
IMPACTS OF SUBSIDIZED SECURITY ON STABILITY AND TOTAL SOCIAL COSTS OF EQUILIBRIUM SOLUTIONS IN AN N -PLAYER GAME WITH ERRORS

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Investment in defense by all agents is a socially optimum equilibrium in many interdependent security scenarios. However, practically, some agents might still choose not to invest in security due to bounded rationality and errors, thus decreasing the total social welfare. Previous work shows that providing subsidies may help induce more agents to invest. Our study suggests that giving subsidies to agents prone to making an erroneous choice could increase the stability of the socially optimum equilibrium, as well as decrease the total social costs.

INTRODUCTION

Homeland security has received a significant amount of attention since September 11, 2001 (Zhuang and Bier, 2007). Many security scenarios (e.g., Internet, transportation, and supply chain) involve interdependence among multiple defenders; that is, one decision-maker's choice can affect the security environment for other decision-makers. For example, lack of security on the part of one computer user, airline, or supply-chain player can increase the risk to other decision-makers. Recently, the risk associated with the attempted terrorist attack on Northwest Airlines Flight 253 was transited from Amsterdam to Detroit. Game theory has already been applied to such interdependent security problems (IDS); see, for example, Kunreuther and Heal (2003); Zhuang et al. (2007).

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In an IDS environment, agents are often at risk of both direct (external) and indirect (internal) threats. Protecting against external threats is critical to reducing the risks of contamination among agents. For example, in order to completely protect a computer network, one must protect all nodes against externally introduced viruses and prevent infected nodes from spreading the virus to the other nodes in the network. Similarly, in public health, the immunized segment of the population may not be completely protected if the risk of transmitted infection from nonimmunized individuals still exists (Heal and Kunreuther, 2007; Anikeeva et al., 2009). Similar arguments apply to the airline industry where failing to properly screen passengers and luggage at one airport or airline may threaten other airports or countries because passengers and luggage may be transported by multiple airlines over the course of a trip (Kunreuther and Heal, 2003; Nikolaev et al., 2007).

Previous game-theoretic models show that in an IDS environment, knowing the decisions of others may influence the security investment choices of individual agents, because risks are essentially shared among agents (Heal and Kunreuther, 2007). The decision to invest also depends on the cost of investment and the agents minimum acceptable rates of return (Grant et al., 1990; Fabrycky et al., 1998). For example, the work of Zhuang et al. (2007) shows that the decision not to invest in security can be a dominant strategy when the investment costs and discount factors are sufficiently high.

On the other side, subsidizing some agents can create a ripple effect of investing by the remaining agents (Dixit, 2003; Kunreuther and Heal, 2003; Heal and Kunreuther, 2007; Zhuang et al., 2007). In other words, providing some incentives to a small number of agents (e.g., government contractors) could make security investment so widespread that it becomes the norm even for firms that are not subject to such incentives. Therefore, though we model incentives for security investment as subsidies, we believe that the same results apply to other indirect subsidies such as bundling of security with other services or making security investment a requirement to be desired for certain contracts.

This article focuses on a game in which some agents choose not to invest and others choose to invest. In general, in this type of game, individuals tend to choose strategies that conform to those of their counterparts (Schelling, 1978). In practice, however, some agents may still choose not to invest even when investing is in their best interest; we denote such behavior as *erroneous choice*. To our knowledge, there is no literature that addresses the stability of equilibrium solutions and the effects of erroneous choice in interdependent security models. The contribution of this article is to develop models to evaluate the effects of erroneous choice on the stability

of the socially optimum equilibrium, and to study the impact of subsidies on the total social cost.

The N -agent model we develop in the following sections serves as a foundation to the section in which we study the stability of equilibrium solutions. Then we discuss the phenomenon of erroneous choice, and we establish a model to evaluate the impact of subsidies on total social cost. Finally, we present our conclusions, and the Appendix provides proofs to theorems in this article.

NOTATION AND MODEL FORMULATION

Let N be the number of interdependent security agents where $N \geq 2$. For simplicity, the pure strategies available to all agents are to invest and not to invest in security. We assume that agents who receive free security would choose to implement security measures. Our model revolves around agents who do not receive free security. We assume that some agents (at random) erroneously choose not to invest when investing is their interest, and the remaining agents choose in accordance with the social equilibrium strategy in light of the number of agents (without subsidy) who make erroneous choices. Following Zhuang et al. (2007), we use the following notation:

- N : Number of agents in the system, where $N \geq 2$.
- h : Number of agents receiving subsidized security, where $0 \leq h \leq N$.
- x : Number of agents making erroneous choices, where $0 \leq x \leq N - h$. There are $N - h - x$ nonsubsidized agents who do not make erroneous choices, where $0 \leq N - h - x \leq N - h$.
- $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none invest}\}_{N-h-x}$: the subequilibrium solutions describing the two possible collective behavior of the $N - h - x$ agents who do not make erroneous choices.
- L : Loss suffered by agent if it is attacked, either directly or indirectly.
- C : Cost of investing in security for an agent. We assume that $0 < C < L$.
- $r \in S_k$: Discount rate of an agent, where $r \geq 0$ and S_k is a range.
- k : The index for of discount rate sets S_k where r belongs to.
- $R_1(k)$ and $R_2(k)$: Minimum and maximum possible discount rates for which an agent would invest, respectively, given that exactly k other agents are not investing.
- s_i : Investment strategy for agent i , for $i = 1, \dots, N$, where $s_i = 1$ if agent i invests in security, and $s_i = 0$ otherwise.

