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How companies and governments react to disasters

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Abstract

A two-stage model is developed between a company and a government. The government, representing the general public, earns taxes on production and chooses the tax rate in stage 1. The company allocates its resources into productive effort and safety effort. The disaster probability is modeled as a contest between the disaster magnitude and the two players' safety efforts. Three new propositions are developed. First, both the government's and the company's safety efforts decrease in the unit safety effort costs, and the company's safety effort increases in the unit production cost and in the company's resources. Second, both players' safety efforts are inverse U shaped in the disaster magnitude. Third, the company's safety effort increases, and the government's safety effort decreases, in taxation. Taxation can thus ameliorate companies' incentive to free ride on governments' provision of safety efforts.

Keywords

Disaster, safety effort, production, profit, taxation, conflict, contest success function

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Introduction

Balancing production against safety is a challenge for any company producing an output when disaster may occur with devastating consequences for the company and society. Examples include energy production (fossil fuels, nuclear power, etc.), travel (air, sea, etc.), mining, farming, fishing, and consumer goods. We consider a government and a representative company within a given industry. A two-stage model is developed in this article. A government chooses taxation of the representative company (Taxation generally varies across industries and is determined by a variety of mechanisms. In this article, we assume that the government chooses taxes and safety, which are two major strategic choices, to maximize profit.) in stage 1, and both the company and the government choose safety effort in stage 2. Safety efforts decrease the disaster impact. The players may free ride on each other's safety efforts.

Companies handle disasters differently. Risk of bankruptcy is large for new companies which may ignore the added risk of disasters. Even well-established companies may discount the possibility of low probability disasters with high consequences. Companies have some officials designated to ensure safety, and other officials designated to maximize production, and these officials routinely fight with each other.

This article argues that production and safety should be analyzed jointly. It becomes increasingly unacceptable for society that the list of disasters increases every year, often caused by companies ignoring safety concerns. We analyze production and disaster preparedness simultaneously. Such a focus is to our knowledge absent in the literature, with two exceptions provided by Hausken and Zhuang.[1,2] Whereas Hausken and Zhuang[1] analyze a model with specific functional forms, in this article general solutions are developed without specific functional forms. This means forfeiting numerical examples to ensure generality. Hausken and Zhuang[2] also analyze a model without specific functional forms, but the sequence of moves is different causing results different from this article. In particular, Hausken and Zhuang[2] assume that the government chooses both the tax rate and the safety effort in stage 1,

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while the company chooses the safety effort in stage 2. In contrast, this article assumes that choosing the tax rate is a more fundamental decision made by the government in stage 1, and in stage 2 both players choose their safety efforts, simultaneously and independently. The realism of this assumption is due to tax rates in many countries often remaining relatively stable over many years, and both the government and company can take this tax rate as given when choosing the optimal safety effort. In decentralized governments, where decision making is not made by one top official but is delegated to lower levels, tax rate and safety effort may be made by different governmental departments. Furthermore, although a company may sometimes know the government's safety effort in advance, this is not necessarily the case. Sometimes the reverse may be true, that is, the government observes the company's safety effort (which may be lax, adequate, etc.) and chooses its own safety effort based on that insight. Even more realistically, according to the argument in this article, neither the government nor the company has sufficient information about each other's safety effort in stage 2. That is, neither player can take the other player's safety effort as given when choosing its own safety effort. This means that the two players would be modeled to choose their safety efforts simultaneously and independently.

Combining safety effort and productive effort is uncommon in the literature. Within industrial organization viewed from economics, Tirole[3] focuses thoroughly on production but has a limited safety focus. Within industrial organization viewed from organizational psychology, Zohar[4] focuses on safety, while deemphasizing the role of production. Zhuang and Bier[5] and Hausken et al.[6] do not consider production, but how defenders allocate resources between terrorism, natural disaster, and all hazards. Osmundsen et al.[7] do not consider disasters but evaluate oil producers' incentives to assure steady supply, accounting for risk. Hausken[8] considers two groups striking a balance between production and contesting each other's production.

Disaster preparedness has been analyzed as a partnership game between the government and the private sector; see, for example, Sadka,[9] Flinders,[10] and Boase.[11] Kunreuther and Heal[12] and Hausken[13] consider the free-riding problem for security investment, Kunreuther and Useem[14] determine reaction strategies for catastrophes, and Kunreuther[15] consider insurance and mitigation related to catastrophic risks.

