Defending Against a Terrorist Who Accumulates Resources

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ABSTRACT

In this paper we study a two-period game between a government and a terrorist, where the terrorist decides whether to stockpile attack resources from the first to the second period. Our results show that the terrorist chooses stockpiling when: (a) the following parameters are in intermediate ranges: the government's asset valuation, the terrorist's first-period resource, the government's unit defense cost, and the terrorist's unit attack cost; (b) the terrorist's second-period resource is small; and (c) the terrorist's resource growth factor or discount factor is large. We also compare our model with the one that does not allow terrorist stockpiling. For moderate growth factors and second-period resources for the terrorist, the terrorist does not prefer the option of stockpiling. The terrorist prefers stockpiling for the more uncommon case that the growth factor for the terrorist's stockpiled resource is very large and the second-period resource is very small. In contrast, the government always prefers that the terrorist has the stockpiling option.

INTRODUCTION

Understanding and defeating terrorist threats over time are important but challenging. In the literature, counterterrorism has been modeled as games between terrorist and government. See Sandler and Siqueira (2009) for a survey of work that examines the strategic interaction between governments and terrorists. However, multiperiod games between terrorist and government have hardly been studied. One exception is Zhuang et al. (2010), who study secrecy and deception in a multiple-period attacker-defender signaling and resource-allocation game.

Assessing the terrorist's capacity to attack over time is essential. In particular, the timing of attack(s) and the terrorist's option of stockpiling attack resources are of particular interest. For example, the lack of major terrorist attacks following 9/11 could possibly be explained by terrorists stockpiling resources and preparing for large future attacks. To our knowledge, this interesting and important scenario has not been studied in the literature. To fill this gap, this paper studies a two-period game between a government and a terrorist, where the terrorist decides whether to stockpile attack resources from the first to the second period.

Terrorists generate resources in multiple ways. They sometimes enjoy benefactors who provide funds, and are often engaged in activities that generate funds. Usually, resources become available over time. A strategic decision for the terrorist is when to use its resources. It might use all of its resources as they become available, or accumulate resources in various ways. In this paper we simplify this complex resource-generating situation by assuming that the terrorist gets one resource in the first period, and a second resource in the second period. The terrorist may either use its resources to attack in both periods, or it may accumulate its resource from the first to the second period. We allow the terrorist to earn interest on, or suffer depreciation of, its first-period resource in terms of a growth factor. Whether the terrorist accumulates resources or not impacts how the government designs its defenses in the first and second periods. On the other hand, the government's first-period defense may indeed impact whether or not the terrorist stockpiles its resource. For example, the U.S. implemented vigorous defenses after 9/11, which likely deterred immediate subsequent attacks, possibly inducing potential terrorists to accumulate resources while waiting for governments to relax their defenses over time, which has been a common trend since 9/11. This strategic interaction between the government's defenses in the two periods, and the terrorist's decision of whether or not to accumulate resources from the first to the second period, is the key focus of this paper.

Whereas earlier research has focused on substitutions between multiple assets (Bier et al. 2007, 2008; Enders and Sandler 2003; Hao et al. 2009; Hausken 2006), this paper confines attention to one asset to focus explicitly on the time dimension. The reason is that the temporal effects (terrorist accumulates resources over time, government allocate resources over time) are much less studied in the literature. One asset is justified when a terrorist bears a grudge against one opponent in general, e.g. the Western World, and is more concerned about whether and when to attack this opponent, rather than about which asset possessed by the defender should be attacked.

Some research has focused on investment substitutions across time. For example, Enders and Sandler (2003) suggest that a terrorist might compile and accumulate resources during times when the government's investments are high, awaiting times when the government might relax his efforts and choose lower investments. Keohane and Zeckhauser (2003: 201, 224) show that "the optimal control of terror stocks will rely on both ongoing abatement and periodic cleanup" of "a terrorist's 'stock of terror capacity.'" Enders and Sandler (2005) use time series to show that little has changed in overall terrorism incidents before and after 9/11. Using 9/11 as a break date, they find that logistically complex hostage-taking events have fallen as a proportion of all events, while logistically simple, but deadly, bombings have increased as a proportion of deadly incidents. Bandyopadhyay and Sandler (2011) consider the interaction between preemption and defense. For example, high-cost defenders might rely on preemption, while too little preemption might give rise to subsequent excessive defense.

Raczynski (2004) simulates the dynamic interactions between terror and antiterror groups. Feichtinger and Novak (2008) use differential game theory to study the intertemporal strategic interactions of Western governments and terror organizations. They illustrate long-run persistent oscillations. Berman and Gavious (2007) study a leader-follower game in which the State provides counterterrorism support across multiple metropolitan areas to minimize losses, while the Terrorist attacks one of the metropolitan areas to maximize his utility. Berrebi and Lakdawalla (2007) consider for 1949–2004 how terrorists seek targets in Israel, responding to costs and benefits, and find that long periods without an attack signal lower risk for most localities, but higher risk for important areas. Barros et al. (2006) apply parametric and semiparametric hazard model specifications to study durations between Euskadi Ta Askatasuna's (ETA, a Spain-based terrorist group) terrorist attacks, which seem to increase in summer and decrease with respect to, e.g., deterrence and political variables. Udwadia et al. (2006) consider the dynamic behavior of terrorists, those susceptible to terrorist and pacifist propaganda, military/police intervention to reduce the terrorist population, and nonviolent, persuasive intervention to influence those susceptible to becoming pacifists. Hausken (2008) considers a terrorist that defends an asset that grows from the first to the second period. The terrorist seeks to eliminate the asset optimally across the two periods. Telesca and Lovallo (2006) find that a terror event is not independent from the time elapsed since the previous event, except for severe attacks that approach a Poisson process. This latter finding suggests that attack and defense decisions are not unit-periodic in nature, but that there are linkages through time. One objective of the current paper is to understand more thoroughly the nature of such linkages through time, affected by changes in resources, unit costs of defense and attack, etc. Brown et al. (2006) consider defender-attacker-defender models. First the defender invests in protecting the infrastructure, subject to a budget constraint. Then, a resource-constrained attack is carried out. Finally, the defender operates the residual system as best possible. They exemplify with border control, the U.S. strategic petroleum reserve, and electric power grids. Sandler and Siqueira (2009) focus on defensive versus proactive countermeasures, the impact of domestic politics, the interaction between militant and political factions within terrorist groups, and fixed budgets.

Azaiez and Bier (2007) consider the optimal resource allocation for security in reliability systems. Bier et al. (2005) and Bier and Abhichandani (2002) assume that the defender minimizes the success probability and expected damage of an attack. Bier et al. (2005) analyze the protection of series and parallel systems with components of different values. Levitin (2007) considers the optimal element separation and protection in complex multistate series-parallel system and suggests an algorithm for determining the expected damage caused by a strategic attacker. Patterson and Apostolakis (2007) introduced importance measures for ranking the system elements in complex systems exposed to terrorist actions. Michaud and Apostolakis (2006) analyzed such measures of damage caused by the terror as impact on people, impact on environment, impact on public image, etc.

Bier et al. (2006) assume that a defender allocates defense to a collection of locations while an attacker chooses a location to attack. They show that the defender allocates resources in a centralized, rather than decentralized, manner, that the optimal allocation of resources can be non-monotonic in the value of the attacker's outside option. Furthermore, the defender prefers its defense to be public rather than secret. Also, the defender sometimes leaves a location undefended and sometimes prefers a higher vulnerability at a particular location even if a lower risk could be achieved at zero cost. Dighe et al. (2009) consider secrecy in defensive allocations as a strategy for achieving more cost-effective attacker deterrence. Zhuang and Bier (2011) model secrecy and deception as a signaling and resource-allocation game between a government and a terrorist.

Hausken et al. (2009) consider a defender that chooses tradeoffs between investments in protection against natural disaster only, protection against terrorism only, and all-hazards protection, allowing sequential or simultaneous moves. Similarly, Zhuang and Bier (2007) study the balance between natural disaster and terrorism, where either the defender moves first (and the attacker second), or they move simultaneously. Levitin and Hausken (2008) consider a two-period model in which the defender, moving first, distributes its resource between deploying redundant elements and protecting them from attacks.

We show the conditions when the government and terrorist prefer versus do not prefer that the terrorist has the option of stockpiling resources. One example in which an agent prefers to limit its strategy set is Schelling's (1960) modeling of burning one's own bridges in war. A second example is the Battle of Julu in 207 B.C., when General Xiang sank all his ships, ensuring that no one could retreat from battle, and destroyed all his food supply and cooking utensils to ensure that his soldiers would fight to survive (Sima ca. 145-ca. 86 B.C.). A third example is Capitan Hernando Cortes' 1519 burning of his ships to motivate his men to adapt to his at-all-costs attitude. A fourth example is Ulysses who wanted to hear the Sirens' song, which would render him irrational. He tied himself to the mast to limit his irrationality (Elster 1984).

