Chapter 4

DEFENDING AGAINST TERRORISM, NATURAL DISASTER, AND ALL HAZARDS

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Abstract: This chapter considers both natural disasters and terrorism as threats. The defender chooses tradeoffs between investments in protection against natural disaster only, protection against terrorism only, and all-hazards protection. The terrorist chooses strategically how fiercely to attack. Three kinds of games are considered: when the agents move simultaneously; when the defender moves first; and when the terrorist moves first. Conditions are shown for when each type of agent prefers each kind of game. Sometimes their preferences for games coincide, but often their preferences are opposite. An agent advantaged with a sufficiently low normalized unit cost of investment relative to that of its opponent prefers to move first, which deters the opponent entirely, causing maximum utility for the first mover and zero utility to the deterred second mover, who prefers to avoid this game. When all-hazards protection is sufficiently cheap, it jointly protects against both the natural disaster and terrorism. As the cost increases, either pure natural disaster protection or pure terrorism protection joins in, dependent on which is more cost effective. As the unit cost of all-hazards protection increases above the sum of the individual unit costs, the extent of such protection drops to zero, and the pure forms of natural disaster protection and terrorism protection take over.

Key words: Terrorism, natural disaster, all hazards protection, unit cost of defense, unit cost of attack, contest success function.

1. INTRODUCTION

Some types of defenses are effective only against terrorism, or only against natural disaster. For example, bollards and other barriers around buildings protect only against terrorism, not against natural disaster. Similarly, improving the wetlands along a coastline protects only against hurricanes (and some other types of natural hazards), not against terrorism. Other kinds of investment—say, emergency response (to minimize damage), or strengthening buildings (to protect against *both* terrorism and natural disaster)—would count as "all hazards" protection. This chapter intends to understand how a defender should allocate its investments between protecting against natural disaster, terrorism, and "all hazards." At first glance, one might expect the unit cost of "all hazards" protection to be high, in which case protection against terrorism and natural disaster individually may be preferable. However, this will not always be the case; for example, one can imagine that improving wetlands might be so costly in some situations that it would be cheaper to harden buildings instead.

Terrorism is a subcategory of intentional attacks, and natural disasters are a subcategory of non-intentional attacks. Other examples of non-intentional attacks are technological hazards such as the Chernobyl nuclear accident, the Piper Alpha accident, etc. Other examples of intentional attacks might include acts of warfare by government actors, or criminal acts (for example, organized crime, which is generally motivated by the desire for economic rewards). Terrorism is often defined as those acts intended to create fear or "terror." Typically, terrorism deliberately targets civilians or "non-combatants," may be practiced either by informal groups or nation states, and is usually perpetrated to reach certain goals (as opposed to a "madman" attack), which may be ideological, political, religious, economic, or of some other nature (such as obtaining glory, prestige, fame, liberty, domination, revenge, or attention for one's cause). For ease of exposition, this chapter refers to the tradeoff between terrorism and natural disasters, but the results could equally apply to tradeoffs between other intentional and non-intentional attacks.

As in Bier et al. (2007), games are considered in which the defender moves either before the terrorist (by implementing observable defenses), or simultaneously with the terrorist (by keeping its defenses secret). Games where the defender moves first are often realistic, since defenders often build up infrastructures over time, which terrorists take as given when they choose their attack strategies. However, games are also considered in which the terrorist acts first, leaving the defender to move second. In general, which agent moves first is likely to depend on the types of threats and defenses being considered.

Examples of cases in which the terrorist moves first are when the terrorist announces (in a manner perceived to be credible) that a new attack will occur at some point in the future, or the terrorist commits resources to such an attack and the defender gains intelligence about those investments. In such cases, the defender can take the terrorist's decision as given when choosing its defensive strategy.

Some past work (e.g., Zhuang and Bier, 2007) concludes that the defender always has a first-mover advantage, but in practice, this cannot always be true. Therefore, this work relaxes that restriction, and makes clear how and when either side can have a first-mover advantage.

The unit costs of attack and defense are essential in determining whether the terrorist or the defender has an advantage in any given instance. The terrorist is at a disadvantage if its unit cost of attack is too high. The defender has three unit costs: one for defense against a natural disaster; one for defense against terrorism; and one for all-hazards protection. These three unit costs taken together determine how weak or strong the defender is relative to the terrorist. (A particular focus of this chapter is on how the defender chooses strategically between these three kinds of investments.)

Clausewitz (1832:6.1.2) argued for the "superiority of defense over attack," which applies for classical warfare: "The defender enjoys optimum lines of communication and retreat, and can choose the place for battle." Surprise is an attacker advantage, but leaving fortresses and depots behind through extended operations also leaves attackers exposed. Examples of features improving defense are the use of trenches (combined with the machine gun) in World War I, castles and fortresses with cannon fire from higher elevations, and the use of checks and guards (in the broad sense of those terms).[1] In World War II, tanks and aviation technology gave some increased advantage to attackers. In the cyber context, the attacker generally has an advantage. In particular, Anderson (2001) argues that "defending a modern information system could … be likened to defending a large, thinly-populated territory like the nineteenth century Wild West: the men in black hats can strike anywhere, while the men in white hats have to defend everywhere."

The need to trade off between protection from terrorism and natural disasters is made clear by the fact that the defender must make decisions about both, in a world of competition for scarce resources. Moreover, these decisions are sometimes made by a single organization (e.g., the Department of Homeland Security in the U.S.). In some cases, defense against both terrorism and natural disasters is possible and cost efficient. In other cases, the focus of defense may appropriately be tilted toward one type of defense, possibly even to the exclusion of the other. In analyzing both terrorism and natural disasters in the same model, we do not intend in any way to neglect the critical differences between these two types of threats. In particular, terrorism is an intentional act, by an intelligent and adaptable adversary, and the purpose of our model is precisely to determine how this fact should be taken into account in making decisions about defensive investments.

[1] The superiority of the defense over the attack appears to be even larger for production facilities and produced goods than for Clausewitz's mobile army (Hausken 2004).

(However, since our model focuses specifically on investment in defenses, we do not consider other stategies for dealing with terrorism, such as negotiation or the threat of retaliation.)

This chapter uses contest success functions to represent the interaction between the defender on the one hand, and terrorism and the natural disaster on the other hand. Contest success functions are commonly used to represent the interactions between intelligent agents. The use of contest success functions in the case of a passive threat (the natural disaster) is perhaps somewhat unorthodox, and basically serves only as a way to specify the intensity of this threat parametrically. Unlike in the case of terrorism, our use of a contest success function for natural disaster does not assume that the disaster has a choice variable over which it can optimize.

Section 2 presents a simple model of the game we formulate to model attacker and defender investments and utilities. Section 3 analyzes the model when the defender and the terrorist move simultaneously; Section 4 lets the defender move first and the terrorist move second, in a two-period game; and Section 5 lets the terrorist move first and the defender move second. Section 6 compares the three games. Section 7 provides sensitivity-analysis results for various numerical examples, and Section 8 concludes.

2. THE MODEL

Consider an asset that the defender values at r. The asset is threatened by a natural disaster, which occurs with probability p, $0 \leq p \leq 1$. If not damaged by natural disaster, the asset can also be targeted by a terrorist. For simplicity, the terrorist is assumed not to attack if a natural disaster occurs. This simplification can be justified by the rare-event approximation (if damage from either terrorism or natural disaster individually is already quite unlikely), or by the assumption that a second incident of damage has at best second-order effects (Kunreuther and Heal, 2003). This latter argument seems plausible in practice—for example, New Orleans may no longer be an interesting target for a terrorist attack after Hurricane Katrina. Of course, a city already devastated by a terrorist attack could still fall victim to a natural disaster afterward, but our model neglects this possibility as a second-order consideration.

In our model, the defender incurs an effort $t_1 \geq 0$ at unit cost b_1 to protect against natural disaster, an effort $t_2 \geq 0$ at unit cost b_2 to prevent a successful terrorist attack, and an effort $t_3 \geq 0$ at unit cost b_3 as "all-hazards" protection. We require $b_3 > \max(b_1, b_2)$, so that all-hazards protection is never optimal for protecting against only one type of hazard (natural disasters or terrorism alone). The unit costs b_i (i=1, 2, 3) are inefficiencies of investment; i.e., $1/b_i$ is the efficiency.

The terrorist values the defender's asset at R, and seeks to destroy the asset (or at least part of it) by incurring an effort $T \geq 0$ at unit cost B. The expenditures $b_i t_i$ and BT can reflect capital costs and/or expenses such as labor costs, while the magnitude of the natural disaster is given by a constant D.