- $s_{-i} \equiv \{s_j, j \neq i\}$: Set of strategies of all agents other than agent i .
- λ : Rate of direct attacks on an agent.
- q_{ij} : Probability that an attack on agent i infects agent j (where we define $q_{ii} = 1$), for $i, j = 1, \dots, N$.
- $\tilde{\lambda}_i$: Total rate of all attacks on agent i (including indirect attacks), for $i = 1, \dots, N$. In particular, we have

$$\tilde{\lambda}_i = (1 - s_i)\lambda_i + \sum_{j \neq i} (1 - s_j)q_{ji}\lambda_j \quad (1)$$

- $P_i(s_i, s_{-i})$: Total expected cost for agent i , for $i = 1, \dots, N$, given its strategy s_i and the strategies of the other agents s_{-i} .

The expected loss for agent i is given by $L_i \int_0^\infty f_i(t) \exp(-rt) dt$, where

$$f_i(t) = \tilde{\lambda}_i \exp(-\tilde{\lambda}_i t)$$

is the probability density function for the time of the first attack on agent i . Hence, the net present value of the expected loss due to attacks experienced by agent i is

$$E(\text{Loss}) = L_i \int_0^\infty \tilde{\lambda}_i \exp(-\tilde{\lambda}_i t - rt) dt = L/(1 + r/\tilde{\lambda}_i) \quad (2)$$

and the total expected cost to agent i is given by

$$P_i(s_i, s_{-i}) = s_i C + L/(1 + r/\tilde{\lambda}_i) \quad (3)$$

Using the total expected costs, we define the equilibrium solution below:

Definition 1. *An equilibrium is a set of strategies $\{s_i, i = 1, \dots, N\}$ such that all agents reach optimum given the other agents' strategies; i.e.,*

$$P_i(s_i, s_{-i}) \leq P_i(1 - s_i, s_{-i}) \quad \forall i = 1, \dots, N \quad (4)$$

Figure 1 shows the scenario where agent i invests in security but agent j does not. We assume that investment in security can protect against direct attacks but does not protect against indirect attacks (Kunreuther and Heal, 2003; Heal and Kunreuther, 2007; Zhuang et al., 2007).

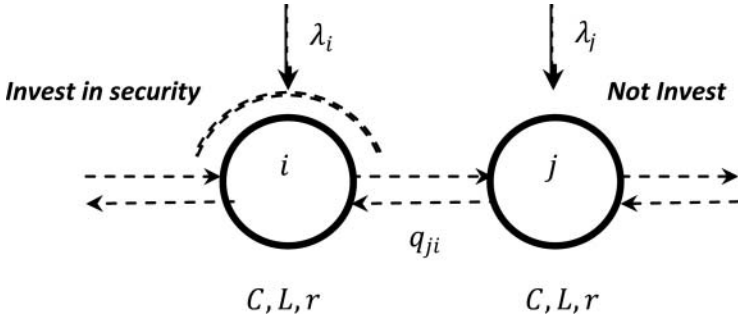


Figure 1. Model structure: Investing by agent i prevents direct attacks λ_i .

N -AGENT HOMOGENOUS MODEL WHERE $r \in S_k$

In this article we focus on a N -agent homogenous model where $r \in S_k, k = 0, 1, \dots, N$, where S_k is defined on the domain of discount rates:

$$S_l = \begin{cases} [R_2(0), \infty] & \text{for } l = 0 \\ [R_1(l - 1), R_1(l)] \cup [R_2(l), R_2(l - 1)] & \text{for } l = 1, \dots, \min(N - 1, \tilde{N}) \\ [R_1(l - 1), R_2(l - 1)] & \text{for } l = \min(N, \tilde{N} + 1) \end{cases} \quad (5)$$

where $\tilde{N} \equiv \lfloor C(L/C - 1)^2 / (4Lq) \rfloor$ and $\lfloor x \rfloor$ is the greatest integer less than or equal to x ; $R_1(k) \equiv \lambda[L/C - 1 - 2kq - \sqrt{(L/C - 1)^2 - 4qkL/C}] / 2$ for $k = 0, \dots, \tilde{N}$; and $R_2(k) \equiv \lambda[L/C - 1 - 2kq + \sqrt{(L/C - 1)^2 - 4qkL/C}] / 2$ for $k = 0, \dots, \tilde{N}$.

Here, \tilde{N} is a bound on the number of agents there can be in a system for certain properties to hold, and $R_1(k)$ and $R_2(k)$ are the minimum and maximum discount rates for which an agent would invest given that exactly others are not doing so, respectively. By taking derivatives, it is easy to show that $R_1(k)$ and $R_2(k)$ are increasing and decreasing in k , respectively. Also, note that $R_1(\tilde{N}) \leq R_2(\tilde{N})$, and $R_1(0) = 0$. Thus, the following relationship holds, as shown in Figure 2: $0 = R_1(0) < R_1(1) < \dots < R_1(\tilde{N}) \leq R_2(\tilde{N}) < \dots < R_2(1) < R_2(0)$.

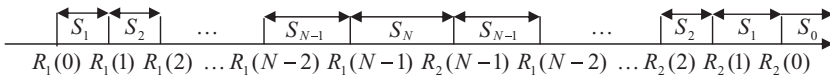


Figure 2. Illustration of the ranges S_l .