The next two sections present and solve the model, respectively. Then, the next section illustrates the model with numerical testing and graphic illustrations. The subsequent section compares with the model when the terrorist is not allowed to stockpile. The last section concludes.

THE MODEL

Notation

t	time period, $t = 1,2$
R_t	terrorist's resource in period t
d_t	government's defense effort protecting the asset in period t
A_t	terrorist's attack effort attacking the asset in period t
P_t	probability of asset damage in period t
b	government's unit defense cost
B	terrorist's unit attack cost
v	government's asset valuation
V	terrorist's asset valuation
u	government's accumulative discounted utility
U	terrorist's accumulative discounted utility
δ	government's discount factor
Δ	terrorist's discount factor
g	the growth factor for the terrorist's stockpiled resource

Assumptions

We assume that the terrorist has an incoming resource R_1 in period 1 and an incoming resource R_2 in period 2. The resource can be converted at unit cost B into an attack effort. If the terrorist does not stockpile, it exerts effort $A_1 = R_1/B$ in period 1 and $A_2 = R_2/B$ in period 2. If the terrorist stockpiles, it exerts no effort in period 1 ($A_1 = 0$) and exerts effort $A_2 = (gR_1 + R_2)/B$ in period 2, where g expresses growth on the terrorist's stockpiled resource: $g = 1$ means no growth, $g > 1$ means growth (e.g., interest on a bank account), and $0 \leq g < 1$ means resource degradation, which occurs when the terrorist cannot preserve its resource over time, due to the resource getting obsolete, stolen, defection, etc. The government exerts effort d_1 in period 1 and effort d_2 in period 2, both at unit cost b, to protect an asset. For the probability P_t of asset damage in period t, we apply the ratio form contest success function (Tullock 1980, Skaperdas 1996) that is commonly used in the attack-defense literature (Zhuang and Bier 2007)—that is,

$$P_t(d_t, A_t) = \frac{A_t}{A_t + d_t} \tag{1}$$

where $\partial P_t/\partial d_t \leq 0$, $\partial P_t/\partial A_t \geq 0$, $t = 1, 2$, and we define $P_t(d_t, 0) = 0$ for all d_t. Note that (1) implies $P_t(d_t, A_t) = 1$ for all A_t, as long as $d_t = 0$. This corresponds to the scenario that there is no inherent (or free) defense. See Zhuang and Bier (2007) for the case that inherent defense is allowed; e.g., $P_t(d_t, A_t) = \frac{A_t}{d_t + A_t + c}$, where the constant c refers to the inherent defense. Inserting the terrorist's attack options into (1) gives

$$\begin{aligned} P_t(d_t, R_t/B) &= \frac{(R_t/B)}{(R_t/B) + d_t} \\ &\qquad \text{if not stockpiling} \\ P_1(d_1, 0) &= 0, \\ P_2(d_2, (gR_1 + R_2)/B) &= \frac{\left((gR_1 + R_2)/B\right)}{\left((gR_1 + R_2)/B\right) + d_2} \\ &\qquad \text{if stockpiling} \end{aligned} \tag{2}$$

When the terrorist stockpiles, the probability of asset damage in the first period is zero. The probability that the asset is not damaged is $1 - P_t$, which the government maximizes, accounting for the asset valuation v, asset defense expenditures bd_t, and attack efforts A_1 and A_2. Conversely, the terrorist maximizes the probability P_t of asset damage, valuing the asset at V, and determining whether or not to stockpile. The government's and terrorist's expected utilities over the two periods are

$$\begin{aligned} u(d_1, d_2, A_1, A_2) &= (1 - P_1)v - bd_1 \\ &\quad + \delta((1 - P_2)v - bd_2) \\ U(d_1, d_2, A_1, A_2) &= P_1 V + \Delta P_2 V \end{aligned} \tag{3}$$

where $0 \leq d \leq 1$ and $0 \leq D \leq 1$ are time discount parameters for the government and terrorist, respectively. For simplicity, if the asset is destroyed in the first period, we assume that the asset gets rebuilt before the second period with costs not accounted for in the model. Inserting (2) into (3) gives[a]

$$u(d_1, d_2, A_1, A_2) = \begin{cases} \dfrac{d_1 v}{(R_1/B) + d_1} - bd_1 + \delta\left(\dfrac{d_2 v}{(R_2/B) + d_2} - bd_2\right) \\ \qquad \text{if not stockpiling} \\ 1v - bd_1 + \delta\left(\dfrac{d_2 v}{[(gR_1 + R_2)/B] + d_2} - bd_2\right) \\ \qquad \text{if stockpiling} \end{cases}$$

$$U(d_1, d_2, A_1, A_2) = \begin{cases} \dfrac{(R_1/B)V}{(R_1/B) + d_1} + \Delta\left(\dfrac{(R_2/B)V}{(R_2/B) + d_2}\right) \\ \qquad \text{if not stockpiling} \\ 0V + \Delta\left(\dfrac{[(gR_1 + R_2)/B]V}{[(gR_1 + R_2)/B] + d_2}\right) \\ \qquad \text{if stockpiling} \end{cases} \tag{4}$$

The structure of the game shown in Figure 1 is such that the government first decides its first period defense effort d_1. Thereafter the terrorist determines whether or not to stockpile, which allows the first contest to be conducted. Finally the government determines its second period defense effort d_2, which allows the second contest to be conducted.

In summary, the game modeled is a two-person, nonzero sum, two-period game of perfect information. Because of our assumption of compactness of decision domains and continuity of functional forms, a unique subgame perfect

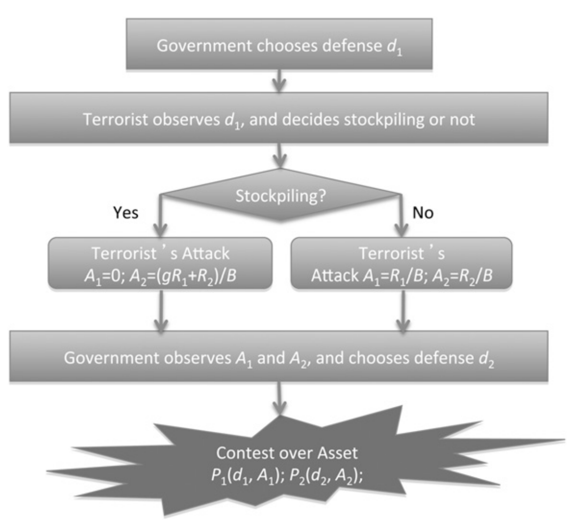

Figure 1. Sequence of moves.

Nash equilibrium exists. In the next section we use backward induction to solve the game. We acknowledge that it is documented in the game theory literature that there might exist some Nash equilibria, which could be interesting, which are not subgame perfect. However, subgame perfectness is a nice refinement of Nash equilibrium in dynamic games. In this paper we only focus on subgame perfect equilibrium, which exists, is unique, and is consistent with our sequence of moves.

SOLVING THE MODEL

Solving the second period

The game is solved with backward induction starting with the second period. The terrorist's effort in the second period is R/B, where $R = R_2$ if not stockpiling and $R = gR_1 + R_2$ if stockpiling. The government's first-order condition in the second period is

$$\frac{\partial u(d_1, d_2, A_1, A_2)}{\partial d_2} = \delta\left(\frac{(R/B)}{((R/B)+d_2)^2}v - b\right) = 0$$
$$\Rightarrow d_2 = \begin{cases} \sqrt{\frac{R_2}{B}}\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^+ \\ \text{if not stockpiling, } A_1 = \frac{R_1}{B}, A_2 = \frac{R_2}{B} \\ \sqrt{\frac{(gR_1+R_2)}{B}}\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{(gR_1+R_2)}{B}}\right)^+ \\ \text{if stockpiling, } A_1 = 0, A_2 = \frac{gR_1+R_2}{B} \end{cases} \tag{5}$$

where $(x)^+ = \max(x,0)$; that is, replacing negative expression (if any) with zero. Equation (5) states that the government defends in the second period if its valuation divided by its unit cost is larger than the terrorist's second-period resource divided by its unit cost.