Asche and Aven[16] consider the business incentives for investing in safety. England[17] determines profit maximization for a firm using a disaster-prone technology. Golbe[18] determines an indeterminate relationship between profit and safety in the US airline industry. Carmichael[19] shows that safety will be underprovided in a competitive labor market with complete information. Hale[20] considers safety management in production. Furthermore, Azaiez and Bier[21] consider the optimal resource allocation for security in reliability systems, and Cheung and Zhuang[22] consider a regulation game between a government that regulates and competing companies that balance production and safety efforts. An incomplete contract analysis of accident prevention has been made by Boyer and Laffont,[23] showing why constitutional constraints on environmental policy can be preferable.

Birsch and Fielder[24] show how the Ford Motor Company "traded off" safety considerations to ensure profitability. Any optimization approach inevitably makes such trade-offs, either explicitly or implicitly. Quantifying trade-offs contributes to transparency which we value in democratic societies. Transparency opens up for criticism by those that disagree, but also provides more facts to explain why disasters occur. For disasters causing injury and death, trade-offs mean valuing life quantitatively, usually handled by considering "the value of a statistical life."[25]

We develop a model with two players, that is, one company and one government. Each player maximizes profit by exerting production effort, but is additionally concerned about safety since a disaster may strike which could be costly for both players. Production effort means applying technology, equipment, and personnel to produce high volumes of a high-quality product. Safety effort means designing, implementing, and ensuring compliance to laws, regulations and procedures ensuring safe production. Safety effort(s) by one or both players decrease(s) the probability of disaster.

Section "The model" presents the model. Section "Analyzing the model" analyzes the model. Section "Numerical examples" presents examples. Section "Conclusion" concludes the article. Appendix 1 provides a notation table. Appendix 2 proves Propositions 1–3.

The model

To purify how the company strikes a balance between profit and safety, we assume that the company has resources R (e.g. a capital good, or labor) which can be converted with unit conversion cost A into productive effort E and with unit cost B into safety effort S, where

$$\begin{aligned} R = AE + BS &\Leftrightarrow E = (R - BS)/A \\ \Leftrightarrow S &= (R - AE)/B \end{aligned} \tag{1}$$

The time duration over which the company allocates the resources R can be short or long. Other cost components such as maintenance, inventory management, and marketing are not included in the resources R. We confine attention to productive effort E and safety effort S since they have distinctly different impact. Investments that may appear to serve both production and safety, for example, reliable production lines, are in our conception divided into one investment into production (which has no emphasis on reliability but merely ensures maximum production) and one investment into safety (which ensures maximum reliability regardless of production). The company's one and only strategic decision variable is its safety effort S, where production E follows from equation (1). The company's production function is $H(S)$, where we assume $\partial H/\partial S < 0$ and hence production decreases in safety effort. It follows from equation (1) that $H(S = R/B) = 0$ and thus maximum safety effort causes zero production.

The disaster probability $p(D, S, s)$ is impacted by the company's safety effort S, the government's safety effort s, and the disaster magnitude D chosen by the nature, $D \geqslant 0$. We assume that $p(D, S, s)$ is continuous and twice differentiable and that $\partial p/\partial S \leqslant 0$ and $\partial p/\partial s \leqslant 0$. Hence, the disaster probability increases in the disaster magnitude and decreases in both safety efforts.

The company's profit is

$$U(S, s, \tau) = (1-\tau)H(S) - F(D, S, s)p(D, S, s) \tag{2}$$

where τ is the taxation percentage chosen by the government, $0 \leqslant \tau \leqslant 1$, and $H(S)$ is the company's production function based on safety effort S in equation (1). The company keeps the remaining fraction $1-\tau$. The company is negatively affected by the disaster as quantified by $F(D, S, s)$. $F(D, S, s)$ is impacted by factors such as costs to businesses and society of disasters that impact the company, public relations costs to ensure a company's reputation when suffering from a disaster, lawyer's fees required to settle disputes and process court cases going to limit the company from financial responsibilities of the disaster. In other words, the company's profit equals the benefit from production after taxes have been paid, subtracting the disaster impact multiplied with the disaster probability. We assume $\partial F/\partial D \geqslant 0$, $\partial F/\partial S \leqslant 0$, and $\partial F/\partial s \leqslant 0$. Hence, the negative impact on the government increases with the disaster magnitude and decreases in both safety efforts.

The government's profit is

$$u(S, s, \tau) = \tau H(S) - f(D, S, s)p(D, S, s) - bs \tag{3}$$

where the government is negatively affected by the disaster as quantified by $f(D, S, s)$, and b is the government's unit cost of safety effort. In other words, the government's profit equals the benefit from taxes from production, subtracting the disaster impact multiplied with the disaster probability, and subtracting the expenditure of safety effort. We assume $\partial F/\partial D \geqslant 0$, $\partial F/\partial S \leqslant 0$, and $\partial F/\partial s \leqslant 0$. Hence, the negative impact on the company increases with the disaster magnitude and decreases in both safety efforts.

