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Subsidizing to disrupt a terrorism supply chain—a four-player game

Xiaojun Shan and Jun Zhuang*

University at Buffalo, State University of New York, NY, USA

Terrorism with weapons of mass destruction (WMDs) is an urgent threat to homeland security. The process of counter-WMD terrorism often involves multiple government and terrorist group players, which is under-studied in the literature. In this paper, first we consider two subgames: a proliferation game between two terrorist groups or cells (where one handling the black market for profits proliferates to the other one to attack, and this is modelled as a terrorism supply chain) and a subsidization game between two governments (where one potential WMD victim government subsidizes the other host government, who can interfere with terrorist activities). Then we integrate these two subgames to study how the victim government can use the strategy of subsidization to induce the host government to disrupt the terrorism supply chain. To our knowledge, this is the first game-theoretic study for modelling and optimally disrupting a terrorism supply chain in a complex four-player scenario. We find that in the integrated game, when proliferation payment is high or low, the practical terrorist group will proliferate and not proliferate, respectively, regardless of government decisions. In contrast, in the subsidization subgame between the two governments, the decision of subsidization depends on its cost. When proliferation payment is medium, the decision of subsidization depends on not only its cost but also the preparation cost and the attacking cost. Findings from our results would assist in government policymaking.

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1. Introduction

Terrorism with weapons of mass destruction (WMDs; ie, chemical, biological, radiological, nuclear weapons, and high explosives) is an urgent threat to the United States. In 1998, al-Qaeda leader Osama bin Laden stated that the acquisition of WMDs is a 'religious duty' for Muslims (McCloud and Osborne, 2001), and the WMD Commission (2005) reported that al-Qaeda has already acquired significant amounts of anthrax. After 9/11/2001, people have become more concerned about the increased likelihood of WMD attacks (Hoffman, 2006). The US has devoted much effort to counter-terrorism with regard to WMDs, such as preventing the importation of illicit nuclear materials (Wein *et al*, 2006).

One significant pattern in the development of terrorism and counter-terrorism is the collaboration both between government groups and between terrorist groups, and the competition between these two groups. For instance, when people or properties of a country are attacked outside the country, the victim country has to count on the host (non-victim) country to protect its property or people (Drakos and Gofas, 2006; Sandler and Siqueira, 2006), as well as to restrict or eliminate transnational terrorism, which is one important characteristic of large international terrorist groups such as al-Qaeda (Mickolus, 2008). Specifically for WMD threats, there might be a proliferation process among terrorist groups, which can be modelled as a terrorism supply chain and involve multiple tasks susceptible to interdiction activities such as destroying the black market for raw nuclear materials. Disrupting the upstream part in this terrorism supply chain could be a more effective mitigation strategy for the governments than fighting with WMD users directly (Brown *et al*, 2009). Using game theory and optimization (eg, stochastic programming), a number of researchers have studied network interdiction as an important method to prevent nuclear smuggling (Golden, 1978; Washburn and Wood, 1995; Cormican *et al*, 1998; Pan *et al*, 2003).

Understanding different terrorist groups is critical for devising optimal mitigation strategies against terrorism. Different terrorist groups may put different weights on religion, politics, finance, reputation, and human life, and can generally be categorized into two major types (Peters, 2001; Arce and Sandler, 2010)—practical groups such as black-market groups, and apocalyptic religion-based

*Correspondence: Jun Zhuang, Department of Industrial and Systems Engineering, University at Buffalo, State University of New York, 317 Bell Hall, Buffalo, NY 14260-2050, USA.
E-mail: jzhuang@buffalo.edu

groups. Practical groups hold specific political or financial goals, and might be relatively prone to renounce WMD development, if some or all of their goals were satisfied. By contrast, religion-based groups are generally more radical, and therefore, might be less likely to renounce their attack plan. We acknowledge that in practice, there may exist neither purely practical nor purely religious terrorist groups. Instead, large terrorist organizations may operate by units of small cells or sub-groups, each of which could be either practical or religious.

Game-theoretic models have been widely utilized in studying optimal strategies in counter-terrorism endeavours (see Sandler and Siqueira, 2009 for an extensive survey). Specifically, game theory has been used to study strategic interactions (such as conflicts and collaboration) among governments (eg, the United States, Afghanistan, China, the European Union, and former Soviet Union countries). Kunreuther and Heal (2003) studied a symmetric interdependent-security game between multiple defenders. Similarly, Zhuang *et al* (2007) investigated a dynamic interdependent-security game among multiple defenders with heterogeneous discount rates. In addition, game theory has also been widely used to study interactions between an attacker and a defender, including Sandler and Lapan (1988), Lapan and Sandler (1993), Major (2002), Woo (2002), Sandler and Arce (2003), Konrad (2004); Brown *et al* (2006), Bier *et al* (2007), Zhuang and Bier (2007), Hausken and Zhuang (2011), He and Zhuang (2012) and Shan and Zhuang (2012, 2013). Enders and Sandler (2005) considered a game between two governments and one terrorist group. However, to our knowledge, very few researchers have studied the collaboration between terrorist groups and no one has studied this complex four-player (two governments and two terrorist groups) game for modelling and optimally disrupting a terrorism supply chain.

Much effort has been devoted to understanding WMD development and mitigation. From the government's perspective, major options for mitigating WMD threats include: (1) prevention (subsidizing a host government to entice it to destroy the black market for WMDs; stopping the development of WMD threats by eliminating or civilizing some terrorist groups); (2) preparing for attacks; (3) real-time response to attacks; and (4) post-attack recovery and retaliation. Other options include media diffusion, society decentralization, and economic sanctions (see Frey, 2004). Arce and Sandler (2005) rigorously compared between defensive anti-terrorism measures (eg, preparing for WMD attacks) and proactive policies (eg, subsidizing a host government to entice it to destroy the black market for WMDs). From the terrorist group's perspective, there are three major stages in WMD threat development: (1) acquisition of crude WMD materials (Salama and Hansell, 2005); (2) production and distribution (ie, proliferation) of WMDs; and (3) the potential use of the final WMDs (ie, WMD attacks).

In this paper, we consider a subsidization game between two governments (where one potential WMD victim government subsidizes the other, who could interfere with terrorist activities) and a proliferation game between two terrorist groups or cells (where one who is handling the black market for profits proliferates to the other one to attack). We allow the two governments the following decisions: destroying the black market of WMDs for the host government and subsidizing and preparing for the victim government. For the terrorist players, the decisions are proliferation for the practical terrorist group, and attacking for the religion-based terrorist group.