Solving the first period

Inserting (5) into (4) gives the players' total utilities as functions of d_1 and A_1,

$$u(d_1, A_1) = \begin{cases} \frac{d_1 v}{(R_1/B)+d_1} - bd_1 + \delta b\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^2 \\ \text{if not stockpiling and } v \geq \frac{bR_2}{B} \\ \frac{d_1 v}{(R_1/B)+d_1} - bd_1 \\ \text{if not stockpiling and } v \leq \frac{bR_2}{B} \\ v - bd_1 + \delta b\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{(gR_1+R_2)}{B}}\right)^2 \\ \text{if stockpiling and } v \geq \frac{b(gR_1+R_2)}{B} \\ v - bd_1 \\ \text{if stockpiling and } v \leq \frac{b(gR_1+R_2)}{B} \end{cases}$$

$$U(d_1, A_1) = \begin{cases} \frac{(R_1/B)V}{(R_1/B)+d_1} + \Delta\left(\frac{\sqrt{\frac{R_2}{B}}V}{\sqrt{\frac{v}{b}}}\right) \\ \text{if not stockpiling and } v \geq \frac{bR_2}{B} \\ \frac{(R_1/B)V}{(R_1/B)+d_1} + \Delta V \\ \text{if not stockpiling and } v \leq \frac{bR_2}{B} \\ \Delta\left(\frac{\sqrt{\frac{(gR_1+R_2)}{B}}V}{\sqrt{\frac{v}{b}}}\right) \\ \text{if stockpiling and } v \geq \frac{b(gR_1+R_2)}{B} \\ \Delta V \\ \text{if stockpiling and } v \leq \frac{b(gR_1+R_2)}{B} \end{cases} \tag{6}$$

To solve the first period, we distinguish between three cases A, B, and C. Case A has two subcases A1 and A2, case B has three subcases B1, B2, B3, and case C has three subcases C1, C2, C3, for a total of eight cases.

Case A: $v \leq bR_2/B$: In this case, the terrorist prefers stockpiling if and only if $\frac{(R_1/B)V}{(R_1/B)+d_1} + \Delta V \leq \Delta V$, which is impossible. This implies that the terrorist always prefers not to stockpile ($A_1 = R_1/B; A_2 = R_2/B$) when the second-period income R_2 alone is sufficiently large to deter government defense. Differentiating the defender's utility in (6) with respect to d_1 gives

$$
\begin{aligned}
&\frac{\partial u(d_1)}{\partial d_1} = \frac{v}{R_1/B + d_1} - \frac{d_1 v}{(R_1/B + d_1)^2} - b = 0 \\
&\Rightarrow d_1^* = \sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_1}{B}}\right)^+ \\
&= \begin{cases} 0 \equiv d_{1A1}^* & \textit{if} \quad \frac{v}{b} \le \frac{R_2}{B} \le \frac{R_1}{B} \quad \textit{or} \quad \frac{v}{b} \le \frac{R_1}{B} \le \frac{R_2}{B} \\ \sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_1}{B}}\right) \equiv d_{1A2}^* & \textit{if} \quad \frac{R_1}{B} \le \frac{v}{b} \le \frac{R_2}{B} \end{cases} \\
&u(d_{1A1}^*) = 0, \quad u(d_{1A2}^*) = b\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_1}{B}}\right)^2
\end{aligned} \tag{7}
$$

where "≡" means "defined to be" for defining notations. The superscript * means optimal value throughout the paper. The variables d_{1A1}^* and d_{1A2}^* mean the optimal value of d_1^* for cases A_1 and A2. The utilities u and U follow from (6).

Case B: $bR_2/B < v \le b(gR_1 + R_2)/B$: In this case, the terrorist prefers stockpiling ($A_1 = 0$ and $A_2 = g(R_1{+}R_2)/B$) if and only if

$$
\begin{aligned}
&\frac{(R_1/B)V}{(R_1/B) + d_1} + \Delta\left(\frac{\sqrt{\frac{R_2}{B}}V}{\sqrt{\frac{v}{b}}}\right) \le \Delta V \\
&\Leftrightarrow d_1 \ge \left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B}
\end{aligned} \tag{8}
$$

The defender's utility is

$$
u(d_1) = \begin{cases} \dfrac{d_1 v}{(R_1/B) + d_1} - bd_1 + \delta b\left(\sqrt{\dfrac{v}{b}} - \sqrt{\dfrac{R_2}{B}}\right)^2 \\ \qquad \text{if } d_1 \le \left(\dfrac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\dfrac{R_1}{B} \\ v - bd_1 \\ \qquad \text{if } d_1 \ge \left(\dfrac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\dfrac{R_1}{B} \end{cases} \tag{9}
$$

As shown in Appendix A, we have three optimum candidates labeled B1, B2, and B3: These are $d_1^* = 0 \equiv d_{1B1}^*$, $d_1^* = \left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B} \equiv d_{1B2}^*$ (second line in (9)) and $d_1^* = \sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_1}{B}}\right) \equiv d_{1B3}^*$ (first line in (9)), which is feasible if and only if $Bv > bR_1$. Note also that $u_{B1} \le u_{B3}$. The true optimal d_1^* is calculated by comparing the utility levels in (9), that is

$$
d_1^* = \begin{cases} d_{1B1}^* & \text{if } u(d_{1B1}^*) \ge u(d_{1B2}^*) \text{ and } Bv \le bR_1 \\ d_{1B2}^* & \text{if } \{u(d_{1B2}^*) \ge u(d_{1B1}^*) \text{ and } Bv \le bR_1\} \\ & \quad \textit{or } \{Bv > bR_1 \text{ and } u(d_{1B2}^*) \ge u(d_{1B3}^*)\} \\ d_{1B3}^* & \text{if } Bv > bR_1 \text{ and } u(d_{1B3}^*) \ge u(d_{1B2}^*) \end{cases} \tag{10}
$$

Once d_1^* is determined in (10), $u\left(d_1^*\right)$ is calculated using (9) as shown in Appendix A. The terrorist attack choices are determined by (8). If $d_1^* = d_{1B2}^*$, the terrorist is indifferent between stockpiling and not stockpiling and we assume that it stockpiles.[b] If $d_1^* = d_{1B3}^*$, we must have the first line of (9) and therefore the terrorist does not stockpile ($A_1 = R_1/B$ and $A_2 = R_2/B$) according to (8). The utilities u and U follow from (6).

Case C: $\nu \ge b(gR_1 + R_2)/B$: In this case, the terrorist prefers stockpiling ($A_1 = 0$ and $A_2 = g(R_1 + R_2)/B$) if and only if

$$
\begin{aligned}
&\Delta\left(\frac{\sqrt{\frac{(gR_1 + R_2)}{B}}V}{\sqrt{\frac{v}{b}}}\right) \ge \frac{(R_1/B)V}{(R_1/B) + d_1} + \Delta\left(\frac{\sqrt{\frac{R_2}{B}}V}{\sqrt{\frac{v}{b}}}\right) \\
&\Leftrightarrow d_1 \ge \left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{(gR_1 + R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B}
\end{aligned} \tag{11}
$$

The defender's utility is

$$u(d_1) = \begin{cases} \dfrac{d_1 \nu}{(R_1/B) + d_1} - bd_1 + \delta b \left(\sqrt{\dfrac{\nu}{b}} - \sqrt{\dfrac{R_2}{B}} \right)^2 \\ \quad \text{if } d_1 \le \left(\dfrac{\sqrt{\dfrac{\nu}{b}}}{\Delta \left(\sqrt{\dfrac{(gR_1 + R_2)}{B}} - \sqrt{\dfrac{R_2}{B}} \right)} - 1 \right) \dfrac{R_1}{B} \\ \nu - bd_1 + \delta b \left(\sqrt{\dfrac{\nu}{b}} - \sqrt{\dfrac{(gR_1 + R_2)}{B}} \right)^2 \\ \quad \text{if } d_1 \ge \left(\dfrac{\sqrt{\dfrac{\nu}{b}}}{\Delta \left(\sqrt{\dfrac{(gR_1 + R_2)}{B}} - \sqrt{\dfrac{R_2}{B}} \right)} - 1 \right) \dfrac{R_1}{B} \end{cases} \tag{12}$$

As shown in Appendix B, we have three optimum candidates labeled C1, C2, and C3. These are $d_1^* = 0 \equiv d_{1C1}^*, d_1^* = \left(\dfrac{\sqrt{\dfrac{\nu}{b}}}{\Delta \left(\sqrt{\dfrac{(gR_1 + R_2)}{B}} - \sqrt{\dfrac{R_2}{B}} \right)} - 1 \right) \times \dfrac{R_1}{B} \equiv d_{1C2}^*$, and $d_1^* = \sqrt{\dfrac{R_1}{B}} \left(\sqrt{\dfrac{\nu}{b}} - \sqrt{\dfrac{R_1}{B}} \right) \equiv d_{1C3}^*$, which is feasible if and only if $B\nu > bR_1$. Note also that $u_{C1} \le u_{C3}$. Again the true optimal $d_1^*$is calculated by comparing the utility levels in (9), that is

$$d_1^* = \begin{cases} d_{1C1}^* & \text{if } u(d_{1C1}^*) \ge u(d_{1C2}^*) \text{ and } Bv \le bR_1 \\ d_{1C2}^* & \text{if } \{u(d_{1C2}^*) \ge u(d_{1C1}^*) \text{ and } Bv \le bR_1\}, \\ & \text{or } \{Bv > bR_1 \text{ and } u(d_{1C2}^*) \ge u(d_{1C3}^*)\} \\ d_{1C3}^* & \text{if } Bv > bR_1 \text{ and } u(d_{1C3}^*) \ge u(d_{1C2}^*) \end{cases} \tag{13}$$

Once d_1^* is determined in (13), $u(d_1^*)$ is calculated using (12) as shown in Appendix B. Analogously to case B, if $d_1^* = d_{1C2}^*$, the terrorist is indifferent between stockpiling and not stockpiling and we assume that it stockpiles. If $d_1^* = d_{1C3}^*$, the terrorist does not stockpile. The utilities u and U follow from (6).