We consider a two-stage game where the government chooses the optimal tax rate τ in stage 1. In stage 2, the company chooses the optimal safety effort S, which according to equation (1) simultaneously determines the optimal productive effort E, and the government chooses the optimal safety efforts.

Analyzing the model

Applying backward induction, we first solve stage 2.

Definition 1. A strategy pair (s, S) is a subgame-perfect Nash Equilibrium if and only if

$$S = S(s) = \arg\max_{S \geqslant 0} U(s, S, \tau), \quad s = \arg\max_{s \geqslant 0} u(s, S, \tau) \tag{4}$$

Using equation (4), for stage 2, the first-order conditions are

$$\begin{cases} s > 0 \Leftrightarrow \frac{\partial u(S,s,\tau)}{\partial s} = -\frac{\partial f(D,S,s)}{\partial s}p(D,S,s) \\ \quad -f(D,S,s)\frac{\partial p(D,S,s)}{\partial s} - b = 0 \\ s = 0 \Leftrightarrow \frac{\partial u(S,s,\tau)}{\partial s} = -\frac{\partial f(D,S,s)}{\partial s}p(D,S,s) \\ \quad -f(D,S,s)\frac{\partial p(D,S,s)}{\partial s} - b < 0 \end{cases}$$
$$\begin{cases} S > 0 \Leftrightarrow \frac{\partial U(S,s,\tau)}{\partial S} = (1-\tau)\frac{\partial H(S)}{\partial S} - \frac{\partial F(D,S,s)}{\partial S} \\ \quad \times p(D,S,s) - F(D,S,s)\frac{\partial p(D,S,s)}{\partial S} = 0 \\ S = 0 \Leftrightarrow \frac{\partial U(S,s,\tau)}{\partial S} = (1-\tau)\frac{\partial H(S)}{\partial S} - \frac{\partial F(D,S,s)}{\partial S} \\ \quad \times p(D,S,s) - F(D,S,s)\frac{\partial p(D,S,s)}{\partial S} < 0 \end{cases} \tag{5}$$

where we assume that $u(S, s=0, \tau)$, $f(D, S, s=0)$, and $p(D, S, s=0)$ are differentiable with respect to s when $s = 0$, and $U(S=0, s, \tau)$, $F(D, S=0, s)$, and $p(D, S=0, s)$ are differentiable with respect to S when $S = 0$. The second-order conditions are (The second-order conditions are sufficient since the government chooses τ in stage 1 and s is stage 2, and thus the Hessian matrix is not needed.)

$$\begin{aligned} \frac{\partial^2 u(S,s,\tau)}{\partial s^2} &= -\frac{\partial^2 f(D,S,s)}{\partial s^2}p(D,S,s) \\ &\quad - 2\frac{\partial f(D,S,s)}{\partial s}\frac{\partial p(D,S,s)}{\partial s} - f(D,S,s)\frac{\partial^2 p(D,S,s)}{\partial s^2}, \\ \frac{\partial^2 U(S,s,\tau)}{\partial S^2} &= (1-\tau)\frac{\partial^2 H(S)}{\partial S^2} - \frac{\partial^2 F(D,S,s)}{\partial S^2}p(D,S,s) \\ &\quad - 2\frac{\partial F(D,S,s)}{\partial S}\frac{\partial p(D,S,s)}{\partial S} - F(D,S,s)\frac{\partial^2 p(D,S,s)}{\partial S^2} \end{aligned} \tag{6}$$

which are negative when $\partial^2 p(D,S,s)/\partial s^2 \geqslant 0$, $\partial^2 f(D,S,s)/\partial s^2 \geqslant 0$, $\partial^2 p(D,S,s)/\partial S^2 \geqslant 0$, $\partial^2 F(D,S,s)/\partial S^2 \geqslant 0$, and $\partial^2 H(S)/\partial S^2 \leqslant 0$.

Optimal safety efforts S and s follow from solving equation (5) and are inserted into equation (3) to give the government's first-stage profit $u(S(\tau), s(\tau), \tau)$. Differentiating gives the first- and second-order conditions

$$\begin{aligned} \frac{du(S(\tau),s(\tau),\tau)}{d\tau} &= \frac{\partial u(S(\tau),s(\tau),\tau)}{\partial \tau} \\ &\quad + \frac{\partial u(S(\tau),s(\tau),\tau)}{\partial s}\frac{\partial s(\tau)}{\partial \tau} + \frac{\partial u(S(\tau),s(\tau),\tau)}{\partial S}\frac{\partial S(\tau)}{\partial \tau} = 0, \\ \frac{d^2 u(S(\tau),s(\tau),\tau)}{d\tau^2} &\leqslant 0 \end{aligned} \tag{7}$$

