement that might be used for control. However we can also obtain Bode diagram data for a controller, whose input is the point normally connected to the measurement from a process, and whose output is a signal to a control valve.

Bode plot general analysis.

If the general system represented by the following transfer function:

$G(s) = \frac{Q(s)}{P(s)}$ (where *Q(s)* and *P(s)* are polynomials is terms of *s*)

is subjected to a sinusoidal input of frequency ω, then the amplitude ratio of the resulting response will be given by:

$AR = modulus\ of\ G(j\omega) = \left|G(j\omega)\right|$

Where $G(j\omega)$ is found of replacing s in $G(s)$ by $j\omega$.

Similarly the phase shift will be given by:

$\phi = phase\ shift = \angle G(j\omega)$.

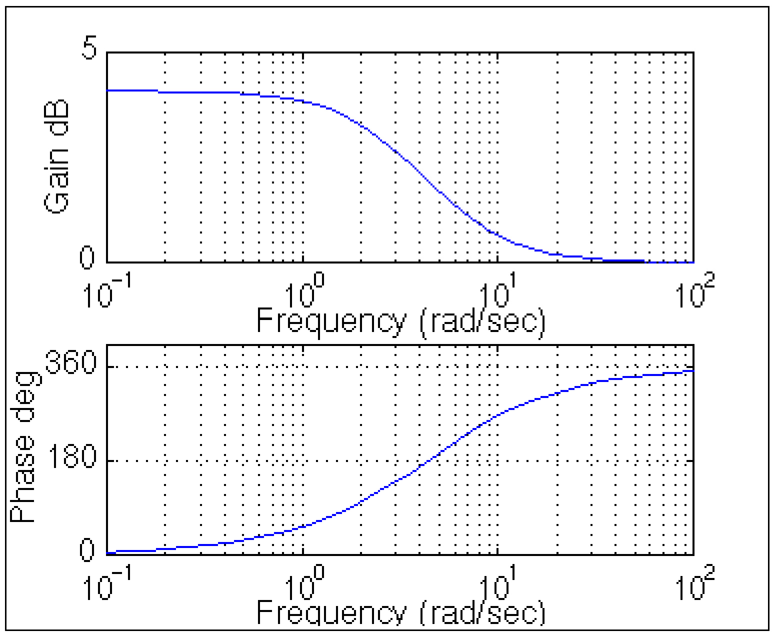
For a given sinusoidal input of $a\sin(\omega t + \vartheta)$, the output will be a sinusoid of:

$$aG(j\omega)\sin(\omega t + \vartheta + \angle G(j\omega)).$$

Bode representation for different cases (in general):

If G(s) is the open loop transfer function of a system and w is the frequency vector, we then plot G(j*w) vs. w. Since G(j*w) is a complex number, we can plot both its magnitude and phase (the Bode plot)

The Bode plots take each one of the points on the above plot and break it down into magnitude and phase. The magnitude is then plotted as gain in decibels and the phase is plotted in degrees. The frequency (on the independent axis) is plotted on a logarithmic scale. Let's take a look at the Bode plots for this function and see if our answers match.



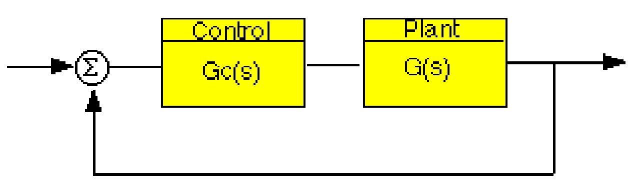
Bode Plot Example.

The frequency is on a logarithmic scale, the phase is given in degrees, and the magnitude is given as the gain in decibels.

Note: a decibel is defined as 20*log10 (|G(j*w|)

Sample Bode Plots:

Let's say that we have the following system:



We can view the open loop Bode plot of this system by looking at the Bode plot of Gc(s)G(s). However, we can also view the Bode plots of G(s) and of Gc(s) and add them graphically. Therefore, if we know the frequency response of simple functions, we can use them to our advantage when we are designing a controller.

To plot the bode plot, we will consider the following four cases:

1. $G(s) = k$ or $G(s) = \frac{1}{k}$
2. $G(s) = (s+1)$ $G(S) = \frac{1}{(s+1)}$
3. $G(s) = (s+1)$ or $G(S) = \frac{1}{(s+1)}$

4. $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2/\omega_n^2 + 2\zeta s/\omega_n + 1$ or

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1}$$

We will examine the above four cases in the following paragraphs.

Case 1: $G(s) = k$ or $G(s) = \frac{1}{k}$

Analysis:

$$G(s) = k$$
$$\Rightarrow G(j\omega) = k$$

Thus, Magnitude ratio:

$$|G(j\omega)| = k$$

in dB, $|G(j\omega)| = 20\log k$

The magnitude will be positive for K value greater than 1 and negative for K value less than 1 and greater than zero.

Phase change:

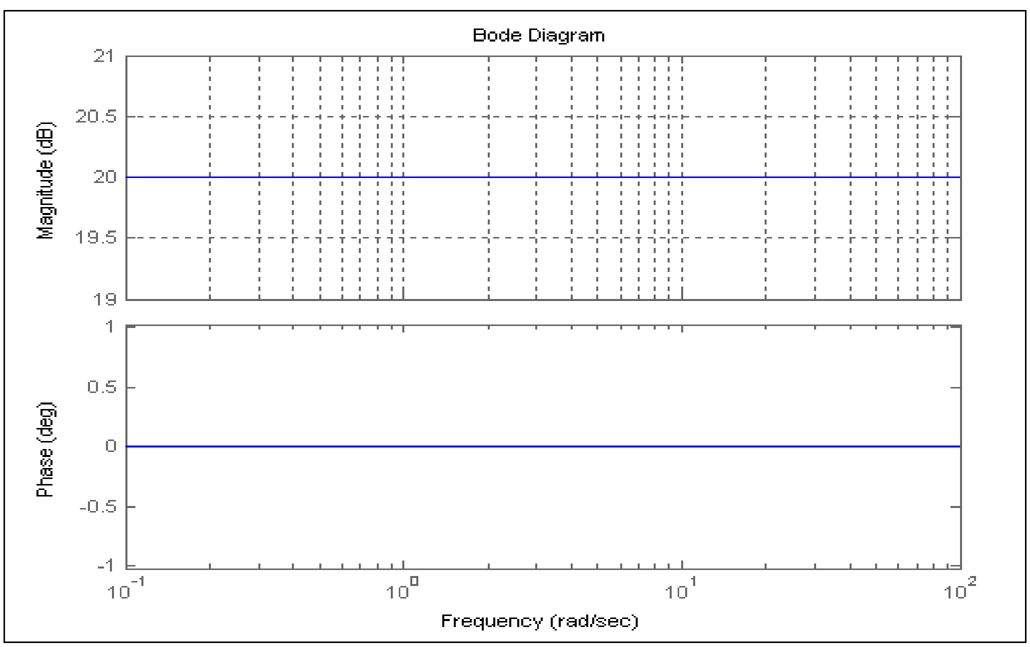
$$\angle|G(s)| = \tan^{-1}(0)$$
$$\angle|G(s)| = 0°$$

Example:

K in the numerator:

$G(s) = 10$

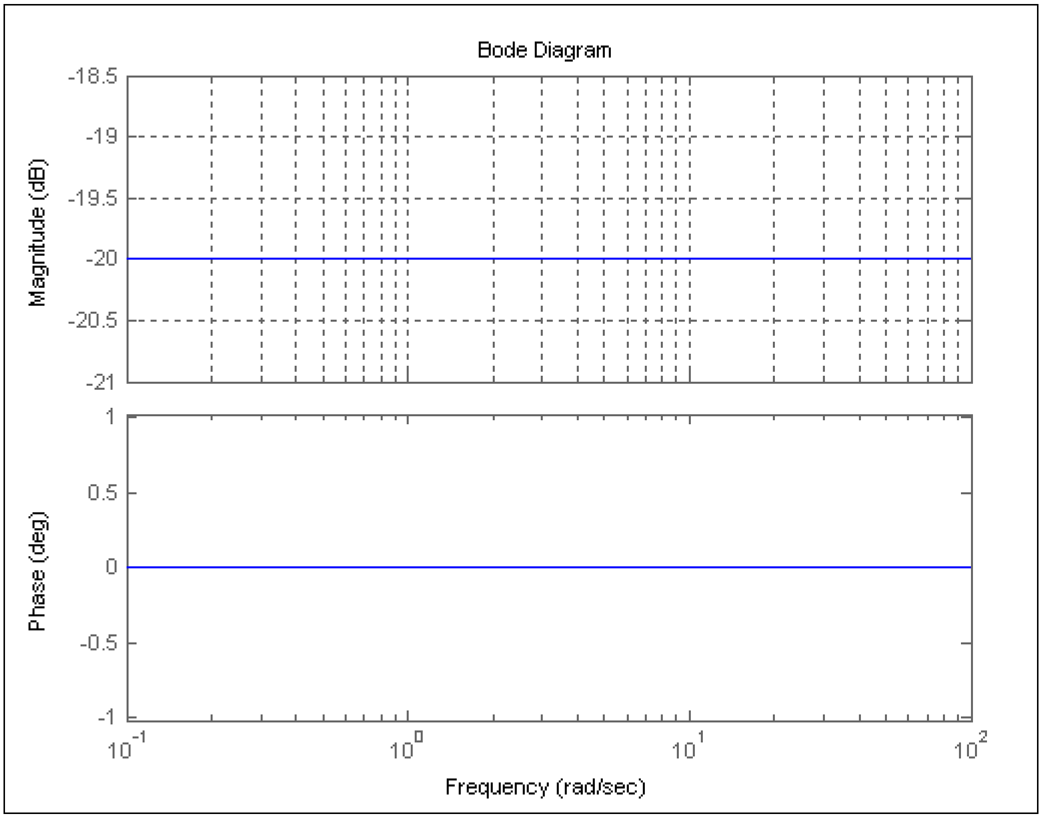
MatLab Command: Bode([10],[1])



k is in the denominator:

$$G(s) = \frac{1}{10}$$

MatLab Command: Bode([1],[10])



Case 1: S or 1/S
Analysis:

Determine the phase angle and AR of the system *1/s*.

$$G(s) = \frac{1}{s}, \quad \text{Replace } s \text{ by } j\omega,$$

$$G(j\omega) = \frac{1}{j\omega} = \frac{1}{j\omega}\frac{j\omega}{j\omega} = 0 - j\frac{1}{\omega}$$

Given a complex number of the form: y = *a+jb*, the modulus and argument of it are as follows:

$$|y| = \sqrt{a^2 + b^2}, \ \angle y = \tan^{-1}\left(\frac{b}{a}\right)$$

Therefore,

$$AR = |G(j\omega)| = \frac{1}{\omega},$$

$$\phi = \tan^{-1} - \infty = -90^o \text{ or } \phi = -\tan^{-1}\infty$$

For Example:

Determine the phase angle and AR of the system $G(s) = \frac{4}{s+1}$ *when subjected to a sinusoidal input R=2sin(3t+60*^{0}*).*

$$G(j\omega) = \frac{4}{j\omega + 1}$$

multiply top and bottom by (-jω + 1)

$$G(j\omega) = \frac{-4j\omega + 4}{\omega^2 + 1} = \frac{4}{\omega^2 + 1} - j\frac{4\omega}{\omega^2 + 1}$$

$$|G(j\omega)| = \sqrt{\left(\frac{4}{\omega^2 + 1}\right)^2 + \left(\frac{4\omega}{\omega^2 + 1}\right)^2} = \frac{4}{\sqrt{\omega^2 + 1}}$$

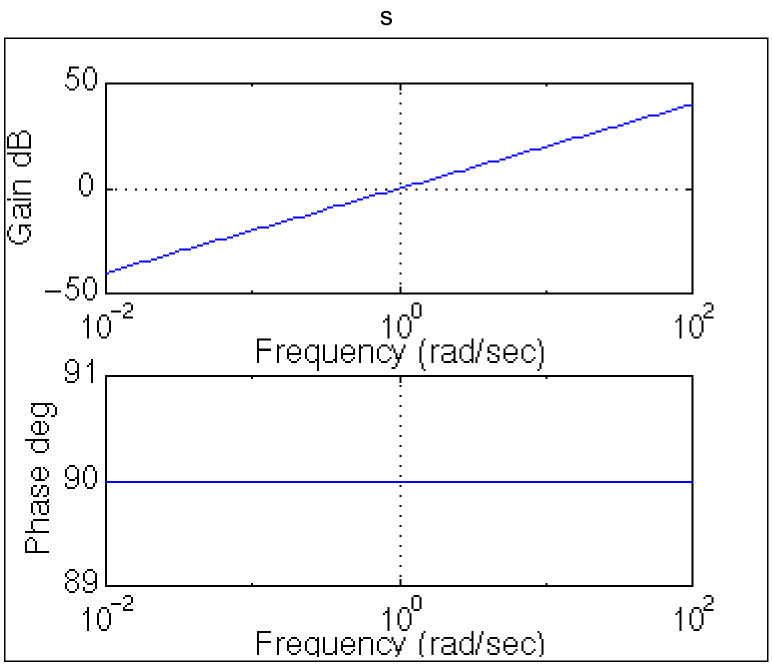
$$\phi = \tan^{-1}\omega$$

$$Y = a|G(j\omega)|\sin(\omega t + \vartheta + \phi)$$

$$Y = 2.6\sin\left(3t - 12^0\right)$$

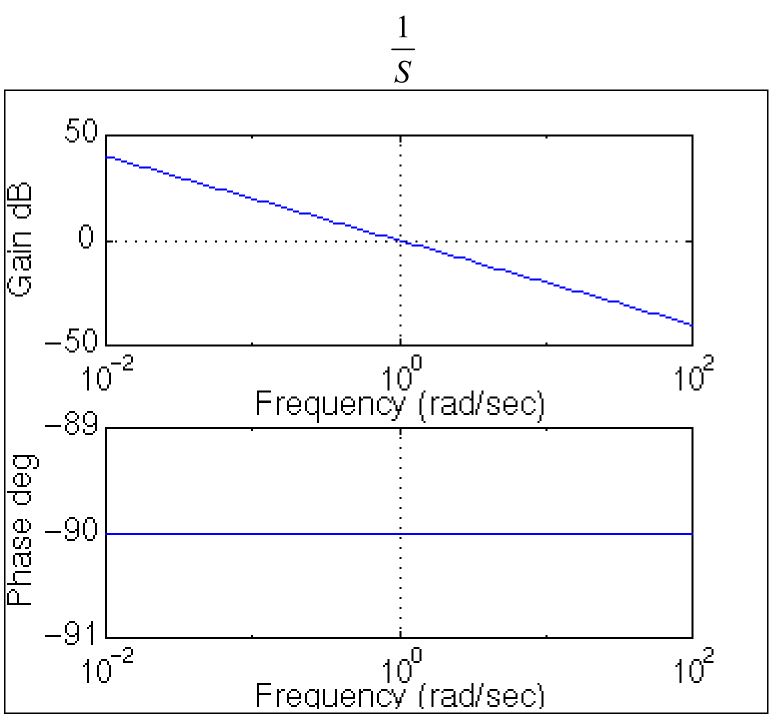
Situation 1: S is in the numerator.

S, MATLAB Command: bode([1 0], 1)



Situation 2: S is in the denominator.