Summarizing the equilibrium solution for the eight cases A1, A2, B1, B2, B3, C1, C2, and C3

Table 1 summarizes the equilibrium solution for the eight cases. The six variables have the same solution for cases B1 and C1, in which the government is inferior and refrains from defending in the first period ($d_1 = 0$). The terrorist exploits this by attacking in the first period ($A_1 = R_1/B$), which means not stockpiling. The six variables also have the same solution for cases B3 and C3. The terrorist does not stockpile, and the government defends in both periods because its valuation divided by its unit cost exceeds the terrorist's resource (in both periods) divided by its unit cost. For case B2, where the asset valuation is moderate, the government deters the terrorist in the first period, leading the terrorist to stockpile; while in the second period, the government gives up because the cost of defending against the accumulated attack is too large. For case C2, where the asset valuation is high, the government deters the government in the first period, leading the terrorist to stockpile, while still competing with the terrorist in the second period. In summary, the government does not defend in any period for case A1, defends only in period 2 for cases B1 and C1, defends only in period 1 for cases A2 and B2, and defends in both periods for cases B3, C2, and C3. The terrorist stockpiles only in cases B2 and C2.

NUMERICAL TESTING OF THE MODEL

We consider the baseline values $r_t = R_1 = b = B = d = D = g = v = V = 1$ and $R_2 = 0.1$, which gives case B2 with stockpiling. Case B2 is especially detrimental for the government, which gets outmaneuvered in period 2 ($d_2 = 0$) by the terrorist's stockpiling. Because case B2 in (10) and case C2 in (13) cannot be easily solved, we illustrate typical ranges of parameter values where stockpiling occurs to gain intuitive understanding. In particular, the next two subsections conduct one-way and two-way sensitivity analysis respectively.

One-way sensitivity analysis

Figure 2 presents the sensitivity analysis for each of the eight parameters v, R_1, R_2, b, B, g, d, and D when the remaining seven parameters are kept to their baselines. The baseline is shown as a dotted vertical line. The vertical stapled lines demarcate the eight cases. Figure 2(a) shows the

Table 1. Solution to Subgame Perfect Nash Equilibrium

Cases	Case A		Case B			Case C		
	$v \le \frac{bR_2}{B}$		$\frac{bR_2}{B} < v \le \frac{b(gR_1+R_2)}{B}$			$v \ge \frac{b(gR_1+R_2)}{B}$		
Cases	A1	A2	B1	B2	B3	C1	C2	C3
Stockpiling	No	No	No	Yes	No	No	Yes	No
$d_1=$	0	$\sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_1}{B}}\right)$	0	$\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)}-1\right)\frac{R_1}{B}$	$\sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_1}{B}}\right)$	Equivalent to case B1	$\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}}-\sqrt{\frac{R_2}{B}}\right)}-1\right)\frac{R_1}{B}$	Equivalent to case B3
$d_2=$	0	0	$\sqrt{\frac{R_2}{B}}\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)$	0	$\sqrt{\frac{R_2}{B}}\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)$		$\sqrt{\frac{(gR_1+R_2)}{B}}\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{(gR_1+R_2)}{B}}\right)$	
$A_1=$	R_1/B	R_1/B	R_1/B	0	R_1/B		0	
$A_2=$	R_2/B	R_2/B	R_2/B	$(gR_1+R_2)/B$	R_2/B		$(gR_1+R_2)/B$	
$u=$	0	$b\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_1}{B}}\right)^2$	$\delta b\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)^2$	$v-b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)}-1\right)\frac{R_1}{B}$	$b\left(\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_1}{B}}\right)^2+\delta\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)^2\right)$		$v-b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}}-\sqrt{\frac{R_2}{B}}\right)}-1\right)\frac{R_1}{B}+\delta b\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{(gR_1+R_2)}{B}}\right)^2$	
$U=$	$(1+\Delta)V$	$\left(\Delta+\sqrt{\frac{bR_1}{Bv}}\right)V$	$\left(1+\Delta\sqrt{\frac{bR_2}{Bv}}\right)V$	ΔV	$\left(\sqrt{R_1}+\Delta\sqrt{R_2}\right)\sqrt{\frac{b}{Bv}}V$		$\Delta V\sqrt{\frac{b(gR_1+R_2)}{Bv}}$	

Note: When $R_1 = 0$, we define $d_1 = A_1 = 0$; when $b = 0$, we define $d_1 = d_2 =$ infinity and the government wins all contests; when $B = 0$, we define $A_1 = A_2 =$ infinity and the terrorist wins all contests.

sensitivity relative to the government's asset valuation v. Case A1 occurs when $v \le 0.1$, case B1 occurs when $0.1 \le v \le 0.9$, case B2 occurs when $0.9 \le v \le 1.1$, case C2 occurs when $1.1 \le v \le 1.4$, and case C3 occurs when $v \ge 1.4$. When v is low, it is not worthwhile for the government to defend (case A1). As v increases, the government does not defend against the first attack, but defends against the second attack (case B1). As v increases further, the government turns this upside down by defending in period 1, inducing the terrorist to stockpile (case B2), while not being able to defend against the large second attack. Observe the sharp drop in the terrorist's utility U from case B1 to case B2 ($v = 0.9$). Further increase of v induces the government to defend in both periods, while the terrorist still stockpiles (case C2). The final increase of v induces the terrorist not to stockpile (case C3). The terrorist enjoys a utility increase in the transition from case C2 to case C3 ($v = 1.4$). The government's utility is always continuously increasing in v.

Figure 2(b) shows the sensitivity relative to the terrorist's first period resource R_1. Case C3 occurs when $0 \le R_1 \le 0.7$, Cases C2 and B2 (stockpiling) occur when $0.7 \le R_1 \le 0.9$ and $0.9 \le R_1 \le 1.2$, respectively. Case B1 occurs when $R_1 \ge 1.2$. Cases A1 or A2 never occur because the condition for case A, $v \le \frac{bR_2}{B}$, does not depend on R_1, and does not hold under baseline parameter values. Observe the low terrorist's utility for cases B2 and C2, as predicted earlier.

Figure 2(c) shows the sensitivity relative to the terrorist's second period resource R_2. Case A2 occurs when $1 \le R_2$, case B occurs when $0 \le R_2 < 1$. For case B, case B2 occurs when $0 \le R_2 \le 0.1$ and case B1 occurs when $0.1 \le R_2 \le 1$.

Figure 2(d) shows the sensitivity relative to the government's unit defense cost b. Case C3 occurs when $0 \le b \le 0.72$; case C2 occurs when $0.72 \le b \le 10/11$; case B2 occurs when $10/11 \le b \le 1.1$; case B1 occurs when $1.1 \le b \le 10$, and case A1 (not shown in Figure 2(d)) occurs when $10 \le b$.

Figure 2(e) shows the sensitivity relative to the terrorist's unit attack cost B. Case C3 occurs when $1.4 \le B$, case C2 occurs when $1.1 \le B \le 1.4$, case B2 occurs when $0.9 \le B \le 1.1$, case B1 occurs when $0.1 \le B \le 0.9$, and case A1 occurs when $B \le 0.1$. Observe that transitioning from right to left in Figure 2(e) gives movement

