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Robustness of Optimal Defensive Resource Allocations in the Face of Less Fully Rational Attacker

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Abstract

Hundreds of billions of U.S. dollars have been spent on homeland security since September 11, 2001, while the optimality and effectiveness of those expenditures remain obscure. In this paper, we develop a numerical model a centralized defender (the government) optimal resource allocation among multiple targets, against an attacker (terrorist) who could be either strategic (i.e. rational) or non-strategic. We also study the sensitivities of the optimal defender budget allocation to: (a) the probability that the attacker is strategic and (b) the choices of non-strategic attackers.

Keywords
homeland security, non-strategic behavior, resource allocation, multiple targets

1. Introduction

Since September 11, 2001, hundreds of billions of U.S. dollars have been spent on homeland security. According to the Office of Management and Budget[1], the total outlay of the U.S. Department of Homeland Security's actual total outlay for 2007 is $38 billions and the expected total outlays for 2008 and 2009 are $42 and $44 billion, respectively. However, allocating those budgets among multiple cities, urban areas, and critical infrastructures (e.g. airports and bridges), remains a challenging task. The optimality and effectiveness of these expenditures remain obscure and have often been criticized. For example, in 2008, Prante and Bohara[2] mentioned that "The distribution of State Homeland Security Grants has been often criticized as pork barrel spending, where political considerations and not terrorism risk are determining the allocation each state receives." Similarly, Paddock[3] pointed out "In 2002, the Homeland Security Grants Program funding was distributed entirely on a formula basis. The result was that, for the next three years, the grant dollars were tied up at the state level and scarcely more than 30 percent of the funding was passed through to local first responders. In some states, that funding has never been spent."

Academic interest in terrorism and counter-terrorism strategies has also been significantly increased since September 11, 2001[4]. Several full-endogenous game-theoretic models (i.e. assuming that both the attacker and the defender are fully strategic, rational and have common knowledge about the rules of the game) in either parallel or series systems have been developed to study the system reliability (Hausken[5], [6], Bier et al.[7]). Applying their model to the real-world data from Willis et al.[8], Bier et al.[9] identifies the attacker and defender equilibrium strategies in a sequential game where the defender moves first, and conclude that the cost effectiveness of defensive investment has a significant impact on the optimal allocation of defensive resources.

In the real world, of course, attackers may not be fully strategic; that is, they may be non-strategic or irrational, for example, picking target to attack randomly. The strategic attacker will adapt his strategies in response to the defensive investment, and therefore may become less interested to carry out attacks if the target is more defended. By contrast, the non-strategic attacker may only strike certain targets (e.g. most valuable assets), regardless of the observed defense levels. Such non-strategic behavior may significantly decrease the robustness of the defender's optimal resource allocation, if the allocation is optimized under the assumption that all attackers are fully strategic.

Flynn[10] considers defending chemical facilities against both chemical accidents and terrorist attacks. Similarly, Chyba[11] points out that many public health measures intended to detect and contain contagious diseases in defending against natural outbreaks as well as deliberate bioterrorism attacks. The optimal balance between defenses against the strategic and non-strategic threats has been studied by Zhuang and Bier[12] who point out that all else equal, it is less cost effective to protect large numbers of targets from strategic terrorists than from natural disasters. Similarly, Powell[13] also studies the allocation of defensive investments between full-strategic and partially strategic attackers. Both Zhuang and Bier[12] and Powell[13] use pre-determined probabilities for non-strategic threats, but allow attack use probabilities for strategic threats to defend on the level of defensive investments.

In this paper, we expand the model from Bier et al.[9] by allowing some probability that the attacker is non-strategic. In particular, we introduce two new types of parameters: (a) the probability of an attacker being strategic; and (b) the probabilities that non-strategic attacker will attack various targets. To our knowledge, ours is the first numerical study to explore how sensitive the defender's optimal budget allocation is to these factors, using realistic data.

The next section of this paper introduces our notation, assumptions, and model. Applying the data introduced in Section 3, Section 4 tests the sensitivity of the defender's optimal budget allocation to these two new parameters (the probability of an attacker is non-strategic, and the probabilities of pre-determined attack strategies). Section 5 summarizes the previous sections, discusses the policy implications of our work, and provides some future research directions.