We assume that the contests between the defender and the natural disaster, and between the defender and the terrorist, take a form that is common in the literature on conflict and rent seeking (Hirshleifer, 1995; Skaperdas, 1996). For the natural-disaster contest, the defender gets to retain an expected fraction h of its asset where h is a contest success function satisfying $\partial h/\partial t_1$>0, $\partial h/\partial t_3$>0, and $\partial h/\partial D$<0. For the terrorist contest, the defender gets to retain an expected fraction g of its asset, and the terrorist gets the remaining fraction $G = 1 - g$, where g is a contest success function satisfying $\partial g/\partial t_2$>0, $\partial g/\partial t_3$>0, and $\partial g/\partial T$<0. The fractions h, g and G can be thought of as either fractions of the asset value (if a natural disaster or terrorist attack results in partial damage), or probabilities of total destruction (if a natural disaster or terrorist attack destroys the entire asset with some probability). We use the common ratio formula (Hausken, 2005; Skaperdas, 1996; Tullock, 1980):

$$h = \frac{t_1 + t_3}{t_1 + t_3 + D}, \quad g = \frac{t_2 + t_3}{t_2 + t_3 + T}, \quad G = \frac{T}{t_2 + t_3 + T} \tag{1}$$

In principle, h, g and G are undefined when nobody spends any effort, but in practice this will never occur. When the terrorist exerts no effort, the status quo is assumed to be preserved, and the defender keeps the asset. Consequently, the defender's and terrorist's utilities are given by u and U, respectively, where:[2]

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[2] Strictly speaking, the first term in the square brackets in the expression for the defender's utility should perhaps be written as $p\ min\left(\frac{t_1 + t_3}{t_1 + t_3 + D}, \frac{t_2 + t_3}{t_2 + t_3 + T}\right)$, which means that the defender's weakest defense, whether against natural disaster or against terrorism, should determine the defender's chance of success. In other words, the "weakest link" is what counts for the defender, and the "best shot" is what counts for the terrorist. In this model, if the defender defends equally well against natural disasters and terrorism (i.e., $t_1 + t_3 = t_2 + t_3$), and T=D, then one of the attacks can be ignored, since this alternative model implicitly assumes that the natural disaster and the terrorist target the same fraction of the asset value r. See Hausken (2002) and Hirshleifer (1983) for "weakest link" versus "best shot" analyses. The min function can be considered as representing a series system, in which both components have to be operative for the system to be operative. Use of the min

$$u = \left[p\frac{t_1 + t_3}{t_1 + t_3 + D} + (1-p)\frac{t_2 + t_3}{t_2 + t_3 + T} \right] r - b_1 t_1 - b_2 t_2 - b_3 t_3,$$
$$U = (1-p)\frac{T}{t_2 + t_3 + T} R - BT \qquad (2)$$

This chapter distinguishes between simultaneous and sequential games. Simultaneous games can be used even if all agents do not move simultaneously, as long as the agents moving later are unaware of any earlier actions. In other words, the crucial feature of a simultaneous game is that no agent knows any other agent's decision at the time of its own decision. By contrast, in sequential games (sometimes also called dynamic games), those agents moving later have some (possibly quite imperfect) knowledge of earlier actions. For convenience, simultaneous games are often represented in normal form (as payoff matrices), while sequential games are usually represented in extensive form (as decision trees with nodes and branches). See e.g. Fudenberg and Tirole (1991) or Rasmusen (2001) for further discussions about games.

3. ANALYZING THE MODEL WHEN DEFENDER AND TERRORIST MOVE SIMULTANEOUSLY

The three first-order conditions for the defender, and the unique first-order condition for the terrorist, are given by

$$\frac{\partial u}{\partial t_1} = \frac{pDr}{(t_1 + t_3 + D)^2} - b_1 = 0,$$
$$\frac{\partial u}{\partial t_2} = \frac{(1-p)Tr}{(t_2 + t_3 + T)^2} - b_2 = 0,$$
$$\frac{\partial u}{\partial t_3} = \frac{pDr}{(t_1 + t_3 + D)^2} + \frac{(1-p)Tr}{(t_2 + t_3 + T)^2} - b_3 = 0, \qquad (3)$$
$$\frac{\partial U}{\partial T} = \frac{(1-p)(t_2 + t_3)R}{(t_2 + t_3 + T)^2} - B = 0$$

See Appendix 1 for the second-order conditions, which are always satisfied, and the Hessian matrix. The four equations in (3) actually have

function assumes that the terrorist undertakes attack effort T regardless of the occurrence of a natural disaster with probability p. This alternative model formulation will be especially realistic when the terrorist moves first (before any possible natural disaster).

only three unknowns, t_1+t_3, t_2+t_3, and T. An interior solution is possible only when $b_1+b_2=b_3$, which causes multiple optima, and which means that investment in all-hazards protection is as effective as protection from both natural disaster and terrorism individually. Although unit costs of such sizes are realistic, the equality is unlikely to hold in practice. Hence an interior solution is unlikely, and we do not consider this case further.

The use of ratio contest success functions implies that in equilibrium, no contestant withdraws from a simultaneous game. (We define equilibrium as a solution from which no agent prefers to deviate unilaterally.) Hence, we never have $t_2=t_3=0$, and we never have T=0; for one of these to occur, the relevant agent would have to choose off-equilibrium behavior, which we do not consider. When B is large, the terrorist chooses low T and earns low utility, but prefers this over choosing $T = U = 0$. Hence, the five relevant corner solutions are $t_1=0$, $t_2=0$, $t_3=0$, $t_1=t_2=0$, and $t_1=t_3=0$, which are illustrated below.

First, consider the case when $b_1+b_2<b_3$. In this case, the more general all-hazards protection has a higher unit cost than the sum of the defender's other two unit costs, yielding $t_3=0$. Solving the first, second, and fourth equations in (3) when $b_1+b_2<b_3$ gives

$$
\begin{aligned}
t_1 &= \begin{cases} \sqrt{\dfrac{pD}{b_1/r}} - D \; when \; \dfrac{p}{b_1/r} \geq D \\ 0 \; otherwise \end{cases}, \\
t_2 &= \frac{(1-p)B/R}{(B/R+b_2/r)^2}, \\
t_3 &= 0,\; T = \frac{(1-p)b_2/r}{(B/R+b_2/r)^2}, \\
u &= \begin{cases} \left(\dfrac{(1-p)(B/R)^2}{(B/R+b_2/r)^2} + \left(\sqrt{p} - \sqrt{Db_1/r}\right)^2 \right) r \; when \; \dfrac{p}{b_1/r} \geq D \\ \dfrac{(1-p)(B/R)^2}{(B/R+b_2/r)^2} r \quad otherwise \end{cases}, \\
U &= \frac{(1-p)(b_2/r)^2}{(B/R+b_2/r)^2} R
\end{aligned}
\tag{4}
$$

In this case, the defender invests in protection against the natural disaster, except when the natural disaster is not sufficiently damaging to justify investment in protection. If the natural disaster had been an intentional agent,

D would have been chosen at a level where t_1 would always be positive in equilibrium. However, since D is exogenously given, t_1=0 is possible. Hence, $t_1 = t_3$=0 is also possible, in contrast to the contest with the terrorist, in which $t_2 = t_3$=0 is not possible.

Observe that t_1 is inverse U shaped in D, and that $t_2 / T = (B/R)/(b_2/r)$. Thus, the defender may choose to invest nothing in protection from natural disaster when the threat is too small, but also when the threat is so overwhelming that it cannot be countered cost-effectively. By contrast, the defender always invests in protection from terrorism, since withdrawal means losing the asset even when the terrorist invests an arbitrarily small effort. Not defending against the terrorist threat thus actually increases the threat.

Now, consider the case when $b_1 + b_2 > b_3$. In this case, the more general all-hazards protection has a lower unit cost than the sum of the defender's other two unit costs. This means that either t_1=0 or t_2=0 at equilibrium. When t_1=0, then t_3 is applied against the disaster. For convenience, let s_1 = t_1 + t_3, and s_2 = t_2 + t_3. Then, solving the second, third, and fourth equations in (3) when $b_1 + b_2 > b_3$ gives

$$t_1 + t_3 = s_1 = \begin{cases} \sqrt{\dfrac{pD}{(b_3 - b_2)/r}} - D \text{ when } \dfrac{p}{(b_3 - b_2)/r} \geq D \\ 0 \text{ otherwise} \end{cases},$$

$$t_2 + t_3 = s_2 = \frac{(1-p)B/R}{(B/R + b_2/r)^2}, T = \frac{(1-p)b_2/r}{(B/R + b_2/r)^2},$$

$$u = \begin{cases} \left(\dfrac{(1-p)(B/R)^2}{(B/R + b_2/r)^2} + \left(\sqrt{p} - \sqrt{D(b_3 - b_2)/r} \right)^2 \right) r \text{ when } \dfrac{p}{(b_3 - b_2)/r} \geq D \\ \dfrac{(1-p)(B/R)^2}{(B/R + b_2/r)^2} r \text{ otherwise} \end{cases}, \tag{5}$$

$$U = \frac{(1-p)(b_2/r)^2}{(B/R + b_2/r)^2} R$$

When $s_1 \leq s_2$, then equation (5) implies that the defender invests in all-hazards protection at level s_1, and t_2 provides the remaining needed defense against the terrorist. If b_2 is sufficiently large, then t_2=0. This can occur when $b_2 < b_3$, and means that all-hazards protection takes care of both the disaster and the terrorist. We do not analyze this case explicitly here, but solving it amounts to setting $t_1=t_2=0$ and solving the third and fourth equations in (3) with respect to t_3 and T (which gives a third-order equation).