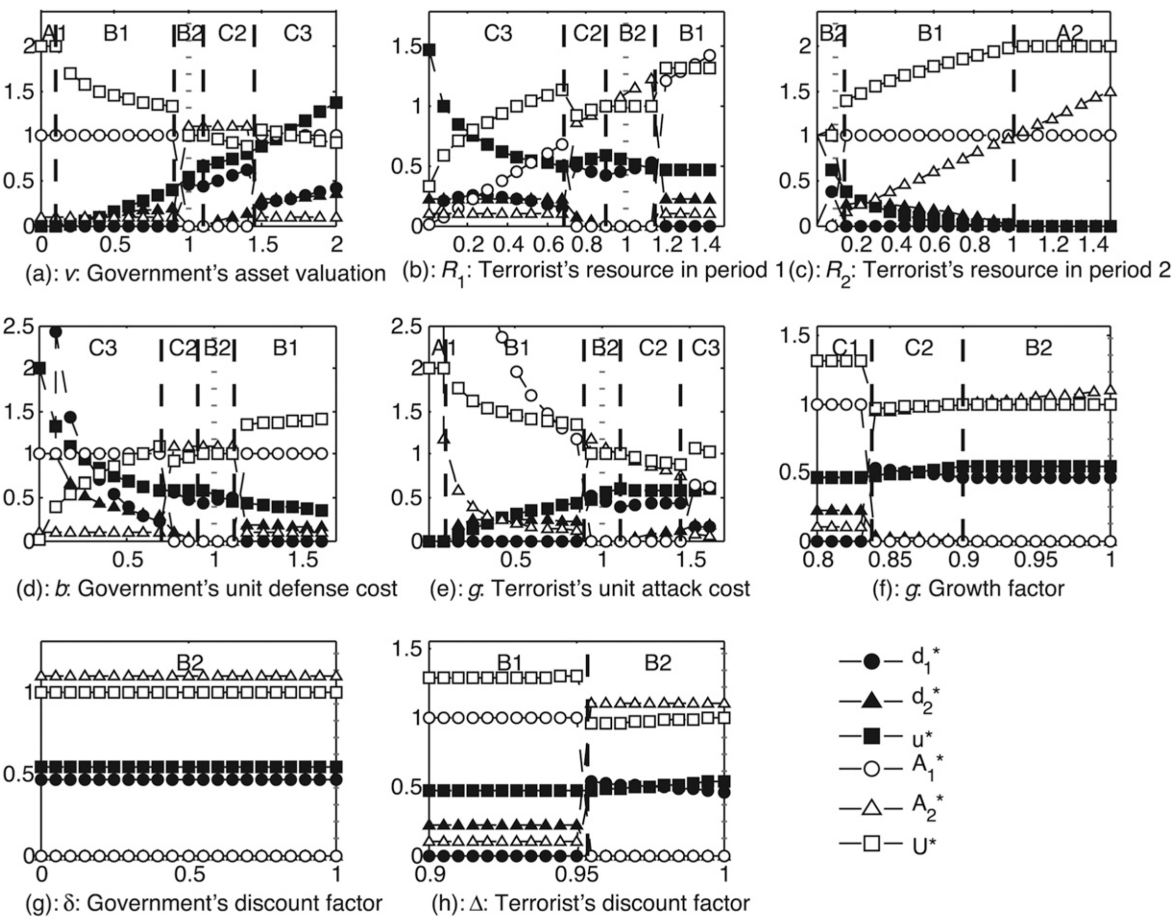

Figure 2. Sensitivity analysis for the eight parameters relative to the baseline when the terrorist is allowed to stockpile.

through the same cases as when transitioning from left to right in Figure 2(d), which illustrates the opposite impact on B and b.

Figure 2(f) shows the sensitivity relative to the growth factor g for the terrorist's stockpiled resource. Case C1 occurs when $0 \leq g \leq 0.84$, case C2 occurs when $0.84 \leq g \leq 0.9$, and case B2 occurs when $g \geq 0.9$. Figure 2(g) shows the sensitivity relative to the government's discount factor d. Case B2 occurs for all $0 \leq d \leq 1$. Figure 2(h) shows the sensitivity relative to the terrorist's discount factor D. Case B1 occurs when $0 \leq D \leq 0.95$, and case B2 occurs when $0.95 \leq D \leq 1$.

Two-way sensitivity analysis

Figure 3 presents the sensitivity analysis for each of the following four pairs of parameters: (R_1, R_2), (b, B), (v, g), and (d, D) when the remaining three pairs of parameters are kept to their baselines. The pentagram in each panel shows the crossing of the vertical and horizontal baselines specified in the beginning of the section.

Figure 3(a) shows the sensitivity relative to the terrorist's resources in periods 1 and 2. We have case A1 for $R_1 \geq R_2 \geq 1$. We have case A2 when $1 \leq R_2 \geq R_1$. Given $R_2 < 1$, we have case C when $gR_1 + R_2 < 1$ and case B otherwise. We find the terrorist stockpiling (cases C2 and B2) when R_1 is large and R_2 is small. Figure 3(b) shows the sensitivity relative to the government's utility defense cost vs. the terrorist's unit attack cost. We observe that only the ratio (B/b) maters when determining the cases. In particular, when B/b increases, we have cases

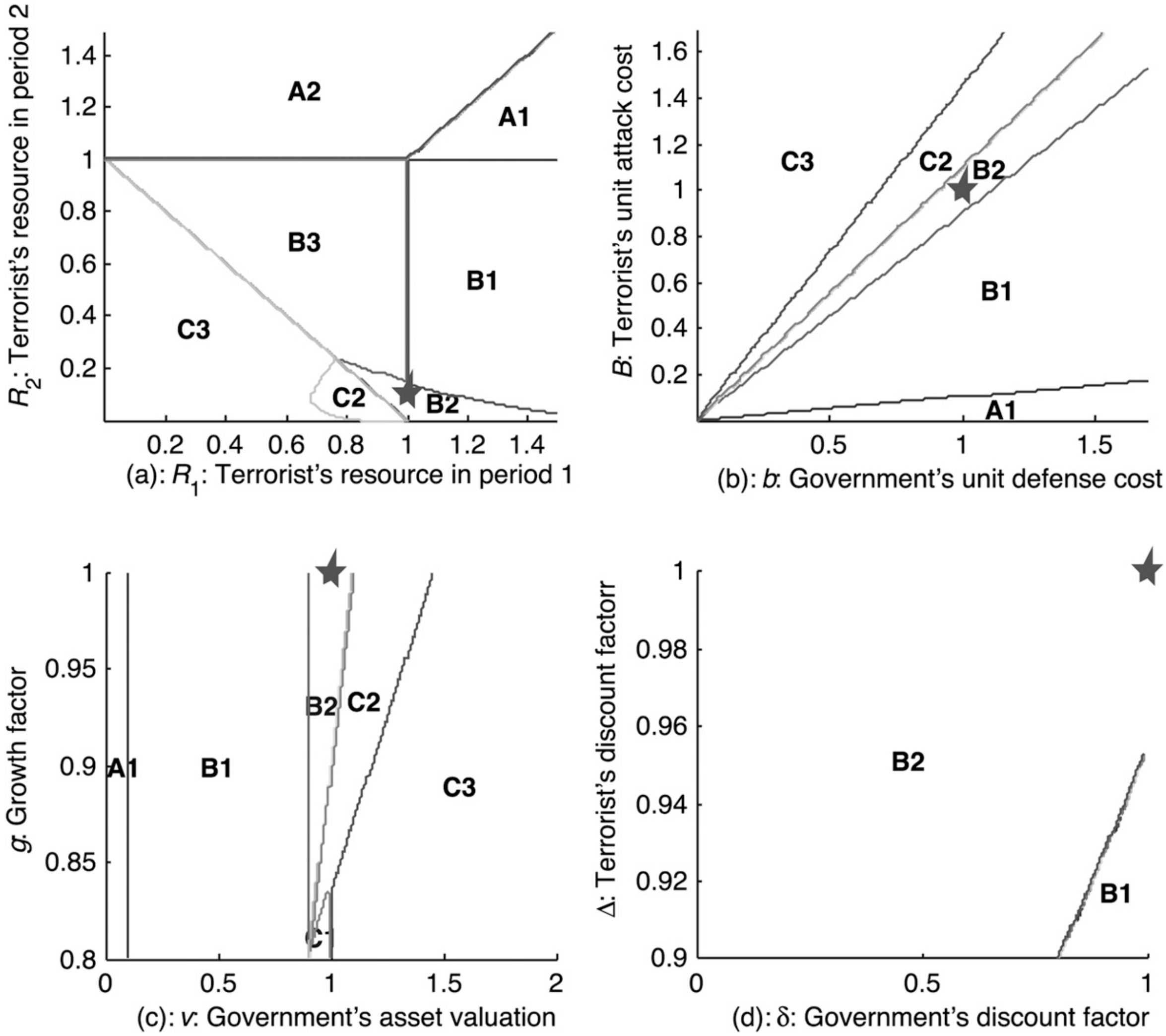

Figure 3. Sensitivity analysis for the eight parameters relative to the baseline.

A, B1, B2, C2, and C3, sequentially. We find the terrorist stockpiling (cases B2 and C2) when the ratio B/b is intermediate. Figure 3(c) shows the sensitivity relative to the government's asset valuation v and the growth factor g. We have case A regardless of the value g when v is smaller than 0.1 due to the condition $v \leq \frac{bR_2}{B}$. When have case B1 regardless of the value g when v is between 0.1 and 0.9. We find the terrorist stockpiling (cases B2 and C2) when $v > 0.9$ and $0.4(v - 0.9) < (g - 0.8)$. Figure 3(d) shows the sensitivity relative to the government's discount factor vs. the terrorist's discount factor. There are only two cases B1 and B2, where B2 (stockpiling) is possible when the government's discount factor is small or the terrorist's discount factor is high.