Intuitively, S_l is the set of those discount rates for which an agent would want to invest if at most $l - 1$ others do not invest but would not want to invest if l or more others do not invest. This notation is represented graphically in Figure 2 for the case where $N \leq \tilde{N} + 1$. When we have $N \geq \tilde{N} + 2$, then $R_1(k)$ and $R_2(k)$ are not defined for $\tilde{N} + 1 \leq k \leq N$, so S_l is empty for $\tilde{N} + 2 \leq l \leq N$. For convenience, we also define $\mathbf{Cl}(S_l)$ to be the closure of the open set S_l .

SUBSIDY AND STABILITY OF EQUILIBRIUM SOLUTIONS

Definition 2. *In an N -agent homogeneous model, let n be the greatest integer such that, even if n agents all change to the opposite strategy, the remaining $N - n$ agents will not want to change their strategies. We then define the stability level of an equilibrium (either $\{\text{all invest}\}$ or $\{\text{none invest}\}$) to be $\alpha = n/(N - 1)$.¹*

Remark 1. If $\alpha = 0$, then the corresponding equilibrium is completely unstable; that is, if even one agent changes strategy, at least one other agent will also prefer to change strategy. If $\alpha = 1$, the corresponding equilibrium is completely stable; that is, no matter how many agents change strategies, no other (rational) agent would want to change strategy. Note that the stability of Nash equilibrium solutions has been defined variously in other research; see, for example, Okada (1981), Kohlberg and Mertens (1986), and Damme (1991).

As indicated previously, let h be the number of agents receiving subsidized (free) security, such that $0 \leq h \leq N$. Because the h agents receiving subsidized security need not incur any cost to invest, we consider only the strategies of the $N - h$ nonsubsidized agents. Let $\{\text{all invest}\}_{N-h}$ and $\{\text{none invest}\}_{N-h}$ be subequilibrium solutions describing the possible behavior of these $N - h$ agents.

Theorem 1. *Consider a model with N homogeneous agents, and assume that a third party offers subsidized security to h agents. Then $\{\text{all invest}\}_{N-h}$ will be a subequilibrium for the $N - h$ nonsubsidized agents for any value of h . This subequilibrium has stability $\alpha = \frac{k-1}{N-h-1}$ if $h \leq N - k - 1$, and $\alpha = 1$ if $h \geq N - k$. By contrast, $\{\text{none invest}\}_{N-h}$ is a subequilibrium only if $h \leq N - k - 1$, in which case its stability is given by $\alpha = \frac{N-h-k-1}{N-h-1}$.*

¹Our definition of the stability α is closely related to what game theorists call p -dominance (see Morris et al., 1995).

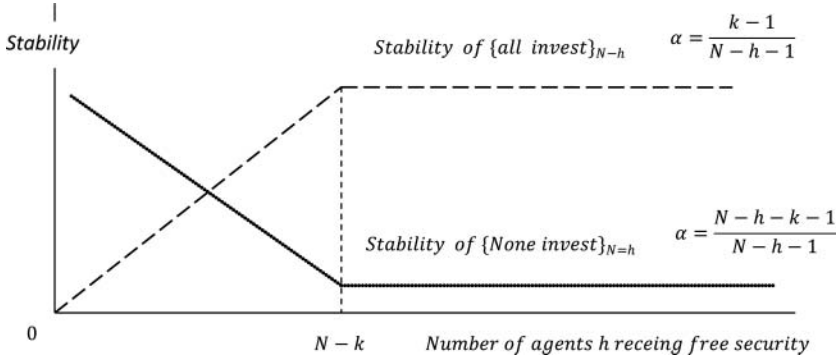


Figure 3. Stability of two subequilibria.

Remark 2. The stability of $\{\text{all invest}\}_{N-h}$ is increasing in h for $h \leq N - k - 1$ and equals 1 (i.e., completely stable) when $h \geq N - k$ agents receive subsidized (free) security. See Figure 3 for an illustration.

Theorem 2. If both $\{\text{all invest}\}_{N-h}$ and $\{\text{none invest}\}_{N-h}$ are possible subequilibrium solutions, then $\{\text{all invest}\}_{N-h}$ will be more stable² than $\{\text{none invest}\}_{N-h}$ when $k > \frac{N-h}{2}$. Conversely, $\{\text{none invest}\}_{N-h}$ will be more stable than $\{\text{all invest}\}_{N-h}$ when $k < \frac{N-h}{2}$. If $N - h$ is even, then the two subequilibrium solutions will be equally stable when $k = \frac{N-h}{2}$. (Proof follows directly from Theorem 1.)

Remark 3. Generally, $\{\text{all invest}\}_{N-h}$ will tend to be more stable than $\{\text{none invest}\}_{N-h}$ when the discount rate of the (homogeneous) agents is close to the region where investing is strictly dominant. Similarly, $\{\text{none invest}\}_{N-h}$ will tend to be more stable than $\{\text{all invest}\}_{N-h}$ when the discount rate is close to the region where not investing is strictly dominant. If $N - h$ is even, then there exists a middle range, $S_{(N-h)/2}$, where the two subequilibrium solutions are equally stable. This is illustrated in Figure 4.

