

State University of New York at Buffalo
Department of Mechanical and Aerospace Engineering.

MAE 431: ENERGY SYSTEM

PROJECT PAPER

**GAS TURBINE CYCLE -
INTRODUCTION TO BRAYTON CYCLE**

SUBMITTED BY: LEE LENG FENG

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Gas Turbine Cycle- Introduction of Brayton Cycle

Abstract:

This project paper gives a brief introduction to Gas Turbine power plant, which mainly focus on Brayton Cycle. The Brayton Cycle is the basic cycle for Gas Turbine power plant. Other cycle models such as Turbojet engine and Combine Gas Turbine-Vapor power cycle involved the use of Brayton Cycle as part of the respective cycle. The topic covered in this project paper including the following:

1. Gas Turbine power Plant.
2. History of Brayton Cycle.
3. Air standard Brayton Cycle.
4. Work and heat transfer in Brayton cycle.
-Ideal Air-Standard Brayton Cycle.
5. Study of pressure ratio effect on the efficiency of Brayton Cycle.
6. Irreversibility and its effect on the efficiency of Brayton Cycle.
7. Regenerative gas turbine.
8. Sample questions related to Brayton Cycle.
9. Using Brayton Cycle in Real life situation.

In this project paper, we focus on the formulation of the mathematical model for Brayton cycle. This included formulate the necessary formulas to find the Work, Heat, and Efficiency of each components involves in Brayton Cycle. The assumptions for deriving each formula are also given. Beside this, this paper will also discuss the pressure ratio effect on efficiency, Irreversibility effect on efficiency, and introduce the idea of regenerative gas turbine. Finally, example and a current development of Brayton Cycle are given to illustrate the use of some formulas that we discussed in this paper.

1. GAS TURBINE POWER PLANT.

Part A: Introduction:

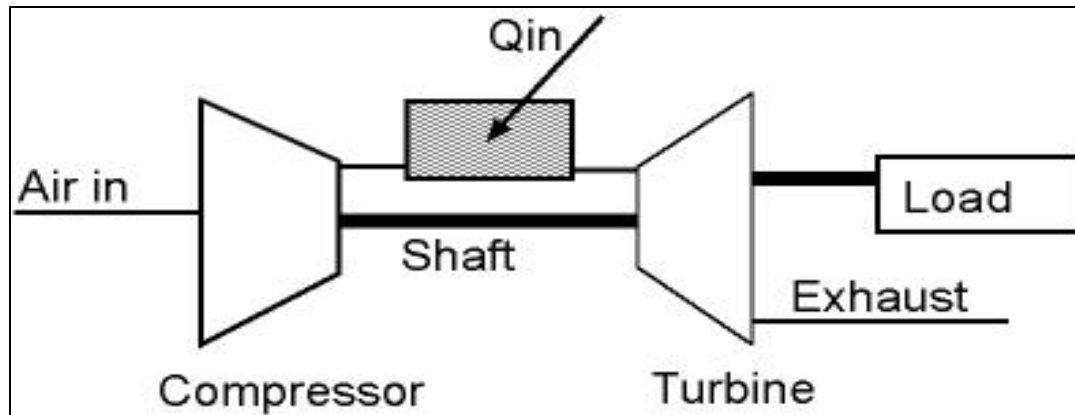


Figure 1. Graphical representation of the basic components for a Gas Turbine Power Plant.

Gas turbine power plants are lighter and more compact than the vapor power plants. Gas turbine power plants are also well suited for the application in transportation industry for their well power-output-to-weight ratio. The main three components of a Gas Turbine power Plant are a compressor, a combustor, and a turbine.

The Gas Turbine Power plant can operate in an open or close basis. These two different modes are shown in Figure 2(a) and 2(b). The open mode in Figure 2 (a) is more common.

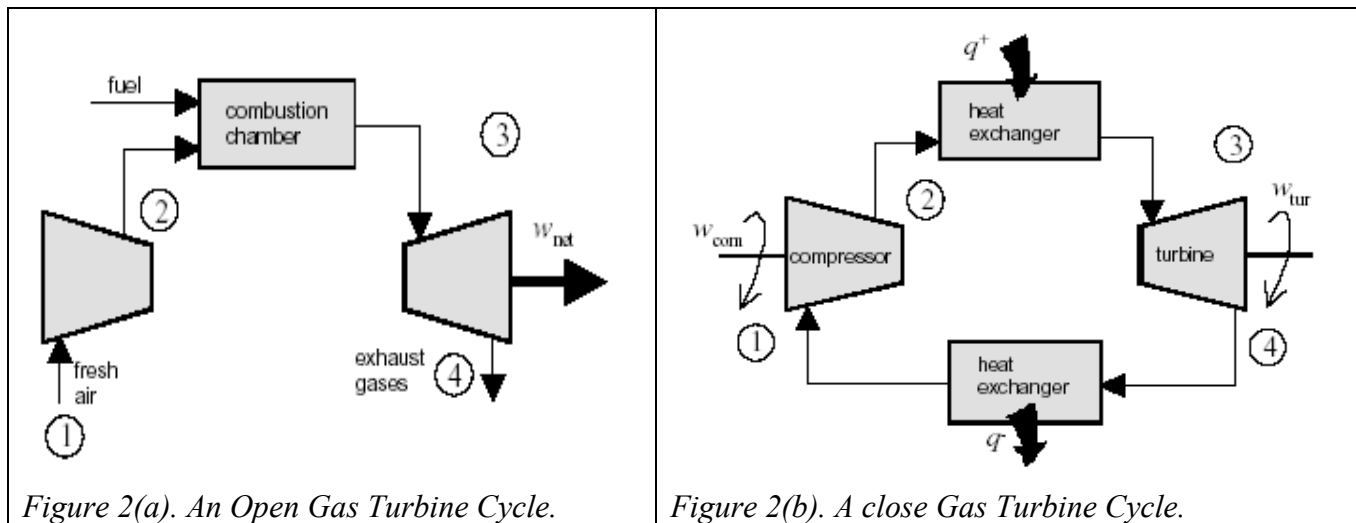


Figure 2(a). An Open Gas Turbine Cycle.

Figure 2(b). A close Gas Turbine Cycle.

Note: The (1), (2), (3) and (4) represent the different state of the air. Where we will use for study in the following paragraph.

Part B: Modeling Gas Turbine Power Plant.

We model the Gas Turbine Power Plant by using the “*air-standard analysis*”. The air standard analysis included following assumptions (Cengel 492):

- I. The working fluid is air, and ideal gas assumption was use for the gas.
- II. All the processes make up the cycle are internally reversible.
- III. The temperature rise that brought by a combustion process was model by a heat transfer process from an external source.
- IV. The exhaust process is replaced by a heat rejection process that restores the working fluid to its initial state.
- V. No composition change in the air during the combustion process.

By doing this, it greatly simplified our analysis because: we avoid using dealing with the complexities of the combustion process and the change of composition during combustion.

However, we also lost some accuracy for which: “The numerical value calculated using this simplification could only provide qualitative indications of power plant performance.” [2]

Now we have some idea of a Gas Turbine Power plant, which base on the theory of Brayton Cycle. And we shall also look for a brief history of Brayton Cycle.

2. HISTORY OF BRAYTON CYCLE.



Invented by George Baily Brayton. His invention of this cycle does not really achieve a commercial successful since they were not as compact and efficient as the silent spark-engines developed in Europe by Nicholas Otto and Eugen Langen.

However, a 22 years old cadet in England name Frank Whittle, who wrote a term paper for his power plants class entitled "*Future Developments in Aircraft Design*"; in the paper he pointed out the inherent limitations of piston driven engines for high-speed flight. Whittle showed that a continuous flow turbine based engine in which combustion occurred at constant pressure could overcome these limitations. In his notebook in which he illustrates on P-v coordinates the thermodynamic cycle of such an engine; it is, of course, a Brayton cycle. [1] With the further study of this cycle by Frank Whittle, the Brayton Cycle was now use in aircraft propulsion, marine power plant, and electric power generation and form the basic cycle for turbojet cycle [2].

The following figures show two example of the application of Brayton Cycle. This is indicated in Figure 3(a), which shows a schematic of a modern Pratt & Whitney FT8 aircraft gas turbine as well as Figure 3(b), a stationary gas turbine from Kawasaki.