$\frac{1}{S}$, [MathLab Command bode(1, [1 0])]



Case 2: $(s+1)$ [A fist order system]

General Analysis:

For a first order system the AR and phase angle can be determined as follows:

$$G(s)=\frac{1}{1+Ts}\Rightarrow G(j\omega)=\frac{1}{1+Tj\omega}$$

$$Gain_{dB}=20\log\left|\frac{1}{1+Tj\omega}\right|$$

$$\frac{1}{1+Tj\omega}=\frac{1}{1+Tj\omega}*\frac{1-Tj\omega}{1-Tj\omega}=\frac{1-Tj\omega}{1+T^2\omega^2}$$

$$\left|\frac{1}{1+Tj\omega}\right|=\sqrt{\frac{1+T^2\omega^2}{\left(1+T^2\omega^2\right)^2}}=\sqrt{\frac{1}{1+T^2\omega^2}}$$

$$Gain_{dB}=20\log\left(\sqrt{\frac{1}{1+T^2\omega^2}}\right)$$

$$Gain_{dB}=-20\log\sqrt{1+T^2\omega^2}$$

$$\angle=\tan^{-1}\omega T$$

For low frequencies, $\omega T<<1$, therefore $\text{Gain}_{\text{dB}}=0dB$

For high frequencies, $\omega T>>1$, therefore $\text{Gain}_{\text{dB}}=-20\log\omega T$

Therefore, the gain plot can be represented by 2 asymptotes: one line at 0dB and another line with -20dB/decade.

$$\omega=1/T \qquad -20\log\sqrt{1+\omega^2T^2}\approx-3dB$$

$$\omega=2/T \qquad -20\log\sqrt{1+\omega^2T^2}\approx-6dB$$

$$\omega=10/T \qquad -20\log\sqrt{1+\omega^2T^2}\approx-20dB$$

For the phase angle:

$$\phi=\angle\left(\frac{1}{1+j\omega T}\right)=-\tan^{-1}\omega T$$

$$\omega=0;\ \phi=0^o \quad \omega=1/T;\ \phi=-45^o$$

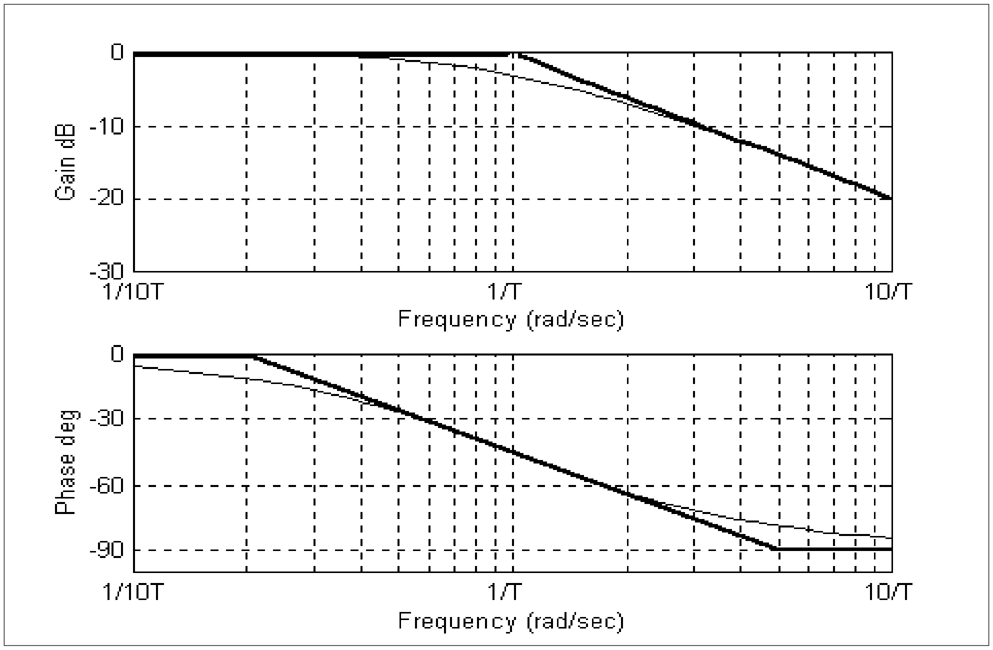
$$\omega\rightarrow\infty;\ \phi\rightarrow-90^o$$

The phase angle can also therefore be approximated by tangents. One tangent for low frequencies where the angle is approx. 0^0, one for high frequencies where the angle is approx. -90^0 and a third tangent linking these other two. The lines can be summarized as follows:

$$0 \le \omega \le \frac{1}{5T} \Rightarrow \phi \approx 0^o$$

$$\frac{5}{T} \le \omega \le \infty \Rightarrow \phi \approx -90^o$$

The Bode diagram consequently is drawn as shown below:



Example: Consider N noninteracting first order systems in series and open loop.

$$G(s) = G_1(s)G_2(s)...G_N(s) = \frac{K_1}{T_1 s+1}\frac{K_2}{T_2 s+1}...\frac{K_N}{T_N s+1}$$

$$Gain_{dB} = 20\log|G(j\omega)|$$
$$= 20\log\left[|G_1(j\omega)| * |G_2(j\omega)| * ... * |G_N(j\omega)|\right]$$
$$= 20\log|G_1(j\omega)| + 20\log|G_2(j\omega)| + ... + 20\log|G_N(j\omega)|$$

$$|G_1(j\omega)| = \left|\frac{K_1}{T_1 j\omega + 1}\right|, \text{ but}$$

$$\frac{K_1}{T_1 j\omega + 1} = \frac{K_1}{T_1 j\omega + 1}\frac{-T_1 j\omega + 1}{-T_1 j\omega + 1} = \frac{K_1 - jK_1T_1\omega}{1 + T_1^2\omega^2}$$

$$|G_1(j\omega)| = \sqrt{\frac{K_1^2\left(1 + T_1^2\omega^2\right)}{\left(1 + T_1^2\omega^2\right)^2}} = \frac{K_1}{\sqrt{1 + T_1^2\omega^2}}$$

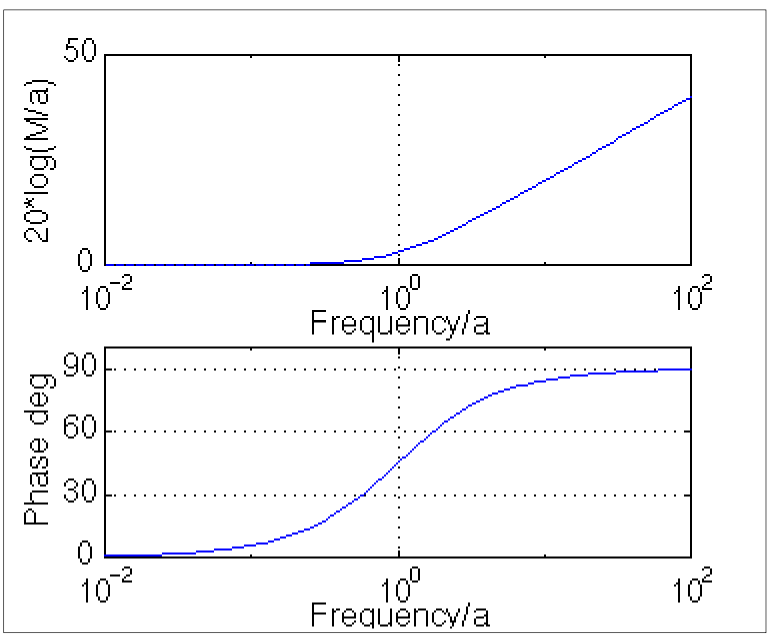
$$\Rightarrow Gain_{dB} = 20\log\left[\frac{K_1}{\sqrt{1 + \tau_1^2\omega^2}}\right] + 20\log\left[\frac{K_2}{\sqrt{1 + T_2^2\omega^2}}\right] + \ldots + 20\log\left[\frac{K_N}{\sqrt{1 + T_N^2\omega^2}}\right]$$

so,

$$\phi = \angle G(j\omega) = \phi_1 + \phi_2 + \ldots + \phi_N$$
$$\phi = -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \ldots - \tan^{-1}\omega T_N$$

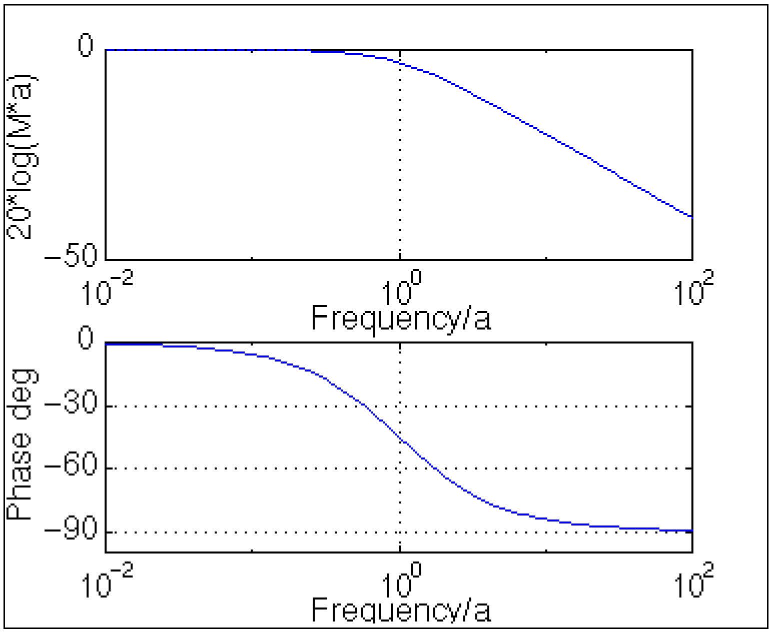
Situation 1: $(s+1)$ is in the numerator.