2. Notation, Assumptions, and Problem Formulation

2.1 Notation

We define the parameters of our model as follows:

- q and $1-q$: Probabilities that an attacker is strategic and non-strategic, respectively.
- n: Number of targets in the system.
- c_i: Defender's budget allocation to target i, for $i = 1, \cdots, n$.
- C: Total budget of the defender.
- $h_i(c_1, \cdots, c_n)$: Probability that a strategic attacker will attack target i, as a function of the defensive resource allocations to all targets, for $i = 1, \cdots, n$.
- h'_i: Probability that a non-strategic attacker will attack target i, for $i = 1, \cdots, n$.
- $L(c_1, \cdots, c_n)$: Total expected loss due to terrorism.
- λ: Cost effectiveness parameter of defensive investment.
- $p_i(c_i)$: Success probability of an attack on target i, as a function of the budget allocated to that target, c_i. Following Bier et al.[14], we assume that is exponentially distributed with parameter λ; i.e. $p_i(c_i) = e^{-\lambda c_i}$, for $i = 1, \cdots, n$.
- x_i: Defender's valuation of target i, for $i = 1, \cdots, n$.
- y_i: Attacker's valuation of target i, for $i = 1, \cdots, n$.

2.2 Assumptions

Following Powell[13] and Bier et al.[9], we assume that a fully-strategic defender wishes to allocate a total budget of C among n targets, $(c_1, \cdots, c_n)$ such that $\sum_{i=1}^{n} c_i = C$. A strategic attacker observes the allocation, and then chooses a set of attack probabilities $(h_1, \cdots, h_n)$, where h_i is the probability of a strategic attacker of launching an attack on target i, such that $\sum_{i=1}^{n} h_i = 1$. Following Powell[13] and Bier et al.[9], we assume that the attacker will choose to attack at most one location. We also assume that the defender is fully strategic, which the attacker may be either strategic or non-strategic, with probabilities q and $1-q$, respectively. A non-strategic attacker is assumed to attack the target i with the pre-determined probabilities h'_i, such that $\sum_{i=1}^{n} h'_i = 1$. We assume that the strategic attacker wants to maximize the expected damage $L(c_1, \cdots, c_n)$, while the defender wants to minimize it.

As in Bier et al.[9] we assume that the attacker's valuation of target i, y_i follows a two-parameter Rayleigh distribution, with its mean value equaling to the defender's valuation x_i. The two-parameter Rayleigh distribution has been used effectively in modeling strength and lifetime data. Importantly for our purposes, the cumulative Rayleigh distribution is not only closed form, but also integrable.

2.3 Problem Formulation

We model the attacker and defender interactions in a sequential game where the defender plays first. The defender's objective is to minimize the total expected loss from an attack by assigning a portion of defensive budget C to each target i , $c_1, \cdots, c_n$, for $i = 1, \cdots, n$. That is,

$$\begin{aligned} \min_{c_1,\cdots,c_n} \quad & L(c_1,\cdots,c_n) && (1)\\ = \quad & q\sum_{i=1}^{n} h_i(c_1,\cdots,c_n)p_i(c_i)x_i + (1-q)\sum_{i=1}^{n} h_i' p_i(c_i)x_i && (2)\\ = \quad & \sum_{i=1}^{n} p_i(c_i)x_i[qh_i(c_1,\cdots,c_n)+(1-q)h_i'] && (3)\\ s.t. \quad & \sum_{i=1}^{n} c_i = C && (4) \end{aligned}$$

where $f_i(y_i) = 2(\frac{y_i-\varepsilon_i}{\sigma_i^2})e^{(\frac{y_i-\varepsilon_i}{\sigma_i})^2}$ and $F_i(y_i) = 1-e^{(\frac{y_i-\varepsilon_i}{\sigma_i})^2}$, and ε_i is the lowest possible value of y_i, which satisfying $\varepsilon_i = x_i(1-\frac{cv\sqrt{\pi}}{\sqrt{4-\pi}})$. Following Bier et al.[9], we assume all the y_i's have the same coefficient of variance (cv), so $\sigma_i = \frac{2x_i cv}{\sqrt{4-\pi}}$. Assuming all y_i are independent, the probability that the attacker will attack target i is given by

$$h_i(c_1,\cdots,c_n) = \int_{\varepsilon_i}^{\infty} f_i(y_i)\prod_{j\neq i} F_i[\frac{p_i(c_i)y_i}{p_j(c_j)}]dy_i \quad (5)$$

For sufficiently large coefficients of variation, $cv > \sqrt{\frac{4}{\pi}-1}$, the minimum value ε_i will be negative. It is assumed that no attack will be made when targets have negative valuation to the attacker.