It turns out that analyzing the model in this article confirms Propositions 1–4 in the different model presented by Hausken and Zhuang,[2] although the assumptions are slightly different. We omit the slightly different assumptions needed to generate the equivalent Propositions 1–4 due to space considerations. In particular, this model assumes that the government chooses tax rate in stage 1, and both players choose their safety efforts, simultaneously and independently, in stage 2. In contrast, Hausken and Zhuang[2] assume that the government chooses both the tax rate and the safety effort in stage 1, while the company chooses the safety effort in stage 2. The equivalent results for Propositions 1–4 reveal a certain robustness. Let us summarize Propositions 1–4 presented by Hausken and Zhuang[2] verbally. First, as the safety effort of one player approaches infinity, the change in the other player's safety effort, with respect to the first player's safety effort, approaches zero. Second, an infinitely large safety effort by any player causes the disaster probability and negative impact of the disaster to decrease toward a constant. Third, the two players' safety efforts are strategic substitutes, that is, as one player's safety effort approaches infinity, the other player's safety effort approaches zero. Fourth, an increase in one player's safety effort decreases the other player's safety effort which enables the players to free ride on each other's safety efforts.

We now proceed with three propositions that differ from those presented by Hausken and Zhuang.[2] The assumptions made for the three propositions are valid for a broad range of parameter values.

Proposition 1. $\partial s/\partial b \leqslant 0$, $\partial S/\partial B \leqslant 0$, $\partial S/\partial A \geqslant 0$, and $\partial S/\partial R \geqslant 0$ if $\partial^2 p(D,S,s)/\partial s^2 \geqslant 0$ and $\partial^2 f(D,S,s)/\partial s^2 \geqslant 0$.

Proposition 1 states that both safety efforts decrease in the unit safety effort costs and that the company's safety effort increases in the unit production cost and in the company's resources. The two second-order conditions specify decreasing marginal effectiveness of the government's safety effort on the disaster probability and how the government is negatively affected by the disaster.

Proposition 2. Assume $F(D,S,s) = F$, $f(D,S,s) = f$, $\partial^2 p(D,S,s)/\partial S^2 \geqslant 0$, $\partial^2 p(D,S,s)/\partial s^2 \geqslant 0$, $\partial^2 p(D,S,s)/\partial S\partial D \leqslant 0$, $\partial^2 p(D,S,s)/\partial s\partial D \leqslant 0$ and

$$\left(\frac{\partial^2 p(D,S,s)}{\partial S\partial D}\right)\left(\frac{\partial^3 p(D,S,s)}{\partial S^2\partial D}\right) \leqslant \left(\frac{\partial^3 p(D,S,s)}{\partial S\partial D^2}\right)\left(\frac{\partial^2 p(D,S,s)}{\partial S^2}\right),$$
$$\left(\frac{\partial^2 p(D,S,s)}{\partial s\partial D}\right)\left(\frac{\partial^3 p(D,S,s)}{\partial s^2\partial D}\right) \leqslant \left(\frac{\partial^3 p(D,S,s)}{\partial s\partial D^2}\right)\left(\frac{\partial^2 p(D,S,s)}{\partial s^2}\right) \tag{8}$$

First, $d^2S/dD^2 \leqslant 0$, and constants $D_c \geqslant 0$ and $D_g \geqslant 0$ exist such that $dS/dD \geqslant 0$ when $D < D_c$, $dS/dD \leqslant 0$ when $D > D_c$, $d^2s/dD^2 \leqslant 0$, $ds/dD \geqslant 0$ when $D < D_g$, and $ds/dD \leqslant 0$ when $D > D_g$. Second, $\lim_{D\to\infty} S(D) = S_D \geqslant 0$ and $\lim_{D\to\infty} s(D) = s_D \geqslant 0$ where S_D and s_D are constants.

Presenting Proposition 2 for general $F(D, S, s) = F$, and $f(D, S, s) = f$ is technically possible but gives two inequalities with 24 terms rather than the two terms in equation (8). Proposition 2, where F and f are constants, states that both players' safety efforts are inverse U shaped in the disaster magnitude D. That is, S and s first increase in D to protect against the escalating disaster magnitude. Eventually, maxima are reached where further increases in S and s are not cost efficiently justified. Thereafter, S and s decrease approaching constants S_D and s_D as the disaster becomes large. This general result is consistent with Hausken and Zhuang,[1] which showed inverse U-shaped safety efforts in the disaster magnitude D for both company and government, adopting specific functional forms. That both players' safety efforts are inverse U shaped in the disaster magnitude D is a stronger result than Hausken and Zhuang's[2] Proposition 3 where it was only possible to prove that the company's safety effort is inverse U shaped in the disaster magnitude D but not possible to prove the same result for the government safety effort. This was due to the sequence of moves, where the government's optimal strategy depends on D not only through the government's profit function but also through the indirect changes to the company's safety effort.