$(s+1)$,MatLab Command: bode([1 1], 1).



Situation 2:

$\frac{1}{(s+1)}$, MatLab Command: bode(1, [1 1])



Case 3: $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2/\omega_n^2 + 2\zeta s/\omega_n + 1$ or

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1}$$

General Analysis:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1}$$

$$G(j\omega) = \frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}$$

$$G(j\omega) = \frac{1}{j\left(\frac{2\zeta\omega}{\omega_n}\right) + \left(1 - \frac{\omega^2}{\omega_n^2}\right)}$$

The magnitude ratio thus is :

$$|G(j\omega)| = 20\log\left|\frac{1}{j\left(\frac{2\zeta\omega}{\omega_n}\right)+\left(1-\frac{\omega^2}{\omega_n^2}\right)}\right|$$

$$G(j\omega) = -20\log\left|j\left(\frac{2\zeta\omega}{\omega_n}\right)+\left(1-\frac{\omega^2}{\omega_n^2}\right)\right|$$

$$|G(j\omega)| = -20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

And the phase shift is then:

$$\angle G(j\omega) = \phi = -\tan^{-1}\left[\frac{\frac{2\zeta\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}\right]$$

To construct the bode diagram that we need, we can use the following information:

Magnitude ratio:
For frequency far less than the natural frequency:

$$\omega << \omega_n \Rightarrow |G(j\omega)| \approx 0dB$$

For frequency far more than the natural frequency:

$$\omega >> \omega_n \Rightarrow |G(j\omega)| \approx -20\log\frac{\omega^2}{\omega_n^2}$$

$$|G(j\omega)| = -40\log\frac{\omega}{\omega_n}dB$$

Thus, the AR curve has two tangents, one with slope of 0dB/decade and the other with slope of -40dB/decade.

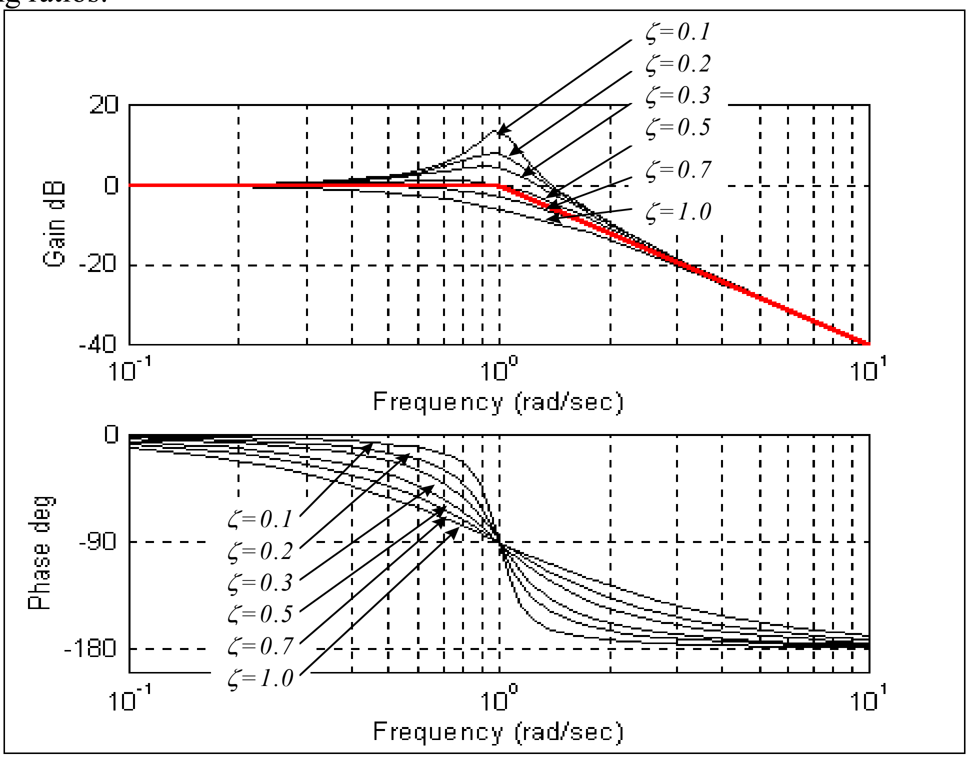
Frequency plot:

For the phase angle:

$$\omega \to 0 \Rightarrow \phi \to 0^o$$
$$\omega = \omega_n \Rightarrow \phi = -90^o$$
$$\omega \to \infty \Rightarrow \phi \to -180^o$$

Thus, the slope begins at 0^0 and tends towards -180^0 as frequency tends towards infinity.

The graph below shows the Bode diagram for a second order system with varying damping ratios:



Bode Stability Criteria:

Bode diagrams are particularly useful for determining the relative stability of a system. The relative stability of a system is calculated by firstly identifying the cross over frequency, ω_{CO}. This is the frequency at which ϕ is -180^0. If the amplitude ratio is greater than 1 at this frequency then the system is unstable. If the amplitude ratio of the system is less than 1 then the system is stable. An AR of 1 indicates a system with neutral stability or critically stable. It also follows that the lower the AR is at the cross over frequency, the greater the relative stability.

This characteristic can be used to tune PID controllers. The parameters of the controller are chosen such that the closed loop system will be some distance away from instability.

Remember that an AR > 1 implies that the $Gain_{dB}>0$.

Gain and Phase Margins

The gain and phase margins are a measure of how far from instability a system is. They are defined as follows:
Gain Margin : the amount of gain in decibels (dB) that is allowed to be increased in the loop before the system becomes unstable.
Thus, at the frequency which causes the phase lag to be -180^0, the extra gain required for:

$$\left|G_{OPENLOOP}(j\omega)\right| = 1$$

Phase Margin: the extra phase lag a system can tolerate before becoming unstable, subject to:

$$\angle G_{OPENLOOP}(j\omega) = 180^o \ and \left|G_{OPENLOOP}(j\omega)\right| = 1$$

Positive gain and phase margins imply that the system is stable.

The gain crossover frequency is the frequency at which the gain margin is equal to 0dB.
The phase crossover frequency is the frequency at which the phase margin is equal to -180^0.

