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Dynamic Forecasting Conditional Probability of Bombing Attacks Based on Time-Series and Intervention Analysis

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In recent years, various types of terrorist attacks occurred, causing worldwide catastrophes. According to the Global Terrorism Database (GTD), among all attack tactics, bombing attacks happened most frequently, followed by armed assaults. In this article, a model for analyzing and forecasting the conditional probability of bombing attacks (CPBAs) based on time-series methods is developed. In addition, intervention analysis is used to analyze the sudden increase in the time-series process. The results show that the CPBA increased dramatically at the end of 2011. During that time, the CPBA increased by 16.0% in a two-month period to reach the peak value, but still stays 9.0% greater than the predicted level after the temporary effect gradually decays. By contrast, no significant fluctuation can be found in the conditional probability process of armed assault. It can be inferred that some social unrest, such as America's troop withdrawal from Afghanistan and Iraq, could have led to the increase of the CPBA in Afghanistan, Iraq, and Pakistan. The integrated time-series and intervention model is used to forecast the monthly CPBA in 2014 and through 2064. The average relative error compared with the real data in 2014 is 3.5%. The model is also applied to the total number of attacks recorded by the GTD between 2004 and 2014.

KEY WORDS: Bombing attack; conditional probability; intervention analysis; terrorist attack; time series

1. INTRODUCTION

There are eight known attack tactics recorded in the Global Terrorism Database (GTD),[(1)] including armed assault, assassination, barricade incident, bombing attack, hijacking, infrastructure attack, kidnapping, and unarmed assault. The most frequently used attack tactic is the bombing attack, followed by armed assault, as shown in Fig. 1.

Over 50% of global terrorist attacks are bombing attacks. A reason for this could be that it is relatively easier to make a bomb than some other weapons, such as guns. For example, bombs used in the 2013 Boston Marathon bombing attack were believed to be homemade improvised explosive devices (IEDs).[(2)] A second reason could be that bombs are easier to transport and more lethal in a large crowd than other weapons. Some bombing attacks have caused a large number of casualties. For example, the 2011 Russia Domodedovo International Airport bombing attack caused 38 deaths and 168 injures; the January 24, 2011 Iraq bombing attacks caused 35 deaths and 65 injures; the 2013 Boston Marathon bombing caused three deaths and 264 injures; and the 2014 Pakistan Jinnah International

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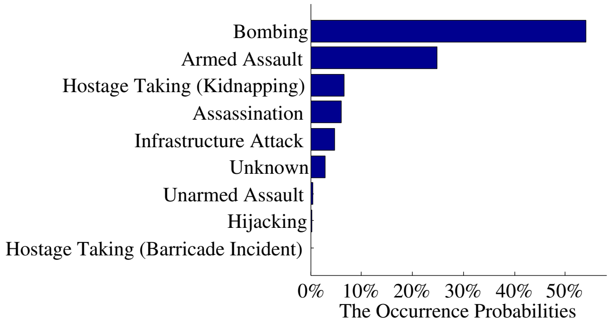

Fig. 1. The occurrence probabilities of different attack tactics from 2000 to 2014.
Source: The GTD.

Airport bombing caused 38 deaths and 22 injuries.[1] Bombing attacks usually cause widespread panic among the public. People injured by bombings often end up disabled and/or with psychic trauma. For example, Mark Fucarile and other survivors of the Boston Marathon bombing lost their limbs and have difficult lives since then.[3] With all of these facts combined, bombing attacks have become one of the most threatening tactics chosen by terrorists.

Deisler and Haimes[4,5] pointed out that risk analysis research is a useful way to reduce the risk of terrorist attacks. Past papers have developed methods to study the risk of bombing attacks happening in different scales of targets; e.g., large regions, infrastructures, or even crowds. For example, Rosoff[6] analyzed the risk of the Ports of Los Angeles and Long Beach being attacked by dirty bombs. Leung[7] studied how to protect bridges from bombing attacks as well as some other attack tactics. Imana[8] studied the damages to bodies in a crowd when suicide bombing attacks happened. These researches mainly focused on the consequence of the bombing attacks. The occurrence probabilities of different types of attacks are also key factors influencing the level of risk. Researches developed on studying the occurrence probabilities of specific types of attacks, such as arson[9] and chemical, biological, radiological, and nuclear (CBRN) attacks.[10] These kinds of studies could help defenders decide the priorities of defensive strategies against different tactics of attacks.

The changes in attack data over time help us better understand and prepare for terrorist attacks. For example, Bogen[11] used data between 1968 and 2004 to forecast the maximum victims and event rates through 2080 in Israel and other regions. Clauset[12] studied the frequency and severity of terrorist attacks worldwide since 1968 to make a forecast of the future attacks. Hausken[13] studied the crime intensity under different situations of imprisonment development in Norway, England and Wales, and the United States and made a forecast through 2103. However, it is not enough to understand the law of severe attacks or crimes in the time domain. The development about global attackers' decision making on choosing different tactics of attack is also an interesting topic needing to be studied. It could contribute to related studies on counterterrorism (e.g., the development of lethal weapons, and the effectiveness of specific weapon policy).

This article fills this gap by modeling and forecasting the conditional probability of bombing attacks (CPBAs) based on time-series and intervention analysis by using data from the GTD. The remainder of this article is structured as follows. Section 2 introduces the methods and related theories used in this research. Section 3 illustrates the process of building the model and shows the results. Section 4 concludes. The Appendix applies the model to the total number of attacks and shows the result.