COMPARING WITH THE MODEL WHEN THE TERRORIST IS NOT ALLOWED TO STOCKPILE

In this section we compare our model that allows stockpiling with the one when the terrorist is not allowed to stockpile. The solution for the latter model is a shortened version of Table 1 after removing the columns of cases B2 and C2. We have thus not included a new table. Table 1 presents three cases A, B and C. For case A, stockpiling does not occur, and thus comparing with it is irrelevant. Hence we consider cases B and C.

Comparing for case B

For case B, $\frac{bR_2}{B} < v \leq \frac{b(gR_1 + R_2)}{B}$, if the terrorist is not allowed to stockpile, (10) simplifies to

$$d_1^* = \begin{cases} d_{1B1}^* & \text{if } Bv \leq bR_1 \\ d_{1B3}^* & \text{if } Bv > bR_1 \end{cases} \tag{14}$$

which restricts the government's strategy set.

For the case where B2 was optimal when stockpiling was feasible, there are two cases:

1. $B\nu \leq bR_1$. From (A5) in Appendix A we must have

$$v - b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B} > \delta b\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^2$$
$$\Leftrightarrow \Delta > \frac{\frac{R_1}{B}\, b\sqrt{\frac{v}{b}}}{\left(\frac{R_1}{B}\, b - \delta b\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^2 + v\right)\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} \tag{15}$$

Therefore, transitioning from allowing stockpiling to not allowing stockpiling: (a) the government's optimal decision in the first period will transition from d_{1B2}* to d_{1B1}*; and the utility will decrease from $v - b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B}$ to $\delta b\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^2$; and (b) the terrorist's utility will increase from ΔV to $\left(1 + \Delta\sqrt{\frac{bR_2}{Bv}}\right)V$. Summing up, the government prefers stockpiling when (15) and

$$\frac{R_2}{B} < \frac{v}{b} \leq Min\left\{\frac{gR_1 + R_2}{B}, \frac{R_1}{B}\right\} \tag{16}$$

The terrorist does not prefer stockpiling when (16) is satisfied. It may sound counter-intuitive that the terrorist may prefer not to have the option of stockpiling. This result follows because the terrorist is the second mover. The second mover may not necessarily prefer to have more options. Many options for the second mover constitute a threat that might lead the first mover to make choices that are detrimental to the second mover. If the second mover's options are limited, the first mover may take this limitation into account and make choices that are less detrimental to the second mover.

2. $B\nu > bR_1$. From (A5) in Appendix A we must have

$$v - b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B} > b\left(\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 + \delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^2\right)$$
$$\Leftrightarrow \Delta > \frac{\frac{R_1}{B} b\sqrt{\frac{v}{b}}}{\left(\frac{R_1}{B} b - b\left(\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 + \delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^2\right) + v\right)\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} \tag{17}$$

Therefore, transitioning from allowing stockpiling to not allowing stockpiling: (a) the government's optimal decision in the first period will transition from d_{1B2}* to d_{1B3}*; and the utility will decrease from $v - b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B}$ to $b\left(\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 + \delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^2\right)$; and (b) the terrorist's utility will transition from ΔV to $\left(\sqrt{R_1} + \Delta\sqrt{R_2}\right)\sqrt{\frac{b}{Bv}}V$. Hence the terrorist prefers stockpiling if and only if

$$\Delta V > \left(\sqrt{R_1} + \Delta\sqrt{R_2}\right)\sqrt{\frac{b}{Bv}}V \Leftrightarrow \Delta > \frac{\sqrt{\frac{R_1}{B}}}{\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}}$$
$$\Leftrightarrow \frac{v}{b} > \frac{1}{\Delta^2}\frac{R_1}{B} + \frac{R_2}{B} + \frac{2}{\Delta}\sqrt{\frac{R_1}{B}}\sqrt{\frac{R_2}{B}} \tag{18}$$

Summing up, the government prefers stockpiling when (17) and

$$\max\left\{\frac{R_1}{B},\frac{R_2}{B}\right\}<\frac{v}{b}\le\frac{gR_1+R_2}{B} \tag{19}$$

The terrorist prefers stockpiling when (19) and (18) are satisfied. Observe that

$$\frac{1}{\Delta^2}\frac{R_1}{B}+\frac{R_2}{B}+\frac{2}{\Delta}\sqrt{\frac{R_1}{B}}\sqrt{\frac{R_2}{B}}>\max\left\{\frac{R_1}{B},\frac{R_2}{B}\right\} \tag{20}$$

because the left-hand side contains both $\frac{1}{\Delta^2}\frac{R_1}{B}\ge\frac{R_1}{B}$ and $\frac{R_2}{B}$ as separate factors. Thus combining (18) and (19), we have

$$\frac{1}{\Delta^2}\frac{R_1}{B}+\frac{R_2}{B}+\frac{2}{\Delta}\sqrt{\frac{R_1}{B}}\sqrt{\frac{R_2}{B}}<\frac{v}{b}\le\frac{gR_1+R_2}{B} \tag{21}$$

In order for the range in (21) to have positive extension, which is the only way in which the terrorist could prefer stockpiling, we must have

$$\frac{1}{\Delta^2}\frac{R_1}{B}+\frac{R_2}{B}+\frac{2}{\Delta}\sqrt{\frac{R_1}{B}}\sqrt{\frac{R_2}{B}}\le\frac{gR_1+R_2}{B}$$
$$\Rightarrow\frac{1}{\Delta^2}+\frac{2}{\Delta}\sqrt{\frac{R_2}{R_1}}\le g \tag{22}$$

Comparing for case C

For case C, $v\ge\frac{b(gR_1+R_2)}{B}$, if the terrorist is not allowed to stockpile, (13) simplifies to

$$d_1^*=\begin{cases} d_{1C1}^* & \text{if } Bv\le bR_1 \\ d_{1C3}^* & \text{if } Bv>bR_1\end{cases} \tag{23}$$

which restricts the government's strategy set.

For the case where C2 was optimal when stockpiling was feasible, there are two cases:

1. $B\nu\le bR_1$. From (B4) in Appendix B we must have

$$v-b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}}-\sqrt{\frac{R_2}{B}}\right)}-1\right)\frac{R_1}{B}$$
$$+\delta b\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{(gR_1+R_2)}{B}}\right)^2$$
$$>\delta b\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)^2 \tag{24}$$

Therefore, transitioning from allowing stockpiling to not allowing stockpiling: (a) the government's optimal decision in the first period will transition from d_{1C2}* to d_{1C1}*; and the utility will decrease from $v-b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}}-\sqrt{\frac{R_2}{B}}\right)}-1\right)\frac{R_1}{B}+\delta b\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{(gR_1+R_2)}{B}}\right)^2$ to $\delta b\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)^2$; and (b) the terrorist's utility will change from $\Delta V\sqrt{\frac{b(gR_1+R_2)}{Bv}}$ to $\left(1+\Delta\sqrt{\frac{bR_2}{Bv}}\right)V$. Summing up, the government prefers stockpiling when (24) and

$$\frac{b(gR_1+R_2)}{B}\le\frac{v}{b}\le\frac{R_1}{B} \tag{25}$$

The terrorist prefers stockpiling if (24), (25) and

$$\Delta V\sqrt{\frac{b(gR_1+R_2)}{Bv}}\ge\left(1+\Delta\sqrt{\frac{bR_2}{Bv}}\right)V$$
$$\Leftrightarrow g\ge\left(\frac{1}{\Delta^2}+\frac{2}{\Delta}\sqrt{\frac{bR_2}{Bv}}\right)\frac{Bv}{bR_1} \tag{26}$$

2. $B\nu>bR_1$. From (B4) in Appendix B we must have

$$v-b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}}-\sqrt{\frac{R_2}{B}}\right)}-1\right)\frac{R_1}{B}$$
$$+\delta b\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{(gR_1+R_2)}{B}}\right)^2$$
$$>b\left(\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_1}{B}}\right)^2+\delta\left(\sqrt{\frac{v}{b}}-\sqrt{\frac{R_2}{B}}\right)^2\right) \tag{27}$$

Therefore, transitioning from allowing stockpiling to not allowing stockpiling: (a) the government's optimal decision in the first period will transition from d_{1C2}* to d_{1C3}*; and the utility will decrease from $v - b\left(\frac{\sqrt{\frac{v}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B} + \delta b\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{(gR_1+R_2)}{B}}\right)^2$ to $b\left(\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 + \delta\left(\sqrt{\frac{v}{b}} - \sqrt{\frac{R_2}{B}}\right)^2\right)$; and (b) the terrorist's utility will transition from $\Delta V\sqrt{\frac{b(gR_1+R_2)}{Bv}}$ to $\left(\sqrt{R_1} + \Delta\sqrt{R_2}\right)\sqrt{\frac{b}{Bv}}V$. Hence the terrorist prefers stockpiling if and only if

$$\Delta V\sqrt{\frac{b(gR_1+R_2)}{Bv}} \geq \left(\sqrt{R_1} + \Delta\sqrt{R_2}\right)\sqrt{\frac{b}{Bv}}V \Leftrightarrow g \geq \frac{1}{\Delta^2} + \frac{2}{\Delta}\sqrt{\frac{R_2}{R_1}} \tag{28}$$

which is equivalent to (22).