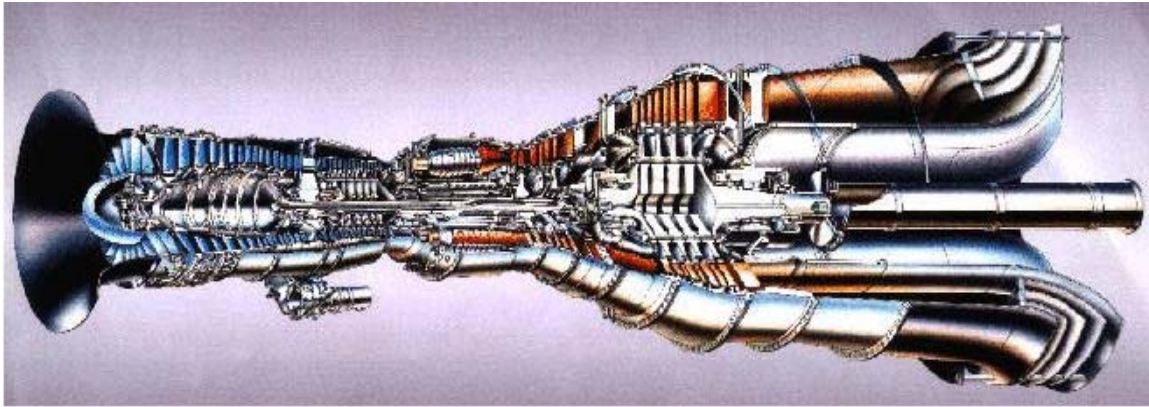


Figure 3(a) A Pratt & Whitney FT8 aircraft propulsion gas turbine.

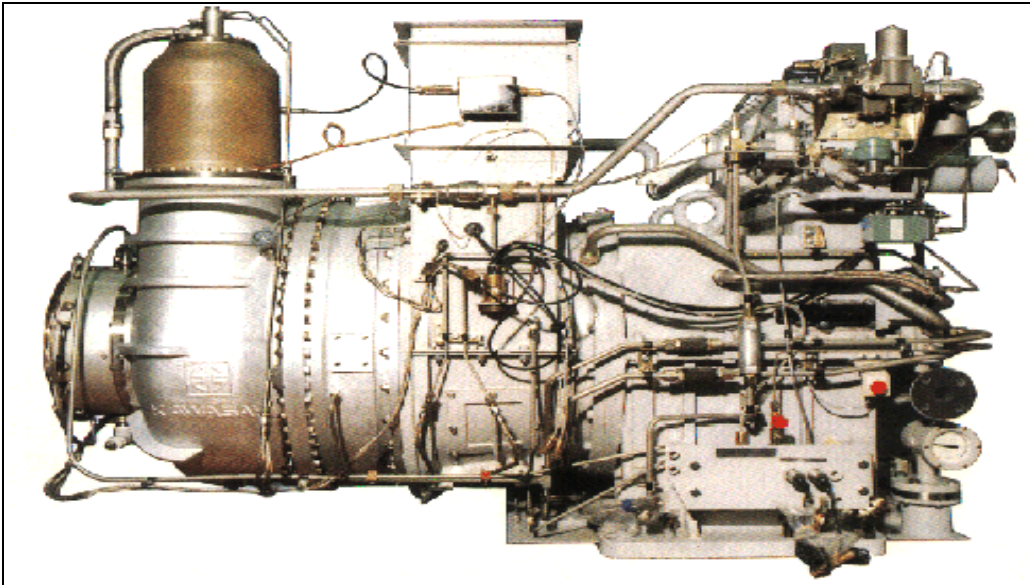


Figure 3(a). A stationary Kawasaki M1A-11 1.2 MW gas turbine used for electricity generation (bottom).

3. AIR STANDARD BRAYTON CYCLE.

By using the air-standard analysis, as shown in our previous paragraph, the diagram of an air-standard gas turbine analysis is shown in Figure 4, which is similar to the close system of the turbine power plant.

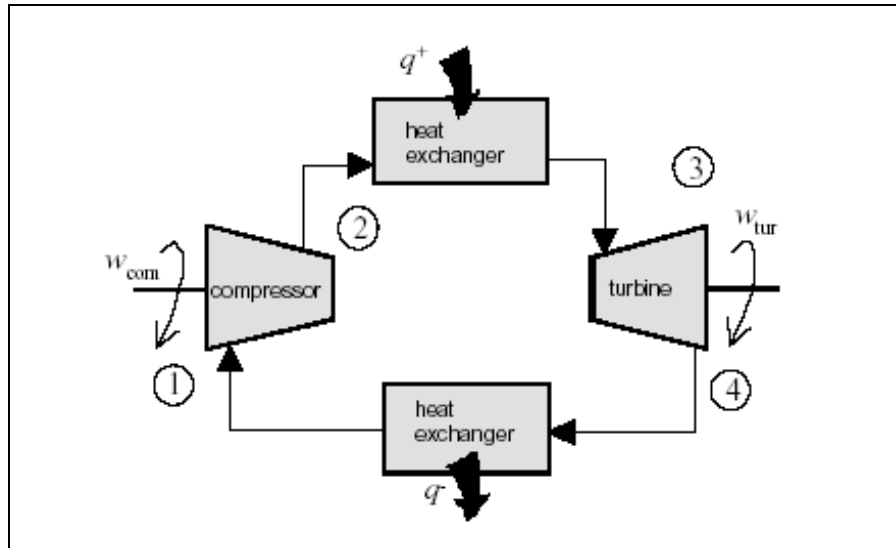


Figure 4. A schematic diagram that shows the Idealized air-standard Brayton Cycle.

The assumptions made for this analysis are:

- (i) The temperature rise that cause by the combustion process is brought by a heat transfer to the working fluid (air) from an external source,
- (ii) The air is consider as an Ideal Gas, and
- (iii) Air is drawn into the compressor and at state 1 from the surrounding and later returned to the surroundings at state 4 with a temperature greater then the ambient temperature. Later, it will eventually return to the same state as the air entering the compressor.

We idealized (iii) assumption by replace this process with a heat exchanger where heat is rejected to the surrounding. This further idealization of (iii) is called the air-standard Brayton cycle. The processes that we shown in the Brayton cycle in Figure 4 can be summarized as follow:

Process 1-2: Isentropic compression in the compressor

Process 2-3: Heat addition at a constant pressure (combustion)

Process 3-4: Isentropic expansion in a turbine

Process 4-1: Heat rejection process at constant pressure.

Like many other cycles, these processes can be shown in a P-v diagram and a T-s diagram as illustrated in Figure 5(a) Brayton Cycle in a P-v diagram and Figure 5(b) Brayton Cycle in a T-s diagram.

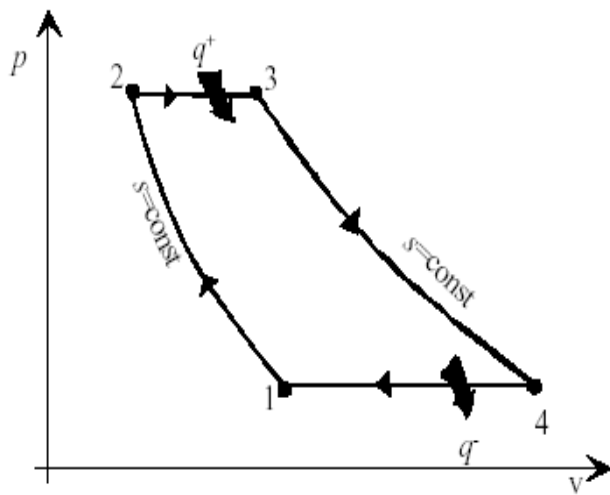


Figure 5(a). Air-standard Brayton Cycle in P-v diagram.