Preparation can take a variety of forms, including generating redundancy in a system composed of multiple elements, increasing protection over these elements and creating false targets to divert attacks (see for example Levitin and Hausken, 2009). Preparation belongs to the category of defensive measures in counter-terrorism efforts, which includes guarding the government's assets. By contrast, prevention by destroying the black market for WMDs can be categorized as proactive or offensive. Other examples of prevention include limiting terrorists' resources, finances, safe havens, infrastructure, or sponsors (Arce and Sandler, 2005).

Figure 1 illustrates our integration of the subgames between governments and terrorist groups. One practical terrorist group T_1 chooses whether to proliferate to the religion-based terrorist group T_2, while T_2 chooses whether to attack G_2 (using WMDs if T_2 successfully obtains them from T_1, or using regular weapons otherwise). This scenario is shown in the right part of Figure 1. On the other hand, as shown in the left part of Figure 1, one country G_1 is not a potential WMD victim but may be a host country for terrorism, and chooses whether to destroy a black market for WMDs (eg, black market for nuclear materials); whereas the country G_2 (potential WMD victim) chooses whether to offer economic incentives or subsidies to G_1 to entice it to destroy the black market and whether to prepare for a WMD attack. The linkage between the two subgames is such that if the host country G_1 chooses to destroy the black market, it becomes more difficult for T_1 to proliferate to T_2, and therefore G_2 might become less threatened by a potential WMD attack from T_2. Otherwise, if G_1 chooses not to destroy the black market, G_2 might have to depend more on its own preparation, which might be costly and ineffective.

We find that in the integrated game, when proliferation payment from T_2 to T_1 is high or low, the victim government will not subsidize the host government to destroy the black market regardless of its cost to G_2. In contrast, in the subsidization subgame between the two governments, the decision of subsidization depends on its cost to G_2. When proliferation payment from T_2 to T_1 is medium, the

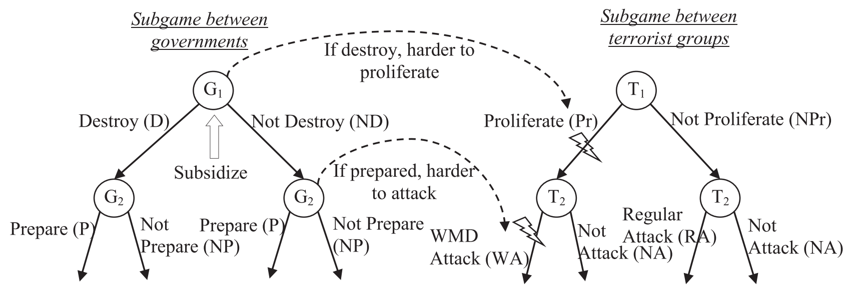

Figure 1 Overall illustration of integration of subgames between governments and terrorist groups.

decision of subsidization depends on not only its cost but also the preparation cost and the attacking cost.

Sections 2 and 3 study the subgames between two terrorist groups and between two governments, respectively. Section 4 integrates the two subgames and studies the whole dynamic process. Section 5 concludes this paper and provides some future research directions. For simplicity, we assume that each player has complete information about the game. We also assume that when the player is indifferent between two alternatives, G_1 will destroy the black market, T_1 will proliferate, G_2 will subsidize and prepare for terrorist attacks, and T_2 will attack.

2. Proliferation game between two terrorist groups

Table 1 lists notation used in this paper. We assume that $B \geqslant C$ since G_1 would not destroy the black market if the subsidy level is not sufficient to cover the cost. We assume that $v \geqslant u$ since the cost of proliferation would increase after the black market is destroyed. We assume that $L > u'$ since a WMD attack would cause more damage than the cost of its development. We assume that $L > l$ since the magnitude of damage caused by a WMD attack would be higher than that caused by a regular attack. Moreover, we assume that $R > C$, reputation loss for G_1 is greater than the cost for G_1 of destroying the black market (or the cost of subsidy from G_2 to G_1).

2.1. Game formulation

We model the subgame between two terrorist groups as a sequential game and the sequence of moves is as follows, and as illustrated in Figure 2. In particular, T_1 first chooses whether to proliferate (Pr or NPr), and T_2 chooses whether and how to attack (WA, RA, or NA). We assume that if T_1 proliferates, T_2 might launch a WMD attack (WA); otherwise, T_2 will be unable to launch a WMD attack and instead launch a regular attack (RA). We have U be the payment from T_2 to T_1 for proliferation and u' be the proliferation cost for T_1.

Figure 2 also shows the payoffs for the four possible outcomes: if T_1 proliferates, its payoff is $U-u'$. Otherwise,

Table 1 Main notation in this paper

Players	
T_1	A practical terrorist group
T_2	A radical religion-based terrorist group
G_1	A non-victim host government
G_2	A potential victim government
Decision variables	
$\{S, NS\}$	G_2 subsidizes G_1 or not
$\{D, ND\}$	G_1 destroys the black market or not
$\{Pr, NPr\}$	T_1 proliferates or not
$\{P, NP\}$	G_2 prepares for terrorist attacks or not
$\{WA, RA, NA\}$	T_2 launches a WMD attack, or a regular attack, or no attack
Parameters	
U	Proliferation payment from T_2 to T_1; we assume that $U \geqslant 0$
B	Cost of subsidy from G_2 to G_1; we assume that $B \geqslant C$
u	Proliferation cost for T_1 if G_1 does not destroy the black market; we assume that $u \geqslant 0$
v	Proliferation cost for T_1 if G_1 destroys the black market; we assume that $v \geqslant u$
u'	Proliferation cost for T_1; we assume that $u' \in \{v, u\}$and $L > u'$
C	Cost for G_1 of destroying the black market; we assume that $C \geqslant 0$
p	Preparation cost for G_2 against either WA or RA, which could include enhanced police patrol, target hardening, critical infrastructure protection; we assume that $p \geqslant 0$
a	Attacking cost for T_2 launching either WA or RA; we assume that $a \geqslant 0$
l	Loss for G_2 from a regular (non-WMD) attack; we assume that $l \geqslant 0$
L	Loss for G_2 from a WMD attack; we assume that $L > l$
R	Reputation loss for G_1 if G_1 is subsidized but does not destroy; we assume that $R > C$
$r \equiv P(L)$ or $P(l)$	Probability that a terrorist attack results in a loss for G_2 if G_2 prepares, we have $r \in [0, 1)$
Other notation	
$\{N, Y\}$	Not a possible equilibrium, and a possible equilibrium, respectively

T_1's payoff is 0. By contrast, if T_2 attacks, its payoff is $L-U-a$, or $l-a$, respectively, depending on T_1's choice of proliferating or not. Otherwise, T_2's payoff is $-U$ or 0, respectively. We assume that $L>U+l$; that is, the loss to G_2 from a WMD attack is greater than the sum of payment from T_2 to T_1 for facilitating proliferation and loss to G_2 from a regular non-WMD attack, and thus, T_2 would prefer WMD attacks if possible. Symbol N denotes a not possible equilibrium while Y denotes a possible equilibrium.