ERRONEOUS CHOICE

As shown in Kunreuther and Heal (2003) and Zhuang et al. (2007), the equilibrium $\{\text{all invest}\}$ is the social optimum and moreover has lower cost than the equilibrium $\{\text{none invest}\}$ for any given agent individually. Therefore, it may be reasonable to expect that any rational agent would choose to invest in this case. However, in practice, some agents may choose

²Also sometimes called *risk dominant* (see Harsanyi and Selten, 1988).

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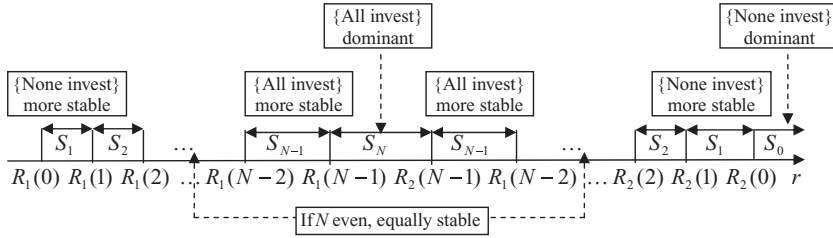


Figure 4. Stability of equilibrium solutions in an N -agent homogeneous model.

not to invest even when it would be in their interests to do so due to bounded rationality and errors. We denote such behavior an *erroneous choice*. In this section, rather than assuming that all of the $N - h$ (nonsubsidized) agents make the same choice (as in the previous section), we assume that a number of x agents erroneously choose not to invest, and the remaining $N - h - x$ agents choose strategy the subequilibrium with the lowest social cost in light of the observed number of erroneous choices. Because the h agents receiving subsidized (free) security need not incur any cost to obtain security, we consider only the strategies of the $N - h - x$ nonsubsidized agents who do not make erroneous choices. Let $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none invest}\}_{N-h-x}$ be subequilibrium solutions describing the possible behavior of the $N - h - x$ nonsubsidized agents not making erroneous choices.

We here examine the effects of erroneous choice on the subequilibrium solutions for the remaining agents who do not make errors. We also examine how subsidization of security investment can help to counteract any adverse effect of erroneous choices.

Theorem 3. *Both $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none invest}\}_{N-h-x}$ are subequilibrium solutions if and only if $x \leq k - 1$ and $h \leq N - k - 1$, respectively. If $x + 1 \leq k \leq N - h - 1$, then $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none invest}\}_{N-h-x}$ are both possible subequilibrium solutions. In this case, the total cost borne by all of the N agents individually in $\{\text{all invest}\}_{N-h-x}$ (when h agents receive subsidized security, x agents make erroneous choices, and $N - h - x$ agents invest) is lower than the corresponding cost when the $N - h - x$ agents do not invest. This implies that $\{\text{all invest}\}_{N-h-x}$ is the socially optimal subequilibrium.*

Both $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none invest}\}_{N-h-x}$ will be subequilibrium solutions if and only if $x \leq k - 1$ and $h \leq N - k - 1$, respectively. Therefore, there will always exist at least one subequilibrium solution. Moreover, if $x + 1 \leq k \leq N - h - 1$, then $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none}$

$invest\}_{N-h-x}$ are both possible subequilibrium solutions. In this case, the total cost borne by any of the N agents individually in $\{all\ invest\}_{N-h-x}$ is lower than the corresponding cost when the $N - h - x$ agents do not invest. Thus, $\{all\ invest\}_{N-h-x}$ is the socially optimal subequilibrium.

Remark 4. If those nonsubsidized agents not making erroneous choices always choose the social optimum, then they will choose to invest whenever $\{all\ invest\}_{N-h-x}$ is a subequilibrium.

Theorem 4. Suppose that each nonsubsidized agent independently makes an erroneous choice with probability ε , where $0 \leq \varepsilon \leq 1$. In this case, the number of agents X making erroneous choices is a random variable with binomial probability mass function given by $P(X = x) = \binom{N-h}{x} \varepsilon^x (1 - \varepsilon)^{N-h-x}$. Let P_{Inv} be the probability that $\{all\ invest\}_{N-h-x}$ is a subequilibrium for those nonsubsidized agents not making erroneous choices. Then, we have $P_{Inv} = 1$ if $h \geq N - k$, and $P_{Inv} = \sum_{x=0}^{k-1} \binom{N-h}{x} \varepsilon^x (1 - \varepsilon)^{N-h-x}$ if $h \leq N - k - 1$.

Remark 5. If fewer than $N - k$ agents receive subsidized (free) security, then P_{Inv} will be increasing in the number of agents h receiving free security (all else constant), in part because provision of free security to a larger number of agents reduces the maximum possible number of agents who could make erroneous choices. P_{Inv} is also increasing in k , where $r \in \mathbf{CI}(S_k)$ is the discount rate of the (homogeneous) agents; that is, as r gets closer to the region where investing is dominant (all else constant), it becomes more likely that investing will be a sub equilibrium for the $N - h - X$ nonsubsidized agents not making erroneous choices. All else constant, P_{Inv} is also decreasing in both the error probability ε and the number of agents N . The above observations are based on known properties of the binomial distribution (Bickel and Doksum, 2001).

TOTAL SOCIAL COST

In this section, we explore the effects of providing subsidized (free) security to a subset of agents on the total (expected) social cost. Let $C_F(h)$ (with $C_F(h) - C_F(h - 1) \leq C$, for $h \geq 1$) be the cost to a third party of providing subsidized (free) security to h agents, $C_x(h)$ be the cost to the x agents who make erroneous choices, $C_O(h)$ be the cost to the other $N - h - x$ agents, and $C_S(h) \equiv C_F(h) + C_x(h) + C_O(h)$ be the total (expected) social cost.