By contrast, when $b_1+b_2>b_3$ but $t_2=0$, then t_3 is applied against terrorism. Solving the first, third, and fourth equations in (3) gives

$$t_1+t_3=s_1=\begin{cases}\sqrt{\dfrac{pD}{b_1/r}}-D \text{ when } \dfrac{p}{b_1/r}\geq D\\ t_3 \text{ otherwise}\end{cases},$$

$$t_2+t_3=s_2=\begin{cases}\dfrac{(1-p)B/R}{(B/R+(b_3-b_1)/r)^2} \text{ when } \dfrac{p}{b_1/r}\geq D\\ \text{third order expression otherwise}\end{cases},$$

$$T=\begin{cases}\dfrac{(1-p)(b_3-b_1)/r}{(B/R+(b_3-b_1)/r)^2} \text{ when } \dfrac{p}{b_1/r}\geq D\\ \text{third order expression otherwise}\end{cases}, \tag{6}$$

$$u=\begin{cases}\left(\dfrac{(1-p)(B/R)^2}{(B/R+(b_3-b_1)/r)^2}+\left(\sqrt{p}-\sqrt{Db_1/r}\right)^2\right)r \text{ when } \dfrac{p}{b_1/r}\geq D\\ \text{third order expression otherwise}\end{cases},$$

$$U=\begin{cases}\dfrac{(1-p)((b_3-b_1)/r)^2}{(B/R+(b_3-b_1)/r)^2}R \text{ when } \dfrac{p}{b_1/r}\geq D\\ \text{third order expression otherwise}\end{cases}$$

When $s_2 \leq s_1$, equation (6) implies that the defender invests in all-hazards protection at level s_2, and t_1 provides the remaining needed defense against the natural disaster. If b_1 is sufficiently large, then we will have $t_1=0$. This can occur when $b_1<b_3$, and means that all-hazards protection takes care of both the disaster and the terrorist. As before, solving this case amounts to setting $t_1=t_2=0$ and solving the third and fourth equations in (3) with respect to t_3 and T (which gives a third-order equation).

4. ANALYZING THE MODEL WHEN DEFENDER MOVES FIRST AND TERRORIST MOVES SECOND

For the two-period game where the defender moves first and the terrorist moves second, the second period is solved first. The first-order condition for the terrorist is

$$\frac{\partial U}{\partial T}=\frac{(1-p)(t_2+t_3)R}{(t_2+t_3+T)^2}-B=0 \quad\Rightarrow\quad T=\sqrt{(1-p)(t_2+t_3)R/B}-(t_2+t_3) \quad (7)$$

The second-order conditions in Appendix 1 remain unchanged. Inserting (7) into (2) gives

$$u=\left[p\frac{t_1+t_3}{t_1+t_3+D}+\sqrt{(1-p)(t_2+t_3)B/R}\right]r-b_1t_1-b_2t_2-b_3t_3 \quad (8)$$

The first-order conditions for the defender in the first period are given by

$$\begin{aligned}
&\frac{\partial u}{\partial t_1}=\frac{pDr}{(t_1+t_3+D)^2}-b_1=0 \quad\Rightarrow\quad t_1+t_3=\sqrt{\frac{pD}{b_1/r}}-D,\\
&\frac{\partial u}{\partial t_2}=\frac{\sqrt{(1-p)B/R}}{2\sqrt{t_2+t_3}}r-b_2=0 \quad\Rightarrow\quad t_2+t_3=\frac{(1-p)B/R}{4(b_2/r)^2}, \qquad (9)\\
&\frac{\partial u}{\partial t_3}=\frac{pDr}{(t_1+t_3+D)^2}+\frac{\sqrt{(1-p)B/R}}{2\sqrt{t_2+t_3}}r-b_3=0
\end{aligned}$$

See Appendix 2 for the second-order conditions, which are always satisfied, and the Hessian matrix. As in the previous section, we distinguish between three cases. First, $b_1+b_2<b_3$ causes $t_3=0$. Solving the first two equations in (10), and inserting into (8) and (2), gives

$$t_1 = \begin{cases} \sqrt{\dfrac{pD}{b_1/r}} - D \ when \ \dfrac{p}{b_1/r} \geq D \\ 0 \ otherwise \end{cases},$$
$$t_2 = \frac{(1-p)B/R}{4(b_2/r)^2}, \ t_3 = 0,$$
$$T = \frac{(1-p)[2b_2/r - B/R]}{4(b_2/r)^2}, \tag{10}$$
$$u = \begin{cases} \left(\dfrac{(1-p)B/R}{4b_2/r} + \left(\sqrt{p} - \sqrt{Db_1/r}\right)^2\right) r \ when \ \dfrac{p}{b_1/r} \geq D \\ \dfrac{(1-p)B/R}{4b_2/r} r \ \ otherwise \end{cases},$$
$$U = \frac{(1-p)(2b_2/r - B/R)^2}{4(b_2/r)^2} R$$

When $2b_2/r - B/R < 0$, the terrorist withdraws from the contest which causes a contest between the defender and the natural disaster, which we do not consider.

Second, consider the case where $b_1 + b_2 > b_3$ and $t_1 = 0$. Solving the second and third equations in (9), and inserting into (7) and (2), gives

$$t_1 + t_3 = s_1 = \begin{cases} \sqrt{\dfrac{pD}{(b_3 - b_2)/r}} - D \ when \ \dfrac{p}{(b_3 - b_2)/r} \geq D \\ 0 \ otherwise \end{cases},$$
$$t_2 + t_3 = s_2 = \frac{(1-p)B/R}{4(b_2/r)^2},$$
$$T = \frac{(1-p)[2b_2/r - B/R]}{4(b_2/r)^2}, \tag{11}$$
$$U = \frac{(1-p)(2b_2/r - B/R)^2}{4(b_2/r)^2} R,$$
$$u = \begin{cases} \left(\dfrac{(1-p)B/R}{4b_2/r} + \left(\sqrt{p} - \sqrt{D(b_3 - b_2)/r}\right)^2\right) r \ when \ \dfrac{p}{(b_3 - b_2)/r} \geq D \\ \dfrac{(1-p)B/R}{4b_2/r} r \ \ otherwise \end{cases}$$

When $2b_2/r - B/R < 0$, the terrorist withdraws from the contest, which causes a contest between the defender and the natural disaster, which we do not consider.

Third, consider the case where $b_1 + b_2 > b_3$ and $t_2 = 0$. Solving the first and third equations in (8), and inserting into (7) and (2), gives

$$t_1 + t_3 = \begin{cases} \sqrt{\dfrac{pD}{b_1/r}} - D \text{ when } \dfrac{p}{b_1/r} \geq D \\ t_3 \text{ otherwise} \end{cases},$$

$$t_2 = 0, t_3 = \begin{cases} \dfrac{(1-p)B/R}{4((b_3 - b_1)/r)^2} \text{ when } \dfrac{p}{b_1/r} \geq D \\ \text{fifth order expression otherwise} \end{cases},$$

$$T = \begin{cases} \dfrac{(1-p)[2(b_3 - b_1)/r - B/R]}{4((b_3 - b_1)/r)^2} \text{ when } \dfrac{p}{b_1/r} \geq D \\ \text{fifth order expression otherwise} \end{cases}, \tag{12}$$

$$u = \begin{cases} \left(\dfrac{(1-p)B/R}{4(b_3 - b_1)/r} + \left(\sqrt{p} - \sqrt{Db_1/r} \right)^2 \right) r \text{ when } \dfrac{p}{b_1/r} \geq D \\ \text{fifth order expression otherwise} \end{cases},$$

$$U = \begin{cases} \dfrac{(1-p)(2(b_3 - b_1)/r - B/R)^2}{4((b_3 - b_1)/r)^2} R \text{ when } \dfrac{p}{b_1/r} \geq D \\ \text{fifth order expression otherwise} \end{cases}$$

The fifth-order equations that result when the natural disaster is highly damaging can be solved numerically, but are too complicated to present here. When $2(b_3 - b_1)/r - B/R < 0$, the terrorist withdraws from the contest, which causes a contest between the defender and the natural disaster, which we again do not consider.

5. ANALYZING THE MODEL WHEN TERRORIST MOVES FIRST AND DEFENDER MOVES SECOND

For the two-period game where the terrorist moves first (say, by committing to a specific plan of attack) and the defender moves second, the second period is again solved first. (We assume that the attacker cannot change its strategy after having chosen it, and that the contest takes place after the defender has chosen its strategy.) The first-order conditions for the defender in this case are

$$\begin{aligned}
\frac{\partial u}{\partial t_1} &= \frac{pDr}{(t_1+t_3+D)^2} - b_1 = 0 \quad \Rightarrow \quad t_1 + t_3 = \sqrt{\frac{pD}{b_1/r}} - D, \\
\frac{\partial u}{\partial t_2} &= \frac{(1-p)Tr}{(t_2+t_3+T)^2} - b_2 = 0 \quad \Rightarrow \quad t_2 + t_3 = \sqrt{(1-p)Tr/b_2} - T, \\
\frac{\partial u}{\partial t_3} &= \frac{pDr}{(t_1+t_3+D)^2} + \frac{(1-p)Tr}{(t_2+t_3+T)^2} - b_3 = 0
\end{aligned} \tag{13}$$

Since the defender's optimization problem remains the same as in Section 3, the second-order condition verification remains the same.