2.2. Solution

Solving the subgame between two terrorist groups presented in Section 2.1, we get the following equilibrium solutions as shown in Table 2 and Figure 3. We have the following three cases:

(1) When the proliferation payment is high ($U \geqslant u'$) and the attacking cost is relatively low ($a \leqslant L-U$), T_1 will proliferate and T_2 will launch a WMD attack.
(2) When the proliferation payment is high ($U \geqslant u'$) and the attacking cost is high ($a>L-U$), or the proliferation payment is low ($U<u'$) and the attacking cost is relatively high ($a>l$), T_1 will not proliferate and T_2 will not attack.
(3) When the proliferation payment is low ($U<u'$) and the attacking cost is low ($a \leqslant l$), T_1 will not proliferate and T_2 will launch a regular attack.

Note that [Pr, NA] is never an equilibrium, since, if T_1 chooses to proliferate, T_2 will prefer to launch a WMD attack to get a higher payoff than otherwise ($L-U-a>-U$). Therefore, an equilibrium where T_1 chooses to proliferate and T_2 chooses to attack is not possible. The equilibrium solutions are shown in Figure 3,

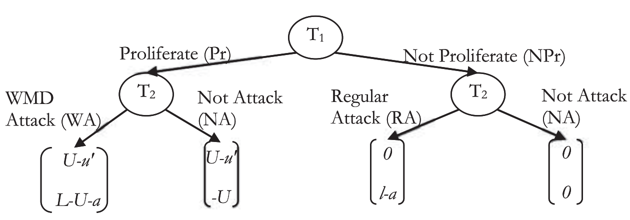

Figure 2 Game tree—subgame between two terrorist groups. (Expected payoffs in brackets are for $[T_1, T_2]$, respectively.)

there are two possibilities: the proliferation cost is (a) high ($u'>L-l$); and (b) low ($u' \leqslant L-l$).

Comparing between Figure 3 (a) and (b), we find that when the proliferation cost is high ($u'>L-l$) as in Figure 3 (a), T_1 is less likely to proliferate and T_2 is less likely to launch a WMD attack, as represented by a smaller triangular area [Pr, WA].

3. Subsidization game between two governments

3.1. Game formulation

We model the subgame between two governments as a sequential game, which is illustrated in Figure 4: G_2 first chooses whether to subsidize G_1 (S or NS); then G_1 chooses whether to destroy the black market (D or ND); and finally, G_2 chooses whether to prepare (P or NP). We assume that, when G_2 subsidizes G_1 ($B \geqslant C$), G_1 will destroy the black market and when G_1 destroys the black market, G_2 will suffer from a regular attack. Otherwise, G_2 will suffer from a WMD attack. The subsidy level B in principle could be modelled as a continuous decision variable for G_2, but its optimal value would always be C when G_2 decides to subsidize G_1, since C is the minimal amount of subsidy required in order to entice G_1 to destroy the black market. We also assume that $C<r(L-l)$, that is, the cost of destroying the black market is less than the increase in loss from a WMD attack compared to a regular attack when G_2 prepares. Similarly, symbol N denotes a not possible equilibrium while Y denotes a possible equilibrium.

Figure 4 shows the payoffs for the eight possible outcomes: given that G_1 is subsidized, if it destroys the black market, its payoff is $B-C$; otherwise, its payoff is $B-R$ since G_1 broke the contract with G_2 and thus, suffers a reputation loss of R. If G_2 prepares for a terrorist attack, its payoff is $-B-p-rL$ for a WMD attack or $-B-p-rl$ for a regular attack, where p is the preparation cost for G_2, r is the probability of loss from an attack when G_2 prepares for attacks. Similarly, if G_2 is not prepared, its payoff is $-B-L$ for a WMD attack or $-B-l$ for a regular attack. Given that G_1 is not subsidized, G_1's payoff is $-C$ if it destroys the black market; otherwise its payoff is 0; if G_2 prepares, its payoff is $-p-rL$ for a WMD attack or $-p-rl$ for a regular attack. Similarly, if G_2 does not prepare, its payoff is $-L$ for a WMD attack or $-l$ for a regular attack.

Table 2 Possible equilibria for subgame between terrorist groups

Cases	*Strategies**	*Payoffs**	*Conditions*
#1.	[Pr, WA]	$[U-u', L-U-a]$	$\{U \geqslant u', a \leqslant L-U\}$
#2.	[NPr, NA]	[0, 0]	$\{U \geqslant u', a>L-U\}$ or $\{U<u', a>l\}$
#3.	[NPr, RA]	$[0, l-a]$	$\{U<u', a \leqslant l\}$

*Strategies and payoffs in brackets are for $[T_1, T_2]$, respectively. Note that [Pr, NA] is never an equilibrium.

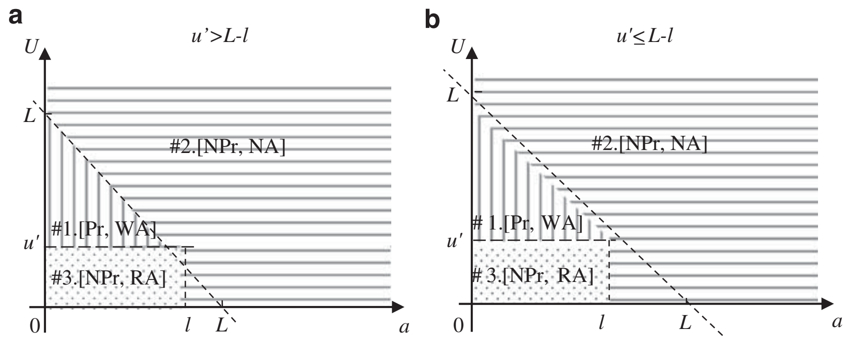

Figure 3 Three possible equilibria (as specified in Table 2) in subgame between terrorist groups as a function of a and U. (Strategies in brackets are for $[T_1, T_2]$, respectively).

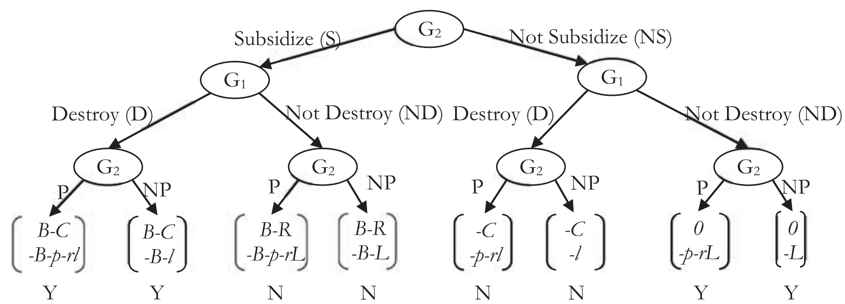

Figure 4 Game tree—subgame between governments. (Expected payoffs in brackets are for $[G_1, G_2]$, respectively.)