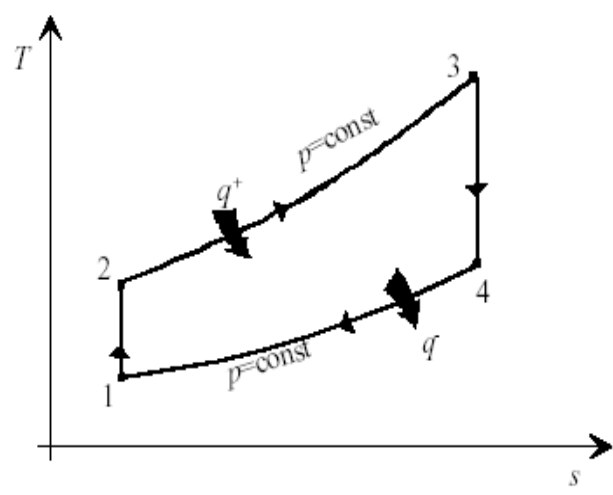


Figure 5(b). Air-standard Brayton Cycle in T-s diagram.

These diagrams clearly show the 4 process that make up the Brayton Cycle. We will discuss the work and heat transfer in the process that shown in Figure 5(a) and (b) in the following paragraph.

4. WORK AND HEAT TRANSFER IN THE BRAYTON CYCLE.

In this section, we will use the P-V diagram and the T-S diagram to explain the process in the compressor (state 1 to state 2), the process in the Heat Exchanger in which heat is transferred in the system (state 2 to state 3), the process in the Turbine (state 3 to state 4), and the process in the heat exchanger in which the heat is transferred out of the system (state 4 to state 1).

Each of these processes is discussed as follow:

State 1 to State 2: Isentropic Compression Process in the Compressor.

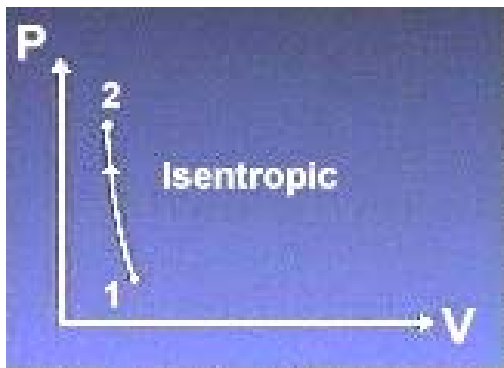


Figure 4-1a. Isentropic process from state 1 to state 2 on P-V diagram.

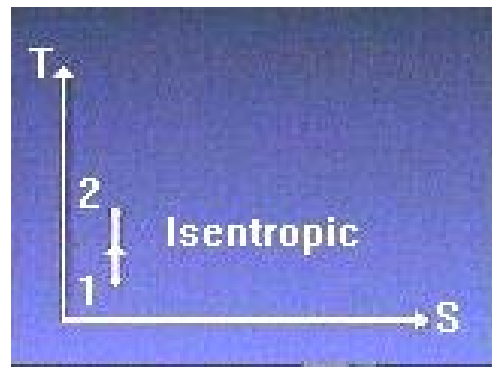


Figure 4-1b. Isentropic Process from state 1 to state 2 on T-S diagram.

We idealized the process as a reversible adiabatic process; so, it is also an isentropic process in the compressor. Thus it shows a straight line on the T-S diagram.

To derived the equation for this process, by using the energy rate balance with the following assumption:

- (1) Steady state exists.
- (2) Kinetic energy and potential energy is negligible.
- (3) No heat transfers during the process, that is, adiabatic.

Apply mass balance; select the boundary of the compressor as the control volume, we have:

$$\sum_{in} \dot{m}_{in} = \sum_{out} \dot{m}_{out}$$

$$\Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$$

The Energy Balance Equation, we have:

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) \quad [\text{Equation 1}]$$

Apply the assumptions given,

$$\Rightarrow \frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$\Rightarrow \dot{W}_{compressor} = \dot{m}(h_1 - h_2)$$

Here, the value of h_2 will be larger than that of h_1 because *state 2* is at a higher temperature, thus, it is prefer to write:

$$\frac{\dot{W}_{compressor}}{\dot{m}} = -(h_2 - h_1)$$

Here, since the when we define the work term in the second law, the work is done by the system, so we minus work from the energy balance equation, to make it a positive value (But keep in mind that for compressor, the work is an input to the system), we can write:

$$\frac{\dot{W}_{compressor}}{\dot{m}} = (h_2 - h_1) \quad [\text{Equation 2}]$$

State 2 to State 3: Isobaric Expansion Process in the Heat Exchanger.

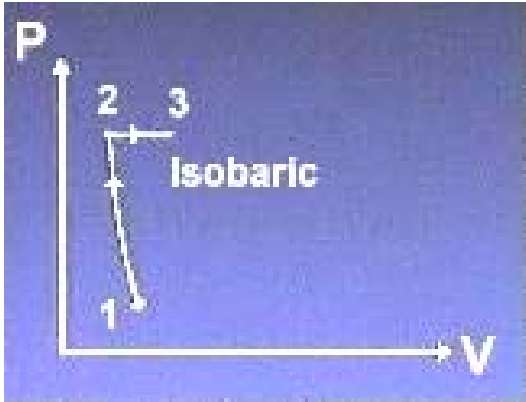


Figure 4-2a. Isobaric process from state 2 to state 3 in the P-V diagram.

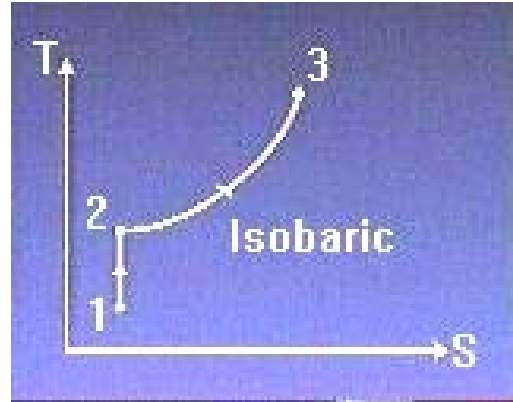


Figure 4-2b. Isobaric process from state 2 to state 3 in the T-S diagram.

From state 2 to state 1, the heat transfer process is maintained at a constant pressure. To calculate the heat transfer rate, the Energy rate balance equation is again used, with the following assumption:

- (1) Steady state exists.
- (2) Kinetic energy and potential energy is negligible.
- (3) No work done during this process.

Apply mass balance; select the boundary of the heat exchanger as the control volume, we have:

$$\sum_{in} \dot{m}_{in} = \sum_{out} \dot{m}_{out}$$

$$\Rightarrow \dot{m}_2 = \dot{m}_3 = \dot{m}$$

The Energy Balance Equation,

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) \quad [\text{Equation 1}]$$

Apply given assumption,

$$\cancel{\frac{dE_{CV}}{dt}} = \dot{Q}_{CV} - \cancel{\dot{W}} + \sum_{in} \dot{m}_{in} \left(h_{in} + \cancel{\frac{V_{in}^2}{2}} + \cancel{gz_{in}} \right) - \sum_e \dot{m}_e \left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e} \right)$$

$$\Rightarrow \dot{Q}_{in} = \dot{m}h_3 - \dot{m}h_2$$

$$\Rightarrow \frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 \quad [\text{Equation 3}]$$

State 3 to State 4: Isentropic Expansion Process in the Turbine.

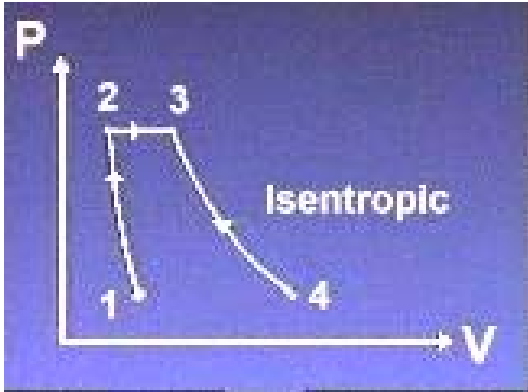


Figure 4-3a. Isentropic Process from state 3 to state 4 on a P-V Diagram.

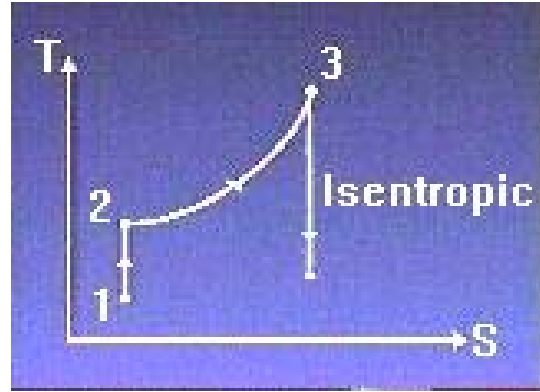


Figure 4-3b Isentropic Process from state 3 to state 4 on a T-S diagram.