Example:

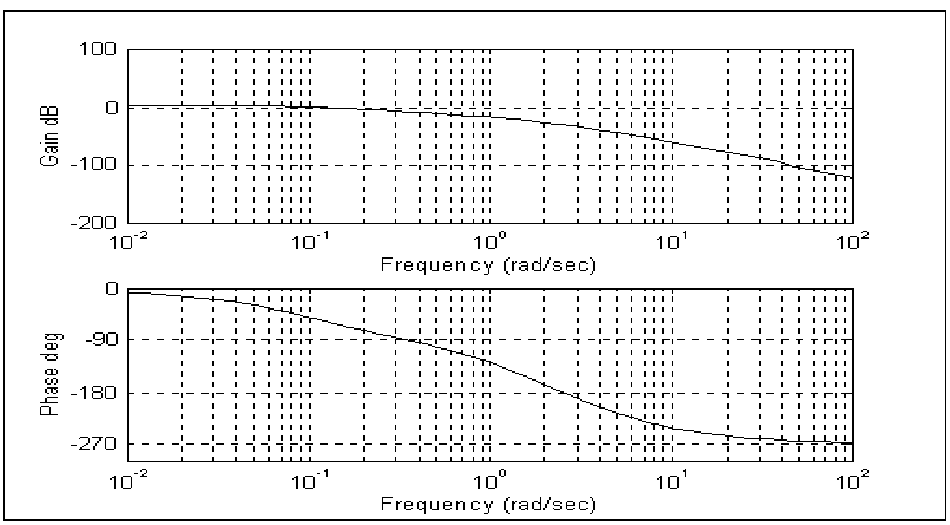
The speed dynamics of an open loop diesel engine can be approximated by the following transfer function:

$$G_{OPENLOOP} = \frac{K_p}{(s+2)(s+3)(s+0.1)}$$

Plot the open loop response on a Bode plot over a frequency range of 0.01 to 100 rads/s when K_p = 1. Also determine the gain and phase margins of the system and the gain required for the phase margin to be 30^0.

$$|G| = 20\log K_p - 20\log|2+j\omega|$$
$$-20\log|3+j\omega| - 20\log|0.1+j\omega|$$
$$|G| = 20\log K_p - 20\log\sqrt{4+\omega^2}$$
$$-20\log\sqrt{9+\omega^2} - 20\log\sqrt{0.01+\omega^2}$$
$$|G| = 20\log K_p + 20\log\frac{1}{\sqrt{4+\omega^2}}$$
$$+20\log\frac{1}{\sqrt{9+\omega^2}} + 20\log\frac{1}{\sqrt{0.01+\omega^2}}$$
$$\angle G = -\tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3} - \tan^{-1}\frac{\omega}{0.1}$$

Determine the AR and phase for a variety of frequencies. The Bode plot is shown below:



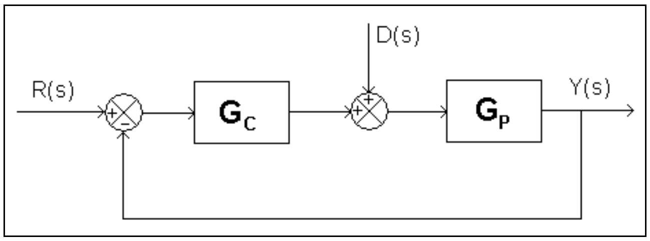
From the curve, the gain margin is 30.25 and the phase margin is 121^0.
For the phase margin to be 30^0 require that ϕ be 150^0. From plot $\phi = 150^0$ at $\omega = 1.5$. At this frequency $AR = -22$. Therefore $20logK_p = 22$, i.e. $K_p = 12.6$.

A6. PROPORTIONAL CONTROL

Proportion control is to add a controller that adjusts the setting in proportion to the deviation of the level from the desired value. We see that proportional control action is described by

$$F(s) = K_p E(s)$$

Where F(s) is the deviation in control signal and K_p is the proportional gain.



When G_C is K_P,

$$PTF = \frac{Y(s)}{R(s)} = \frac{K_P G_P}{1 + K_P G_P}$$

$$DTF = \frac{Y(s)}{D(s)} = \frac{G_P}{1 + K_p G_P}$$

Explain with an example

$$Let \qquad G_p = \frac{1}{s-3}$$

$$PTF = \frac{k_p}{s - 3 + k_p}, DTF = \frac{1}{s - 3 + k_p}$$

1) Stability

$$PTF = \frac{k_p}{s - 3 + k_p}$$

IF k_p is bigger than 3, pole will be left hand side so it makes system stable. Namely, proportion controller can control stability of system.

2) Performance(speed)

$$\tau = \frac{1}{k_p - 3}$$

Because proportional controller magnifies the error between input and output, it can change system response. In example, when k_p increases, time constant will decrease and it makes system response faster.

3) Accuracy and robustness

$$\lim_{s\to 0}(PTF) = \frac{k_p}{k_3 - 3} \neq 1$$

$$\lim_{s\to 0}(DTF) = \frac{1}{k_3 - 3} \neq 0$$

So it is impossible to control accuracy and robustness by proportional controller.

A7. INTEGRAL CONTROL.

Proportional control produces a constant error because the control signal does not change as system reaches equilibrium. An integral control is thus modifying control signal as long as long as error exists. Change in control is proportional to the integral of the error.

Thus: In Frquency domain,

$$F(s) = K_I(\frac{1}{s})E(s)$$

In Time domain,

$$f(t) = K_I \int_0^t e(t)dt, f(0) = 0$$

Integral Control, $G_C(s) = \frac{K_I}{s}$ where K_I = “Intergral Control gain”

Integral control have the possibility to give a perfect control, however, it may keep increase the momentum in the system causing unstable. Thus, we never use integral control by itself. It was usually used with propotional control which called the P.I controller.

For example,

PTF (Primary Transfer Function)= $\frac{G_C}{(s-3)+G_C}$

If we use a intergral controller, $G_C = K_I / s$

PTF= $\frac{K_I / s}{(s-3)+K_I / s}$

$$\Rightarrow \frac{K_I}{s^2 - 3s + K_I}$$

There is a negative term in the characteristic Equation, thus the system cannot be stable.

However, if we use a P.I controller, $G_C = K_I / s + K_P$

$$PTF = \frac{K_P + K_I / s}{(s-3) + K_P + K_I / s}$$
$$\Rightarrow \frac{K_P s + K_I}{s^2 - 3s + K_P s + K_I}$$

The system will stable if $K_P > 3, K_I > 0$.

Example:

Let $G_c = \frac{k_I}{s}$ and $G_p = \frac{1}{s-3}$,

$$PTF = \frac{k_I}{s^2 - 3s + k_I}, DTF = \frac{1}{s^2 - 3s + k_I}$$

and poles are $s_{1,2} = \frac{3 \pm \sqrt{9 - 4k_I}}{2}$ and $\tau = \frac{2}{3 \pm \sqrt{9 - 4k_I}}$

If $\sqrt{9 - 4k_I}$ is smaller than 0, it is impossible to move pole to left half plane on the real axis so We can not control stability and performance (speed).

A8. DERIVATIVE CONTROL

The derivative controller reacts to the rate of change of the error. This is the basis of derivative control action.

$$F(s) = K_D s E(s)$$

Where K_D is the derivative gain. This algorithm is also called rate action. It is used to damp out oscillations. Because it depends on only the error rate, derivative control should never be used alone. When used with proportional action, the following PD-control algorithm results

$$F(s) = (K_P + K_D s) E(s)$$

Explain with an example

Let $G_c = k_P + k_D s$ and $G_p = \frac{1}{s-3}$

$$PTF = \frac{sk_D + k_p}{(1 + k_D)s + (k_P - 3)}, DTF = \frac{1}{(1 + k_D)s + (k_P - 3)}$$

1) Stability and Speed

$s = \frac{1+k_D}{k_P - 3}$ and $\tau = \frac{k_P - 3}{1+k_D}$ Stability and Time constant is depend on both k_p and k_D

2) Accuracy and robustness

$$\lim_{s\to 0}(PTF) = \frac{k_p}{k_P - 3} \neq 1$$

$$\lim_{s\to 0}(DTF) = \frac{1}{k_P - 3} \neq 0$$

So it is impossible to control both accuracy and robustness with PD controller.

The characteristics of P, I, and D controllers:

A proportional controller (Kp) will have the effect of reducing the rise time and will reduce ,but never eliminate, the steady-state error. An integral control (Ki) will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative control (Kd) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response. Effects of each of controllers Kp, Kd, and Ki on a closed-loop system are summarized in the table shown below.

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

Note that these correlations may not be exactly accurate, because Kp, Ki, and Kd are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for Ki, Kp and Kd.

A9. OBSERVER.