2. METHODOLOGY

2.1. Time-Series and Intervention Analysis

The data in the GTD are used to calculate the conditional probabilities of different attack tactics based on the assumption that the intentions of different attack tactics have the same probabilities of being recorded. The conditional probability C_m of the attack tactic m is calculated in Equation (1), where N_i is the event number of tactic i recorded in the database.

$$C_m = \frac{N_m}{\sum_{i=1}^{8} N_i}, \quad i = 1, \ldots, 8 \tag{1}$$

In particular, the conditional probabilities in a certain period should remain statistically stable if no influential exogenous events occurred. It will not be easily influenced by the completeness of the data collection. Thus, changes to the conditional probabilities may indicate a series of attacks, the development of more effective attack weapons, or the influence of some exogenous events. That is, when the conditional probability of a certain attack tactic goes up, or deterministic influential events happen, it is time for the government to allocate more targeted defensive resources; e.g., using more bomb-sniffing dogs to search for suspicious bombs.

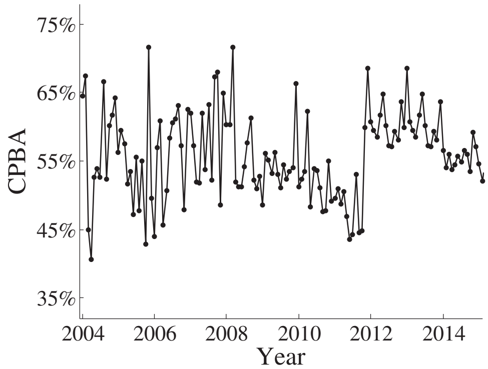

Fig. 2. Time-series plot of the conditional probability of bombing attack (CPBA) worldwide from 2004 to 2014.

When looking at data in the time domain, trends and sudden changes can be analyzed, showing the dynamic information of terrorist attacks. Time-series methods have often been used to analyze the information in time-related data by finding out the statistical regularity inside the chronological ordered random data set.(17) For example, Enders and Sandler(14–16) used time-series and other econometric methods to study the difference between domestic and transnational terrorism, especially how transnational terrorism affects global economics and political environments. The formal and structural method of time-series analysis is put forward by Box and Jenkins in 1976. Useful models include the autoregressive-moving average (ARMA) model, the autoregressive integrated moving average (ARIMA) model, etc.(18) When analyzing the sudden changes in the process, intervention analysis is used to analyze the influence of exogenous events.(19)

2.2. Data and Analysis Process

Supported by the National Consortium for the Study of Terrorism and Responses to Terrorism (START), the GTD has collected information on over 140,000 terrorist attacks worldwide from 1970 to 2014. Considering data that are more relevant to today's global political and economic climates, the monthly CPBA data between 2004 and 2014 are used to form the fluctuant time series, as shown in Fig. 2. Our research includes four steps to build the integrated forecast model: (1) build a time-series model to fit the CPBA process between January 2004 and October 2011; (2) make forecasts between November 2011 and December 2013, fit the residuals between the predicted and observed data with the intervention model; (3) integrate the time-series model in step 1 and intervention model in step 2 to generate a new model; and (4) validate the new model by forecasting the CPBA in 2014. Compare the fitted and observed data. Make a forecast of the CPBA through 2064.

3. DATA PROCESS AND RESULTS

3.1. Time-Series Modeling of the CPBA Process

Fig. 2 shows that the CPBA data before and after October 2011 differ significantly. First, the observed CPBA data before October 2011 (presented as process $\{X_t\}$) are analyzed using the time-series model.

The ARMA(p,q) model integrates the autoregressive model and the moving average model. Considering a time-series process $\{M_t\}$, the moving average model only considers a finite number of disturbances fitted to the current value, so in the moving average model, the current disturbance and the disturbances q periods ahead are used for regression; that is, $\hat{M}_t = \mu + e_t - \sum_{i=1}^{q} \theta_i e_{t-i}$. In the autoregressive model, contributions of all disturbances in the past are considered. After derivation, variables p periods ahead in the series are used as regression variables of the current variable; that is, $\hat{M}_t = \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + e_t$. The parameters include δ, ϕ, μ, and θ. The process $\{e_t\}$ is the series of disturbances, also known as white noise. The ARMA model can only be applied to stationary series(20) whose statistical properties do not change over time. An ARIMA (p,d,q) model(21) can be used to deal with the nonstationary but homogeneous process, which means that the first or higher order differences of the original nonstationary process could produce a stationary ARMA process. The new notation d denotes the orders of difference required by the original process. However, the excessive difference procedure should be avoided when using the ARIMA model.