Table 3 Possible equilibria for subgame between two governments

Cases	*Strategies**	*Payoffs**	*Conditions*
#1.	[D, (S, P)]	$[B-C, -B-p-rl]$	$\{B \leqslant r(L-l),\ p \leqslant l(1-r)\}$
#2.	[D, (S, NP)]	$[B-C, -B-l]$	$\{B \leqslant L-l,\ p > l(1-r),\ B \leqslant p+rL-l\}$
#3.	[ND, (NS, P)]	$[0, -p-rL]$	$\{B > r(L-l),\ p \leqslant L(1-r),\ B > p+rL-l\}$
#4.	[ND, (NS, NP)]	$[0, -L]$	$\{B > L-l,\ p > L(1-r)\}$

*Strategies and payoffs in brackets are for $[G_1, G_2]$, respectively.

3.2. Solution

Solving the sequential game specified in Section 3.1, we get the following equilibrium solutions as shown in Table 3 and Figure 5. There are four cases:

(1) When the subsidy level is low ($B \leqslant r(L-l)$) and the preparation cost is low ($p \leqslant l(1-r)$), G_2 will subsidize G_1, who will destroy the black market, and G_2 will prepare for a WMD attack.
(2) When the subsidy level is relatively low ($B \leqslant \min\{L-l, p+rL-l\}$), and the preparation cost is relatively high ($p > l(1-r)$), G_2 will subsidize G_1, who will destroy the black market, and G_2 will not prepare for a WMD attack.
(3) When the subsidy level is relatively high ($B > \max\{r(L-l),\ p+rL-l\}$), and the preparation cost is relatively low ($p \leqslant L(1-r)$), G_2 will not subsidize G_1, who will not destroy the black market, and G_2 will prepare for a WMD attack.
(4) When the subsidy level is high ($B > L-l$) and the preparation cost is high ($p > L(1-r)$), G_2 will not subsidize G_1, who will not destroy the black market, and G_2 will not prepare for a WMD attack.

When the subsidy level is low ($B \leqslant r(L-l)$) or high ($B > L-l$) (Cases 1 and 4 in Table 3), G_2's decision of subsidizing G_1 does not directly depend on the preparation cost. However, the threshold for G_2's decision of preparation increases to $L(1-r)$ in Case 4 from $l(1-r)$ in Case 1. This implies that when G_2 subsidizes G_1, they are less likely to invest in the preparedness. Comparing Cases 2 and 3, when the subsidy level is medium ($r(L-l) < B \leqslant L-l$), there is interdependency between the decisions of subsidization and preparation. That is, if G_2 does not subsidize G_1, at equilibrium they will prepare. If G_2 does subsidize G_1, at equilibrium they will not prepare.

4. Integrated game

4.1. Game formulation

There are several interactions between two subgames studied in Sections 2–3. For example, the loss for G_2 may depend on T_1's decision of proliferating. The reverse is also true: T_1's payoff may depend on G_1's decision of destroying the black market. In particular, if G_1 does not destroy the black market, it will be easier for T_1 to proliferate WMDs to T_2 (ie, proliferation cost is $u<v$), thus making G_2 more threatened by a WMD attack from T_2. Moreover, G_2 could prepare for terrorist attacks and lower the expected loss from attacks and thus lower the expected gain for T_2 (eg, when T_2 launches a regular attack, G_2 will suffer a loss of l). However, if G_2 prepares for a terrorist attack, he will only suffer a loss of rl. Similarly, when T_2 launches a WMD attack, G_2 will suffer a loss of L. However, if G_2 prepares for a terrorist attack, he will only suffer a loss of rL). Therefore, we integrate these two subgames and compute overall equilibria.

The sequence of moves of the integrated game is shown in Figure 6: G_2 chooses whether to subsidize G_1 (S or NS), G_1 chooses whether to destroy the black market (D or ND), T_1 chooses whether to proliferate (Pr or NPr), G_2 chooses whether to prepare (P or NP), and finally, T_2 chooses whether to attack (WA or RA or NA). Besides assuming $B \geqslant C$, we assume that $L-U>l>rL-U>rl$. The left part of the inequality is equivalent to $L>U+l$; that is, the loss from a WMD attack is greater than the sum of payment for facilitating proliferation and loss from a regular non-WMD attack. In other words, terrorist groups would prefer WMD attacks if possible. It is reasonable to assume that the gain for T_2 from a regular attack is more than the gain for T_2 from a WMD attack when G_2 prepares ($l>rL$), which implies the middle inequality $l>rL-U$. The right inequality $rL-U>rl$ implies that the net gain for T_2 from a WMD attack when G_2 prepares after paying for proliferation is more than the gain for T_2 from a regular attack when G_2 prepares. Similarly, symbol N denotes a not possible equilibrium while Y denotes a possible equilibrium.

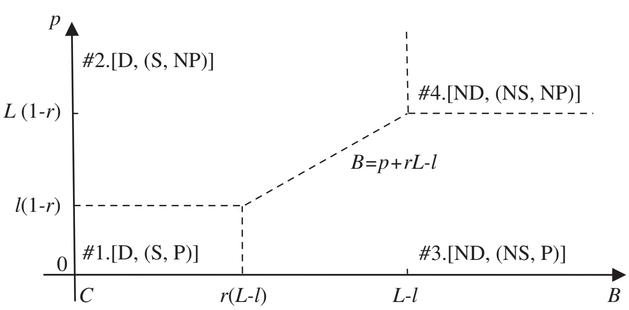

Figure 5 Four possible equilibria (as specified in Table 3) in subgame between governments as a function of B and p. (Strategies in brackets are for $[G_1, G_2]$, respectively.)