Similarly, the process from state 3 to state 4 is idealized as a isentropic process. As in the compressor, apply the Energy Rate balance equation with the following assumption:

- (1) Steady state exists.
- (2) Kinetic energy and potential energy is negligible.
- (3) No heat transfers during the process, that is, the process is adiabatic.

Apply mass balance; select the boundary of the turbine as the control volume, we have:

$$\sum_{in} \dot{m}_{in} = \sum_{out} \dot{m}_{out}$$

$$\Rightarrow \dot{m}_4 = \dot{m}_3 = \dot{m}$$

The Energy Balance Equation,

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) \quad [\text{Equation 1}]$$

$$\Rightarrow \frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$\Rightarrow \dot{W}_{turbine} = \dot{m}(h_3 - h_4)$$

$$\Rightarrow \frac{\dot{W}_{turbine}}{\dot{m}} = (h_3 - h_4) \quad [\text{Equation 4}]$$

State 4 to state 1: Isobaric Heat Rejection Process in the Idealized heat exchanger.

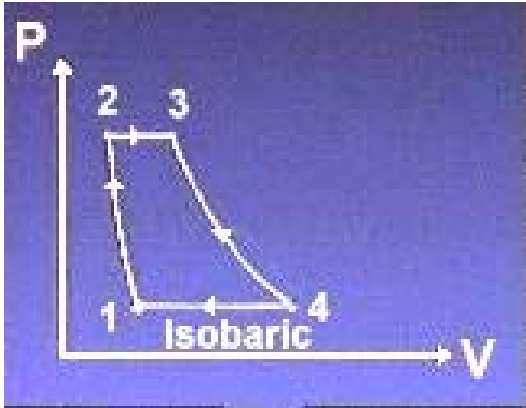


Figure 4-4a. Isobaric Process from state 4 to state 1 on a P-V diagram.

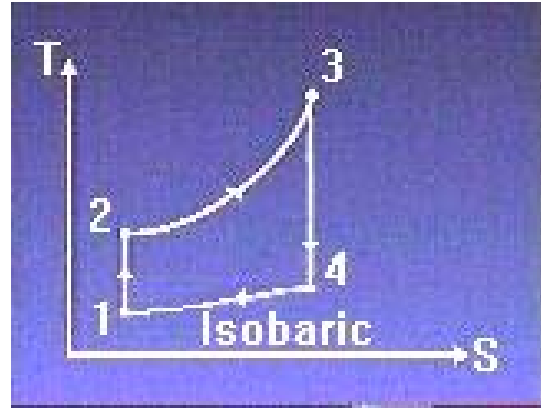


Figure 4-4b. Isobaric Process from state 4 to state 1 on a T-S diagram.

Similar with process from state 2 to state 3, the process is isobaric. Using the Energy rate balance equation with the following assumptions:

- (1) Steady state exists.
- (2) Kinetic energy and potential energy is negligible.
- (3) No work done during this process.

Apply mass balance; select the boundary of the heat exchanger as the control volume, we have:

$$\sum_{in} \dot{m}_{in} = \sum_{out} \dot{m}_{out}$$

$$\Rightarrow \dot{m}_4 = \dot{m}_1 = \dot{m}$$

The amount of heat lost is calculated by applying Energy Rate balance:

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) \quad [\text{Equation 1}]$$

$$\Rightarrow \frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$\Rightarrow \dot{Q}_{out} = \dot{m}h_4 - \dot{m}h_1$$

$$\Rightarrow \frac{\dot{Q}_{in}}{\dot{m}} = h_4 - h_3$$

[Equation 5]

Thermal efficiency of Brayton Cycle:

We have derived the equation for finding the work done and heat transfer for each of the process in the Brayton Cycle, now we are able to derive the equation for the thermal efficiency for the Brayton Cycle.

The thermal efficiency of a system is defined as (Cengel 488):

$$\text{Thermal Efficiency, } \eta = \frac{\text{net - work - output}}{\text{total - heat - input}} \quad [\text{Equation 6}]$$

$$\Rightarrow \eta = \frac{\dot{W}_{\text{turbine}} / \dot{m} - \dot{W}_{\text{compressor}} / \dot{m}}{\dot{Q}_{\text{in}} / \dot{m}}$$

In which desired work out put is $(\dot{W}_{\text{turbine}} / \dot{m} - \dot{W}_{\text{compressor}} / \dot{m})$, since some turbine work is used to drive the compressor

$$\Rightarrow \eta = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} \quad [\text{Equation 7}]$$

Back work Ratio of a Brayton Cycle:

$$\text{Back work ratio} = \frac{\dot{W}_{\text{compressor}} / \dot{m}}{\dot{W}_{\text{turbine}} / \dot{m}}$$

$$\Rightarrow \text{Back work Ratio} = \frac{(h_2 - h_3)}{(h_3 - h_4)} \quad [\text{Equation 8}]$$

Note: For a same pressure rise, a gas turbine compressor would require a much greater work per unit of mass flow than a vapor power plant. This is because the specific volume that passes through the plant will be much higher than required for the vapor power plant. For this reason, a large portion of the work produced by the turbine was used to drive the compressor. As a comparison, the typical back work ratio of a gas turbine power plant is 40-80%. Where the back work ratio for a vapor power plant is only 1-2%. [2]

Ideal Air-Standard Brayton Cycle.

We have derived the equation to calculate the work and heat that involved in the Brayton Cycle. Now we need to further idealize the cycle by using the ideal gas equations. This is called the “*cold-air standard analysis*”. The assumptions that follow are:

- (1) The enthalpy value for finding work and heat in equations [Eq 1], [Eq 2], [Eq 3] and [Eq 4] can be found on the air table. Here we further simplifying saying that the specific heat of air is not a function of temperature. This does not lost a lot of accuracy if the temperature variation is only of few hundred degrees. This is called the “cold air-standard analysis”
- (2) The irreversibility associated in the system was being ignored. For this, the performance of this idealized cycle will provide an upper limit of the efficiency of a Brayton Cycle can achieved.
- (3) There are no frictional pressure drops.
- (4) Airflow at constant pressure at the heat exchanger.
- (5) Heat Transfer to the surrounding is being ignored.

In this case, we can apply the ideal gas equation, in which, for the isentropic process from state 1 to state 2 and from state 3 to state 4, the pressure relations are:

For isentropic process (Moran 277),

$$P_1 V_1^k = P_2 V_2^k \quad \text{or} \quad \frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^k \quad \text{[Equation 9]}$$

$$\text{And, } P_3 V_3^k = P_4 V_4^k \quad \text{or} \quad \frac{P_4}{P_3} = \left(\frac{V_3}{V_4} \right)^k \quad \text{[Equation 10]}$$

Where $k =$ Specific heat constant ratio, C_p/C_v .

Divide Equation 9 by Equation 10, we have,

$$\frac{P_1 V_1^k}{P_4 V_4^k} = \frac{P_2 V_2^k}{P_3 V_3^k}$$

Since $P_1 = P_4$ and $P_2 = P_3$, we have,

$$\Rightarrow \frac{P_1 V_1^k}{P_4 V_4^k} = \frac{P_2 V_2^k}{P_3 V_3^k}$$

$$\Rightarrow \frac{V_1^k}{V_4^k} = \frac{V_2^k}{V_3^k}$$

$$\Rightarrow \frac{V_1}{V_4} = \frac{V_2}{V_3} \quad \text{[Equation 11]}$$

Substitute Equation 11 into Equation 9, we have,

$$\begin{aligned} \frac{P_1}{P_2} &= \left(\frac{V_2}{V_1} \right)^k \\ \Rightarrow \frac{P_1}{P_2} &= \left(\frac{V_3}{V_4} \right)^k \end{aligned}$$

From Equation 10,

$$\Rightarrow \frac{P_1}{P_2} = \frac{P_4}{P_3} \quad \text{[Equation 12]}$$

Also, by applying ideal gas law, we have,

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{(k-1)}{k}} \quad \text{[Equation 13]}$$

and

$$T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{(k-1)}{k}} \quad \text{or} \quad T_4 = T_3 \left(\frac{P_1}{P_2} \right)^{\frac{(k-1)}{k}} \quad \text{[Equation 14]}$$

Or, combine Equation 12 and Equation 13, we have,

$$T_4 = T_3 \left(\frac{P_1}{P_2} \right)^{\frac{(k-1)}{k}} \quad \text{and} \quad T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{(k-1)}{k}}$$

$$\Rightarrow T_4 = T_3 \left(\frac{T_1}{T_2} \right)$$

$$\Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2} \quad \text{[Equation 15]}$$

Interpretation of the Ideal Air-standard Brayton Cycle.