A time-series process that has a unit root (meaning that 1 is a root of the process's characteristic equation) is considered to be nonstationary.(22) So, a unit root test is often conducted before the model is built. In this article, an augmented Dickey–Fuller (ADF) test(23) is used to determine the fiducial probability of a unit root existence in the time-series sample. The more negative the t value is, the stronger to reject the hypothesis that there exists a unit root at some level of confidence. The process $\{X_t\}$ is tested by the ADF test. The result in Table I shows that fiducial probability of the t-value is much less

Table I. Unit Roots Test of Process $\{X_t\}$

	t Value	Fiducial Probability
ADF test value	−8.08	<0.1%
Reference value	−3.50	1%

Table II. Unit Roots Test of Process $\{Y_t\}$

	t Value	Fiducial Probability
ADF test value	−10.59	<0.1%
Reference value	−3.50	= 1%

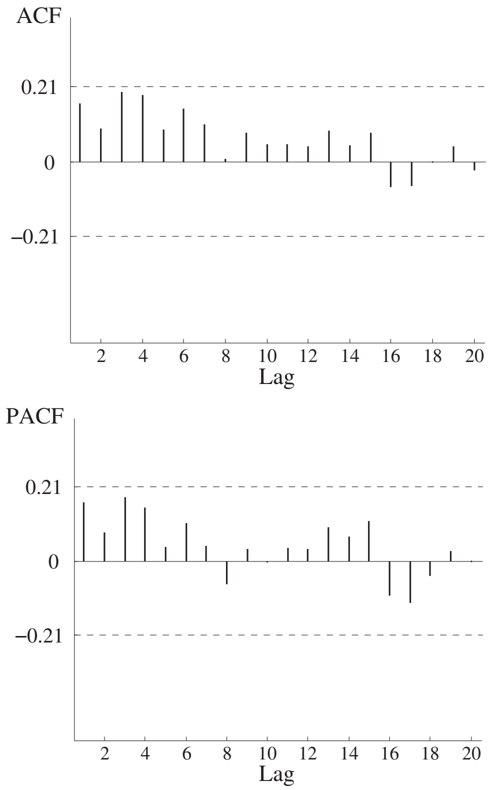

Fig. 3. The ACF and the PACF of the process $\{X_t\}$ (with 5% significance limits for the autocorrelations).

than 0.1%, which indicates that the process $\{X_t\}$ is at least trend stationary (i.e., the process can generate a stationary one after removing the underlying trend).

The autocorrelation function (ACF),[18] which is also known as serial correlation, means that there is correlative relationship between two different random variables in the series. If variables in the series are significantly correlated with the variables p periods ahead, the series is p-order autocorrelative (self-regression). However, the ACF only shows the simple correlative relationship between the two variables without removing the influence of data between these two variables, which makes it difficult to distinguish series of different orders. The partial ACF (PACF) solves this problem by removing the impact of other random variables, except the two variables being analyzed.

The ACF and PACF of the process $\{X_t\}$ are shown in Fig. 3. The 5% significance levels[24] shown by the two horizontal dotted lines are used to distinguish the lag values that are statistically significant from zero. Fig. 3 shows that the lag values of the ACF and the PACF cannot be extracted clearly for process $\{X_t\}$. Therefore, it is not acceptable to build an ARMA model directly.

The process $\{Y_t\}$ is used to denote the first difference series of the $\{X_t\}$. And the process $\{\hat{Y}_t\}$ and the $\{\hat{X}_t\}$ are used to denote the fitted series of $\{Y_t\}$ and $\{X_t\}$, respectively, where the process $\{\hat{X}_t\}$ is derived from the $\{\hat{Y}_t\}$. The unit roots of the process $\{Y_t\}$ are examined by the ADF test, with the results shown in Table II. The hypothesis that the process $\{Y_t\}$ does not have a unit root is accepted. The ACF and PACF of the $\{Y_t\}$ together with the 5% significance levels are shown in Fig. 4. It shows that the ACF of the $\{Y_t\}$ cuts off after lag 1 and then follows a sinusoidal pattern about 0, while the PACF tails off near 0. According to the first-order negative correlation features, the ARMA(0,1) model is chosen to fit the process $\{Y_t\}$.[25]

The process $\{e_t\}$ calculated by the fitted process $\{\hat{Y}_t\}$ is examined. The ACF and the PACF plots (Fig. 5) of the $\{e_t\}$ show that there is no significant autocorrelation or partial autocorrelation. Data in the process $\{e_t\}$ have an average value of −0.32%, which is close to 0. Due to these findings, the process $\{e_t\}$ can be defined as white noise. Thus, the first-order difference is sufficient for modeling, and Equation (2) shows the fitted function of the process $\{\hat{Y}_t\}$:

$$\hat{Y}_t = e_t - 0.891e_{t-1} - 0.093 \qquad (2)$$

Consequently, the fitted ARIMA(0,1,1) function of the process $\{X_t\}$ is shown in Equation (3).

$$\hat{X}_t = \hat{X}_{t-1} + e_t - 0.891e_{t-1} - 0.093 \qquad (3)$$

The process $\{\hat{X}_t\}$ contains a deterministic drift, so a downward trend with the slope of −0.09% is expected when doing the forecast. The fitted values in Fig. 6 show that the CPBA gradually increased