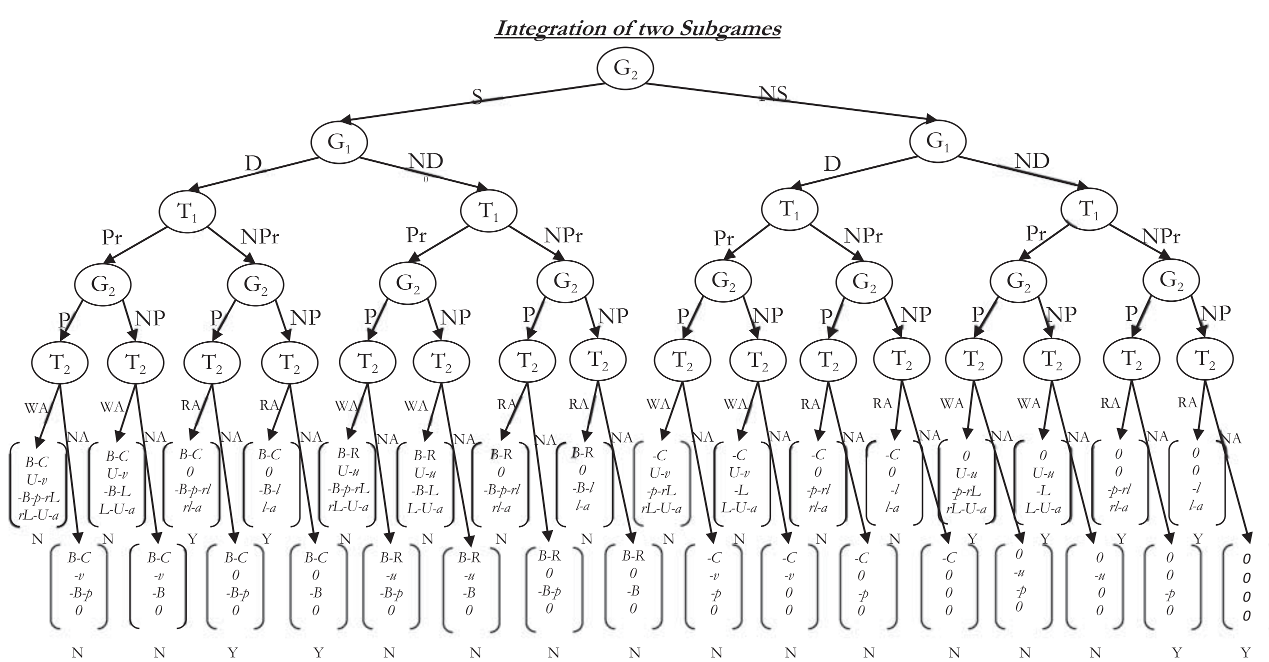

Figure 6 Game tree—integration of subgame between governments and subgame between terrorist groups. (Expected payoffs in brackets are for $[G_1, T_1, G_2, T_2]$, respectively.)

Table 4 Expected payoffs* for integrated game under eight conditions

Case 1: G_2 Subsidize G_1 (S), G_1 Destroy (D), and T_1 Proliferate (Pr)

$G_2\backslash T_2$	A WMD attack (WA)	Not attack (NA)
Prepare (P)	$[B-C, U-v, -B-p-rL, rL-U-a]$	$[B-C, -v, -B-p, 0]$
Not prepare (NP)	$[B-C, U-v, -B-L, L-U-a]$	$[B-C, -v, -B, 0]$

Case 2: G_2 Subsidize G_1 (S), G_1 Destroy (D), and T_1 Not proliferate (NPr)

$G_2\backslash T_2$	A regular attack (RA)	Not attack (NA)
Prepare (P)	$[B-C, 0, -B-p-rl, rl-a]$	$[B-C, 0, -B-p, 0]$
Not prepare (NP)	$[B-C, 0, -B-l, l-a]$	$[B-C, 0, -B, 0]$

Case 3: G_2 Subsidize G_1 (S), G_1 Not destroy (ND), and T_1 Proliferate (Pr)

$G_2\backslash T_2$	A WMD attack (WA)	Not attack (NA)
Prepare (P)	$[B-R, U-u, -B-p-rL, rL-U-a]$	$[B-R, -u, -B-p, 0]$
Not prepare (NP)	$[B-R, U-u, -B-L, L-U-a]$	$[B-R, -u, -B, 0]$

Case 4: G_2 Subsidize G_1 (S), G_1 Not destroy (ND), and T_1 Not proliferate (NPr)

$G_2\backslash T_2$	A regular attack (RA)	Not attack (NA)
Prepare (P)	$[B-R, 0, -B-p-rl, rl-a]$	$[B-R, 0, -B-p, 0]$
Not prepare (NP)	$[B-R, 0, -B-l, l-a]$	$[B-R, 0, -B, 0]$

Case 5: G_2 Not subsidize G_1 (NS), G_1 Destroy (D), and T_1 Proliferate (Pr)

$G_2\backslash T_2$	A WMD attack (WA)	Not attack (NA)
Prepare (P)	$[-C, U-v, -p-rL, rL-U-a]$	$[-C, -v, -p, 0]$
Not prepare (NP)	$[-C, U-v, -L, L-U-a]$	$[-C, -v, 0, 0]$

Case 6: G_2 Not subsidize G_1 (NS), G_1 Destroy (D), and T_1 Not proliferate (NPr)

$G_2\backslash T_2$	A regular attack (RA)	Not attack (NA)
Prepare (P)	$[-C, 0, -p-rl, rl-a]$	$[-C, 0, -p, 0]$
Not prepare (NP)	$[-C, 0, -l, l-a]$	$[-C, 0, 0, 0]$

Case 7: G_2 Not subsidize G_1 (NS), G_1 Not destroy (ND), and T_1 Proliferate (Pr)

$G_2\backslash T_2$	A WMD attack (WA)	Not attack (NA)
Prepare (P)	$[0, U-u, -p-rL, rL-U-a]$	$[0, -u, -p, 0]$
Not prepare (NP)	$[0, U-u, -L, L-U-a]$	$[0, -u, 0, 0]$

Case 8: G_2 Not subsidize G_1 (NS), G_1 Not destroy (ND), and T_1 Not proliferate (NPr)

$G_2\backslash T_2$	A regular attack (RA)	Not attack (NA)
Prepare (P)	$[0, 0, -p-rl, rl-a]$	$[0, 0, -p, 0]$
Not prepare (NP)	$[0, 0, -l, l-a]$	$[0, 0, 0, 0]$

*Expected payoffs in brackets are for $[G_1, T_1, G_2, T_2]$, respectively.

Table 4 and Figure 6 illustrate the expected payoffs for this game. For example, as in Case 1 of Table 4 and indicated by the left-most branch in Figure 6, given G_2 subsidizes G_1, G_1 destroys the black market and T_1 proliferates, if G_2 chooses to prepare and T_2 chooses a WMD attack, the payoffs for $[G_1, T_1, G_2, T_2]$ are $[B-C, U-v, -B-p-rL, rL-U-a]$.