For the next proposition we interpret $S(\tau)$ and $s(\tau)$ in stage 2 as best response functions to τ chosen by the government in stage 1. When we here and later write one variable as a function of another variable, it is to indicate best responses.

Proposition 3. Assume $\partial^2 H/\partial S^2 \geqslant 0$, $\partial^2 p(D,S,s)/\partial S^2 \geqslant 0$, $\partial^2 F(D,S,s)/\partial S^2 \geqslant 0$, $S > 0$. Then, $dS/d\tau = \partial S/\partial\tau > 0$,

$$\frac{du(S,s,\tau)}{d\tau} \geqslant 0 \text{ if } H(S) + \tau\frac{\partial H}{\partial S}\frac{\partial S}{\partial \tau} - f(D,S,s) \times \frac{\partial p(D,S,s)}{\partial S}\frac{\partial S}{\partial \tau} - p(D,S,s)\frac{\partial f(D,S,s)}{\partial S}\frac{\partial S}{\partial \tau} \geqslant 0, \tag{9}$$

$$\frac{dU(S,s,\tau)}{d\tau} \geqslant 0 \text{ if } -H(S) + (1-\tau)\frac{\partial H}{\partial S}\frac{\partial S}{\partial \tau} - F(D,S,s) \times \frac{\partial p(D,S,s)}{\partial S}\frac{\partial S}{\partial \tau} - p(D,S,s)\frac{\partial F(D,S,s)}{\partial S}\frac{\partial S}{\partial \tau} \geqslant 0. \tag{10}$$

If, additionally, $\partial^2 f(D,S,s)/\partial S\partial s \geqslant 0$, then $ds/d\tau \leqslant 0$.

We trivially have $\partial u(S,s,\tau)/\partial\tau = H(S) \geqslant 0$, $\partial U(S,s,\tau)/\partial\tau = -H(S) \leqslant 0$. $\partial s/\partial\tau = 0$, $ds/d\tau = (\partial s/\partial S)(\partial S/\partial\tau)$; $dS/d\tau = \partial S/\partial\tau$. Proposition 3 assumes decreasing marginal effectiveness of the company's safety effort on the disaster probability and how the company is negatively affected by the disaster. The assumption $\partial^2 H/\partial S^2 \geqslant 0$ expresses a concave production function, and hence production decreases convexly in safety effort. (The common concave production functions are a good approximation in highly developed economies, for example, when the ratio of capital to labor is large.[26]) With these assumptions, first, the company's safety effort increases in taxation. Taxation can thus ameliorate the companies' incentive to free ride on the governments' provision of safety effort. From the company's perspective, increasing taxation to some extent corresponds to increasing unit cost of production, and thus this result corresponds to $\partial S/\partial A \geqslant 0$ in Proposition 1. Second, equation (9) is satisfied when $\tau = 0$ since the first, third, and fourth terms on the left-hand side are positive, and the second term is zero. Hence, the government can increase its profit by increasing taxation above $\tau = 0$. The inequality (9) expresses a possible upper limit above which taxation cannot be profitably increased. Third, in equation (10), the first and second terms on the left-hand side are negative, and the third and fourth terms are positive. With sufficiently large production, $H(S)$ is large and equation (10) is not satisfied which intuitively means that the company does not prefer increasing taxation. This is especially true when τ increases from $\tau = 0$ when the second term is most negative.

Numerical examples

This section provides numerical examples in which the influence of the game parameters on the strategic variables and the game outcome is analyzed. We assume the ratio from disaster probability,[27] $p(D, S, s) = D^2/[(D + S)(D + s)]$, scaling functions $F(D, S, s) = F$, $f(D, S, s) = f$, and production function $H(E) = E$. Solving as in section "Analyzing the model" gives six cases illustrated in Figures 1 and 2:

Case 1A: $S = R/B$, $s > 0$. The company allocates all its resources R/B into safety effort, causing bankruptcy. This theoretical case emerges when the disaster D is small, and the company's resources divided by its unit cost of safety effort, R/B, is low.
Case 1B: $R/B > S > 0$, $s > 0$; the interior solution.
Case 2: $S = 0$, $s > 0$. The government bails out the company which focuses exclusively on production.
Case 3A: $S = R/B$, $s = 0$.
Case 3B: $R/B > S > 0$, $s = 0$. The government provides no safety effort, while the company strikes a balance between production and safety effort, which is possible if the government has a large unit cost b of safety effort, or the disaster D is large.
Case 4: $S = 0$, $s = 0$. Both the government and company have large unit costs b and B of safety effort, or the disaster is large.