In the case described previously, (where all nonsubsidized agents make the same choice), Theorem 3 shows that there are only two possible

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subequilibrium solutions. The total cost paid by the N agents for the subequilibrium $\{\text{all invest}\}_{N-h}$ is given by

$$C_A(h) = (N - h)C \quad (6)$$

Similarly, for $\{\text{none invest}\}_{N-h}$, we have

$$C_A(h) = hC_{Sub2} + (N - h)C_{Non} \quad (7)$$

where C_{Sub2} and C_{Non} are as defined in the proof of Theorem 3. In the case described in the previous section (involving erroneous choices), if $h \geq N - k$, then we have

$$C_A(h) = \sum_{x=0}^{N-h} P(X = x) [(N - h - x)C_{Inv} + xC_{Err} + hC_{Sub1}] \quad (8)$$

and, if $h \leq N - k - 1$, then we will have

$$C_A(h) = \sum_{x=0}^{k-1} P(X = x) [(N - h - x)C_{Inv} + xC_{Err} + hC_{Sub1}] \quad (9)$$

$$+ (1 - P_{Inv}) [(N - h)C_{Non} + hC_{Sub2}]$$

where, C_{Inv} , C_{Err} , C_{Non} , C_{Sub1} , and C_{Sub2} are as defined in the proof of Theorem 3, and P_{Inv} and $P(X = x)$ are as given in Theorem 4.

Theorem 5. *In both subequilibrium solutions $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none invest}\}_{N-h-x}$ (if they exist), the total social cost $C_S(h)$ is nonincreasing in the number h of agents receiving subsidized (free) security if $C_F(h)$ satisfies $C_F(h) - C_F(h - 1) \leq C$, for $h \geq 1$. That is, agents will in general benefit from subsidized (free) security.*

Remark 6. From Theorems 3 and 5, we know that subsidization of security will decrease in general decrease the total social cost $C_S(h)$ in both subequilibrium $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none invest}\}_{N-h-x}$ and moreover will ensure that $\{\text{all invest}\}_{N-h-x}$ is the unique subequilibrium whenever $h \geq N - k$. Provided that the rate of attacks is so large that $\{\text{all invest}\}_{N-h}$ would be an equilibrium solution in the absence of errors, then extensive numerical results suggest that the total social costs $C_S(h)$ will be nondecreasing in h (as shown in Figure 5) for both the basic case and the case of erroneous choice discussed previously. However, we have not been able to prove this. In order to prove this speculation, it would be sufficient to prove that $\frac{dC_A(h)}{dh} \leq -C$, because we already know that $C_S(h) = C_F(h) + C_A(h)$.

If our speculation is true, this would suggest that in order to minimize the total (expected) social costs, all agents should receive subsidized (free) security (i.e., $h = N$) in cases where investing in security is the social optimum and security can be provided at a lower cost by a third party (such as a government) than by the agents themselves.

We now explore the effects of providing subsidized (free) security to a subset of agents on the total (expected) social cost, by considering three possible functional forms for $C_F(h)$, all of which satisfy $\frac{dC_F(h)}{dh} \leq C$, for $h \geq 1$ (for simplicity, we treat h as a continuous variable):

1. $C_F(h) = hC$; i.e., the cost to a third party of providing security is the same as the cost to the agents themselves.
2. $C_F(h)$ is increasing and concave (e.g., $C_F(h) = Ch^{0.9}$); i.e., a third party can provide security at lower cost than the individual agents could (e.g., due to economies of scale).
3. $C_F(h)$ is constant in h (e.g., $C_F(h) = 200C$); in this case, we will have $C_F(h) \leq hC$ for h sufficiently large; i.e., provision of

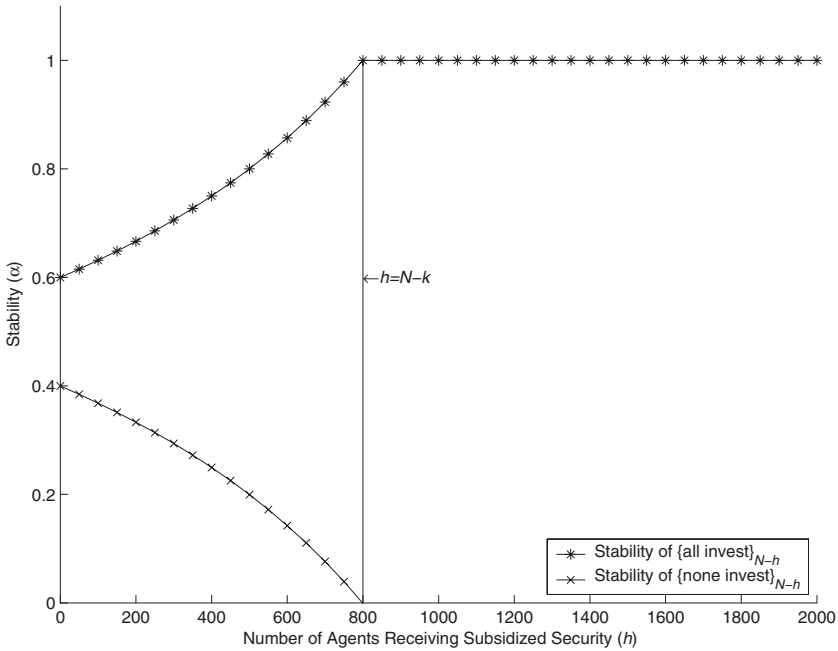


Figure 5. Total social costs of subequilibrium solutions $\{\text{all invest}\}_{N-h}$ and $\{\text{none invest}\}_{N-h}$.