If there is no irreversibility, the area under the P-V diagram represents the net work output (or net heat added) and the area under the T-S diagram represents the net Heat added (or net work output). This can be shown in the P-V diagram and the T-s Diagram of the Brayton Cycle.

On the T-s diagram, area 2-3-a-b-2 represent heat added per unit of mass and area 1-4-a-b-1 is the heat rejected per unit of mass. On the P-v diagram, area 1-2-a-b-1 represents the compressor work input per unit of mass and area 3-4-b-1-3 is the turbine work output per unit of mass. This is shown in Figure 5 below.

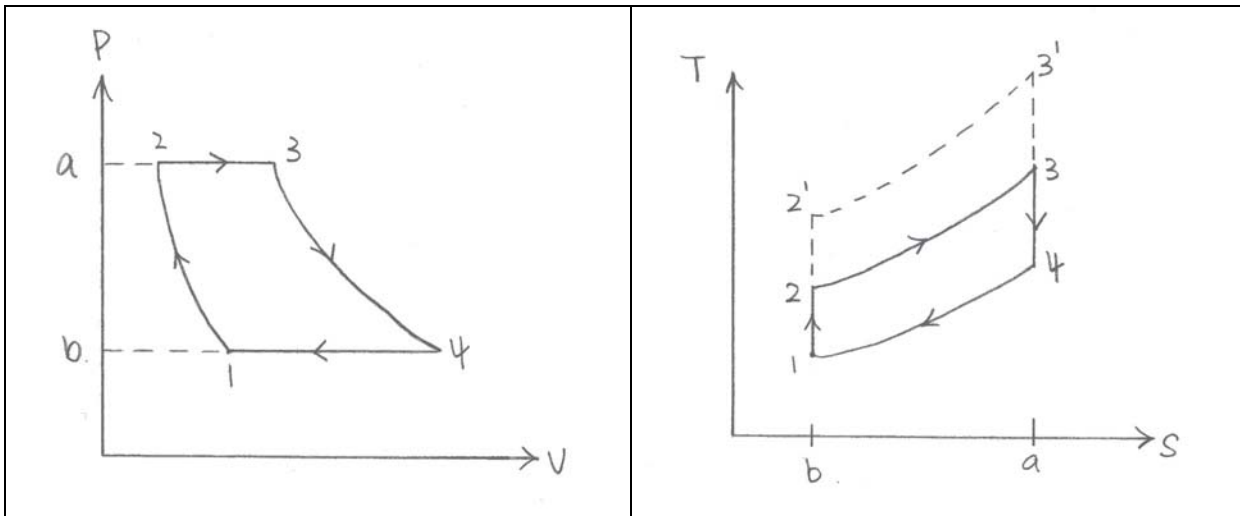


Figure 5. Air standard ideal Brayton Cycle on a P-v diagram and T-s diagram.

5. STUDY OF PRESSURE RATIO EFFECT ON THE EFFICIENCY OF BRAYTON CYCLE.

Using our cold-air standard analysis, we found that the thermal efficiency of the Brayton Cycle increase with the increase of the pressure ratio. There are two ways we can get this conclusion. The first method is from the T-s diagram. With a higher-pressure ratio across the compressor (that is, the air is compressed to a higher temperature.), the T-s diagram change from 1-2-3-4-1 to 1-2'-3'-4-1. "Since the same amount of heat is being rejected, cycle 1-2'-3'-4-1 would have greater thermal efficiency."(Moran 446).

Now, we need to derive an Equation that shows the pressure effect on the thermal efficiency.

From the equation of the Specific Heat for Ideal Gas (Cengel 142), we have,

$$dh = C_p(T) dT$$

$$\Rightarrow \Delta h = (h_i - h_j) = C_p(T) (T_i - T_j) \quad \text{[Equation 16]}$$

Assuming constant specific heat for air, the thermal efficiency for the Brayton cycle become (substitute into Equation 7):

$$\Rightarrow \eta = \frac{C_p(T_3 - T_4) - C_p(T_2 - T_1)}{C_p(T_3 - T_2)}$$

$$\Rightarrow \eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2)}$$

$$\Rightarrow \eta = \frac{(T_3 - T_2) - (T_1 - T_4)}{(T_3 - T_2)}$$

$$\Rightarrow \eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

Take the T_1 and T_2 out from the bracket, we have,

$$\Rightarrow \eta = 1 - \frac{T_1}{T_2} \frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)}$$

Since, from Equation 13,

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}, \quad \text{[Equation 13]}$$

The Thermal efficiency now become

$$\Rightarrow \eta = 1 - \frac{T_1}{T_2}$$

Then, we substitute Equation 11 in this equation,

$$\Rightarrow \eta = 1 - \frac{1}{\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}} \quad [\text{Equation 17}]$$

This result was plotted in Figure 6, where we can see that for the cold-air standard ideal Brayton Cycle, the thermal efficiency is a function of the pressure ratio across the compressor.

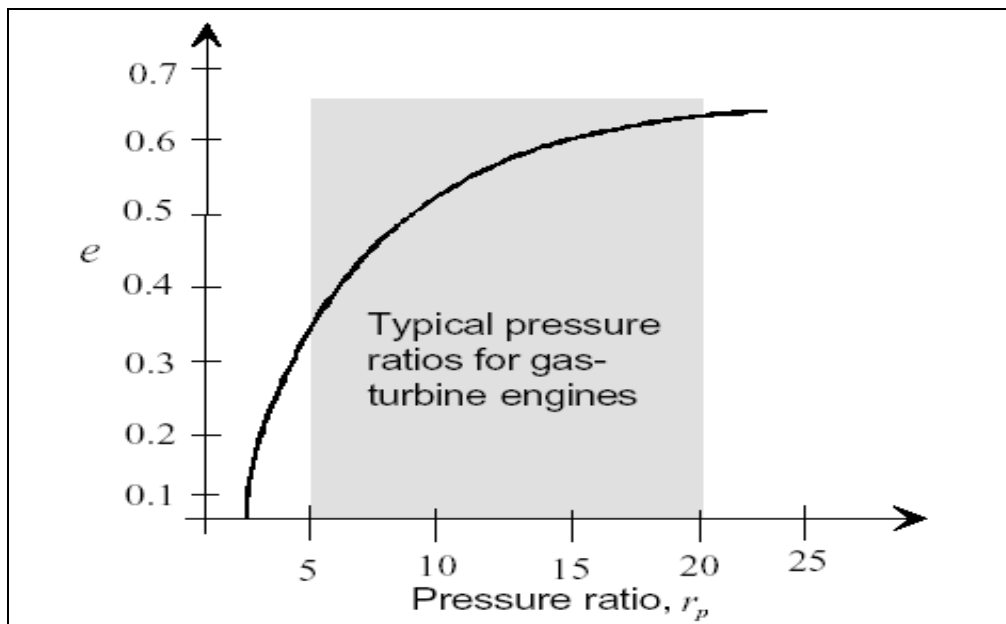


Figure 6. Graph shows the change in the efficiency with respect to the increase in pressure ratio.

Thus, now our consideration will be: is there any restriction for the pressure ratio and thus the thermal efficiency on the cold-air standard ideal Brayton Cycle?

The answer is yes. There is a temperature limit of 1700 K on the turbine base on the metallurgical considerations. Therefore we need to consider that effect of the pressure ratio on the temperature so that the temperature will not exceed this limit when the air passes through the turbine inlet.

There is an interesting diagram that shows two cycles having the same turbine inlet temperature but having different pressure ratio across the compressor. As shown in Figure 7 below.