3. Data Sources

According to Willis et al.[8], the ten urban areas with the highest expected damage from terrorism are: New York; Chicago; San Francisco; the Washington DC, area (including parts of Maryland, Virginia, and West Virginia); Los Angeles and Long Beach; the Philadelphia area (including parts of New Jersey); the Boston area (including parts of New Hampshire); Houston; Newark; and the Seattle area (including Bellevue and Everett). Following Bier et al.[9], we restrict our analysis to these ten urban areas for the purpose of computational tractability. Table 1 shows the expected damages from Willis et al.[8] and the budget allocated to those ten areas from the Office of Grants and Training, U.S. Department of Homeland Security [15]. Since the data on expected damages from Willis et al.[8] are from 2004, we use the FY2004 UASI Grant Allocation as the budget to be allocated. However, data from 2008 is available at [1] and ready to be used.

We assume that the defender valuations of these ten cities, x_i, are given first by the expected property losses (column 2 in Table 1), and then by the expected fatalities (column 3 in Table 1).

4. Sensitivity of Percentage of Strategic Attacker on Optimal Budget Allocation

We apply the model developed in Section 2 to the data source discussed in Section 3 and consider different levels of the probabilities of an attacker being non-strategic and their corresponding pre-determined attack choices. In particular, we let the $cv = 0.1$, following Bier et al.[9]. And we let the cost effectiveness of defensive investment $\lambda = 0.05$. (for the sensitivities of λ, see Bier et al.[9]). We use the data provided in Tables 1 and assume this information is common knowledge to non-strategic attacker. Furthermore, we consider two scenarios describing the behavior of the non-strategic attacker:

- Scenario (a) 100% probability of attacking the city with the highest attacker valuation;
- Scenario (b) 50% probability each of attacking two cities with the two highest expected property losses or fatalities.

Each case is combined with the percentage of strategic attacker q at levels of 0, 10%, 20%, 30%, ..., and 100%, respectively.

Table 1: Expected property losses, fatalities, and UASI budget allocations for the ten urban areas with the highest losses

Urban Area	Expected Property Losses [8] ($ million)	Expected Fatalities [8]	FY 2004 UASI Grant Allocations [15] ($ million)
New York	413	304	47
Chicago	115	54	34
San Francisco	57	24	26
Washington, DC-MD-VA-WV	36	29	29
Los Angeles-Long Beach	34	17	40
Philadelphia, PA-NJ	21	9	23
Boston, MA-NH	18	12	19
Houston	11	9	20
Newark	7.3	4	15
Seattle-Bellevue-Everett	6.7	4	17
Total	719	466	270

4.1 Using Expected Property Losses as the Measure of Target Attractiveness

Figure 1(a) shows the result of case (a), when the non-strategic attacker is assumed to attack the city with the highest expected property loss, which is New York, with the probability of 100%. As showing in Figure 1(a), when $1-q=0$, the attacker is fully strategic and the optimal defense allocation is well spread over the ten cities. As the value of $1-q$ is increasing, we can see there is more money being transferred to New York from other cities at optimality. Eventually when $1-q=1$, the defender knows that attacker is surely non-strategic and will only attack New York, all the money goes to New York.

Figure 1(b) shows the result of scenario (b), when the non-strategic attacker is assumed to attack the first two cities with the highest expected property losses, which are New York and Chicago, with the probability of 50% of each. Similar to case (a), as value of $1-q$ is increases, the optimal defense allocation eventually goes to New York and Chicago.

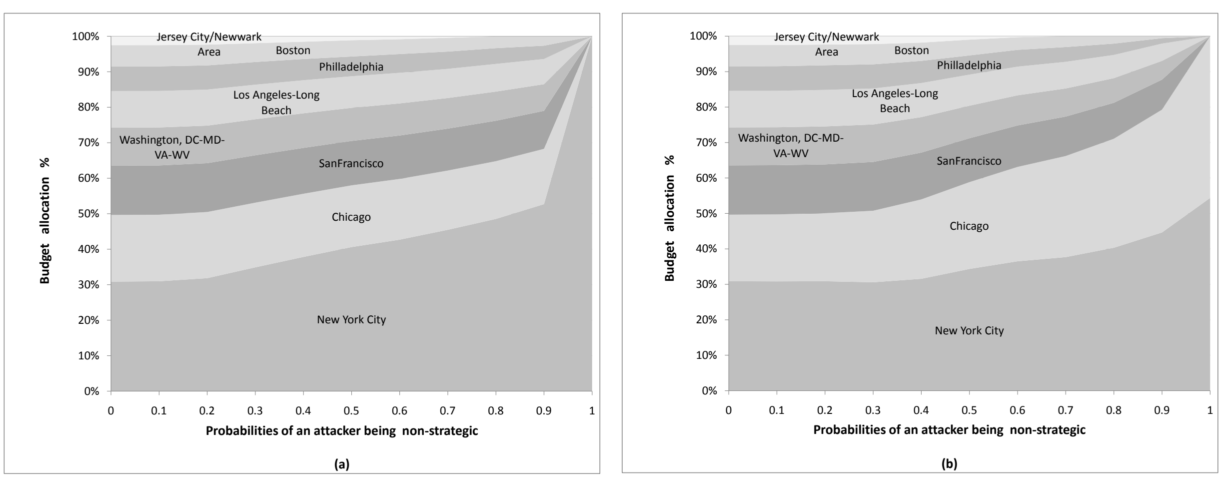

Figure 1: Optimal budget allocation as a function of the probability for an attacker to be non-strategic (using property losses as the measure of target attractiveness)