Initially, we assume all the state variables are available for feedback in our study of close loop control system. In practice, however, not all state variables are available for feedback. Then we need to estimate unavailable state variables. It is important to note that we should avoid differentiating a state variable to generate another one. There are methods available to estimate unmeasurable state variables without a differentiating process. Estimation of unmeasurable state variables is commonly called Observation. A device or computer programs that estimate or observes all state variables are called a state observer, or simply observer. There are two type of observer, which includes full-order state observer and minimum order observer.

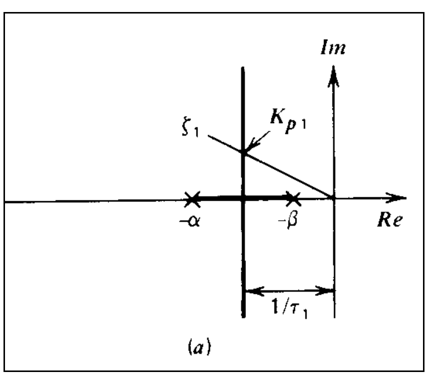
A state observer thus estimates the state variables based on the measurements of the output and control variables. In many practical problems, a state observer vector is used to generate the desired control vector.

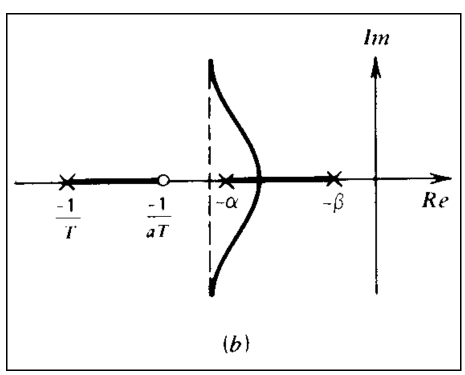
A10. LEAD COMPENSATION

The primary function of the lead compensator is to reshape the frequency – response curve to provide sufficient phase lead angle to offset the excessive phase lag associated with the components of the fixed system.

Root locus design of compensators:

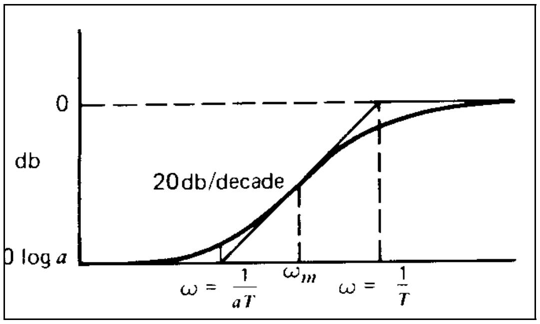
The effects of the compensator in terms of time domain specifications (characteristic roots) can be shown with the root locus plot. Consider the second-order plant with the real distinct roots s = -α,-β.The root locus for this system with proportional controller is shown Figure a. With lead compensation, the root locus becomes that shown in Figure b.

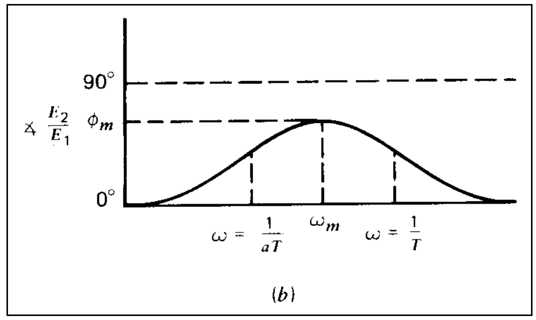




Bode design of compensators

For any particular value of the parameter a, the lead compensator can provide a maximum phase lead φ_m. This value, and the frequency ω_m at which it occurs, can be found as a function of a and T from bode plot





A11. LAG COMPENSATION.

Lead and lag compensators are used quite extensively in control. A lead compensator can increase the stability or speed of response of a system; a lag compensator can reduce (but not eliminate) the steady state error. Depending on the effect desired, one or more lead and lag compensators may be used in various combinations.

A variety of compensator circuits have been developed, but the lead and lag compensators are the simplest ways of adjusting gains and phases in a variety of frequency ranges. In addition to the PID compensators, they are the most widely used.

Lead, lag, and lead/lag compensators are usually designed for a system in transfer function form. A first-order lag compensator can be designed using the root locus. A lag compensator in root locus form is given by:

$$G(s) = \frac{(s - z_o)}{(s - p_o)}$$

Where the magnitude of zo is greater than the magnitude of p_o. A phase-lag compensator tends to shift the root locus to the right, which is undesirable. For this reason, the pole and zero of a lag compensator must be placed close together (usually near the origin) so they do not appreciably change the transient response or stability characteristics of the system.

How does the lag controller shift the root locus to the right? If you recall finding the asymptotes of the root locus that lead to the zeros at infinity, the equation to determine the intersection of the asymptotes along the real axis is:

$$\alpha = \frac{\sum(\mathbf{poles}) - \sum(\mathbf{zeros})}{(\#\mathbf{poles}) - (\#\mathbf{zeros})}$$

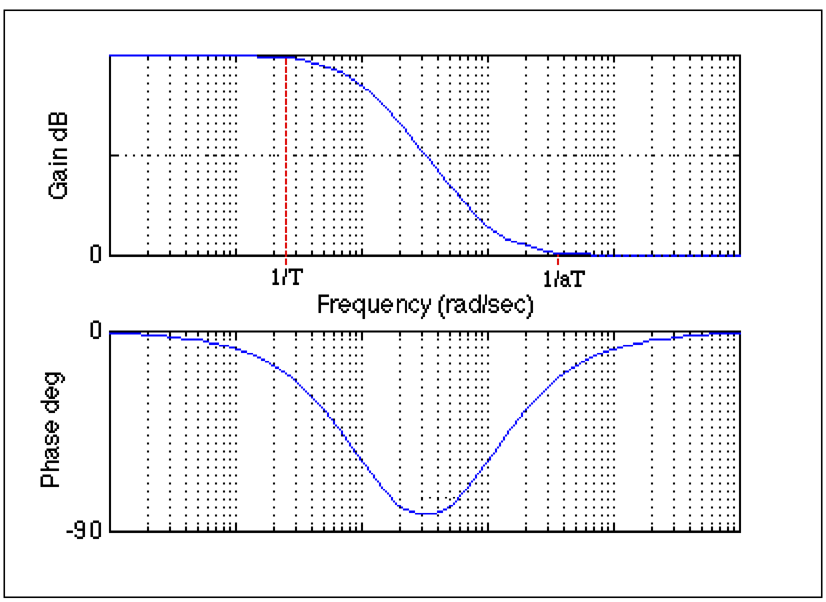
When a lag compensator is added to a system, the value of this intersection will be a smaller negative number than it was before. The net number of zeros and poles will be the same (one zero and one pole are added), but the added pole is a smaller negative number than the added zero. Thus, the result of a lag compensator is that the asymptotes' intersection is moved closer to the right half plane, and the entire root locus will be shifted to the right.

Lag or Phase-Lag Compensator using Frequency Response.

A first-order phase-lag compensator can be designed using the frequency response. A lag compensator in frequency response form is given by:

$$G(s) = \frac{1}{a}\left(\frac{1+aTs}{1+Ts}\right) \quad (a<1)$$

The phase-lag compensator looks similar to a phase-lead compensator, except that a is now less than 1. The main difference is that the lag compensator adds negative phase to the system over the specified frequency range, while a lead compensator adds positive phase over the specified frequency. A bode plot of a phase-lag compensator looks like the following



The two corner frequencies are at 1/T and 1/aT. The main effect of the lag compensator is shown in the magnitude plot. The lag compensator adds gain at low frequencies; the magnitude of this gain is equal to **a**. The effect of this gain is to cause the steady-state error of the closed-loop system to be decreased by a factor of **a**. Because the gain of the lag compensator is unity at middle and high frequencies, the transient response and stability are not impacted too much.

The side effect of the lag compensator is the negative phase that is added to the system between the two corner frequencies. Depending on the value **a**, up to -90 degrees of phase can be added. Care must be taken that the phase margin of the system with lag compensation is still satisfactory.