Table 5 Equilibrium solutions for the integrated game

Cases	*Strategies**	*Payoffs**	*Conditions*
#1.	[ND, Pr, (NS, P), WA] (Case 7 in Table 4)	$[0, U-u, -p-rL, rL-U-a]$	$\{U\geqslant v, p\leqslant L(1-r), a\leqslant rL-U\}$ or $\{u\leqslant U<v, p\leqslant l(1-r), a\leqslant rl, B\geqslant r(L-l)\}$ or $\{u\leqslant U<v, l(1-r)\leqslant p\leqslant B+l-rL, a\leqslant rl\}$ or $\{u\leqslant U<v, p\leqslant l, rl\leqslant a\leqslant rL-U, B\geqslant rL\}$
#2.	[ND, Pr, (NS, NP), WA] (Case 7 in Table 4)	$[0, U-u, -L, L-U-a]$	$\{U\geqslant v, p>L(1-r), a\leqslant rL-U\}$ or $\{U\geqslant v, p>L, rL-U\leqslant a\leqslant L-U\}$ or $\{u\leqslant U<v, p>L, rL-U\leqslant a\leqslant l, B\geqslant L-l\}$ or $\{u\leqslant U<v, p>L(1-r), a\leqslant rL-U, B\geqslant L-l\}$ or $\{u\leqslant U<v, p\geqslant L, l\leqslant a\leqslant L-U, B\geqslant L\}$
#3.	[ND, NPr, (NS, P), NA] (Case 8 in Table 4)	$[0, 0, -p, 0]$	$\{U\geqslant v, p\leqslant L, rL-U\leqslant a\leqslant L-U\}$ or $\{U<u, p\leqslant l, rl<a<l\}$ or $\{U\geqslant v, p>l, rL-U<a<l\}$
#4.	[ND, NPr, (NS, NP), RA] (Case 8 in Table 4)	$[0, 0, -l, l-a]$	$\{U<u, p>l, rl\leqslant a\leqslant l\}$ or $\{U<u, p>l(1-r), a\leqslant rl\}$ or $\{u\leqslant U<v, l<p<L, rL-u<a<l\}$ or $\{U\geqslant v, l<p\leqslant L, rL-U<a<l\}$
#5.	[ND, NPr, (NS, NP), NA] (Case 8 in Table 4)	$[0, 0, 0, 0]$	$\{u\leqslant U<v, a>L-U\}$ or $\{U\geqslant v, a>L-U\}$, or $\{U<u, a>l\}$, or $\{u\leqslant U<v, p<L, l<a<L-U\}$
#6.	[D, NPr, (S, P), RA] (Case 2 in Table 4)	$[B-C, 0, -B-p-rl, rl-a]$	$\{u\leqslant U<v, p<l(1-r), a\leqslant rl, B\leqslant r(L-l)\}$
#7.	[D, NPr, (S, NP), RA] (Case 2 in Table 4)	$[B-C, 0, -B-l, l-a]$	$\{u\leqslant U<v, B+l-rL<p<L(1-r), a\leqslant rl\}$ or $\{u\leqslant U<v, p>L(1-r), a\leqslant rL-U, B\leqslant L-l\}$ or $\{u\leqslant U<v, l<p<L(1-r), rl\leqslant a\leqslant rL-U\}$ or $\{u\leqslant U<v, p>L, rL-U\leqslant a\leqslant l, B\leqslant L-l\}$
#8.	[D, NPr, (S, P), NA] (Case 2 in Table 4)	$[B-C, 0, -B-p, 0]$	$\{u\leqslant U<v, p\leqslant l, rl\leqslant a\leqslant rL-U, B\leqslant rL\}$ or $\{u\leqslant U<v, p\leqslant l, rL-U\leqslant a\leqslant l\}$
#9.	[D, NPr, (S, NP), NA] (Case 2 in Table 4)	$[B-C, 0, -B, 0]$	$\{u\leqslant U<v, p\geqslant L, l\leqslant a\leqslant L-U, B\leqslant L\}$
#10.	[ND, NPr, (NS, P), RA] (Case 8 in Table 4)	$[0, 0, -p-rl, rl-a]$	$\{U<u, p\leqslant l(1-r), a\leqslant rl\}$

*Strategies and payoffs in brackets are for $[G_1, T_1, G_2, T_2]$, respectively.

4.2. Solution

Solving the sequential game specified in Section 4.1, we get the following solution as shown in Table 5 and illustrated in Figure 7 (we only show the case of $L(1-r)>l$, the other case $L(1-r)\leqslant l$ is analogous to Figure 7 and is omitted for space consideration). There are three possibilities: (a) when the proliferation payment from T_2 to T_1 is higher than or equal to its elevated cost given the black market is destroyed ($U\geqslant v$); (b) when the proliferation payment is lower than its cost given the black market is not destroyed ($U<u$); and (c) when the proliferation payment is medium ($u\leqslant U<v$).

4.2.1. Possibility (a): $U\geqslant v$. From Table 5 and Figure 7 (a), we see that when the proliferation payment from T_2 to T_1 is higher than or equal to its elevated cost given the black market is destroyed ($U\geqslant v$), T_1 will proliferate when T_2 chooses to attack. Therefore, at equilibria, G_2 will always choose not to subsidize, G_1 will always choose not to destroy the black market and T_1 will choose to proliferate (Cases 1 and 2 in Table 5). Then, considering T_2's equilibrium strategies, we have three cases: (1) when the attacking cost is low ($a\leqslant rL-U$), T_2 will always attack. When the preparation cost is low ($p\leqslant L(1-r)$), G_2 will prepare (Case 1). (2) When the attacking cost is medium ($rL-U<a\leqslant l$), T_2's decision depends on G_2's decision. When the preparation cost is low ($p\leqslant l$), G_2 will prepare and thus T_2 will not attack (Case 3). In contrast, when the preparation cost is high ($p>l$), G_2 will not prepare and thus T_2 will attack (Case 2). (3) When the attacking cost is high ($a>l$ and $p<L$, or $a>L-U$), G_2 will not prepare and T_2 will not attack (Case 5).

4.2.2. Possibility (b): $U<u$. From Table 5 and Figure 7 (b), we see that when the proliferation payment is lower than its cost given the black market is not destroyed ($U<u$), T_1 will not proliferate. Therefore, at equilibria, G_2 will always choose not to subsidize, G_1 will always choose not to destroy the black market and T_1 will always choose not to proliferate (Cases 3, 4, 5, and 10 in Table 5). Then, considering T_2's equilibrium strategies, we have three cases: (1) when the attacking cost is low ($a\leqslant rl$), T_2 will always attack (Cases 4 and 10). When the preparation cost is low ($p\leqslant l(1-r)$), G_2 will prepare (Case 10). (2) When the attacking cost is medium ($rl<a\leqslant l$), T_2's decision depends on G_2's decision. When the preparation cost is low ($p\leqslant l$), G_2 will prepare and T_2 will not attack (Case 3). (3) When the attacking cost is high ($a>l$), G_2 will not prepare and T_2 will not attack (Case 5).