We distinguish between three cases. First, $b_1+b_2<b_3$ causes $t_3=0$. Inserting the second equation in (13) into (2) gives

$$U = R\sqrt{(1-p)Tb_2/r} - BT \tag{14}$$

The first-order condition for the terrorist in the first period is

$$\frac{\partial U}{\partial T} = \frac{\sqrt{(1-p)b_2/r}}{2\sqrt{T}}R - B = 0 \quad \Rightarrow \quad T = \frac{(1-p)b_2/r}{4(B/R)^2} \tag{15}$$

The second-order condition is satisfied; i.e.

$$\frac{\partial^2 U}{\partial T^2} = -\frac{\sqrt{(1-p)b_2/r}}{4T^{3/2}}R < 0 \tag{16}$$

Inserting (15) into (13) and (2) when $b_1+b_2<b_3$ gives

$$
\begin{aligned}
&t_1 = \begin{cases} \sqrt{\dfrac{pD}{b_1 / r}} - D \ when \ \dfrac{p}{b_1 / r} \geq D \\ 0 \ otherwise \end{cases}, \\
&t_2 = \frac{(1-p)[2B/R - b_2/r]}{4(B/R)^2}, \ t_3 = 0, \ T = \frac{(1-p)b_2/r}{4(B/R)^2}, \\
&u = \begin{cases} \left(\dfrac{(1-p)(2B/R - b_2/r)^2}{4(B/R)^2} + \left(\sqrt{p} - \sqrt{Db_1/r}\right)^2 \right) r \ when \ \dfrac{p}{b_1/r} \geq D \\ \dfrac{(1-p)(2B/R - b_2/r)^2}{4(B/R)^2} r \ \ otherwise \end{cases}, \\
&U = \frac{(1-p)b_2/r}{4B/R} R
\end{aligned}
\tag{17}
$$

When $2B/R - b_2/r < 0$, the defender withdraws from the contest with the terrorist, setting $t_2 = t_3 = 0$. The terrorist earns $U = (1-p) R - B\,T$, with investment T defined in (17). The contest with the natural disaster remains, and the defender invests t_1 in this contest.

Second, we consider $b_1 + b_2 > b_3$ and $t_1 = 0$. Equations (14)–(16) remain as before. Inserting (15) into (13) and (2) gives

$$
\begin{aligned}
&t_1 + t_3 = s_1 = \begin{cases} \sqrt{\dfrac{pD}{(b_3 - b_2)/r}} - D \ when \ \dfrac{p}{(b_3 - b_2)/r} \geq D \\ 0 \ otherwise \end{cases}, \\
&t_2 + t_3 = s_2 = \frac{(1-p)[2B/R - b_2/r]}{4(B/R)^2}, \\
&T = \frac{(1-p)b_2/r}{4(B/R)^2}, \quad U = \frac{(1-p)b_2/r}{4B/R} R, \\
&u = \begin{cases} \left(\dfrac{(1-p)(2B/R - b_2/r)^2}{4(B/R)^2} + \left(\sqrt{p} - \sqrt{D(b_3 - b_2)/r}\right)^2 \right) r \\ \qquad when \ \dfrac{p}{(b_3 - b_2)/r} \geq D \\ \dfrac{(1-p)(2B/R - b_2/r)^2}{4(B/R)^2} r \ \ otherwise \end{cases}
\end{aligned}
\tag{18}
$$

When $2B/R - b_2/r < 0$, the defender withdraws from investing directly in the contest with the terrorist, setting $t_2 = 0$, while t_3 provides protection against both the natural disaster and the terrorist.

Third, we consider $b_1 + b_2 > b_3$ and $t_2 = 0$. Solving the first and third equations in (13) gives

$$t_3 = \sqrt{(1-p)Tr/(b_3 - b_1)} - T \tag{19}$$

Observe that (19) is similar to the second equation in (15), but with b_2 replaced by $b_3 - b_1$. Inserting (19) into (2) gives

$$U = R\sqrt{(1-p)T(b_3 - b_1)/r} - BT \tag{20}$$

The first-order condition for the terrorist in the first period is

$$\frac{\partial U}{\partial T} = \frac{\sqrt{(1-p)(b_3 - b_1)/r}}{2\sqrt{T}} R - B = 0 \quad \Rightarrow \quad T = \frac{(1-p)(b_3 - b_1)/r}{4(B/R)^2} \tag{21}$$

The second-order condition is always negative; i.e.

$$\frac{\partial^2 U}{\partial T^2} = -\frac{\sqrt{(1-p)(b_3 - b_1)/r}}{4T^{3/2}} R < 0 \tag{22}$$

Observe that (20)–(22) are equivalent to (14)–(16), but with b_2 again replaced by $b_3 - b_1$. Inserting (21) into (13) and (2) gives

$$t_1 + t_3 = s_1 = \begin{cases} \sqrt{\dfrac{pD}{b_1 / r}} - D \text{ when } \dfrac{p}{b_1 / r} \geq D \\ t_3 \text{ otherwise} \end{cases},$$

$$t_2 + t_3 = s_2 = \begin{cases} \dfrac{(1-p)[2B/R - (b_3 - b_1)/r]}{4(B/R)^2} \text{ when } \dfrac{p}{b_1 / r} \geq D \\ \text{higher order expression otherwise} \end{cases},$$

$$T = \begin{cases} \dfrac{(1-p)(b_3 - b_1)/r}{4(B/R)^2} \text{ when } \dfrac{p}{b_1 / r} \geq D \\ \text{higher order expression otherwise} \end{cases}, \tag{23}$$

$$U = \begin{cases} \dfrac{(1-p)(b_3 - b_1)/r}{4B/R} R \text{ when } \dfrac{p}{b_1 / r} \geq D \\ \text{higher order expression otherwise} \end{cases},$$

$$u = \begin{cases} \left(\dfrac{(1-p)(2B/R - (b_3 - b_1)/r)^2}{4(B/R)^2} + \left(\sqrt{p} - \sqrt{Db_1 / r} \right)^2 \right) r \\ \qquad \text{when } \dfrac{p}{b_1 / r} \geq D \\ \text{higher order expression otherwise} \end{cases}$$

(As before, the higher-order expressions can be evaluated numerically.)

When $2B/R - (b_3 - b_1)/r < 0$, the defender withdraws from the contest with the terrorist, setting $t_2 = t_3 = 0$. The terrorist earns $U = (1 - p) R - B T$, where T is determined by (23). The contest with the natural disaster remains, and the defender invests t_1. In the special case where $t_1 = t_2 = 0$, inserting $t_1 = t_2 = 0$ into the third equation in (13) and solving with respect to t_3 gives a fourth-order equation for t_3 as a function of T. Inserting this value of t_3 into (2) gives the terrorist's first-period utility.

6. COMPARING THE THREE GAMES

Tables 4-1 to 4-3 below compare the equilibrium levels of effort and utilities, respectively, for the three games outlined above. For simplicity, these tables do not show the expressions that apply when one agent is completely deterred. Table 4-3 assumes $p/(b_1 / r) \geq D$ since the higher order expressions in (6), (12), (23) are either voluminous or intractable; hence, the max operator is not needed in the expressions for s_1 and u.

Table 4-1. Equilibrium efforts and utilities for the three games when $b_1 + b_2 < b_3, t_3 = 0$, $0 \le p \le 1$

	t_1	t_2	T
Simultaneous game		$\frac{(1-p)B/R}{(B/R+b_2/r)^2}$	$\frac{(1-p)b_2/r}{(B/R+b_2/r)^2}$
Defender moves first	$\max\left\{0, \sqrt{\frac{pD}{b_1/r}} - D\right\}$	$\frac{(1-p)B/R}{4(b_2/r)^2}$	$\frac{(1-p)[2b_2/r - B/R]}{4(b_2/r)^2}$
Terrorist moves first		$\frac{(1-p)[2B/R - b_2/r]}{4(B/R)^2}$	$\frac{(1-p)b_2/r}{4(B/R)^2}$
	u		U
Simultaneous game	$\left(\frac{(1-p)(B/R)^2}{(B/R+b_2/r)^2} + \left(\max\left\{0, \sqrt{p} - \sqrt{Db_1/r}\right\}\right)^2\right)r$		$\frac{(1-p)(b_2/r)^2}{(B/R+b_2/r)^2}R$
Defender moves first	$\left(\frac{(1-p)B/R}{4b_2/r} + \left(\max\left\{0, \sqrt{p} - \sqrt{Db_1/r}\right\}\right)^2\right)r$		$\frac{(1-p)(2b_2/r - B/R)^2}{4(b_2/r)^2}R$
Terrorist moves first	$\left(\frac{(1-p)(2B/R - b_2/r)^2}{4(B/R)^2} + \left(\max\left\{0, \sqrt{p} - \sqrt{Db_1/r}\right\}\right)^2\right)r$		$\frac{(1-p)b_2/r}{4B/R}R$

Table 4-2. Equilibrium efforts and utilities for the three games when $b_1 + b_2 < b_3, t_1 = 0$, $0 \le p \le 1$

	t_3	$s_2 = t_2 + t_3$	T
Simultaneous game		$\frac{(1-p)B/R}{(B/R+b_2/r)^2}$	$\frac{(1-p)b_2/r}{(B/R+b_2/r)^2}$
Defender moves first	$\max\left\{0, \sqrt{\frac{pD}{(b_3-b_2)/r}} - D\right\}$	$\frac{(1-p)B/R}{4(b_2/r)^2}$	$\frac{(1-p)[2b_2/r - B/R]}{4(b_2/r)^2}$
Terrorist moves first		$\frac{(1-p)[2B/R - b_2/r]}{4(B/R)^2}$	$\frac{(1-p)b_2/r}{4(B/R)^2}$
	u		U
Simultaneous game	$\left(\frac{(1-p)(B/R)^2}{(B/R+b_2/r)^2} + \left(\max\left\{0, \sqrt{p} - \sqrt{D(b_3-b_2)/r}\right\}\right)^2\right)r$		$\frac{(1-p)(b_2/r)^2}{(B/R+b_2/r)^2}R$
Defender moves first	$\left(\frac{(1-p)B/R}{4b_2/r} + \left(\max\left\{0, \sqrt{p} - \sqrt{D(b_3-b_2)/r}\right\}\right)^2\right)r$		$\frac{(1-p)(2b_2/r - B/R)^2}{4(b_2/r)^2}R$
Terrorist moves first	$\left(\frac{(1-p)(2B/R - b_2/r)^2}{4(B/R)^2} + \left(\max\left\{0, \sqrt{p} - \sqrt{D(b_3-b_2)/r}\right\}\right)^2\right)r$		$\frac{(1-p)b_2/r}{4B/R}R$

Since $b_1 + b_2 < b_3$, which causes t_3=0 in Table 4-1, no variables depend on b_3.