Figure 1 plots the three optimal strategic choice variables which are the optimal safety efforts s and S and tax rate τ and also plots the productive effort E, the disaster probability p, and the profits u and U, relative to the baseline $R = 0.5$, $D = B = F = f = b = 1$, and $A = 0.1$ which causes case 4 where both players withdraw safety effort, $S = s = 0$. Division with 5, that is, $u/5$ and $E/5$, is done for scaling purposes. In panel (a), as the disaster D decreases below $D = 1.00$, a transition to case 2 occurs where only the government exerts safety effort $s > 0$, and the company continues to withdraws safety effort, $S = 0$, in order to focus on production. Both players' profits and the tax rate increase, and the disaster probability decreases. As D decreases below $D = 0.42$, a transition to case 1B occurs where both players exert safety effort, and the disaster probability decreases. Taxation increases inducing the company to decrease productive effort, and both players' profits decrease. As D decreases below $D = 0.10$, a transition to case 1A occurs where the company allocates all its resources to safety effort.

In panel (b), profits, productive effort, and taxation increase within case 4 as the company's resources R increase within $0 < R < 5.33$. When R is extremely large $R > 5.33$ (not plotted), we have case 3B where the government free rides without exerting safety effort.

In panel (c), as the company suffers higher unit cost A of production, the company exerts lower productive effort E. As A increases above $A = 0.36$, a transition to case 3B occurs where the government continues to withdraw safety effort, $s = 0$, and imposes lower taxation. That is, the government free rides on the company's safety effort, while both players earn negative and decreasing profits. As A increases above $A = 1.25$, a transition to case 3A occurs where the company allocates all its resources to safety effort, causing bankruptcy.

In panel (d), as the unit cost B of company safety effort decreases below $B = 0.40$, a transition to case 3B occurs where the company finds it worthwhile to exert safety effort and the government free rides. Company profits increase, the company's productive effort decreases, and the government profits decrease.

In panel (e), as the government's unit cost b of safety effort decreases below $b = 1.00$, a transition to case 2 occurs where only the government exerts safety effort $s > 0$. That is, now the company takes advantage and free rides on the government's safety effort. Both players' profits and taxation increase.

In panel (f), as the scaling function F for how the company is negatively affected by the disaster decreases, the government's profit and taxation increase, while the company's profit is inverse U shaped in case 4 for the range $0 < F < 5.15$. For extremely large $F > 5.15$ (not plotted), the company exerts safety effort, while the government does not, that is, case 3B.

In panel (g), as the scaling function f for how the government is negatively affected by the disaster increases above $f = 1.00$, a transition to case 2 occurs where only the government exerts safety effort $s > 0$, suffering lower profit, while the company earns higher profit despite higher taxation.

Figure 1 illustrates all the six cases by making the company resource constrained ($R = 0.5$) which gives case 4 in all seven panels. In order to illustrate