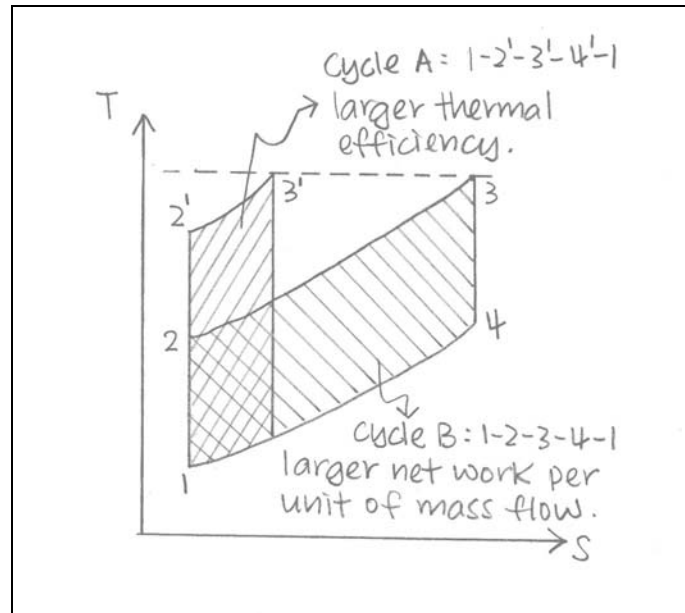


Figure 7. Ideal Brayton cycle with different pressure ratio and the same turbine inlet temperature.

Cycle A has a greater pressure ratio and thus a better thermal efficiency. On the other hand, cycle B has a larger enclosed area (shown in the shaded area) and thus has a greater *net work* developed per unit of mass flow. For cycle A to have a same net power output as cycle B, we will need a larger mass flow required. This means that we need a larger system to operate the cycle. For gas turbine power plant that used in transportation, this will be an important consideration because weight must be kept small for transportation purpose. So, as a trade off, it is usually desirable to design the Brayton cycle such that it operate near the compressor ratio that yield the most work per unit of mass flow and not the just to increase the pressure ratio to achieve the highest thermal efficiency.

6. EFFECT OF IRREVERSIBILITY ON THERMAL EFFICIENCY OF BRAYTON CYCLE.

In reality, neither the compressor nor the turbine will follow a perfect isentropic or adiabatic path. They are real machines and, therefore, a certain degree of irreversibility is involved in their operation. Here, when take into account this irreversibility and study the effect on the thermal efficiency of the Brayton Cycle.

This is indicated in the T-s diagram shown in Figure 8. Because of frictional effects within the compressor and turbine, the working fluid would experience increase in specific entropy across these component. Note that the level of entropy increases during operation of the compressor as well as of the turbine as shown in path 1-2' instead of 1-2 and path 3-4' instead of 3-4. (Moran 449)

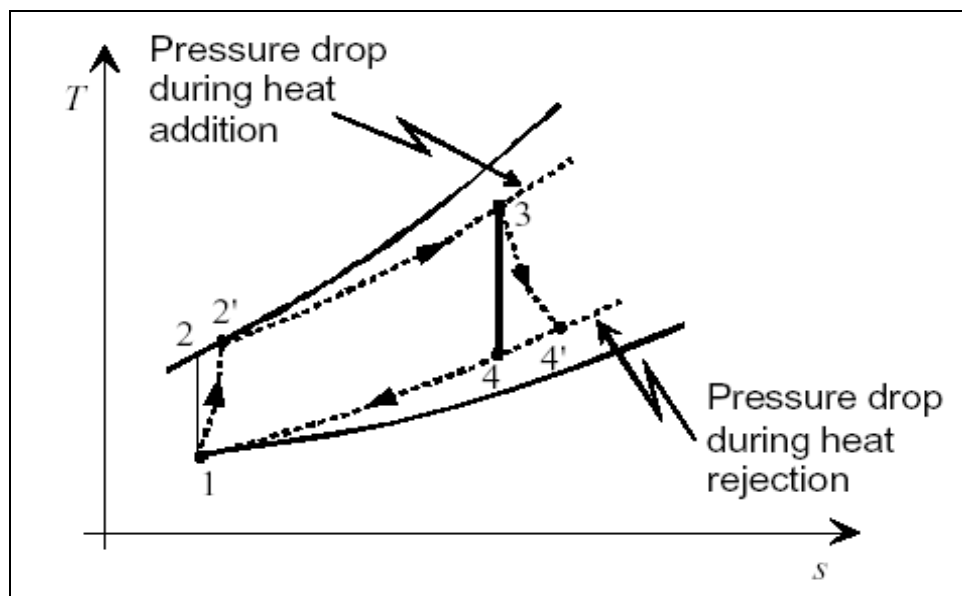


Figure 8. A more realistic Brayton Cycle that shows the pressure drops when air pass through the heat exchanger and the increase in entropy across the compressor and turbine.

The pressure drops in the air when the air pass through the heat exchanger is mainly due to the friction. However, because frictional pressure drop are less significant source of irreversibility (Moran 449), we will ignore them for our study of the effect of irreversibility on thermal efficiency of the Brayton Cycle.

Due to the effect of irreversibility, the work that developed by the turbine decrease and the work input to the compressor increase. Thus, we can present these effects as the isentropic efficiency for the turbine and compressor.

For the turbine, since the actual work out put is less than the isentropic work output,

$$\eta_{\text{turbine}} = \frac{(\dot{W}_{\text{turbine}} / \dot{m})}{(\dot{W}_{\text{turbine}} / \dot{m})_s} = \frac{h_3 - h_4}{h_3 - h_{4,s}} \quad [\text{Equation 18}]$$

And for the compressor, the actual work input is more than the isentropic work input,

$$\eta_{\text{compressor}} = \frac{(\dot{W}_{\text{compressor}} / \dot{m})}{(\dot{W}_{\text{Compressor}} / \dot{m})_s} = \frac{h_{2,s} - h_1}{h_2 - h_1} \quad [\text{Equation 19}]$$

Currently, an efficiency of 80% to 90% can now be achieved. Here, however, note that actually among the irreversibility of actual gas turbine power plant, combustion irreversibility is the most significant by far. However, the air-standard analysis limited this irreversibility to be evaluated using current knowledge.

The heat required to add to the air in the heat exchanger can now be calculated as follow:

Using the same approach, the mass balance equation,

$$\sum_{in} \dot{m}_{in} = \sum_{out} \dot{m}_{out}$$

$$\Rightarrow \dot{m}_2 = \dot{m}_3 = \dot{m}$$

The Energy Balance Equation,

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) \quad [\text{Equation 1}]$$

$$\Rightarrow \frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + gz_{in} \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$\Rightarrow \dot{Q}_{iout} = \dot{m}h_3 - \dot{m}h_5$$

$$\Rightarrow \frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_5 \quad [\text{Equation 20}]$$

Which is the heat needed to add in the heat exchanger when a regenerator is added. From this analysis, we can see that the net work that developed by the turbine will not be affected with the addition of the regenerator. However, since the required heat is reduced, from equation 7 that we have derived, the thermal efficiency of the Brayton Cycle is

$$\eta_{\text{without regenerator}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} \quad [\text{Equation 7}]$$

Now the term $h_3 - h_2$ is replace by $h_3 - h_5$, which has a smaller value, the efficiency of the overall Brayton Cycle system increased.

$$\eta_{\text{with regenerator}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_5} \quad [\text{Equation 21}]$$

8. SAMPLE QUESTION FOR BRAYTON CYCLE.

Question (modified from Example 8-5, [2]):

A stationary power plant operating on an ideal Brayton Cycle has a pressure ratio of 8. The gas temperature is 300K at the compressor inlet and 1300K at the turbine inlet. Utilizing the air-standard assumptions, determine:

- The gas temperature at the exit of the compressor and the turbine,
 - The back work ratio, and
 - The thermal efficiency.
- (d) Compare the thermal efficiency using cold-air standard analysis.

Solution:

First draw the T-s and P-v diagram of the Brayton and indicate each given value on the diagrams. As shown in Figure 11 below.