Since $b_1 + b_2 > b_3$ and t_1=0 in Table 4-2, no variables depend on b_1. The defender substitutes optimally between t_2 and t_3 so that $s_2 = t_2 + t_3$, and the terrorist's investment T and utility U do not depend on b_3. In this substitution, only t_3 affects the risk from the natural disaster. Hence, s_2, T,

Table 4-3. Equilibrium efforts and utilities for the three games when $b_1+b_2<b_3, t_2=0,$ and $p/(b_1/r)\geq D$

	$s_1=t_1+t_3$	t_3	T
Simultaneous game	$\sqrt{\frac{pD}{b_1/r}}-D$	$\frac{(1-p)B/R}{(B/R+(b_3-b_1)/r)^2}$	$\frac{(1-p)(b_3-b_1)/r}{(B/R+(b_3-b_1)/r)^2}$
Defender moves first		$\frac{(1-p)B/R}{4((b_3-b_1)/r)^2}$	$\frac{(1-p)[2(b_3-b_1)/r-B/R]}{4((b_3-b_1)/r)^2}$
Terrorist moves first		$\frac{(1-p)[2B/R-(b_3-b_1)/r]}{4(B/R)^2}$	$\frac{(1-p)(b_3-b_1)/r}{4(B/R)^2}$
	u		U
Simultaneous game	$\left(\frac{(1-p)(B/R)^2}{(B/R+(b_3-b_1)/r)^2}+\left(\sqrt{p}-\sqrt{Db_1/r}\right)^2\right)r$		$\frac{(1-p)((b_3-b_1)/r)^2}{(B/R+(b_3-b_1)/r)^2}R$
Defender moves first	$\left(\frac{(1-p)B/R}{4(b_3-b_1)/r}+\left(\sqrt{p}-\sqrt{Db_1/r}\right)^2\right)r$		$\frac{(1-p)(2(b_3-b_1)/r-B/R)^2}{4((b_3-b_1)/r)^2}R$
Terrorist moves first	$\left(\frac{(1-p)(2B/R-(b_3-b_1)/r)^2}{4(B/R)^2}+\left(\sqrt{p}-\sqrt{Db_1/r}\right)^2\right)r$		$\frac{(1-p)(b_3-b_1)/r}{4B/R}R$

and U depend only on b_2, B, r, R, and p. However, the defender's investment t_3 in all-hazards protection decreases as b_3 increases above b_2, which causes the defender's utility also to decrease as b_3 increases above b_2, since T is independent of b_3.

Since $b_1+b_2>b_3$ and $t_2=0$ in Table 4-3, no variables depend on b_2. Roles are changed compared with Table 4-2. The defender substitutes optimally between t_1 and t_3 so that t_1+t_3 does not depend on b_3 (although t_3, T, and U depend on the extent to which $b_3>b_2$). In this substitution, only t_3 affects the terrorist. Both the defender's investment t_3 in all-hazards protection and the defender's utility u decrease as b_3 increases above b_1. When the terrorist moves first, its investment and utility increase in b_3. In other words, the terrorist benefits from a high unit cost of all-hazards defense. The first-order derivatives $\partial T/\partial b_3$ and $\partial U/\partial b_3$ in Table 4-3 are straightforward to set up, but are not discussed here, since they can be either positive or negative (depending on the parameter values), and their interpretation is complicated to explain.

Table 4-4 compresses Tables 4-1 to 4-3 into one table. In particular, Table 4-4 is equivalent to Table 4-1 ($b_1+b_2<b_3$, t_3=0) when $b_v=b_1$, $b_w=b_2$, $x=t_1$, and $y=t_2$. Table 4-4 gives Table 4-2 ($b_1+b_2>b_3$, t_1=0) when $b_v=b_3-b_2$, $b_w=b_2$, $x=t_3$, and $y=t_2+t_3$. Finally, Table 4-4 gives Table 4-3 ($b_1+b_2>b_3$, t_2=0) when $b_v=b_1$, $b_w=b_3-b_1$, $x=t_1+t_3$, and $y=t_3$, assuming $p/(b_1/r)\geq D$. In this notation, x is the defense against the natural disaster (t_1, t_3, or t_1+t_3), and y is the defense against terrorism (t_2, t_3, or t_2+t_3).

We first consider the efforts. For the simultaneous game, the defender's effort y in defense against terrorism increases in the terrorist's unit cost B divided by the terrorist's valuation R when the normalized marginal cost of terrorism defense, b_w/r, is greater than B/R, and otherwise decreases in B/R. Analogously, the terrorist's effort T increases in the defender's normalized marginal cost of terrorism defense b_w/r is greater than B/R, and otherwise decreases in b_w/r.

Evidently, when all-hazards protection is sufficiently cheap, it replaces both pure natural disaster protection and pure terrorism protection.

Table 4-4. Equilibrium efforts and utilities for the three games

	x	y	T
Simultaneous game	$\max\left\{0, \sqrt{\frac{pD}{b_v/r}} - D\right\}$	$\frac{(1-p)B/R}{(B/R+b_w/r)^2}$	$\frac{(1-p)b_w/r}{(B/R+b_w/r)^2}$
Defender moves first		$\frac{(1-p)B/R}{4(b_w/r)^2}$	$\frac{(1-p)[2b_w/r-B/R]}{4(b_w/r)^2}$
Terrorist moves first		$\frac{(1-p)[2B/R-b_w/r]}{4(B/R)^2}$	$\frac{(1-p)b_w/r}{4(B/R)^2}$
	u		U
Simultaneous Game	$\left(\frac{(1-p)(B/R)^2}{(B/R+b_w/r)^2} + \left(\max\left\{0, \sqrt{p} - \sqrt{Db_v/r}\right\}\right)^2\right)r$		$\frac{(1-p)(b_w/r)^2}{(B/R+b_w/r)^2}R$
Defender moves first	$\left(\frac{(1-p)B/R}{4b_w/r} + \left(\max\left\{0, \sqrt{p} - \sqrt{Db_v/r}\right\}\right)^2\right)r$		$\frac{(1-p)(2b_w/r-B/R)^2}{4(b_w/r)^2}R$
Terrorist moves first	$\left(\frac{(1-p)(2B/R-b_w/r)^2}{4(B/R)^2} + \left(\max\left\{0, \sqrt{p} - \sqrt{Db_v/r}\right\}\right)^2\right)r$		$\frac{(1-p)b_w/r}{4B/R}R$

PROPOSITION 1:

When the defender moves first, its effort y is higher than in the simultaneous game when

$$b_w/r < B/R \tag{24}$$

When the defender moves first, the terrorist is deterred from incurring effort when $2b_w/r < B/R$, in which case the terrorist chooses $T = 0$ and earns zero utility.

PROOF:

Follows from Table 4-4.

In other words, with a sufficiently low unit cost of defense, or a sufficiently high asset value, the defender can deter the terrorist altogether.

PROPOSITION 2:

When the terrorist moves first, its effort is higher than in the simultaneous game when the inequality in (24) is reversed; that is, when

$$B / R < b_w / r \tag{25}$$

Here again, when the terrorist moves first, the defender is deterred from incurring effort when $B/R < b_w/(2r)$, in which case the defender chooses $t_2 = 0$, loses its asset, and earns zero utility.

PROOF:

Follows from Table 4-4.

As with the defender, if the terrorist has a sufficiently low unit cost of attack or a sufficiently high asset value, it can deter the defender from investing in protection from terrorism altogether.

Let us now consider the utilities of the two agents.

PROPOSITION 3:

(a) Over the three games, both the defender and the terrorist always prefer the game in which they move first rather than a simultaneous game. (b) The defender prefers the terrorist to move first rather than herself to move first if and only if $1 < \frac{B / R}{b_w / r} < 2.62$. (c) The terrorist prefers the defender to move first rather than moving first itself if and only if $0.38 < \frac{B / R}{b_w / r} < 1$. (d) The defender prefers a simultaneous game rather than allowing the terrorist to move first if and only if $0 < \frac{B / R}{b_w / r} < 1$. (e) The terrorist prefers a simultaneous game rather than allowing the defender to move first if and only if $\frac{B / R}{b_w / r} > 1$.