4.2.3. Possibility (c): $u \leqslant U < v$. From Table 5 and Figure 7 (c–e), there are three possibilities depending on the magnitude of B. We see that when the proliferation payment is medium ($u \leqslant U < v$), T_1's decision of proliferation depends on G_2's decision of subsidizing G_1 to destroy the black market. If G_2 chooses to subsidize G_1 to destroy the black market, T_1 will not proliferate. If T_2 will not attack, T_1 will not proliferate. Otherwise, T_1 will proliferate. One main difference among Figure 7 (c–e) is when p is low (eg, $p < l(1-r)$ or $p < B + l - rL$) and the attacking cost is low (eg, $a \leqslant rl$), the equilibrium strategies are either [D, NPr, (S, P), RA] if B is low ($B < r(L-l)$) or [ND, Pr, (NS, P), WA] if B is high ($B \geqslant r(L-l)$). Another difference is when p is low (eg, $p \leqslant l$) and the attacking cost is low (ie, $rl \leqslant a \leqslant rL - U$), the equilibrium strategies are either [D, NPr, (S, P), NA] if B is low ($B < rL$) or [ND, Pr, (NS, P), WA] if B is high ($B \geqslant rL$). Considering T_2's equilibrium strategies, we have five cases, which are illustrated in Figure 7.

(c_1) The attacking cost is extremely low ($a \leqslant rl$). In this case, T_2 will always attack. When the preparation cost is extremely low ($p \leqslant l(1-r)$), G_2 will prepare, and if the subsidy level is low ($B < r(L-l)$), G_2 will subsidize G_1 and G_1 will destroy the black market (Case 6 in Table 5). Otherwise, G_2 will not subsidize G_1 and G_1 will not destroy the black market (Case 1). When the preparation cost is relatively low ($l(1-r) < p \leqslant L(1-r)$ or $B + l - rL < p \leqslant L(1-r)$), G_2 will not prepare and in the meantime, G_2 will subsidize G_1 and G_1 will destroy the black market (Case 7). When the preparation cost is relatively high ($p > L(1-r)$), G_2 will not prepare and if the subsidy level is low ($B \leqslant L - l$), G_2 will subsidize G_1 and G_1 will destroy the black market (Case 7). Otherwise, G_2 will not subsidize G_1 and G_1 will not destroy the black market (Case 2).

(c_2) The attacking cost is relatively low ($rl < a \leqslant rL - U$). When the preparation cost is extremely or relatively low ($p \leqslant l$), G_2 will prepare, G_2 will subsidize G_1, G_1 will destroy the black market, and T_2 will not attack (Case 8 in Table 5). When the preparation cost is medium ($l < p < L(1-r)$), G_2 will not prepare, G_2 will subsidize G_1, G_1 will destroy the black market, T_1 will not proliferate and T_2 will attack (Case 7). When the preparation cost is relatively high ($p \geqslant L(1-r)$), G_2 will not prepare and if the subsidy level is low ($B \leqslant L - l$), G_2 will subsidize G_1 and G_1 will destroy the black market and T_2 will attack (Case 7). Otherwise, G_2 will not subsidize G_1 and G_1 will not destroy the black market and T_2 will attack (Case 2).

(c_3) The attacking cost is medium ($rL - U < a \leqslant l$). When the preparation cost is relatively low ($p \leqslant l$), G_2 will

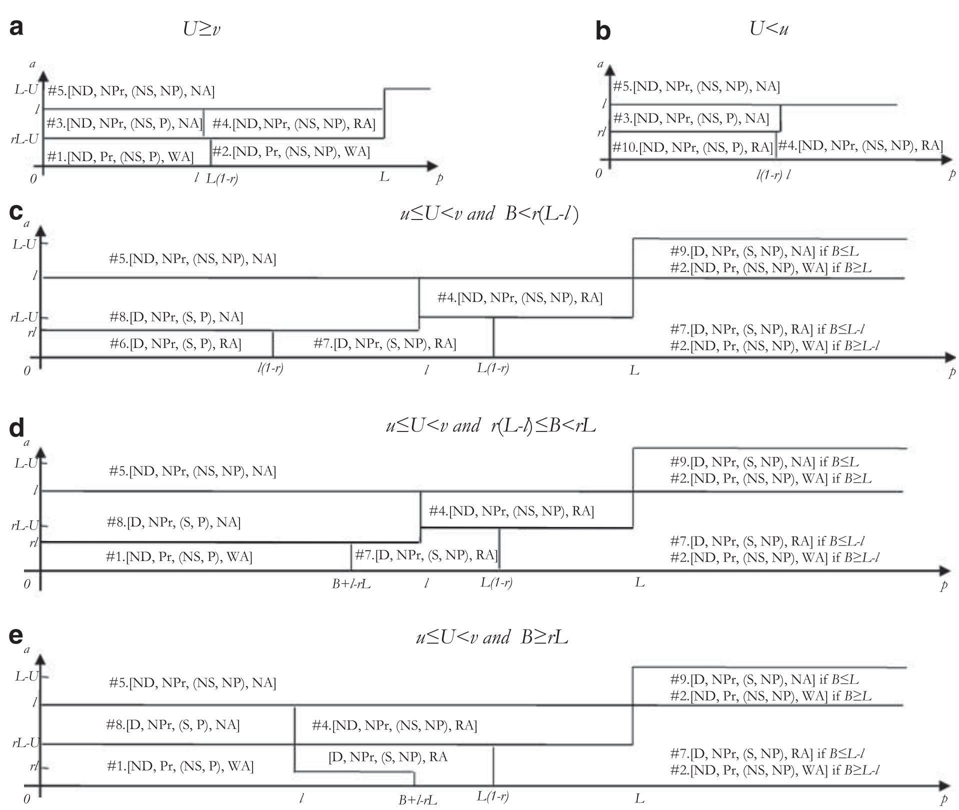

Figure 7 Possible equilibria (as specified in Table 5) in integrated game as a function of p and a when $L(1-r) > l$.

subsidize and prepare, G_1 will destroy the black market, and T_2 will not attack (Case 8 in Table 5). When the preparation cost is medium ($l<p\leqslant L$), G_2 will not prepare, G_2 will not subsidize G_1, G_1 will not destroy the black market, and T_2 will attack (Case 4). When the preparation cost is high ($p>L$), G_2 will not prepare and if the subsidy level is low ($B\leqslant L-l$), G_2 will subsidize G_1, G_1 will destroy the black market, T_2 will attack (Case 7). Otherwise, G_2 will not subsidize G_1, G_1 will not destroy the black market, T_2 will attack (Case 2).