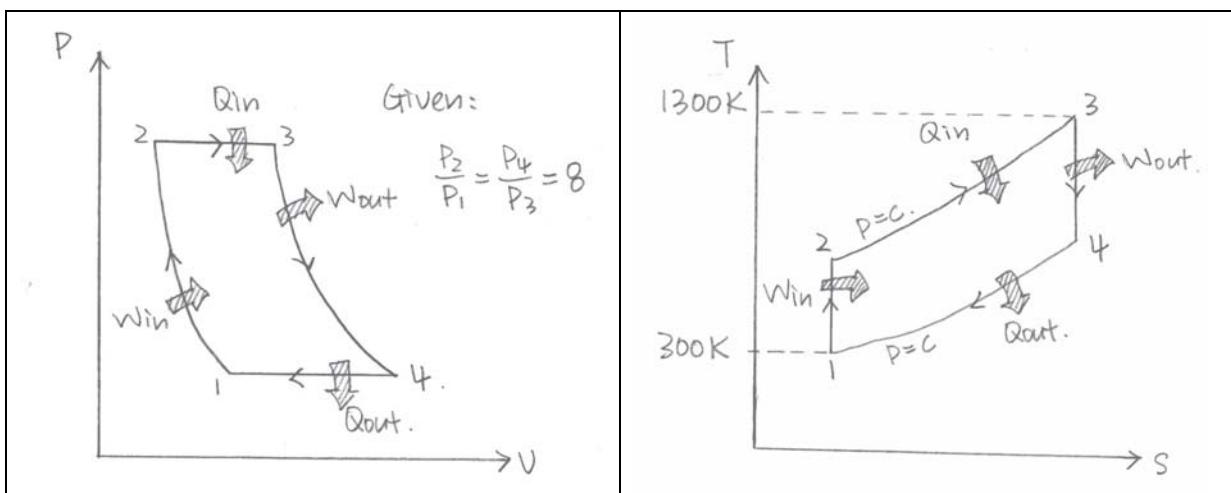


Figure 11. T-s diagram and P-v diagram of standard ideal Brayton Cycle.

Assumptions:

- Steady state exists.
- The air standard analysis is applicable.
- Kinetic and potential energy changes are negligible.

(a) The information that we have is:

State 1: $T_1=300\text{K}$

From air table, $h_1 = 300.19\text{kJ/Kg}$ and $P_{r1} = 1.386$

So, for ideal gas, $P_{r2} = \frac{P_2}{P_1} P_{r1}$

$$\begin{aligned}\therefore P_{r2} &= (8)(1.386) \\ &= 11.09\end{aligned}$$

From table, for $P_{r2} = 11.09$,

$$\begin{aligned}\text{We can determine state 2: } T_2 &= 540 \text{ K} \\ h_2 &= 544.35 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{State 3: } T_3 &= 1300 \text{ K, from table, we can find } h_3 = 1395.97 \text{ kJ/kg} \\ P_{r3} &= 330.9\end{aligned}$$

We can then determine state 4:

$$\begin{aligned}P_{r4} &= \frac{P_4}{P_3} P_{r3} \\ \therefore P_{r2} &= \left(\frac{1}{8}\right)(1.386) \\ &= 41.36\end{aligned}$$

For $P_{r4} = 41.36$, we can find $T_4 = 770 \text{ K}$ and $h_4 = 789.11 \text{ kJ/kg}$.

So, the gas temperature at the exits of compressor is $T_2 = \underline{540 \text{ K}}$
And the gas temperature at the exits of turbine is $T_4 = \underline{770 \text{ K}}$

(b) The back work ratio,

From [Equation 8],

$$\begin{aligned}\text{Back work ratio} &= \frac{\dot{W}_{\text{compressor}} / \dot{m}}{\dot{W}_{\text{turbine}} / \dot{m}} \\ \Rightarrow \text{Back work Ratio} &= \frac{(h_2 - h_3)}{(h_3 - h_4)} \\ \Rightarrow \text{Back work Ratio} &= \frac{(544.35 - 300.19)}{(1395.97 - 789.11)} \\ \Rightarrow \text{Back Work Ratio} &= 0.402 = \underline{40.2\%}\end{aligned}$$

So, in this example, 40.2% of the turbine work is use to drive the compressor.

(c) To determine the Thermal efficiency, use Equation 6

$$\text{Thermal Efficiency, } \eta = \frac{\text{net - work - output}}{\text{total - heat - input}}$$

$$\Rightarrow \eta = \frac{\dot{W}_{\text{turbine}} / \dot{m} - \dot{W}_{\text{compressor}} / \dot{m}}{\dot{Q}_{\text{in}} / \dot{m}}$$

$$\Rightarrow \eta = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

$$\Rightarrow \eta = \frac{(1395.97 - 789.11) - (544.35 - 300.19) \text{ kJ / kg}}{(1395.97 - 544.35) \text{ kJ / kg}}$$

$$\Rightarrow \eta = 0.426 = \underline{42.6\%}$$

(d) Using the cold-air-standard analysis, the thermal efficiency is determine from equation 17

$$\eta = 1 - \frac{1}{(P_2 / P_1)^{\frac{(k-1)}{k}}}$$

$$\Rightarrow \eta = 1 - \frac{1}{(8_1)^{\frac{(1.4-1)}{1.4}}} \quad (k = 1.4 \text{ for air})$$

$$\Rightarrow \eta = 0.448 = \underline{44.8\%}$$

By a comparison, by using constant specific heat for air, the thermal efficiency of the Brayton cycle is higher.

9. REAL LIFE EXAMPLE OF USING THE BRAYTON CYCLE (ADAPTED FROM WEBSITE [5]).

Introduction:

This is an application of the Brayton cycle in nuclear power plant. This engine is called the Adams engine. It uses the concept of Brayton cycle and modifies it by using the nuclear source replacing the usual combustion process in the Brayton cycle. In this section, we will introduce this engine and also discuss some of the solution to this engine, which is currently under development.

Overview:

The concept of Brayton is used in the development of Nuclear Power Plant that called the Adams engine. Adams Engines' philosophy begins with a simple thought: Nuclear power plants do not have to be large to be economical. These engines will have the benefits that will not have problem of pollution as in other engines. Adams Engines has developed a reasonable and practical way to produce nuclear engines with power outputs of between 1 and 100 MWe.

History:

The idea of nuclear gas turbines is not new; it dates back to the mid 1940s. The main early commercial supporter was Esher Wyss Ltd. of Zurich. The first U.S. plant using the concept was the Army's ML-1 demonstration reactor, a 300 kWe nuclear heated, nitrogen cooled machine designed to be truck transportable. Thus machines was the first closed cycle gas turbine of any kind built in the U.S. and it operated for several years in the early 1960s before losing its funding as the Vietnam War intensified.

Currently, there are two gas turbine Nuclear power plant projects taking place. One is the GT-MHR (Gas Turbine Modular Reactor), a joint effort with participation from General Atomics, the US Department of Energy, Framatome, MINATOM, and Fuji Electric. The second active project is the PBMR (Pebble Bed Modular Reactor) which is primarily a South African project led by ESKOM, the fourth largest electric utility in the world.

Importance:

Gas turbines have become mature power sources. Gas cooled reactors have been designed and tested through full scale operating programs. The operating temperature available from the gas cooled reactors matches will with simple cycle gas turbines using uncooled blades. The impact of this design is significant. The source of operating this gas turbine is less and also environmentally friendly than those power plants that required using fuel in combustion process. When we consider the amount of fuel left in this world, the development of this gas turbine will become a good solution.

Solution:

The concept of Adams Engines is base on closed Brayton cycle. They combine proven compressor and turbine designs with fully tested gas cooled reactor concepts. They use the inherent stability of a core with a negative temperature coefficient combined with a poppet type throttle valve to control power output by controlling coolant flow.

Because of the nature of the fuel design and the geometry of the core, fuel melting is not credible even in the event of complete loss of coolant flow at full power. The engines use a low-pressure coolant, allowing savings in pressure vessel and containment construction. It appears that the ultimate capital cost of the machine will be competitive with combustion gas turbines.

Why the Brayton cycle is being used in this Adams Engine is because of the following reasons:

1. The Brayton cycle is one of the simplest heat cycles available. In its most basic form, all that is needed is a compressor, a heat source, a turbine, and a heat sink
2. The size for implementing a Brayton Cycle is more compact and the weight of a Brayton cycle engines is suitable for transportation purpose.
3. For land-based applications, heat recovery steam generators can be added to Adams Engines to increase total cycle efficiency.
4. The exhaust heat can also be used for district heating, industrial process heat or desalination. The high temperature gas could even be used as the heat source for a thermoelectric device that directly converts heat into electricity.