B PART. APPLICATIONS:

B1. STABILITY

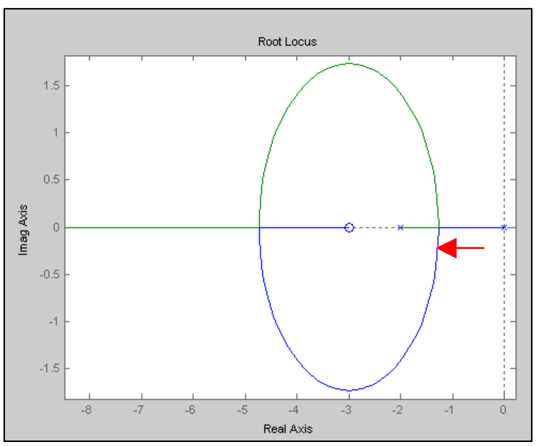
Stability of a system implies that if disturbed away from its equilibrium it will return to equilibrium. Namely, If the system is stable, it will return to and remain at the equilibrium.

Routh-Hurwitz

Stability properties of linear system are determined by the roots of its characteristic equation. If any root lies in the right half of the complex plane, the system is unstable.

The Routh-Hurwitz criterion tells us how many roots lie in the right-half plane.

Root locus:



If dominant pole is on the right half plane, this system is unstable so the value of k which can move dominant pole to left half plane must be chosen.

PID control:

Proportional controller and derivative controller can control stability by moving poles to the left half plane but integral controller can not move pole to the left but move on the imaginary axis and change damping ratio.

B2. TIME CONSTANT:

The time constant is the time required for a system to response by 63% of its initial value. For a system that has higher order than one, there exist several roots, all with negative real parts, then every root will have a time constant. We then have to determine the largest time constant, which is the toots whose exponential term dominates the response. This root was then call the dominant roots and its time constant is called the dominant time constant.

For a first order system, if the roots of the characteristic equation is s=-1, the time constant tau will be:

$$\tau = 1/s$$

For a second order system which has the form:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

The time constant is given by $\tau = 1/\xi\omega_n$

Note:
The time constant is related to the following topics:

1. See “Routh Hurwitz” part for how to find a controller that specify a specific time constant requirement.
2. See “Root Locus” part for how to determine the fastest time constant from root locus plot.
3. See “Integral Control”, “Proportional Control” and “Derivative Control” on how a PI, PD, or a PID controller affect the characteristic equation and thus affect the time constant of the system.

B3. DAMPING COEFFICIENT

The damping coefficient, or alternatively, the damping ratio, is the most difficult quantity to determine. Both mass and stiffness can be determined by static test; however, damping requires a dynamic test to measure. A record of the displacement response of an underdamped system can be used system to determine the damping ratio.

The dynamic behavior of the second order system can then be described in terms of two parameters ζ and ω_n, If $0 < \zeta < 1$, the closed loop poles are complex conjugate and lie in the left-half s plane. The system is then called underdamped, and the transient response is oscillatory. If $\zeta = 1$, the system is called critically damped. Overdamped systems correspond to $\zeta > 1$, the transient response of critically damped and overdamped systems do not oscillate. If $\zeta = 0$, the transient response does not die out.

Root locus

For a second-order system,

$$S^2 + 2\zeta\omega_n S + \omega_n^2 = 0$$

$$S_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

In the case of under damped system, $\zeta < 1$, $S = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$

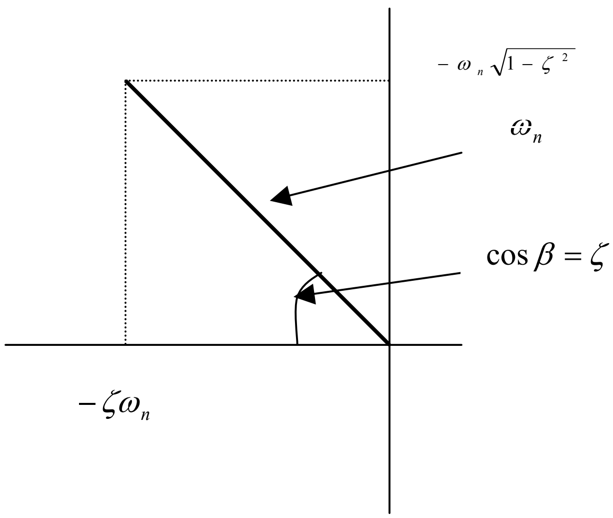
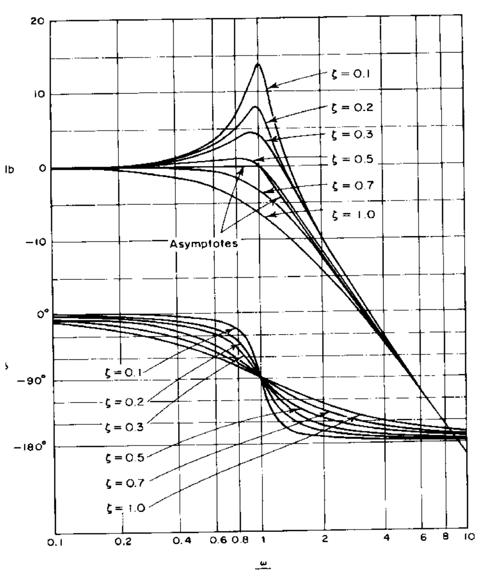


Figure shows that the cosine of angle is damping ratio.

Bode diagram



The damping ratio ζ determines the magnitude of this resonant peak.

B4. CRITICAL DAMPING AND OVER DAMPING.

The second order system's free response for the stable case can be conveniently characterized by the damping ratio ξ. The damping ratio is the ratio of the actual damping constant c to the critical value c_c.

For a critically damped system, the damping ratio is equal to one.

For overdamped system, the damping ratio is greater than one. In this case, exponential behavior occurs.

For underdamped system, the damping ratio is less then one.

The damping ratio will only have meaning on a stable system. For an unstable system, the damping ratio is not defined. If we have a damping ratio of negative value, this means that the system is not stable and the damping ratio is not define.

Note:

1. Damping ratio can be used for a quick check for oscillatory behavior. For example, if overdamped occurs, we can know that the system will have no oscillation in the system's free response.
2. See more topic on damping ratio in "Bode plot", "Transfer function", "Root locus", and "P,I,D controller".

B5. NATURAL FREQUENCY AND DAMPED NATURAL FREQUENCY.

For a second order system, when there is no damping, the characteristic roots are purely imaginary. The imaginary part, and therefore the frequency of oscillation, is termed the undamped natural frequency or simply the natural frequency. For a system having a characteristic equation of the form:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

where ω_n is the natural frequency.

On the other hand, the frequency of the oscillation is called the damped natural frequency. Denoted as ω_d

The damped natural frequency is determined by:

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

Thus, the frequencies ω_d and ω_n will only have meaning for the underdamped system case, where the damping ratio is less than 1. the ω_d, damped natural frequency is always less than the undamped natural frequency.

When there is no damping, the characteristic roots are purely imaginary. The imaginary part, and therefore the frequency of oscillation, for this case is $b = \sqrt{k/m}$ .This frequency is termed the undamped natural frequency, or simply the natural frequency, and is denoted ω_n Thus

$$\omega_n = \sqrt{\frac{k}{m}}$$

Root locus

For a second-order system,

$$S^2 + 2\zeta\omega_n S + \omega_n^2 = 0$$

$$S_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2},$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

In the case of under damped system, $\zeta < 1$, $S = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$