(c_4) The attacking cost is relatively high ($l<a\leqslant L-U$). G_2 will always not prepare. When the preparation cost is relatively low ($p\leqslant L$), G_1 will not subsidize G_1, G_1 will not destroy the black market and T_2 will not attack (Case 5 in Table 5). When the preparation cost is high ($p\geqslant L$), if $B\geqslant L$, G_2 will not prepare and will subsidize G_1, G_1 will destroy the black market and T_2 will not attack (Case 9). Otherwise, if $B<L$, G_2 will not subsidize G_1, G_1 will not destroy the black market, and T_2 will attack (Case 2).

(c_5) The attacking cost is extremely high ($a>L-U$). In this case, T_2 will not attack, G_2 will not subsidize G_1 and will not prepare, and G_1 will not destroy the black market (Case 5 in Table 5).

We notice that in general, when p is high, G_2 will not prepare; when p is low, G_2 will prepare. When p is medium, G_2's decision of preparation depends on its decision of subsidization. The desired equilibrium [D, NPr, (S, NP), NA] for the victim government can be obtained when the proliferation payment is medium ($u\leqslant U<v$), the attacking cost is relatively high ($l<a\leqslant L-U$) and the preparation cost is high ($B<L\leqslant p$, Case 9 in Table 5). In other words, under certain circumstances such as $u\leqslant U<v$, $l<a\leqslant L-U$ and $B<L\leqslant p$, the victim government could use the strategy of subsidy to deter the terrorist group from attacking.

4.2.4. Branches that are not possible equilibria. There are 22 branches that are not possible equilibriums (branches marked 'N' in Figure 6). The four branches on the left (when G_2 chooses to subsidize G_1, G_1 chooses to destroy the black market, and T_1 chooses to proliferate) are not possible equilibria because when G_1 chooses to destroy, it is generally not profitable/feasible for T_1 to proliferate; and even if it was still profitable/feasible for T_1 to proliferate, it would make G_2's subsidy not worthwhile and thus G_2 would not have subsidized G_1 in the first place. Note that the middle 16 branches in Figure 6 (when G_2 chooses to subsidize and G_1 chooses not to destroy the black market, or when G_2 chooses not to subsidize and G_1 chooses to destroy the black market) are also not possible equilibria. This is because we assume that the reputation loss for G_1 is higher than the cost of destroying the black market, and as a result, if G_2 subsidizes G_1, G_1 will only choose to destroy the black market. On the other hand, if G_1 is not subsidized, G_1 has no incentive to destroy the black market. Finally, the two branches on the right with negative payoff ($-u$) for T_1 can never be reached because when it is T_1's turn to make a decision, T_1 will not choose to proliferate if that leads to a negative payoff for T_1; while choosing not to proliferate will lead to payoff 0.

5. Conclusion and future research directions

5.1. Conclusion

Terrorism with WMDs is an urgent threat to homeland security. One significant pattern in the development of terrorism and counter-terrorism is the intra-group collaboration between both governments (one potential WMD victim government subsidizes the host non-victim government, who could interfere with terrorist activities) and terrorist groups (one practical group who is handling the black market for profits proliferates to the other religion-based group to attack), and the inter-group competition between these two groups. The WMD proliferation process between terrorist groups is essentially a supply chain. Disrupting the upstream part in this terrorism supply chain could be a more effective mitigation strategy for the two governments than fighting with a WMD user (downstream terrorism supply chain player) directly. Game theory has been widely used to study strategic interactions between governments, and between one government and one terrorist group, but very few researchers have integrated them. In this paper, we studied a subsidization game between two governments and a proliferation game between two terrorist groups, and also integrated these two subgames to study how the victim government can mitigate WMD threats by subsidizing a host government to disrupt the terrorism supply chain in comparison to each subgame.

From the subsidization game between two governments, we find that when the condition is met such that the victim government subsidizes the host government for counter-terrorism activities at equilibrium, the victim government is less likely to prepare. Specifically, when the preparation cost or the subsidy level is medium, there is inter-dependency between the decisions of subsidizing and preparing. That is, given the subsidy level or the preparation cost is medium, if the victim government subsidizes the other government at equilibrium, they will not choose preparation as part of the equilibrium strategy. By contrast, from the proliferation game between the two terrorist groups, we find that the decisions of proliferation and attack depend on each other. The decision of proliferation not only depends on its reward and cost but also relates to the expected damage caused by the proliferated WMD. If the expected damage is higher, the religion-based terrorist group is more likely to attack and thus is motivated to pay the practical terrorist group to proliferate.

From the integrated game, when proliferation profit is high or low, the practical terrorist group will proliferate and not proliferate, respectively, regardless of government decisions. Note that this result could not be found from the subsidization subgame between two governments, where the decision of subsidization merely depends on its cost. By contrast, when proliferation payment is medium, subsidizing the host government (to destroy the black market) directly determines whether proliferation will occur.

When the attacking cost is low, subsidization or preparation alone is never sufficient to deter attacks. The optimal strategy for the potential victim government is to both subsidize the host government and prepare for a WMD attack given these are not too costly. Therefore, subsidization is not an entire substitute for preparation. However, when the attacking cost is relatively high and the preparation cost is medium or high, subsidization alone can effectively deter attacks. Findings from our results may assist in government policymaking.

5.2. Discussion and future research directions

As a first step towards modelling this complex scenario involving strategic interactions among four players, this paper first employs a sequential game with a certain order of movements, assuming perfect and complete information. The next step might be to allow a continuous level of costs for destroying the black market. We could also model the possible retaliation of the victim government if the host country breaks the contract and fails to destroy the black market after receiving subsidy.

For simplicity, we assume the same preparation cost and the same success probability for both a WMD attack and a regular attack. In reality, the preparation cost would be much higher against a WMD attack than that against a regular attack. Similarly, the success probability of a WMD attack will be significantly lower than that of a regular attack. Moreover, games of incomplete information where asymmetric information of cost between governments and terrorist groups could be studied to provide additional insights and better reflect the real and complicated situations of cooperation and conflict among different parties of interest in terrorism and counter-terrorism.

In this paper we mostly use binary decision variables for all the players as a first step towards tackling this important problem. However, in reality, players may make choices in a continuous manner (eg, Zhuang and Bier, 2007). For example, levels of proliferation, attack, defense, and subsidy could be continuous to model more realistic and interesting scenarios. Instead of subsidizing or not, the victim government might use a two-stage subsidy scheme (ie, first provide small amounts of subsidies and second provide additional amounts according to the subsidized government's efforts in destroying the black market). This extension might be interesting because the victim government might be able to achieve more efficient cooperation with the host government through such a two-step subsidy mechanism. Finally, we could also consider a more complex terrorism supply chain network involving more than two terrorist groups.

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