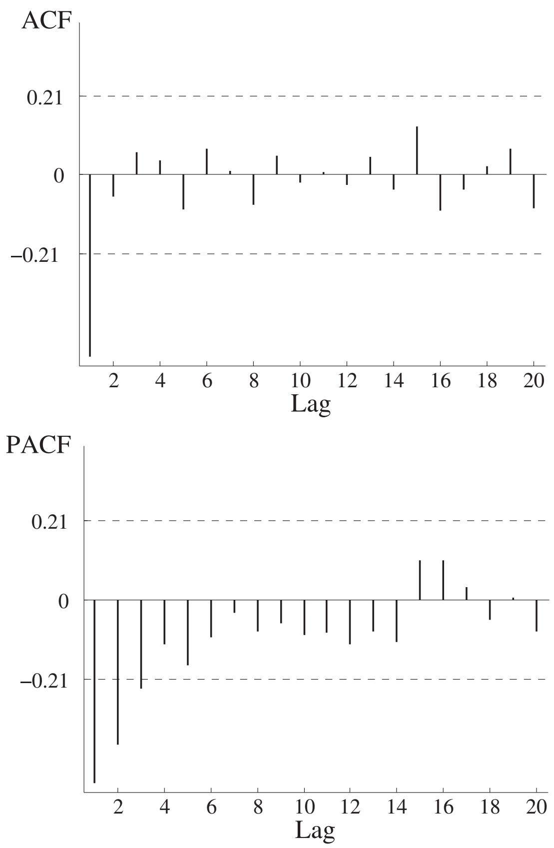

Fig. 4. The ACF and the PACF of the process $\{Y_t\}$ (with 5% significance limits for the autocorrelations).

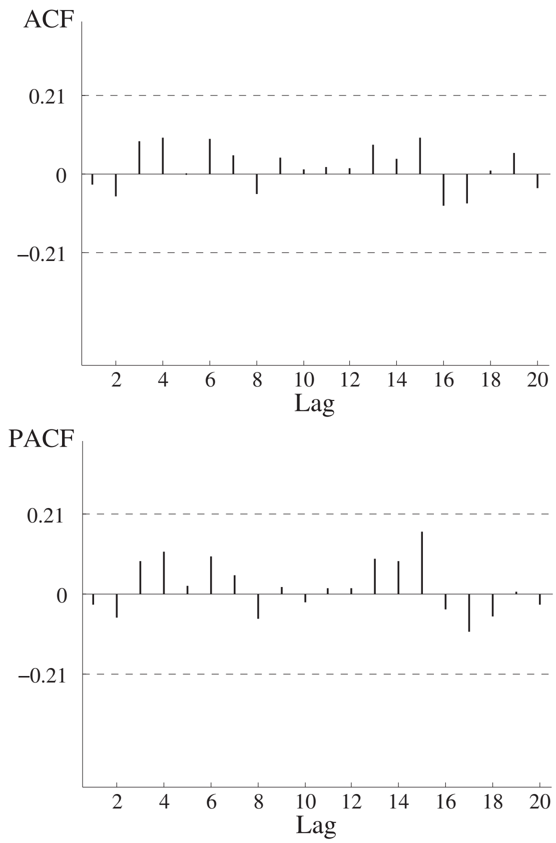

Fig. 5. The ACF and the PACF of $\{e_t\}$ (with 5% significance limits for the autocorrelations).

from 52.9% to 58.8% between 2006 and 2008, and then decreased to 52.0% till the end of 2011. At that time, there is a significant jump in the CPBA process.

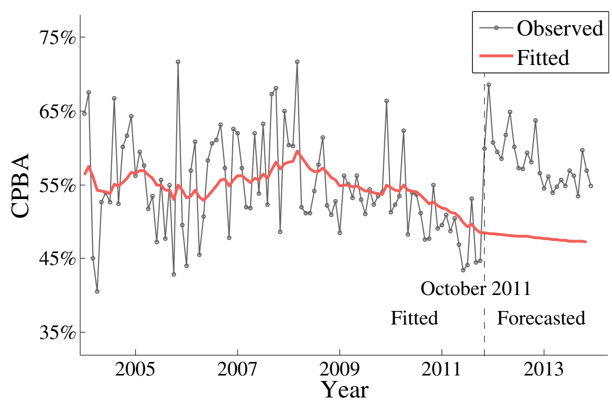

Fig. 6. Fitted and forecasted values of the CPBA from 2004 to 2013 by the time-series model.

3.2. Intervention Modeling of the CPBA Process

At the end of 2011, there is a sudden jump in the CPBA process over a two-month period, as shown in Fig. 6. An intervention analysis is done to understand how the data changed under external intervention.

There are two general types of indicator variables used to transfer the two kinds of basic interventions, a step variable (denoted by $S_{t-1}^{(T)}$) for step intervention and a pulse variable (denoted by $P_{t-1}^{(T)}$) for pulse intervention, as defined in Equation (4).

$$S_t^{(T)} = \begin{cases} 1, \; t \geq T \\ 0, \; t < T \end{cases}, \quad P_t^{(T)} = \begin{cases} 1, \; t = T \\ 0, \; t \neq T \end{cases} \tag{4}$$

The step intervention means that the effect lasts for a period of time after the intervention appears at time T. It describes a lasting effect in the process. For example, if a technique has been developed to fight against a certain attack tactic efficiently, the conditional probability of this attack tactic may have a negative step intervention. The pulse intervention describes a relatively temporary

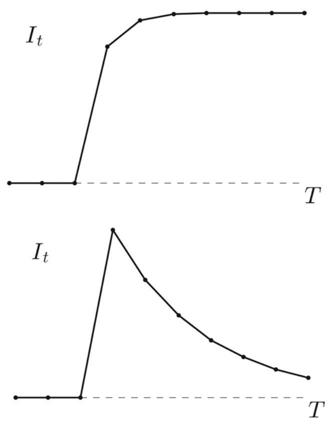

Fig. 7. Examples of step intervention and pulse intervention.