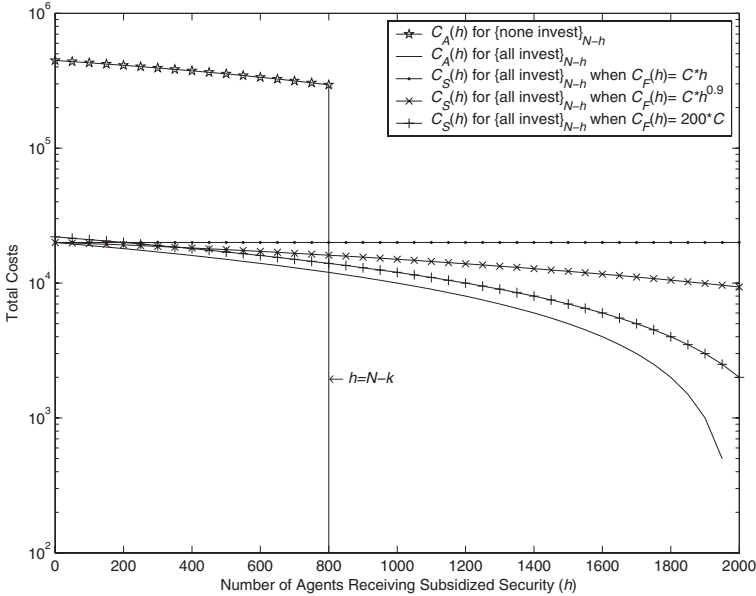


Figure 6. Stability of the subequilibrium solutions $\{\text{all invest}\}_{N-h}$ and $\{\text{none invest}\}_{N-h}$.

security by a third party is cost-effective only for systems with a large number of agents. This is a bounding case, in which once security technology (e.g., antivirus software) has been developed, it can be provided to any number of agents at no additional cost. In this case, it would be clearly optimal to give free security to all agents, provided that the constant $C_F(h)$ is sufficiently small relative to $C_A(0)$.

Example. In this example, we now numerically explore the effects of offering subsidized (free) security for the case where all nonsubsidized agents make the same choice, using the following parameters: $C = 10$, $L = 1000$, $q = 0.01$, $\lambda = 0.01$, $k = 1200$, and $N = 2000$. Figure 6 shows stability of the subequilibrium solutions $\{\text{all invest}\}_{N-h}$ and $\{\text{none invest}\}_{N-h}$ as a function of h . When no agents receive subsidies security ($h = 0$), the stabilities of the subequilibrium solutions $\{\text{all invest}\}_{N-h}$ and $\{\text{none invest}\}_{N-h}$ equal 0.6 and 0.4, respectively. For $0 < h \leq N - k - 1 = 799$, providing h subsidized (free) security convexly increases the stability of $\{\text{all invest}\}_{N-h}$ ($\alpha = \frac{k-1}{N-h-1} = \frac{1199}{1999-h}$) and concavely decreases the

stability of $\{\text{none invest}\}_{N-h}$ ($\alpha = \frac{N-h-k-1}{N-h-1} = \frac{799-h}{1999-h}$), as predicted in Theorem 1. When $h \geq N - k = 800$, the stability of $\{\text{all invest}\}_{N-h}$ reaches its maximum 1 and the stability of $\{\text{all invest}\}_{N-h}$ reaches its minimum 0 (actually $\{\text{all invest}\}_{N-h}$ is no longer an equilibrium in this case).

Figure 5 shows the cost $C_A(h)$ actually borne by the agents as a function of h for two subequilibrium solutions $\{\text{none invest}\}_{N-h}$ and $\{\text{all invest}\}_{N-h}$. In particular, such total costs are extremely high for unfavorable subequilibrium $\{\text{none invest}\}_{N-h}$ if it exists under the condition $h \leq 799$. Figure 5 also shows the total social cost $C_S(h)$ as a function of h in $\{\text{all invest}\}_{N-h}$ for three different assumptions about $C_F(h)$. Note that $C_S(h)$ is nonincreasing in all three cases. (By contrast, for the subequilibrium $\{\text{none invest}\}_{N-h}$, $C_S(h)$ is decreasing in all three cases. However, the results for $\{\text{none invest}\}_{N-h}$ are not shown in Figure 5, because for $\{\text{none invest}\}_{N-h}$, $C_S(h)$ is approximately equal to $C_A(h)$ in all three cases, so it would not be clearly visible in the figure.) When $C_F(h) = C * h$, the total social cost $C_S(h)$ keeps constant because the third party's cost is exactly the subsidy-receiver's benefits. In the other two cases ($C_F(h) = C * h^{0.9}$ and $C_F(h) = 200 * C$) where the marginal subsidy costs for the third party decrease in h , the total social costs concavely decrease in h .

CONCLUSIONS

Providing subsidies for some agents to invest in security may increase the stability of socially optimum equilibrium solutions and decrease the total social cost of security. We investigate the effects of providing subsidized (free) security on both the stability of equilibrium solutions and the total social cost, in the case of homogeneous agents. Results show that subsidization can increase the stability of the socially optimum equilibrium solution in which all agents invest, reduce or eliminate the adverse effect of erroneous choices or the behavior of agents not making erroneous choices, and decrease the total (expected) social cost of achieving the social optimum.