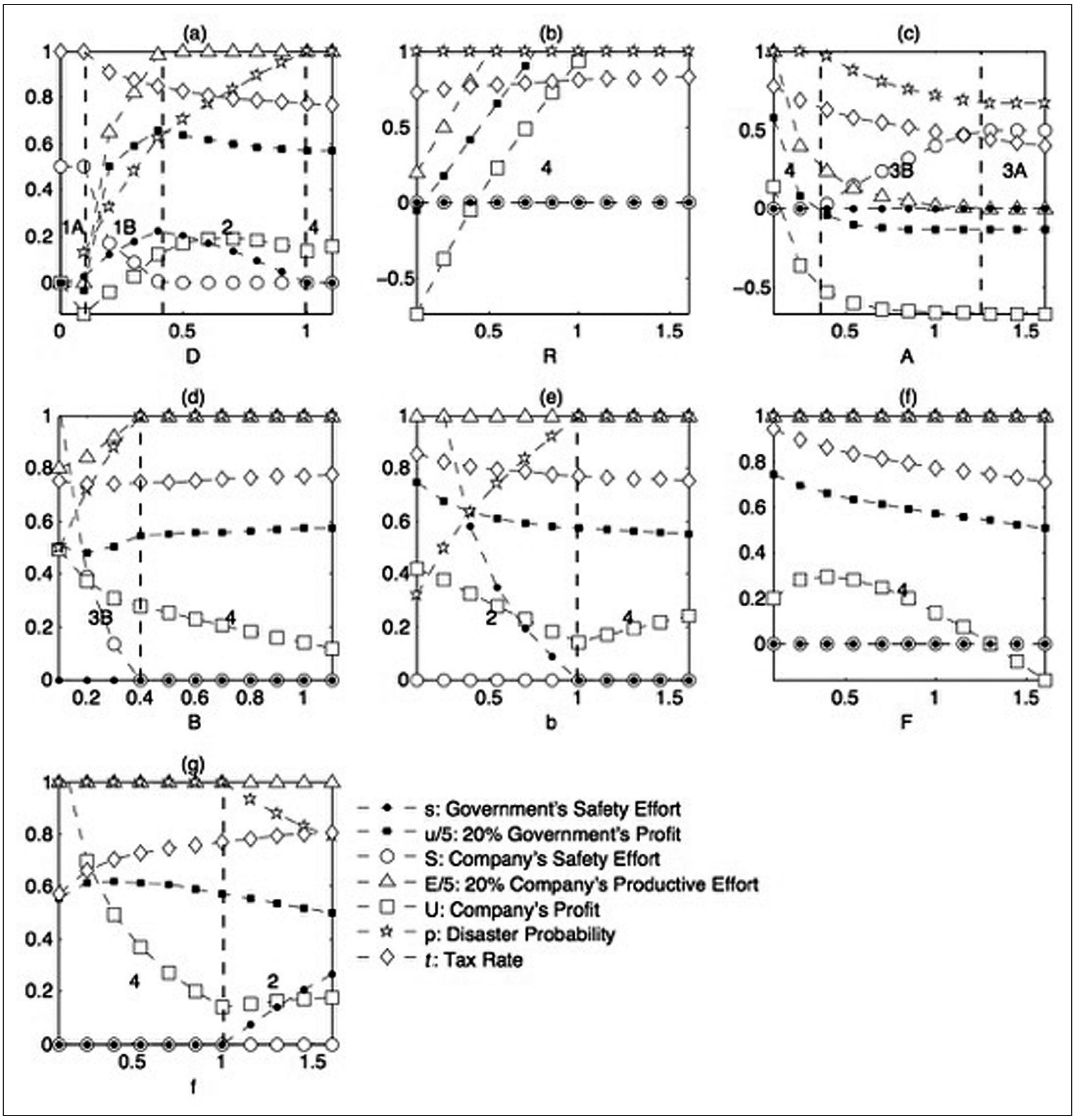

Figure 1. Equilibrium behavior of s, S, τ, E, p, u, and U as functions of (a) D, (b) R, (c) A, (d) B, (e) b, (f) F, and (g) f with baseline values $R = 0.5$, $D = B = F = f = b = 1$, and $A = 0.1$. Division of u and E with 5 is for scaling purposes.

additional or other cases, Figure 2 makes three changes to the baseline. First, the company's resources are quadrupled to $R = 2$. Second, the company's unit cost of production is increased to $A = 0.25$. Third, the government's unit cost b of safety effort is cut in half to $b = 0.5$. This gives case 2 at the baseline where the government provides safety effort, $s > 0$, and the company withdraws safety effort, $S = 0$. More specifically, Figure 2 plots the three optimal strategic choice variables which are the optimal safety efforts s and S and tax rate τ and also plots the productive effort E, the disaster probability p, and the profits u and U, relative to the baseline $R = 2$, $D = B = F = f = 1$, $b = 0.5$, and $A = 0.25$. Division with 10, that is, $u/10$ and $E/10$, is done for scaling purposes.

In panel (a), as the disaster D increases, the optimal tax rate τ decreases. Intermediate D around the baseline $D = 1$ is accompanied with intermediate taxation causing the company in case 2 to withdraw safety effort. Low D, $0.2 < D < 0.93$, causes the interior solution case 1B with higher taxation τ inducing the company to exert positive safety effort $S > 0$. Extremely low $D < 0.2$ leads the government in case 3A to exert zero safety effort, $s = 0$, while imposing high taxation to force the company to allocate all its resources R/B into safety effort. Conversely, high $D > 2.00$ overwhelms