Summing up, the government prefers stockpiling when (27) and

$$\frac{R_1}{B} < \frac{v}{b} \text{ and } \frac{v}{b} \geq \frac{gR_1 + R_2}{B} \tag{29}$$

The terrorist prefers stockpiling when (29) and (28) are satisfied.

One-way sensitivity analysis

Figure 4 replicates Figure 2 when the terrorist is not allowed to stockpile. Observe that for all the ranges where the terrorist uses stockpiling in Figure 2, the terrorist's and government's utilities in Figure 4 are higher and lower, respectively, than those in Figure 2. In other words, with our baseline parameter values, the terrorist does not benefit from the option of stockpiling, while the government benefits from stockpiling. In Appendix C we provide another example with less realistic parameter values (high growth factor g and extremely low second period resource R_2), where the terrorist could benefit from the option of stockpiling.

CONCLUSION

In this paper we study a two-period game in which the terrorist decides whether to stockpile attack resources from the first to the second period. In the first period the government decides its first period defense effort followed by the terrorist determining whether or not to stockpile. In the second period the government determines its second period defense effort.

We first assume that stockpiling is an option for the terrorist. That option influences the government's first period defense. Our results show that the terrorist chooses stockpiling when: (a) the following parameters are in intermediate ranges: the government's asset valuation, the terrorist's first-period resource, the government's unit defense cost, and the terrorist's unit attack cost; (b) the terrorist's second-period resource is small; and (c) the terrorist's resource growth factor or discount factor. The terrorist's decision does not depend on the government's discount factor.

We secondly compare with the model where the terrorist is not allowed to stockpile. It turns out that for moderate growth factors and second-period resources for the terrorist, the terrorist does not prefer the option of stockpiling. We show that when the growth factor, from the first to the second period, for the terrorist's stockpiled resource is very large, and the terrorist's second-period resource is very small, then it is possible for the terrorist to prefer stockpiling. Similar cases in which a player prefers to restrict its strategy set are found in warfare (e.g. burning one's bridges or destroying one's resources). In contrast, our analysis shows that for all cases in which stockpiling happens at equilibrium, not allowing stockpiling would decrease the government's utility. This implies that the government always prefers that the terrorist has the stockpiling option. It might sound counterintuitive that the terrorist might prefer not to have the option of stockpiling, which follows because the terrorist is the second mover. Many options for the second mover constitute a threat that might lead the first mover to make choices that are detrimental to the second mover, and conversely when the second mover has fewer options.

Finally, we acknowledge that, like many other academic papers in the field of operations

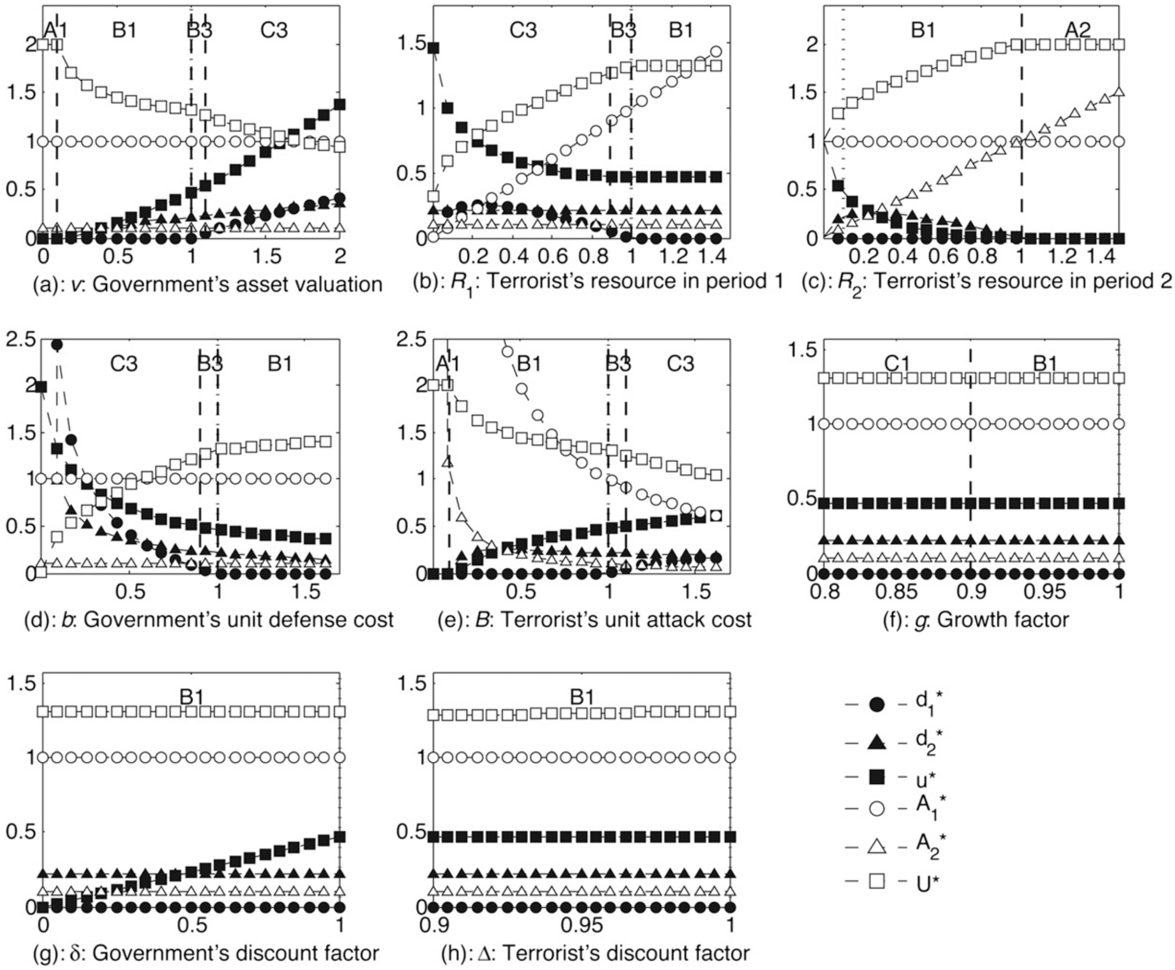

Figure 4. Sensitivity analysis for the eight parameters relative to the baseline when the terrorist is not allowed to stockpile.

research, the parameter values in this paper might be challenging to quantify in practice. Such theoretical papers might provide building blocks and insights for future practical work. We believe that the parameters in this paper (resource, unit costs, asset valuation, etc.) do have real-world counterparts. We propose two methods for future research to handle the quantification challenge. First, the parameters most challenging to quantify can be defined as a fuzzy variables and fuzzy logic model can be studied. Second, the range of possible variation of the parameters that are challenging to quantify can be determined and the most conservative ''worst case'' government's strategy can be obtained under the assumption that the parameters take the values that are most favorable for the terrorist. These parameters can be considered as additional strategic variables that the terrorist can choose within the specified ranges.

NOTES

[a] The authors have looked at the zero-sum game, corresponding to (4), where bd_1 and bd_2 are included as positive terms for the terrorist, and $\delta = \Delta$. The solutions d_1 and d_2 are the same as when (4) is applied, and the same eight subcases arise for when the terrorist prefers to stockpile, and for ranges of parameter values that are qualitatively similar to the ranges determined in this paper.