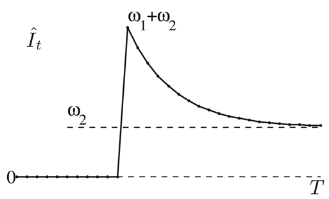

Fig. 8. Examples of one intervention response.

change that only affects the very moment after the intervention appears. For example, the sudden change of regime in a country may cause a pulse intervention in the terrorist attacks. The examples of two basic interventions are shown in Fig. 7. There are several patterns of response that can be transferred with the two basic interventions. Fig. 8 is one of the typical responses having both the temporary and permanent effects. It represents a sudden pulse change with magnitude $(\omega_1 + \omega_2)$ (the total strength of the temporary and permanent effects) after time T, and then gradually decays with rate δ (decay rate of temporary effect) and goes back to magnitude ω_2 (the strength of the permanent effect). Equation (5) shows the model of Fig. 8. It is chosen because of the close pattern shown in Fig. 9. When doing a long-term forecast, the intervention events are assumed to follow a Poisson distribution with a rate of λ per month in the prediction model. The strength of the temporary effect and permanent effect follows the normal distribution with mean of μ_1 and μ_2 and standard deviation of σ_1 and σ_2. The decay rate of temporary effect also follows the normal distribution with mean of μ_δ and standard deviation of σ_δ:

$$\hat{I}_t = \left(\frac{\omega_1 B}{1 - \delta B} + \frac{\omega_2 B}{1 - B} \right) P_t^{(T)}. \tag{5}$$

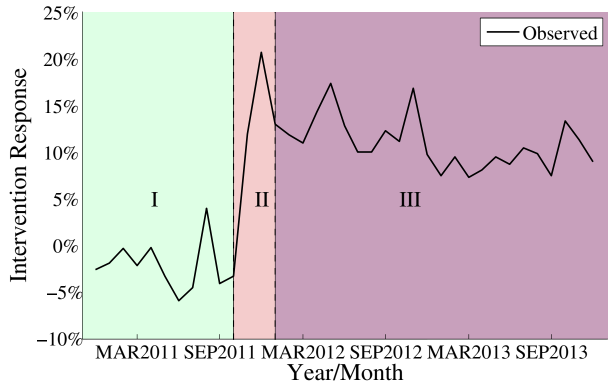

Fig. 9. Partitions of intervention response from 2011 to 2013.

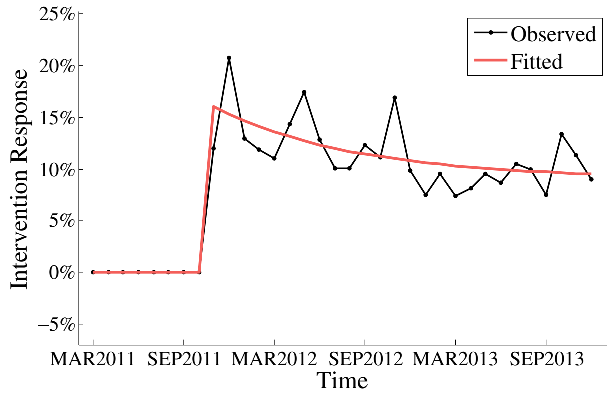

Fig. 10. Fitted values of intervention response from 2011 to 2013.

The residuals between the observed and fitted values are calculated to form the intervention response series. The process $\{\hat{I}_t\}$ is used to denote the fitted intervention response series, which includes three parts: (1) the preintervention response (part I in Fig. 9), which can be fitted by the model in Section 3.1, (2) midintervention (part II in Fig. 9), where the CPBA suddenly increased significantly and reached the peak value in a two-month period, and (3) postintervention (part III in Fig. 9), where the temporary effect of the intervention gradually fades away.

The least-square method is used to assess parameters ω_1, ω_2, and δ. The most suitable values are $\omega_1 = 0.07$, $\omega_2 = 0.09$, and $\delta = 0.90$. The smallest least variance is 0.018, which shows the model fits well. The values of ω_1 and ω_2 show that the CPBA has increased by 16.0% to reach the peak value, but still stays 9.0% greater than the predicted level after the temporary effect gradually decays. The intervention response fitting is shown in Fig. 10. Consequently,

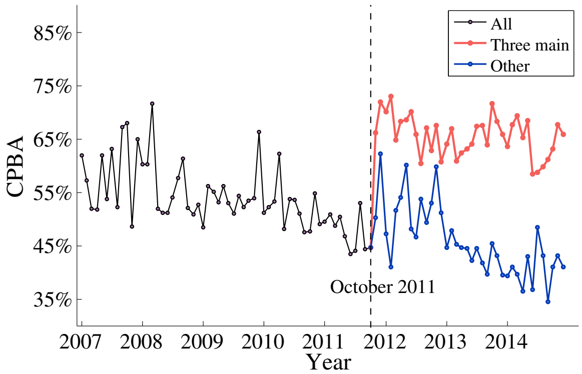

Fig. 11. The CPBA comparison between Iraq, Afghanistan, Pakistan, and other countries from 2004 to 2014.