PROOF:

See Appendix 3.

Proposition 3 is illustrated in Figure 4-1. When the terrorist is advantaged with a low unit cost, it prefers to move first due to its relative strength, which the defender seeks to avoid. Conversely, when the terrorist is disadvantaged with a high unit cost, it prefers to move first to prevent being deterred from attacking at all, while the defender prefers to deter an attack through its first-mover advantage. When $0.38 < (B/R)/(b_w/r) < 1$, both agents prefer that the defender moves first, and when $1 < (B/R)/(b_w/r) < 2.62$, both agents prefer that the terrorist moves first. At the transition points 0.38, 1, 2.62, the agents are indifferent between the two neighboring strategies.

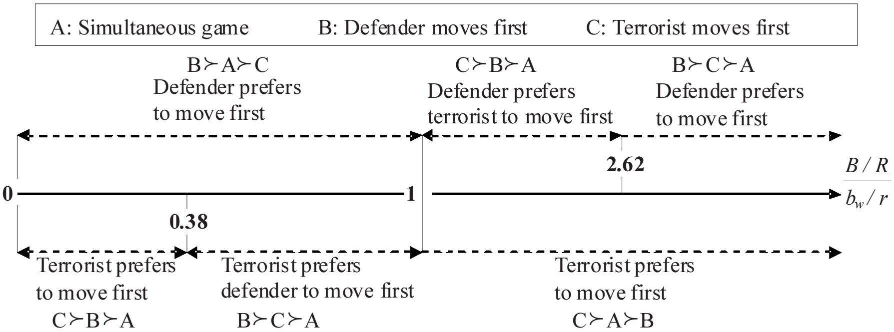

Figure 4-1. Defender and terrorist preferences when accounting for all preference orders

7. SENSITIVITY ANALYSIS AS PARAMETERS VARY

The base-case parameter values for the sensitivity analyses given in this section are $b_1=b_2=B=0.5$, $b_3=r=R=1$, $p=0.2$, $D=0.05$. This means that the unit costs of defense against the natural disaster and terrorism are equal, and equal to the terrorist's unit cost, while all-hazards protection is twice as expensive. While the case with equal unit costs may be unlikely to occur in practice (just as any other choice may be unlikely), it makes it easy to show the effects of changing any one parameter. The probability of a natural disaster is 20%, the defense against it is fixed at 0.05, and the defender and terrorist value the asset equally at one.

Figure 4-2 shows t_1, t_2, t_3, T, u, and U for all three games, as functions of b_1. The defender's investment t_1 against the natural disaster and its utility u decrease convexly in b_1 when $b_1 < 0.5$. These variables are determined by (4). At $b_1=b_2=0.5$, t_1 becomes too expensive and drops from 0.09 to zero, t_2 drops from 0.4 to 0.31, and all-hazards protection t_3 takes over.

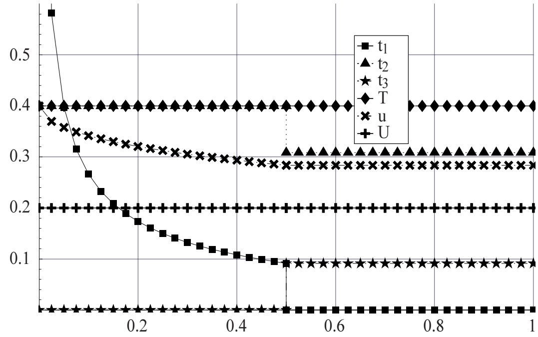

Figure 4-2. t_1, t_2, t_3, T, u, U as functions of b_1 for all three games.

Figure 4-3 shows the same six variables as functions of b_2 for the simultaneous game. The defender's investment t_2 against terrorism and utility u decrease convexly in b_2 when $b_2 < 0.5$. At $b_2 = b_1 = 0.5$, t_2 and t_1 make downward shifts such that $t_1 = 0$ when $b_2 > 0.5$, while t_3 makes an upwards shift. As b_2 increases above 0.5, defense against terrorism becomes increasingly expensive, and t_2 decreases, reaching zero at $b_2 = 0.86$. For $0.86 < b_2 < 1$, all-hazard protection single-handedly takes care of both the natural disaster and terrorism. The inverse U shape for the terrorist's investment T is commonly observed in contests of this kind (Hausken 2006). When b_2 is low, the terrorist is overwhelmed by the solid defense and withdraws due to weakness. When b_2 is high, there is no need for the terrorist to invest heavily, since the defense is weak. As we might expect, the defender's utility u decreases while the terrorist's utility U increases as b_2 increases.

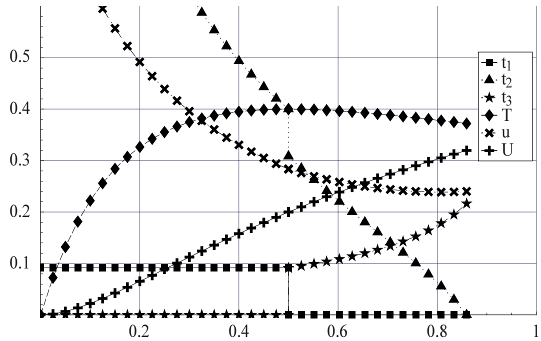

Figure 4-3. t_1, t_2, t_3, T, u, U as functions of b_2 for the simultaneous move game

Figure 4-4, for the game in which the defender moves first, is roughly similar, except that when b_2<0.25, the terrorist is completely deterred (in accord with Proposition 1), since $2b_2/r$ < B/R. Figure 4-5 shows the six variables as functions of b_2 for the game when the terrorist moves first. For the particular parameter values used in Figure 4-5, the defender is not deterred from investing in protection from terrorism, in contrast to Figure 4-4, in which the terrorist is deterred. Like Figure 4-4, Figure 4-5 is roughly similar to Figure 4-3, except that T is not inverse U shaped, and instead is always increasing in b_2. The terrorist's utility also increases in b_2. This means that when the terrorist moves first and b_2 is large, it must invest heavily to prevent being countered by heavy defensive investment after the fact. The terrorist has a first mover disadvantage when 0.5<b_2<0.79 in Figure 4-5. The range 0.5<b_2=b_w<0.79 in Table 4-2, which with the given parameters can be written as 0.63< $\frac{B/R}{b_w/r}$ <1, is thus also such, according to Figure 4-1, that both agents prefer the defender to move first.

Figure 4-6 shows the six variables as functions of b_3 for the simultaneous game when b_1=B=0.5, b_2=0.72, r=R=1, p=0.2, D=0.05. When b_3<0.82, all-hazard protection takes care of all needed protection. As b_3 increases above 0.82, all-hazard protection starts getting expensive, and it becomes preferable to use some pure terrorism protection. Hence, t_2 increases, while t_3 decreases. As b_3 increases above b_1+b_2 = 1.22, all-hazard protection vanishes as being too expensive, causing t_3=0, pure natural disaster protection t_1 jumps from 0 to 0.09, and pure terrorism protection t_2 jumps from 0.18 to 0.27. The terrorist's investment and utility

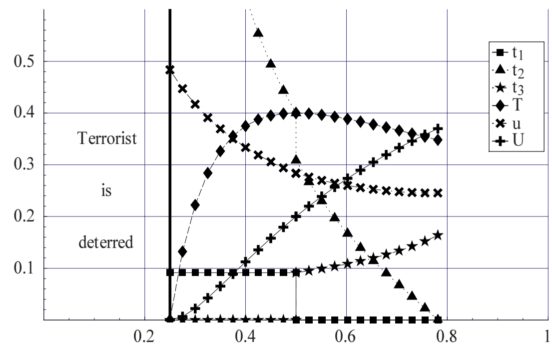

Figure 4-4. t_1, t_2, t_3, T, u, U as functions of b_2 when defender moves first

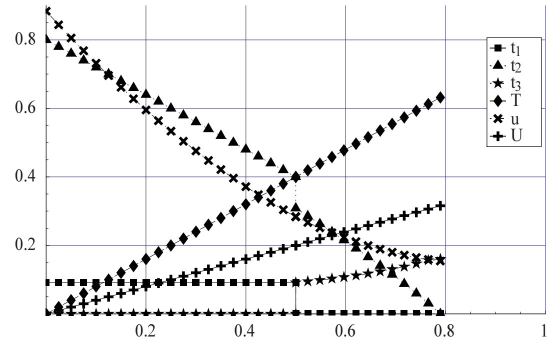

Figure 4-5. t_1, t_2, t_3, T, u, U as functions of b_2 when terrorist moves first

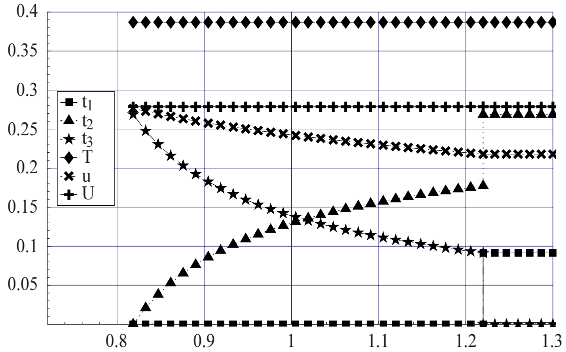

Figure 4-6. t_1, t_2, t_3, T, u, U as functions of b_3 for the simultaneous move game. No natural disaster protection when $b_3 < 1.22$

are constant, and the defender's utility decreases in b_3 over the range $0.82 < b_3 < 1.22$.