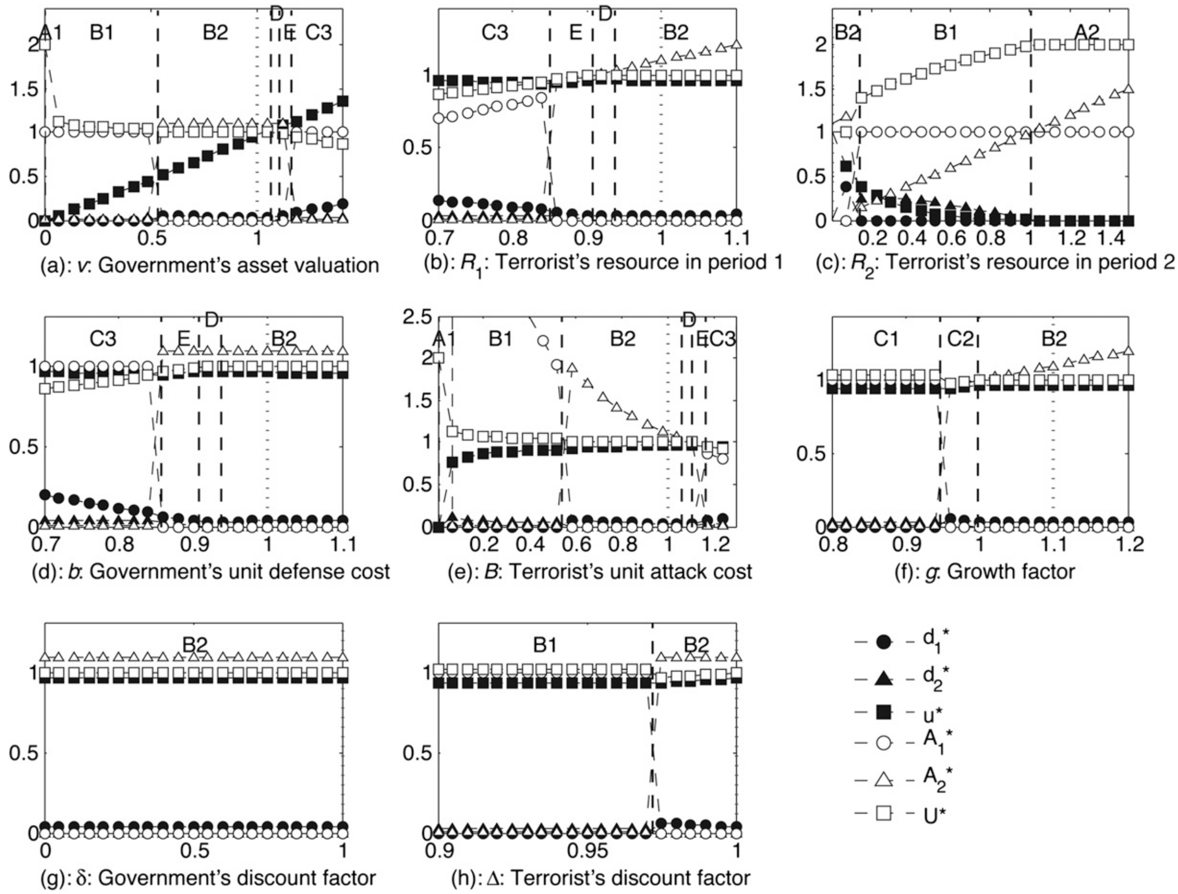

Figure 5. Sensitivity analysis for the eight parameters relative to the baseline when the terrorist is allowed to stockpile for $R_2 = 0.001$, $g = 1.1$.

[b] The indifference result follows because the government makes continuous choices. When d^*_{1B2} applies, the government chooses d^*_{1B2} optimally in the first period to make the terrorist indifferent between stockpiling and not stockpiling, and thereafter chooses d_2 optimally in the second period.

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APPENDIX A: CASE B IN SECTION 3

For the second line in (9), we have the corner solution $d_1^* = \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B}$, which is strictly positive. For the first line in (9), differentiating with respect to d_1 gives

$$\frac{\partial u(d_1)}{\partial d_1} = \frac{\nu}{R_1/B + d_1} - \frac{d_1 \nu}{(R_1/B + d_1)^2} - b = 0 => d_1^* = \sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right), \tag{A1}$$

which is feasible if and only if $B\nu > bR_1$). In summary, we have three possible candidates of d_1^*, that is $d_1^* = 0 \equiv d_{1B1}^*$ (first line in (9)), $d_1^* = \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1\right)\frac{R_1}{B} \equiv d_{1B2}^*$ (second line in (9)), and $d_1^* = \sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right) \equiv d_{1B3}^*$ (first line in (9)), and the true optimal d_1^* is calculated by comparing the utility levels in (9), which gives (10). Inserting $d_1^* = 0 \equiv d_{1B1}^*$ into the first line in (9) gives

$$u(d_{1B1}^*) = \delta b\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)^2 \tag{A2}$$

Inserting $d_1^* = \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} \equiv d_{1B2}^*$ into either the first or the second line in (9) gives

$$u(d_{1B2}^*) = \nu - b\left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} \tag{A3}$$

Inserting $d_1^* = \sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right) \equiv d_{1B3}^*$ into the first line in (9) gives

$$u(d_{1B3}^*) = b\left(\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 + \delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 \right) \tag{A4}$$

Note that we always have $u(d_{1B3}^*) \geq u(d_{1B1}^*)$, when both feasible. Therefore, we have:

$$d_1^* = \begin{cases} 0 & \text{if } \delta b\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 \geq \nu - b\left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right)\frac{R_1}{B} \\ & \text{and } B\nu \leq bR_1 \\ \hline \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} & \text{if} \left\{ \nu - b\left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right)\frac{R_1}{B} \geq \delta b\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)^2 \right. \\ & \left. \text{and } B\nu \leq bR_1 \right\}, \text{or} \\ & \left\{ B\nu > bR_1, \text{and } \nu - b\left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right)\frac{R_1}{B} \right. \\ & \left. \geq b\left(\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 + \delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)^2 \right) \right\} \\ \hline \sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right) & \text{if } B\nu > bR_1 \text{ and } b\left(\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right)^2 + \delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)^2 \right) \\ & \geq \nu - b\left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right)\frac{R_1}{B} \end{cases} \tag{A5}$$

APPENDIX B: CASE C IN SECTION 3

Similar to our analysis in Case B, we have three candidates of $d_1^* = 0 \equiv d_{1C1}^*$, $d_1^* \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{(gR_1 + R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} \equiv d_{1C2}^*$, and $d_1^* = \sqrt{\frac{R_1}{B}}\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}}\right) \equiv d_{1C3}^*$, where d_{1C3}^* is feasible if and only if $B\nu > bR_1$. Again the true optimal d_1^* is calculated by comparing the utility levels in (12), which gives (13). Inserting $d_1^* = 0 \equiv d_{1C1}^*$, into the first line of (12) we have

$$u(d_{1C1}^*) = \delta b\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}}\right)^2 \tag{B1}$$

Inserting $d_1^* \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} \equiv d_{1C2}^*$ into either the first or the second line of (12) we have

$$u(d_{1C2}^*) = \nu - b \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} + \delta b \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{(gR_1+R_2)}{B}} \right)^2 \tag{B2}$$

Inserting $d_1^* = \sqrt{\frac{R_1}{B}} \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}} \right) \equiv d_{1C3}^*$ into the first line of (12) we have

$$u(d_{1C1}^*) = b \left(\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}} \right)^2 + \delta \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}} \right)^2 \right) \tag{B3}$$

Note that we always have $u(d_{1C3}^*) \geq u(d_{1C1}^*)$ when both feasible. Therefore we have

$$d_1^* = \begin{cases} 0 & \text{if } \delta b \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}} \right)^2 \geq \nu - b \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} + \delta b \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{(gR_1+R_2)}{B}} \right)^2 \text{ and } B\nu \leq bR_1 \\ \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} & \begin{array}{l} \text{if} \left\{ \nu - b \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} + \delta b \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{(gR_1+R_2)}{B}} \right)^2 \geq \delta b \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}} \right)^2 \text{ and } B\nu \leq bR_1 \right\}, \text{or} \\ \left\{ B\nu > bR_1, \text{and } \nu - b \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} + \delta b \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{(gR_1+R_2)}{B}} \right)^2 \geq b \left(\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}} \right)^2 + \delta \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}} \right)^2 \right) \right\} \end{array} \\ \sqrt{\frac{R_1}{B}} \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}} \right) & \text{if } B\nu > bR_1 \text{ and } b \left(\left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_1}{B}} \right)^2 + \delta \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{R_2}{B}} \right)^2 \right) \geq \nu - b \left(\frac{\sqrt{\frac{\nu}{b}}}{\Delta\left(\sqrt{\frac{(gR_1+R_2)}{B}} - \sqrt{\frac{R_2}{B}}\right)} - 1 \right) \frac{R_1}{B} + \delta b \left(\sqrt{\frac{\nu}{b}} - \sqrt{\frac{(gR_1+R_2)}{B}} \right)^2 \end{cases} \tag{B4}$$

APPENDIX C: CONSIDERING THE CASE WHEN THE TERRORIST PREFERS STOCKPILING

We define case D as a subcase of case B2 where (18) and (19) are satisfied. We define case E as a subcase of case C2 where (24) and (25), or (28) and (29), are satisfied. Finding parameter values where the terrorist prefers stockpiling is not easy, but Figure 5 shows an example of such cases (D and E) where we keep the same parameter values as earlier, i.e. $r_t = R_1 = b = B = d = D = \nu = V = 1$, except increasing g with 10% to $g = 1.1$, and decreasing R_2 with a factor 100 to $R_2 = 0.001$. This means that the terrorist has virtually no resource in the second period, and enjoys growth on its first period resource. Cases D and E occur in panels a, b, d, e, not embracing the baseline value of each parameter. In Figure 5(a), D and E occur for ν strictly larger than 1. In Figure 5(b), D and E occur for R_1 strictly lower than 1. In Figure 5(d), D and E occur for b strictly lower than 1. In Figure 5(e), D and E occur for B strictly larger than 1. Cases D and E do not occur when we vary the other four parameter values.