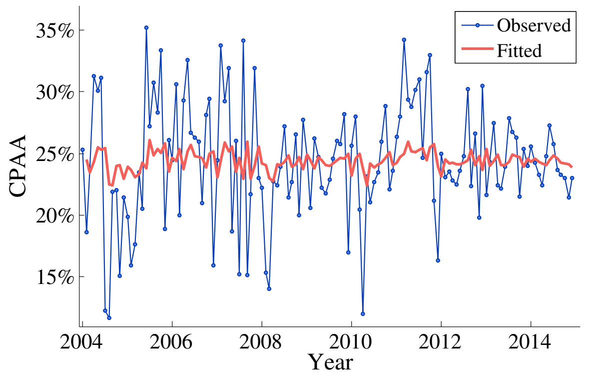

Fig. 12. The fitted CPAA from 2004 to 2014.

Table III. The Comparison of the Terrorist Attacks between Iraq, Afghanistan, Pakistan, and Other Countries from November 2011 to December 2013

Region	Total	Proportion	Bombing	Proportion
Three countries	10,757	52.2%	7,775	62.5%
Others	9,842	47.8%	4,666	37.5%

Equation (6) is used to represent the intervention response:

$$\hat{I}_t = 1.90\hat{I}_{t-1} - 0.90\hat{I}_{t-2} + 0.16P_{t-1}^{(T)} - 0.15P_{t-2}^{(T)}. (6)$$

A search of the time node is performed to find out what exogenous event influenced the CPBA process. At the end of 2011, America withdrew all troops from Iraq[26] and some troops from Afghanistan,[27] left unstable governments in these two countries, and unsolved conflicts between Afghanistan and Pakistan. Social unrest became a big problem in these countries. According to the performed analysis, these events may have led to a sudden increase in the number of bombing attacks in these three countries. The CPBA of these three countries was separated from other countries in the world, as shown in Fig. 11, and it can be seen that bombing attacks happened much more frequently in Iraq, Afghanistan, and Pakistan after October 2011. The bombing attacks in other countries continued a downtrend with larger fluctuation. Additionally, 62.3% of global bombing attacks from November 2011 to December 2013 were located in these three countries,[1] as shown in Table III.

3.3. Integrated Time-Series and Intervention Model

Comparisons between the conditional probabilities of bombing attacks and armed assaults is done to check the significance of bombing attacks data. The same method is used to build a time-series model (ARMA(1,0)) to fit the conditional probability of armed assaults (CPAAs) (shown in Fig. 12), which contributed to a quarter of all attacks over the world. The results show that the CPAA process remains stable (around 25%) showing no trend, and no significant fluctuation can be observed at the end of 2011. The big difference in these two data patterns of attack tactics provides the proof that previously mentioned political events did not affect the CPAA significantly like the CPBA.

The integrated time-series and intervention model is used to forecast the CPBA in 2014 and through 2064, as shown in Fig. 12. When doing the forecast through 2064, the baseline values of the average event number per month λ are set as $\lambda = 1/96$ or $\lambda = 1/144$, indicating the interventions have an average interval of 8 years and 12 years, respectively. Other baseline values of parameters are set as follows: $\mu_1 = 0.07$, $\mu_2 = 0.09$, $\sigma_1 = 0.01$, $\sigma_2 = 0.01$, $\mu_\delta = 0.90$, and $\sigma_\delta = 0.10$. The means of the parameters are referred to the observed intervention response that happened at the end of year 2011, as discussed in Section 3.2. Fig. 13 shows that when the average interval of the interventions is larger (i.e., 12 years, or $\lambda = 1/144$), the downtrend of the CPBA is easier to maintain.

Four residual plots in Figs. 14(a)–(d) are used to analyze the residuals of fitness among the 120 CPBA data between 2004 and 2013. From these four plots, we note that the sequence of residuals is close to a normal distribution and has an average value

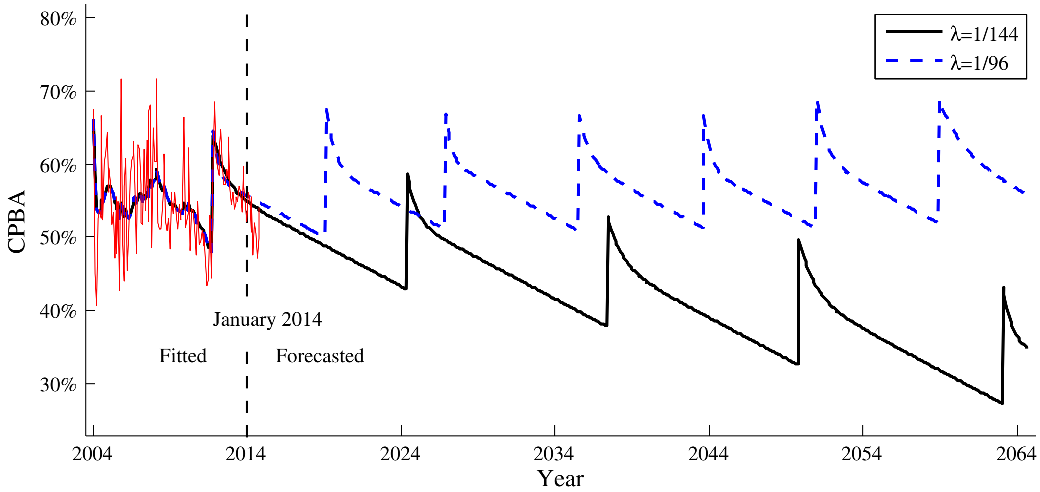

Fig. 13. The fitted and forecasted CPBA from 2004 to 2064 with different average intervals of interventions.