Figure 4-7 shows the six variables as functions of b_3 for the simultaneous game when $b_1=0.5$, $b_2=0.6$, $B=1.5$, $r=R=1$, $p=0.5$, $D=0.2$. (Note that the defender's utility has been divided by 3 in this figure, for scaling purposes.) When $b_3 < 0.74$, all-hazard protection takes care of all protection. As b_3 increases above 0.74, all-hazard protection starts getting expensive, and it is preferable for an alternative means of protection to take over. Whereas pure terrorism protection starts taking over in Figure 4-6, in Figure 4-7 it is preferable for pure natural disaster protection to take over. Hence, t_1 increases, while t_2 remains at $t_2=0$. As b_3 increases above b_1+b_2 = 1.1, the same logic as in Figure 4-6 applies. All-hazard protection vanishes

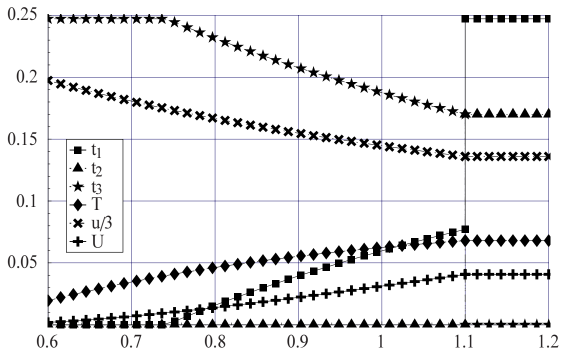

Figure 4-7. t_1, t_2, t_3, T, u, U as functions of b_3 for the simultaneous move game. No pure terrorism protection when b_3 <1.1

as being too expensive, causing t_3=0; natural disaster protection t_1 jumps from 0.08 to 0.25; and pure terrorism protection t_2 jumps from 0 to 0.17. The terrorist's investment and utility now do depend on b_3, and increase as b_3 increases, while the defender's utility decreases in b_3, as we might expect.

8. CONCLUSIONS

This chapter considers two threats, natural disaster and terrorism, from which a defender can protect through three kinds of investments. These defenses are against the disaster only, against terrorism only, or against all hazards. The defender makes tradeoffs between these three kinds of investments, under the assumption that the terrorist chooses optimally how fiercely to attack, there is a fixed probability of a natural disaster of an exogenously determined magnitude, and the defender and terrorist can have different evaluations of the asset that the defender seeks to protect.

Three kinds of games are considered: when the agents move simultaneously; when the defender moves first; and when the terrorist moves first. Conditions are shown for when each agent prefers each kind of game. Sometimes their preferences coincide, but often their preferences are opposite.

A crucial insight is that an agent advantaged with a low unit cost of investment prefers to move first, which deters its opponent from investing at all, causing maximum utility for the first mover and zero utility to the deterred second mover, who prefers to avoid this game. However, perhaps surprisingly, there are also cases in which an agent prefers to force its opponent to pre-commit to a given level of investment.

When all-hazards protection is sufficiently cheap, it jointly protects against both the natural disaster and terrorism, with no need for either pure natural disaster protection or pure terrorism protection. As the cost of all-hazards protection increases above a certain level, either pure natural disaster protection or pure terrorism protection (but not both) joins in as supportive of all-hazards protection. As the unit cost of all-hazards protection increases further, it eventually reaches a level where pure natural disaster protection and pure terrorism protection become more cost effective, at which point all-hazards protection drops to zero.

To understand the implications of these results , when protecting targets that have relatively low value to potential terrorists (if only because the terrorists can easily substitute some other target of comparable value), defenders will more often wish to invest in protection from natural disasters and/or all-hazards protection, and less often wish to invest in protection against terrorism alone. For the same reason, protections that are effective against only a relatively narrow range of terrorist threats (such as defending a specific target) may be less desirable than investments that protect against a broader range of threats (such as protecting an entire country through improved border security). However, one caveat to this conclusion is that the history of large-scale natural disasters is much longer than the history of large-scale terrorism. This suggests that any one terrorist attack contains much more "signal value" than any one natural disaster, and argues for protecting against potentially catastrophic forms of terrorism (but not necessarily against terrorist threats with lesser consequences).

A key caveat in this work is that throughout this chapter, the expenditures increase linearly in the investments. Future research may allow the expenditures to depend non-linearly on the investments, or let the unit costs depend on the levels of investment; e.g., showing diminishing marginal returns to investment in both attack and defense.

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APPENDIX 1: VERIFICATION OF SECOND-ORDER CONDITIONS FOR SECTION 3

For the terrorist's optimization problem, the second-order condition is trivial; there is only one decision variable, T, so the fact that $\frac{\partial^2 U}{\partial T^2} = -\frac{2(1-p)(t_2+t_3)R}{(t_2+t_3+T)^2} < 0$ is sufficient. For the defender's optimization problem, since there are three decision variables (t_1, t_2, and t_3), we want to see that the corresponding Hessian matrix is negative semi-definite:

$$H = \begin{bmatrix} \frac{\partial^2 u}{\partial t_1^2} & \frac{\partial^2 u}{\partial t_1 \partial t_2} & \frac{\partial^2 u}{\partial t_1 \partial t_3} \\ \frac{\partial^2 u}{\partial t_2 \partial t_1} & \frac{\partial^2 u}{\partial t_2^2} & \frac{\partial^2 u}{\partial t_2 \partial t_3} \\ \frac{\partial^2 u}{\partial t_3 \partial t_1} & \frac{\partial^2 u}{\partial t_3 \partial t_2} & \frac{\partial^2 u}{\partial t_3^2} \end{bmatrix}$$
$$= \begin{bmatrix} -\frac{2pDr}{(t_1+t_3+D)^3} & 0 & -\frac{2pDr}{(t_1+t_3+D)^3} \\ 0 & -\frac{2(1-p)Tr}{(t_2+t_3+T)^3} & -\frac{2(1-p)Tr}{(t_2+t_3+T)^3} \\ -\frac{2pDr}{(t_1+t_3+D)^3} & -\frac{2(1-p)Tr}{(t_2+t_3+T)^3} & -\frac{2pDr}{(t_1+t_3+D)^3} - \frac{2(1-p)Tr}{(t_2+t_3+T)^3} \end{bmatrix} \tag{A1}$$

In order to show that H is negative semi-definite, it is sufficient to show the following three conditions: (1) $|H_{11}| \le 0$; (2) $\begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix} \ge 0$, where $\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \equiv H$; and (3) $|H| \le 0$. The first condition obviously holds, because $|H_{11}| = H_{11} = -\frac{2pDr}{(t_1+t_3+D)^3} < 0$. The second condition also holds, since $\begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix} = H_{11}H_{22} - H_{12}H_{21} = \left[\frac{2pDr}{(t_1+t_3+D)^3}\right]\left[\frac{2(1-p)Tr}{(t_2+t_3+T)^3}\right] > 0$.
Finally, the third condition holds, since we have

$$
\begin{aligned}
|H| &= H_{11}\begin{vmatrix} H_{22} & H_{23} \\ H_{32} & H_{33} \end{vmatrix} - H_{12}\begin{vmatrix} H_{21} & H_{23} \\ H_{31} & H_{33} \end{vmatrix} + H_{13}\begin{vmatrix} H_{21} & H_{23} \\ H_{31} & H_{33} \end{vmatrix} \\
&= -\frac{2pDr}{(t_1+t_3+D)^3}\begin{bmatrix} \frac{2(1-p)Tr}{(t_2+t_3+T)^3}\left(\frac{2pDr}{(t_1+t_3+D)^3}+\frac{2(1-p)Tr}{(t_2+t_3+T)^3}\right) - \\ \left(\frac{2(1-p)Tr}{(t_2+t_3+T)^3}\right)\left(\frac{2(1-p)Tr}{(t_2+t_3+T)^3}\right) \end{bmatrix} \\
&\quad -\frac{2pDr}{(t_1+t_3+D)^3}\left[-\left(\frac{2(1-p)Tr}{(t_2+t_3+T)^3}\right)\left(\frac{2pDr}{(t_1+t_3+D)^3}\right)\right] = 0
\end{aligned}
\tag{A2}
$$

This completes the verification.