Pressure consideration in the design of Adams Engines was also being discussed. In Adams Engines, Nitrogen is used. Nitrogen is a gas with aerodynamic and thermodynamic properties that are very similar to those of air. Turbomachinery designed to work with air will function well with nitrogen as the working fluid. Nitrogen is chemically stable at the temperatures used in Adams Engines and is not significantly affected by a neutron flux. Furthermore, Nitrogen is a single-phase fluid over the temperature range of interest so there is no need to raise system pressure in order to keep thermodynamic characteristics predictable.

Adams Engines will operate with the compressor suction at approximately one atmosphere (one bar). The operating pressure ratio will depend on the limiting temperature of the reactor. For the first system, it is likely to be approximately 10:1. Figure 1 is a process diagram of an Adams Engine using moderate temperature limits. Projected thermodynamic efficiency is approximately 35%. Some changes to the Compressor and the Turbine are necessary.

The Compressor used in the Adams Engine is being modified such that it can with a wide variety of system flow rates. They might be centrifugal compressors or machines with combinations of axial stages and centrifugal stages. There is an adequate selection of such machines available that are currently in service in aircraft or marine power applications.

The turbine will be a multi-stage axial flow turbine. It will use conventional gas turbine design principles. A split shaft turbine with a separate power turbine offers several advantages for system designed for potential vehicle applications. The turbines in normal combustion engines operate in a high temperature, corrosive exhaust stream that can include sand and soot. In contrast the nitrogen turbine will operate in an inert environment that can use a purification system to eliminate even very small particulate

matter. This should increase the life of the turbine by reducing corrosion, erosion, and the formation of deposits on turbine blades.

Furthermore, the nature of the heat sink will depend on the application where the engine is used. Marine power plants will probably want to take advantage of the ocean to provide cooling. The cooler in this case will resemble a conventional condenser, probably using double tube-sheet design to prevent salt water from entering into the primary coolant system. It may be advantageous to provide an intermediate heat transfer loop between the sea and the primary system; this would also allow the use of the waste heat for space heating. Beside this, for a land based power plant, water-cooling may be a luxury. Since the exit temperature of the turbine is on the order of several hundred degrees Celsius, a reasonably sized air cooler can provide the necessary heat sink. There are also the previously mentioned options of using the waste heat for area space heating, desalination, or industrial process heat.

Beside the modification for these basic components of the Brayton Cycle, when designing a Adams Engine, we need to consider also the other components such as the shielding, the core, heat control, and the containment. The Adams engine is still not yet commercially available but we could see its potential and its advantages over other cycle. This engine which basically using the concept of Brayton cycle by modifies the way of combustion and using Nitrogen instead of air may become a source of power supply in the future. The diagram showing the Adams engine is given in Figure 11.

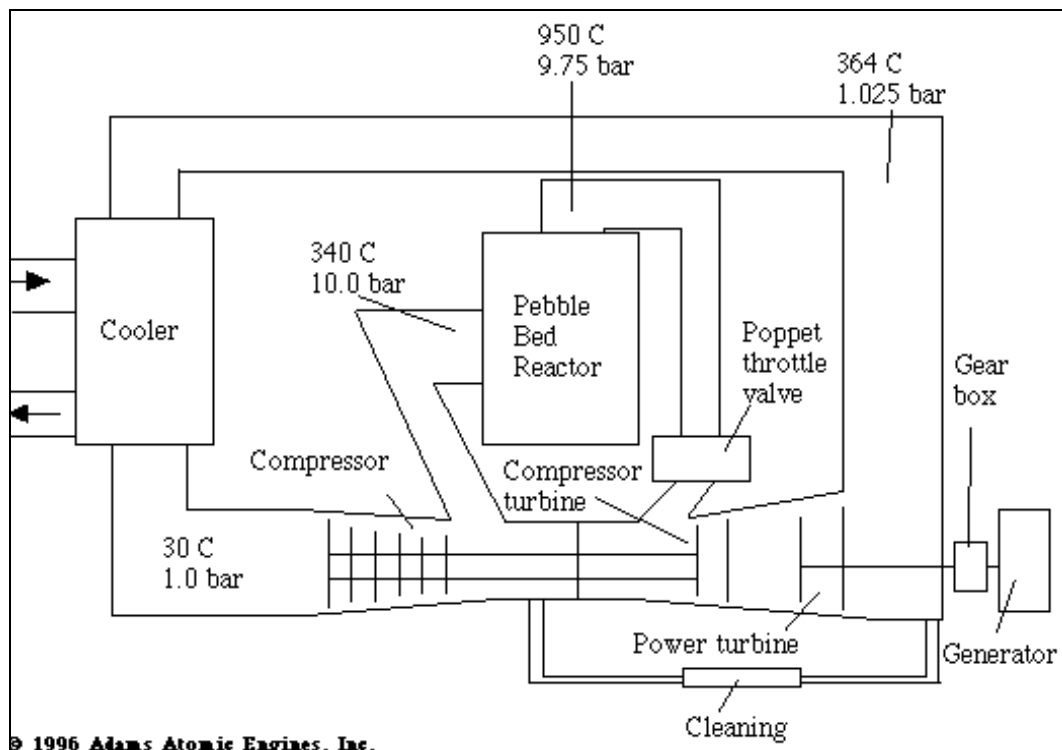


Figure 11: Adams engine process diagram.

Summary:

Adams Engines build on the strengths of well-tested and developed technologies. They use a defense in depth strategy to ensure public protection. The fuel designed for gas-cooled reactors has been fully tested and shown to keep the coolant circuit essentially free from fission product contamination. Radiation levels in gas-cooled reactors are typically lower than the levels found in water-cooled reactors because of the clean coolant circuit and the lack of corrosion products. The clean coolant circuit also reduces a major cost item and political hurdle to nuclear power acceptance by reducing the need for disposal of low-level waste. Any waste generated will be dry, rather than liquid, which tends to be more difficult to handle.

The high temperature capability of the core and its ability to withstand accidents without operator action or automated safety systems make the engines well suited for operation as distributed power systems and on board merchant ships. The public will be protected even if the engines are operated in densely populated areas. The reduced need for engineered safety systems allows plant construction to be simple and focused on maintaining high quality for those few components that remain. Because the plants are smaller and less complex, operators will be able to know their plant in a way that is difficult in large-scale steam plants. The knowledgeable operators will be able to deliberately react to any problem based on clear, easily understood indications.

Heavy metals (uranium, plutonium, and thorium) are compact primary energy sources available in almost unlimited quantities here on earth. They offer an energy choice that eliminates much of the environmental damage done by burning fossil fuel without requiring the sacrifices that would be required in order to abandon an energy based economy.

Fuel costs for a given amount of heat have always been lower in nuclear plants than in fossil fuel plants. This was true even in the 1960s when oil cost \$3.00 per barrel. The engines described above will retain the fuel cost advantage of nuclear fuel while attacking capital, operations, and maintenance costs in a way that actually improves safety margins. In summary, Adams Engines:

1. Use a low pressure, single phase, and chemically non-active coolant.
2. Use a direct heat transfer cycle that eliminates steam generators.
3. Use a high temperature fuel design that eliminates the need for forced convection decay heat removal systems.
4. Use a fuel design that is ready for long-term dry storage.
5. Use an internal shield to protect the pressure vessel from neutron damage.
6. Use compressor and turbine designs that can be mass-produced.
7. Can be sized to fit a variety of applications.
8. Can be sized to allow economical prototyping.
9. Are small enough to decommission without dismantling.
10. Are small enough to be produced in a factory and delivered tested and ready for power production.

Above we have discussed the utilization of Brayton cycle in the development of the Adams engine. It shows that the simple Brayton cycle has it potential of not only

useful in current power generation but also will be useful in future power generation with the further development of the cycle.

CONCLUSION:

In this project paper, we have discussed the background of Brayton Cycle, the formulas that related to calculate the work and heat for the Brayton Cycle. Beside this, the effect of pressure ratio and irreversibility on the thermal efficiency of the Brayton Cycle is also discussed. One example problem was used to illustrate the use of the formulas and one real life problem was also being discussed. In doing this project paper, it helps me to understand the formulation of those equations used for Brayton Cycle, and also gives me an opportunity to explore the current development in the field of gas turbine power plant. I would like to take this opportunity to thank the instructor, David Blekhman, in helping me for preparing these materials.

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