Table IV. Relative Error (RE) of Forecast in 2014

Month	RE	Month	RE	Month	RE	Month	RE
Jan.	−5.46%	Apr.	−7.67%	July	−4.01%	Oct.	−8.00%
Feb.	2.35%	May	1.96%	Aug.	−7.16%	Nov.	0.47%
Mar.	1.21%	June	−1.38%	Sept.	−0.14%	Dec.	−2.21%

close to 0. The slightly lower tails are due to the reservation of the extreme data in the process. There is no significant correlation between the residuals and fitted values, regardless of whether by fitted values or by order. These observations indicate that the fitness of the integrated model is acceptable.

Observed data in 2014 are also used to validate the model. The relative errors between the observed and fitted values from 2014 are calculated in Table IV. Notably, the data from 2014 are not used when doing the forecast. The average relative error is 3.50%, showing that the influence of a long-term decreasing trend and the intervention still works in 2014. However, the data decreased a little faster than forecast, which indicates the situations in these countries may be improved.

4. CONCLUSIONS

Among over 140,000 terrorist attacks collected by the GTD, over half are bombing attacks, and about a quarter are armed assaults. After removing the noise, we noted that between 2006 and 2008, the CPBA increased from 52.9% to 58.8%. After 2008, the CPBA went into a downtrend of 0.09% per month to 52.0% until the end of 2011. The ARIMA model fails to forecast a jump in the CPBA process at the end of 2011.

The intervention analysis shows that the CPBA increased 16.0% at the end of 2011, and keeps 9.0% larger than the predicted level after the temporary effect gradually decays. During that period, the United States withdrew all troops from Iraq and some from Afghanistan. At the same time, the relationship between Pakistan and Afghanistan worsened. Data show that the increased CPBA mainly comes from these three countries. This research infers that the chaotic situation of society may cause terrorists to choose bombing attacks more frequently. By contrast, the CPAA remained stable in the last 10 years and did not fluctuate significantly at the end of 2011.

The integrated time-series and intervention model is used to forecast the monthly CPBA in 2014 and through 2064. The average relative error compared with the real data in 2014 is 3.5%, showing the good performance of the forecast model. For prediction in the next 50 years, the interventions are assumed to follow a Poisson distribution with average intervals of 8 years or 12 years.

Time-series analysis can be used to analyze the dynamic information of terrorist attacks in the time domain by removing noise in the data. Intervention analysis can be used to understand the influence of exogenous events on global terrorism activities. In this research, the integrated time-series and intervention model is used to study long-term tendency in the sequence of the CPBA all over the world, as well as the influence of social unrest

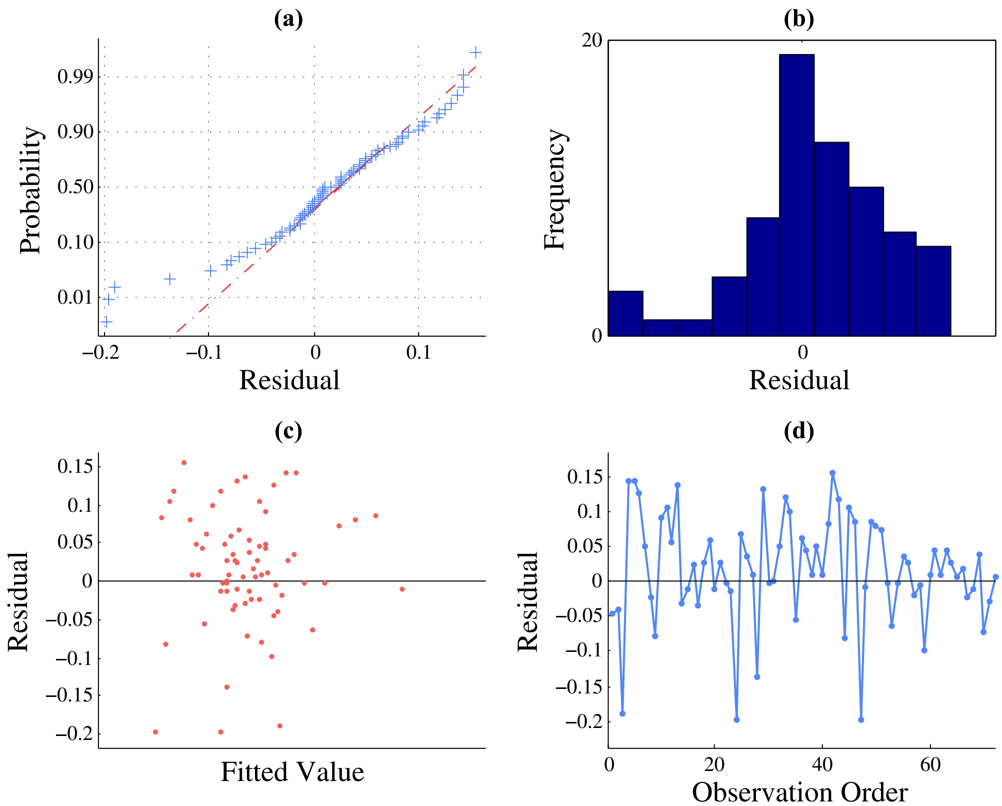

Fig. 14. Residual plots of integrated model of $\{X_t\}$: (a) normal probability plot, (b) histogram of the residuals, (c) residuals by fitted value, and (d) residuals by order.

in Iraq, Afghanistan, and Pakistan, to the sudden change in the CPBA process. It should be noticed that, similar to all statistical models trained with past data, this model can only offer good predictions if the future conditions and relationships are in line with the historic conditions and relationships. The intervention part of the model was used to account for a historical artifact. The forecast model cannot predict the future interventions themselves.