Our work suggests that under appropriate circumstances, providing subsidized security to some agents will (1) ensure that even agents for whom not investing would otherwise be dominant do actually invest (through careful targeting of the subsidies to those agents); (2) lead to tipping and cascading, thereby causing additional agents to invest; (3) increase the stability of the socially optimum equilibrium in which all agents invest; (4) counteract the effects of erroneous choices; and (5) decrease (expected) total social costs. Thus, it might sometimes be worthwhile for third parties (such as governments) to subsidize the provision of security or otherwise

ensure that the strategy of investing in security is adopted when it is the social optimum, because that might not otherwise occur. Our study demonstrates that giving those subsidies to agents prone to making erroneous choices could increase the stability of the socially optimum equilibrium solutions, as well as decrease the total social costs.

We acknowledge that our model is static; i.e., assuming that all agents move simultaneously, or at least no agent knows the others' decision at the time she makes her own decision. However, in practice, the sequence of moves and information sharing (Zhuang and Bier, 2009; Zhuang et al., 2010) do play important roles, especially in the situations where multiple equilibrium solutions exist and agents compete for some common resources. For example, agents may strategically pretend to be error-making agents in order to receive free security and thus the third party would have to be more selective when choosing an agent to subsidize. Future research in more dynamic settings of interdependent security games would be interesting.

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APPENDIX

Proof of Theorem 1

Let $Inv(\cdot)$ and $Non(\cdot)$ represent the set of possible discount rates for investing and noninvesting agents. $h \leq N - k - 1$, then after $k - 1$ agents change from investing (in the subequilibrium $\{\text{all invest}\}_{N-h}$) to not investing, $\{\text{all invest}\}_{N-h-k+1}$ is still a subequilibrium for the remaining $N - h - k + 1$ agents, because $r \in \mathbf{CI}(S_k) \subset Inv(N - k + 1)$. However, after k agents change from investing to not investing, $\{\text{all invest}\}_{N-h-k}$ is no longer a subequilibrium for the remaining $N - h - k$ agents, because $r \in \mathbf{CI}(S_k) \not\subset Inv(N - k)$. So, $n = k - 1$ is the largest number of agents that can change strategies such that the remaining agents will want to continue investing at subequilibrium. Therefore, $\{\text{all invest}\}_{N-h}$ has stability $\alpha = (k - 1)/(N - h - 1)$. Similarly, it can be shown that $\{\text{none invest}\}_{N-h}$ has stability $\alpha = (N - h - k - 1)/(N - h - 1)$ if $h \leq N - k - 1$. Now we consider the case $h \geq N - k$. In this case, $\{\text{all invest}\}_{N-h}$ is a

subequilibrium for the nonsubsidized $N - h$ agents and has stability $\alpha = 1$ (because for $h \geq N - k$, it is no longer possible to have k of the $N - h$ nonsubsidized agents change strategies). To see why $\{\text{none invest}\}_{N-h}$ is not a subequilibrium in this case, note that if all of the $N - h$ nonsubsidized agents choose not to invest, there will be only h agents having security measures. From table 3 in Zhuang et al. (2007), we know that $\{\text{none invest}\}_{N-h}$ will be a subequilibrium for the $N - h$ nonsubsidized agents if and only if $r \in \mathbf{CI}(S_k) \subset \text{Non}(h)$ or, equivalently, $h \leq N - k - 1$.

Proof of Theorem 3

Because the agents are assumed to be homogeneous, by the similar argument as in Theorem 4 in Zhuang et al. (2007), we know that all of the nonsubsidized $N - h - x$ agents not making erroneous choices will choose the same strategy in any subequilibrium. If all of these $N - h - x$ agents choose to invest, then there will be a total of $N - x$ agents having security measures. From Table 3 in Zhuang et al. (2007), we know that $\{\text{all invest}\}_{N-h-x}$ will be a subequilibrium for the $N - h - x$ nonsubsidized agents not making erroneous choices if and only if $r \in \mathbf{CI}(S_k) \subset \text{Inv}(N - x)$ or, equivalently,

$$k \geq x + 1 \quad (\text{A1})$$

In this case, table 1 in Zhuang et al. (2007) indicates that the total cost borne by each of the agents receiving subsidized (free) security is given by

$$C_{\text{Subl}} \equiv L / [1 + r / (xq\lambda)].$$

The total cost borne by each of the nonsubsidized $N - h - x$ agents not making erroneous choices is given by $C_{\text{Inv}} = C + L / [1 + r / (xq\lambda)]$, where $C > 0$ is the cost for any one agent to invest in security and the total cost borne by each of the x agents making erroneous choices is given by

$$C_{\text{Err}} \equiv L / \{1 + r / [\lambda + (x - 1)q\lambda]\}.$$

Similarly, if all of the remaining $N - h - x$ agents choose not to invest, then there will be only h agents having security measures. Again, from table 3 in Zhuang et al. (2007), $\{\text{none invest}\}_{N-h-x}$ will be a subequilibrium for these $N - h - x$ agents if and only if $r \in \mathbf{CI}(S_k) \subset \text{Non}(h)$ or, equivalently,

$$k \leq N - h - 1 \quad (\text{A2})$$

In this case, table 1 in Zhuang et al. (2007) shows that the total cost borne by each of the agents receiving free security is given by $C_{Sub2} \equiv L/\{1 + r/[(N - h)q\lambda]\}$, and the total cost borne by any of the $N - h$ nonsubsidized agents is given by $C_{Non} \equiv L/\{1 + r/[\lambda + (N - h - 1)q\lambda]\}$.