4.2 Using Expected Fatalities as the Measure of Target Attractiveness

Figure 2(a) shows the result of scenario (a), when the non-strategic attacker is assumed to attack the city with the highest fatalities, which is New York City, with the probability of 100%. Similar to Figure 1(a), as value of $1-q$ is increasing, all the money eventually goes to New York City.

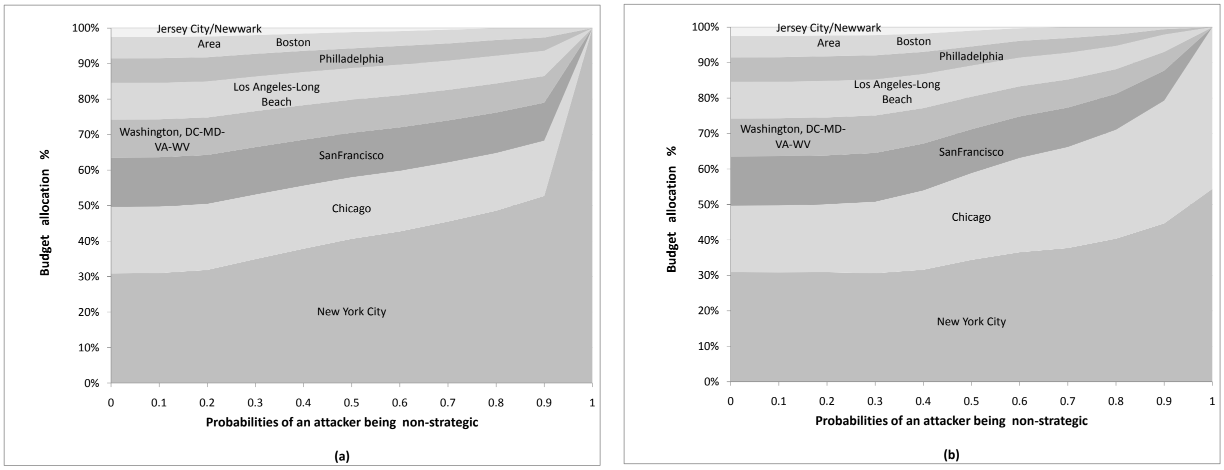

Figure 2: Optimal budget allocation as a function of the probability for an attacker to be non-strategic (using expected fatalities as the measure of target attractiveness)

Figure 2(b) shows the result of scenario (b), when the non-strategic attacker is assumed to attack the first two cities with the highest expected fatalities, which are New York City and Chicago, with the probability of 50% of each. Analogous to Figure 2(a), as value of $1-q$ is increasing, all the money eventually goes to New York City and Chicago.

In summary, from Figures 1-2, we see that the defender's optimal budget allocation is sensitive to the probability for attacker to be non-strategic. As this probability increases, the defender's optimal budget allocation eventually goes to targets being considered to be pre-determined chosen to attack.

5. Summary and Future Research Directions

Hundreds of billions of U.S. dollars have been spent on homeland security since September 11, 2001, while the optimality and effectiveness of those expenditures remain obscure. In this paper, we develop a numerical model to determine the centralized defender (government) optimal resource allocation among multiple targets, against an attacker (terrorist) who could be either strategic (endogenous or rational) or non-strategic. We also study the sensitivities of (a) the probability of an attacker being strategic and (b) the non-strategic attacker's corresponding pre-determined choices, on the optimal defender budget allocation.

We find that the defender's optimal budget allocation is sensitive to the percentage of non-strategic attacker. As the probability for an attacker being non-strategic increases, the defender's optimal budget allocation eventually goes to cities being considered to be pre-determined chosen to attack.

In the near future, this model could be extended to study the sensitivity of the percentage of non-strategic attacker upon the real defender payoffs, if the defender believes that the attacker is fully strategic, or fully non-strategic. These extensions would help evaluate the robustness of the many game-theoretical resource allocation model and the practically-used non-game-therectical resource allocation model.

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