Future research would focus on analyzing the data in some regions where terrorist attacks happen frequently. This would help study the influence of some local changes of social environment to the data (e.g., new policy, or the stage change of political parties).

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APPENDIX: TIME-SERIES MODEL OF TOTAL ATTACKS RECORDED BY THE GTD

The time-series model is used to fit the total number of attacks recorded by the GTD by month. After doing the ADF test, the original process proves to be nonstationary (as shown in Table AI). The

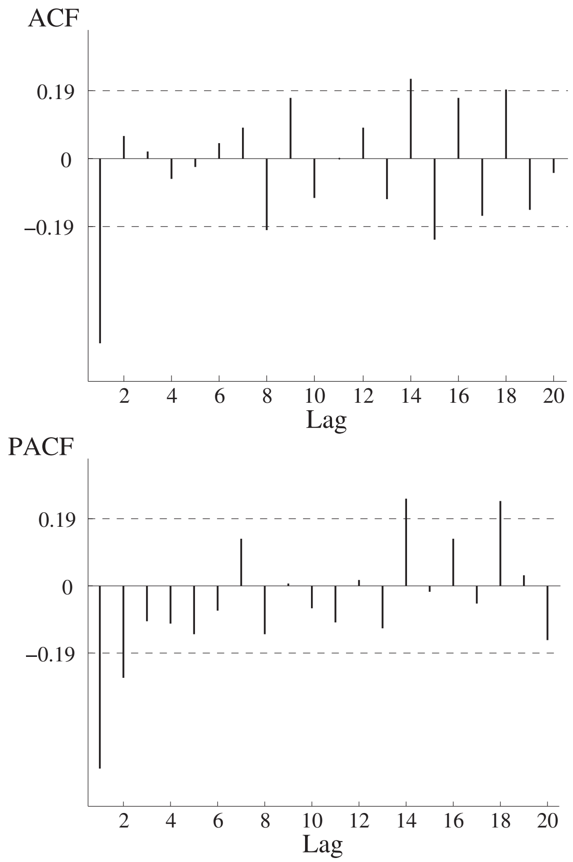

Fig. A1. The ACF and the PACF of total attacks' difference process.

first difference of the original process has been calculated, and its stationary properties are shown in Table AI. The ACF and PACF of the first difference process are shown in Fig. A1 (as in Fig. 4 in the body). We can see the same pattern shown in the ACF and PACF graphs of first difference process of the CPBA. The time series of total attacks is fitted with an ARIMA(0,1,1) model too. The fitted function of total attacks (denoted by $\{\hat{A}_t\}$) is shown in Equation (A1). The observed and fitted values of the total attacks between 2004 and 2014 are shown in Fig. A2.

Table AI. Unit Roots Test of Total Attacks Process and Its Difference Process

	t Value	Fiducial Probability	
Original process	−0.766	0.825	Nonstationary
Difference process	−19.969	<0.001	Trend stationary

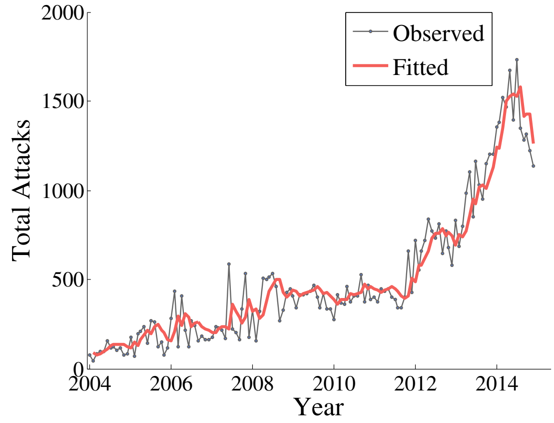

Fig. A2. Observed and fitted values of total attacks from 2004 to 2014.

$$\hat{A}_t = \hat{A}_{t-1} + e_t - 0.541e_{t-1} + 8.949. \qquad \text{(A1)}$$

After removing the noise, we see a clear uptrend in the total attacks after 2004, and the increase speeds up from the end of 2011 until the end of 2013, which may be the effect of social unrest analyzed in Section 3.2. The total number of attacks went down in 2014. From 2004 to 2014, there were about 8.95 more attacks per month when compared to the previous month. Although both the processes of total attacks and CPBA are fitted by the ARIMA (0,1,1) model, information is different. The constant term in Equation (A1) represents the uptrend in the total attacks, which cannot be found in Equation (3) showing the CPBA. Lastly, there is no significant intervention in the time-series plot of total attacks, which could help find out the effect of exogenous events.

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