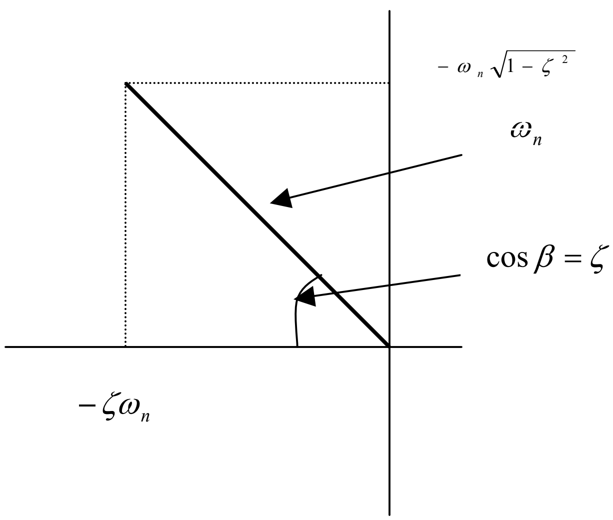
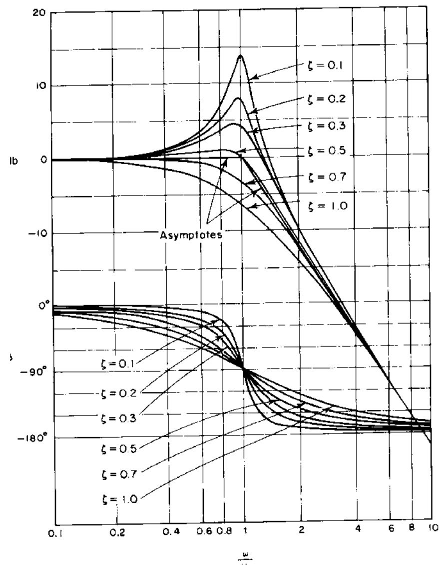


Figure shows that the distance from origin is natural frequency.

Bode plot



Near the frequency $\omega = \omega_n$, a resonant peak occurs, as may be expected from

$$-20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}$$

The damping ratio ζ determine the magnitude of this resonant peak.

To obtain the frequency-response curves of a given quadratic transfer function, we must first determine the value of the corner frequency ω_n and that of the damping ratio ζ. Then, by using the family of curves can be plotted.

Notes:

1. See more related topic on Natural frequency on “Bode plot” on how to determine the natural frequency from a bode plot of a second order system.
2. See more topics on Natural Frequency on “Lead compensator” and “lag compensator” on how to shift the corner frequency of a band filter and a notch filter.
3. See “Root Locus” on how frequency is related to the plot.

B6. ACCURACY AT STEADY-STATE RESPONSE

Steady-state response

The time response of a control system consists of two parts: the transient and steady-state response. By transient response, we mean that which goes from the initial to the final state. By steady-state response, we mean the manner in which the system output behaves as t approaches infinity.

PID control

For system being accurate, the stead-state of output/input must be 1 such as,

$$\lim_{s\to 0}(PTF) = 1$$

In the case of PID controller, $\lim_{s\to 0}(PTF)$ is 1 but if P or D controller is used alone it can make system accurate also PD controller can not do it but I controller can make $\lim_{s\to 0}(PTF)$ to 1.

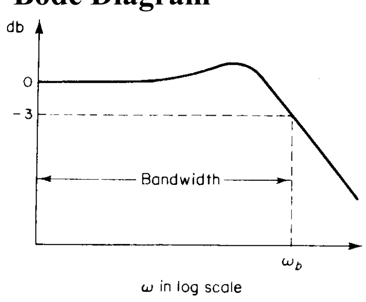
In example, $Gc = k_P + \frac{k_I}{S} + k_D S$ and $G_p = \frac{1}{s-3}$ **(PID controller)**

Accuracy is $\lim_{s\to 0}\frac{S^2 k_D + S k_P + k_I}{(1+k_D)S^2 + (k_P - 3)S + k_I} = 1$

In the case of $G_c = k_P + k_D s$ and $G_p = \frac{1}{s-3}$ **(PD controller)**

$$\lim_{s\to 0}(PTF) = \frac{k_p}{k_P - 3} \neq 1$$

Bode Diagram



The definition of db is $20\log_{10}\left\|\frac{Output}{Input}\right\|$, so in a certain frequency band, if db is 0, in the frequency band, output is same with input. Namely , when input frequency is in the band, input and output is same.

B7. ROBUSTNESS WITH RESPECT TO DISTURBANCE.

For achieving the control goal, we try to make a system that is:

1. Stable.
2. Speed (Performance).
3. Accuracy. (Response to command).
4. Robustness. (Response to Distubance).

After we found the criteria for designing a controller that is stable, having desired speed, and accuracy, we would like to know how to the robustness of the system, which is the response of the system with respect to disturbances. To study this part of response for the system, we determine the system's robustness from its Disturbance Transfer Function, which is define by:

For a close loop system:

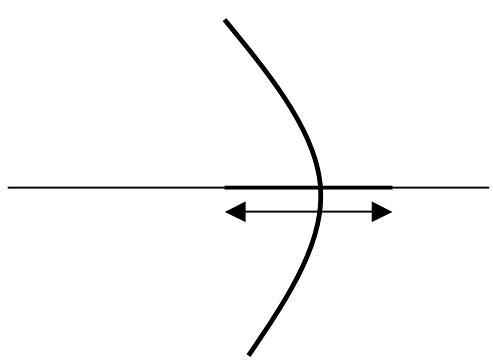
$$\text{DTF}=\frac{G_P}{1+G_P G_C}$$

To determine the system's robustness,

$$\text{Robustness}=\lim_{s\to 0}(DTF)$$

We prefer the robustness value to be as small as possible such that the system did not response much due to the disturbances. Obviously a zero is preferred.

Root locus



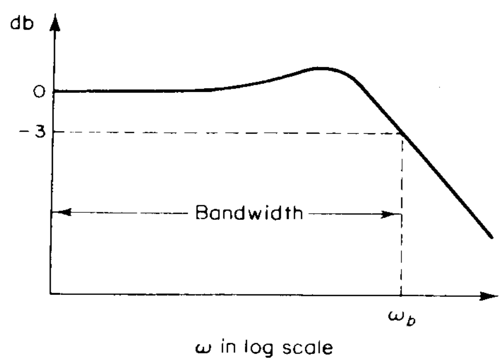
If controller increases arrowed area, the system has more robustness with respect to disturbance. Because poles can move on the real axis without changing system characteristic, such as oscillation.

Notes:

1. The Robustness of the system is important in designing a controller for the system. Refer to "P,I,D controller" for more information.
2. The priority in designing a controller for a system is Stable, Speed, Robustness, and then Accuracy.

B8. BANDWIDTH

Definition 1:

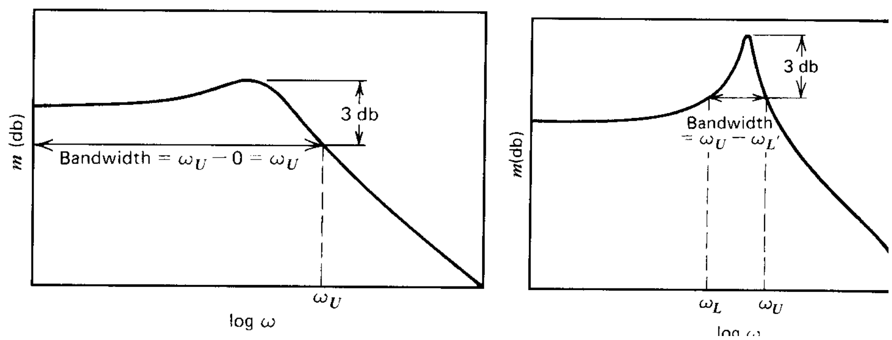


Referring the figure, the frequency ω_b at which the magnitude of the closed-loop frequency response is 3db below its zero-frequency value is called the cutoff frequency

The frequency range $0 \le \omega \le \omega_b$ in which the magnitude of the closed loop does not drop -3 db is called the bandwidth of the system. The bandwidth indicates the frequency where the gain starts to fall off from its low-frequency value. Thus, the bandwidth indicates how well the system will track an input sinusoid. Note that for a given ω_n, the rise time increases with increasing damping ratio. On the other hand, the bandwidth decrease with the increase of damping ratio.

The ability to respond the input signal. A large bandwidth correspond to a small rise time, or fast response. Roughly speaking, we can say that bandwidth is proportional to the speed of response.

Definition 2:



The bandwidth definition is arbitrary based on the half-power points. Bandwidth is a measure of the range of frequencies for which a significant portion of the system's input is felt by the output. The system filtered out to a greater extent those input components whose frequency lies outside the bandwidth.