APPENDIX 2: VERIFICATION OF SECOND-ORDER CONDITIONS FOR SECTION 4

Since the terrorist's optimization problem remains the same as in Section 3, the second-order condition is still satisfied. For the defender's optimization problem in Equation (8), again we want to see that the corresponding Hessian matrix is negative semi-definite:

$$
\begin{aligned}
H &= \begin{bmatrix} \frac{\partial^2 u}{\partial t_1^2} & \frac{\partial^2 u}{\partial t_1 \partial t_2} & \frac{\partial^2 u}{\partial t_1 \partial t_3} \\ \frac{\partial^2 u}{\partial t_2 \partial t_1} & \frac{\partial^2 u}{\partial t_2^2} & \frac{\partial^2 u}{\partial t_2 \partial t_3} \\ \frac{\partial^2 u}{\partial t_3 \partial t_1} & \frac{\partial^2 u}{\partial t_3 \partial t_2} & \frac{\partial^2 u}{\partial t_3^2} \end{bmatrix} \\
&= \begin{bmatrix} -\frac{2pDr}{(t_1+t_3+D)^3} & 0 & -\frac{2pDr}{(t_1+t_3+D)^3} \\ 0 & -\frac{\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}}r & -\frac{r\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}} \\ -\frac{2pDr}{(t_1+t_3+D)^3} & -\frac{r\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}} & -\frac{2pDr}{(t_1+t_3+D)^3}-\frac{\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}}r \end{bmatrix}
\end{aligned}
\tag{A3}
$$

In order to show that H is negative semi-definite, it is sufficient to show the following three conditions: (1) $|H_{11}| \leq 0$; (2) $\begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix} \geq 0$; and (3) $|H| \leq 0$. The first condition obviously holds, because $|H_{11}| = H_{11} = -\frac{2pDr}{(t_1+t_3+D)^3} < 0$. The second condition also holds, since $\begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix} = H_{11}H_{22} - H_{12}H_{21} = \left(\frac{2pDr}{(t_1+t_3+D)^3}\right)\frac{\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}}r > 0$. Finally, the third condition also holds, since we have

$$
\begin{aligned}
|H| &= H_{11}\begin{vmatrix} H_{22} & H_{23} \\ H_{32} & H_{33} \end{vmatrix} - H_{12}\begin{vmatrix} H_{21} & H_{23} \\ H_{31} & H_{33} \end{vmatrix} + H_{13}\begin{vmatrix} H_{21} & H_{23} \\ H_{31} & H_{33} \end{vmatrix} \\
&= -\frac{2pDr}{(t_1+t_3+D)^3}\left[\begin{array}{l} \frac{\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}}r\left(\frac{2pDr}{(t_1+t_3+D)^3} + \frac{\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}}r\right) \\ -\frac{r\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}}\left(\frac{r\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}}\right) \end{array}\right] \\
&\quad -\frac{2pDr}{(t_1+t_3+D)^3}\left[-\frac{\sqrt{(1-p)B/R}}{4(t_2+t_3)^{3/2}}r\left(\frac{2pDr}{(t_1+t_3+D)^3}\right)\right] = 0
\end{aligned}
\tag{A4}
$$

APPENDIX 3: PROOF OF PROPOSITION 3

(a) The defender (weakly) prefers to move first rather than playing the simultaneous game when

$$
\begin{aligned}
&\frac{(1-p)B/R}{4b_w/r} > \frac{(1-p)(B/R)^2}{(B/R+b_w/r)^2} \Leftrightarrow \frac{1}{4b_w/r} > \frac{B/R}{(B/R+b_w/r)^2} \\
&\Leftrightarrow (B/R-b_w/r)^2 > 0
\end{aligned}
\tag{A5}
$$

which is always satisfied. Similarly, the terrorist prefers to move first rather than playing the simultaneous game when

$$
\begin{aligned}
&\frac{(1-p)b_w/r}{4B/R} > \frac{(1-p)(b_w/r)^2}{(B/R+b_w/r)^2} \Leftrightarrow \frac{1}{4B/R} > \frac{b_w/r}{(B/R+b_w/r)^2} \\
&\Leftrightarrow (B/R-b_w/r)^2 > 0
\end{aligned}
\tag{A6}
$$

This is always (weakly) satisfied. Therefore, we have shown that over the three games, both the defender and the terrorist always prefer the game in which they move first rather than a simultaneous game.

(b) When $0 < \frac{B/R}{b_w/r} < 0.5$, by Proposition 2 the defender is deterred from exerting effort, and earns zero utility against an overwhelming terrorist that moves first. Therefore the defender prefers herself moving first rather than allowing the terrorist to move first. When $\frac{B/R}{b_w/r} \geq 0.5$, the defender is not deterred in a game in which the terrorist moves first and therefore we can use Table 4-4. By Table 4-4, the defender prefers the terrorist to move first rather than herself moving first if and only if

$$\frac{(1-p)B/R}{4b_w/r} < \frac{(1-p)(2B/R - b_w/r)^2}{4(B/R)^2}$$

$$\Leftrightarrow \quad \left(\frac{B}{R} - \frac{b_w}{r}\right)\left(\frac{B}{R} - \frac{(3-\sqrt{5})b_w}{2r}\right)\left(\frac{B}{R} - \frac{(3+\sqrt{5})b_w}{2r}\right) < 0 \qquad \text{(A7)}$$

$$\Leftrightarrow \left(\frac{B}{R} - \frac{b_w}{r}\right)\left(\frac{B}{R} - 2.62\frac{b_w}{r}\right) < 0 \text{ (note that we have } \frac{B/R}{b_w/r} \geq 0.5 \text{ here)}$$

$$\Leftrightarrow 1 < \frac{B/R}{b_w/r} < 2.62$$

(c) Analogously, when $\frac{B/R}{b_w/r} > 2$, by Proposition 1 the terrorist is deterred from exerting effort, and earns zero utility against an overwhelming defender that moves first. Therefore the terrorist prefers moving first rather than allowing the defender to move first. When $\frac{B/R}{b_w/r} \leq 2$, the terrorist is not deterred in a game in which the defender moves first and therefore we can use Table 4-4. By Table 4-4, the terrorist prefers the defender to move first rather than moving first if and only if

$$\begin{aligned}
&\frac{(1-p)b_w/r}{4B/R} < \frac{(1-p)(2b_w/r-B/R)^2}{4(b_w/r)^2} \\
&\Leftrightarrow \left(\frac{B}{R}-\frac{b_w}{r}\right)\left(\frac{B}{R}-\frac{(3-\sqrt{5})b_w}{2r}\right)\left(\frac{B}{R}-\frac{(3+\sqrt{5})b_w}{2r}\right) > 0 \\
&\Leftrightarrow \left(\frac{B}{R}-\frac{b_w}{r}\right)\left(\frac{B}{R}-0.38\frac{b_w}{r}\right) < 0 \text{ (note we have } \frac{B/R}{b_w/r} \le 2 \text{ here)} \\
&\Leftrightarrow 0.38 < \frac{B/R}{b_w/r} < 1
\end{aligned} \tag{A8}$$

(d) When $0 < \frac{B/R}{b_w/r} < 0.5$, by Proposition 2 the defender is deterred from exerting effort, and earns zero utility against an overwhelming terrorist that moves first. Therefore the defender prefers the simultaneous game rather than allowing the terrorist to move first. When $\frac{B/R}{b_w/r} \ge 0.5$, the defender is not deterred in a game in which the terrorist moves first and therefore we can use Table 4-4. By Table 4-4, the defender prefers the simultaneous game rather than allowing the terrorist to move first if and only if

$$\begin{aligned}
&\frac{(1-p)(B/R)^2}{(B/R+b_w/r)^2} > \frac{(1-p)(2B/R-b_w/r)^2}{4(B/R)^2} \\
&\Leftrightarrow \quad \frac{(B/R)^2}{(B/R+b_w/r)^2} > \frac{(2B/R-b_w/r)^2}{4(B/R)^2} \\
&\Leftrightarrow \quad \frac{(B/R)}{(B/R+b_w/r)} > \frac{(2B/R-b_w/r)}{2(B/R)} \text{ (note we have } \frac{B/R}{b_w/r} \ge 0.5 \text{ here)} \\
&\Leftrightarrow \quad (B/R) < (b_w/r)
\end{aligned} \tag{A9}$$

Equation (A9) is satisfied if and only if $0.5 \le \frac{B/R}{b_w/r} < 1$. In summation we have shown that the defender prefers a simultaneous game rather than allowing the terrorist to move first if and only if $0 < \frac{B/R}{b_w/r} < 1$.

(e) When $\frac{B/R}{b_w/r} > 2$, Proposition 1 gives that the terrorist is deterred if the defender moves first, which the terrorist seeks to avoid and therefore the terrorist prefers a simultaneous game rather than allowing the defender to move first. When $\frac{B/R}{b_w/r} \le 2$, the terrorist is not deterred if the defender moves first, and then we can use Table 4-4. By Table 4-4, the terrorist prefers the simultaneous game rather than allowing the defender to move first if and only if

$$\begin{aligned}
&\frac{(1-p)(b_w/r)^2}{(B/R+b_w/r)^2} > \frac{(1-p)(2b_w/r-B/R)^2}{4(b_w/r)^2} \\
\Leftrightarrow \quad &\frac{(b_w/r)^2}{(B/R+b_w/r)^2} > \frac{(2b_w/r-B/R)^2}{4(b_w/r)^2} \\
\Leftrightarrow &\frac{(b_w/r)}{(B/R+b_w/r)} > \frac{(2b_w/r-B/R)}{2(b_w/r)} \text{ (note we have } \frac{B/R}{b_w/r} \le 2 \text{ here)} \\
\Leftrightarrow &(B/R) > (b_w/r)
\end{aligned} \tag{A10}$$

In summation, we have shown that the terrorist prefers a simultaneous game rather than allowing the defender to move first if and only if $\frac{B/R}{b_w/r} > 1$.