There will always exist at least one subequilibrium, because at least one of inequalities (A1) and (A2) will hold (by the assumption that $0 \leq x \leq N - h$, and the fact that x, h, k , and N are all integers). If $h \geq N - k$, then $\{\text{none invest}\}_{N-h-x}$ will not be a possible subequilibrium solution, so $\{\text{all invest}\}_{N-h-x}$ will be the unique subequilibrium for all values of $x \leq N - h$. Conversely, if $x \geq k$, then $\{\text{all invest}\}_{N-h-x}$ will not be a subequilibrium, so $\{\text{none invest}\}_{N-h-x}$ will be the unique subequilibrium for all values of $h \leq N - k - 1$. Finally, if $x + 1 \leq k \leq N - h - 1$, then both $\{\text{all invest}\}_{N-h-x}$ and $\{\text{none invest}\}_{N-h-x}$ will be subequilibrium solutions. In this case, straightforward algebra shows that we will have $C_{Inv} < C_{Non}$, $C_{Err} \leq C_{Non}$, and $C_{Sub1} \leq C_{Sub2}$. Thus, the costs borne by any of the N agents individually in the subequilibrium $\{\text{all invest}\}_{N-h-x}$ will be less than or equal to the corresponding costs in the subequilibrium $\{\text{none invest}\}_{N-h-x}$.

Proof of Theorem 4

From Theorem 3, if $h \geq N - k$, then we must have $P_{Inv} = 1$, because $\{\text{all invest}\}_{N-h-x}$ is the unique subequilibrium for any value of x in that case. From inequality (A1), if $h \geq N - k$, then the probability that $\{\text{all invest}\}_{N-h-x}$ is a subequilibrium is given by $P_{Inv} = P(X \leq k - 1)$.

Proof of Theorem 5

For the subequilibrium solution $\{\text{all invest}\}_{N-h-x}$ (if it exists), the number of agents having security measures will be $N - x$. Then using table 1 in Zhuang et al. (2007) (but subtracting the investment cost C for those h agents that receive subsidized security), we will have $C_x(h) = xL/(1 + r/\tilde{\lambda})$ for $\tilde{\lambda} = \lambda + (x - 1)q\lambda$, and note that $C_x(h)$ is independent of h in this particular case. We also have $C_o(h) = (N - h - x)[C + L/(1 + r/\tilde{\lambda})]$ for $\tilde{\lambda} = xq\lambda$. Similarly, for the subequilibrium solution $\{\text{none invest}\}_{N-h-x}$ (if it exists), the number of agents having security measures in place will be given by h . Then using table 1 in Zhuang et al. (2007) (but again subtracting the investment cost C for the subsidized agents), we will have $C_x(h) = xL/(1 + r/\tilde{\lambda})$ for $\tilde{\lambda} = \lambda + (N - h - 1)q\lambda$, and $C_o(h) = (N - h - x)L/(1 + r/\tilde{\lambda})$ for $\tilde{\lambda} = \lambda + (N - h - 1)q\lambda$.

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For the subequilibrium $\{\text{all invest}\}_{N-h-x}$ (if it exists), $C_x(h) = C_x(h - 1)$. Therefore, for $h \geq 1$, we will have:

$$\begin{aligned} C_S(h) - C_S(h - 1) &= C_F(h) + C_x(h) + C_O(h) - C_F(h - 1) - C_x(h - 1) - C_O(h - 1) \\ &= [C_F(h) - C_F(h - 1)] - \left[C + \frac{L}{1 + r/xq\lambda} \right] \\ &\leq C - C \\ &= 0 \end{aligned}$$

Therefore, we will have $C_S(h)$ nonincreasing in h in this case. For the subequilibrium $\{\text{none invest}\}_{N-h-x}$ (if it exists), similar to the proof of Theorem 3 in Zhuang et al. (2007), we can show that $r \leq R_2(0)$ or, equivalently, $C - L/(1 + r/\lambda) \leq 0$. Therefore, for $h \geq 1$, we will have:

$$\begin{aligned} C_S(h) - C_S(h - 1) &= C_F(h) + C_x(h) + C_O(h) - C_F(h - 1) - C_x(h - 1) - C_O(h - 1) \\ &= (C_F(h) - C_F(h - 1)) + \frac{(N - h)L}{1 + r/[\lambda + (N - h - 1)q\lambda]} \\ &\quad - \frac{(N - h + 1)L}{1 + r/[\lambda + (N - h)q\lambda]} \\ &\leq C + \frac{(N - h)L}{1 + r/[\lambda + (N - h)q\lambda]} - \frac{(N - h + 1)L}{1 + r/[\lambda + (N - h)q\lambda]} \\ &= C - \frac{L}{1 + r/[\lambda + (N - h)q\lambda]} \\ &\leq C - \frac{L}{1 + r/\lambda} \\ &\leq 0 \end{aligned}$$

Therefore, we will have $C_S(h)$ nonincreasing in h in this case.

BIOGRAPHICAL SKETCH

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