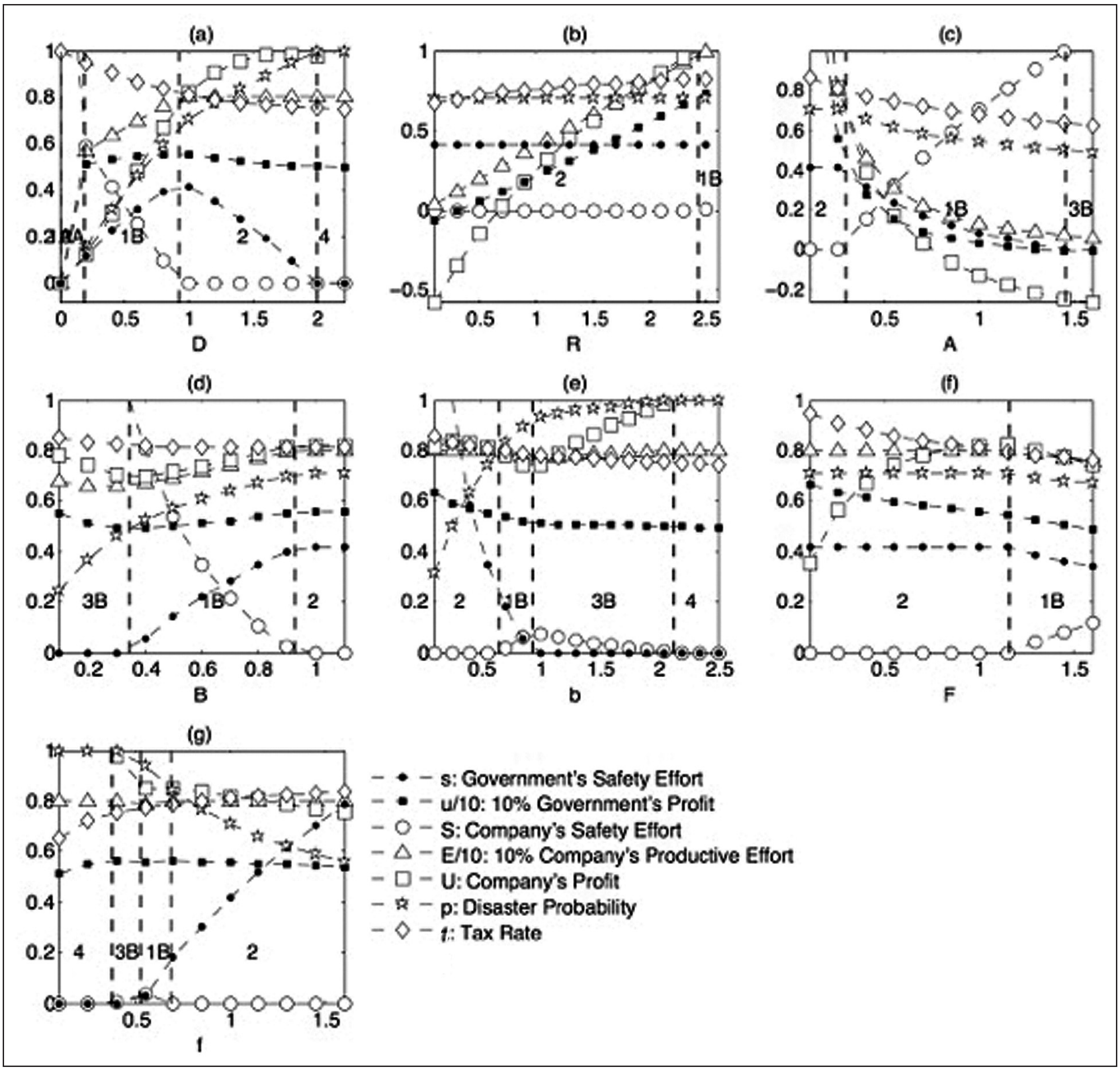

Figure 2. Equilibrium behavior of s, S, τ, E, p, u, and U as functions of (a) D, (b) R, (c) A, (d) B, (e) b, (f) F, and (g) f with baseline values $R = 2$, $D = B = F = f = 1$, $b = 0.5$, and $A = 0.25$. Division of u and E with 10 is for scaling purposes.

both players causing both players in case 4 to withdraw safety efforts which are not cost efficient.

In panel (b), both players benefit in case 2 from a more resourceful company as R increases and suffer negative profit for low R. As R increases above $R = 2.43$, the company transitions to exerting positive safety effort S in case 1B.

In panel (c), with company unit production cost $A < 0.29$, both players earn high profits and the company exerts no safety effort in case 2. As the company suffers higher unit production cost, $0.29 < A < 1.46$, the company exerts higher safety effort S in case 1B, facing lower taxation, and eventually earns negative profit. For $1.46 < A < 5.00$, the company's high safety effort induces the government in case 3B to withdraw safety effort, $s = 0$. For $A > 5.00$ (not plotted) case 3A emerges where the company allocates all its resources to safety effort.

In panel (d), unit cost $B > 0.92$ of company safety effort induces the company to withdraw safety effort in case 2. When $0.34 < B < 0.92$, the company's safety effort S becomes worthwhile causing case 1B. When $B < 0.34$, the company efficiently exerts safety effort in case 3B while the government free rides.

In panel (e), as the government's unit cost b of safety effort increases above $b = 0.65$, the company exerts positive safety effort $S > 0$ causing transition from case 2 to case 1B. As b increases above 0.93, the costly safety effort causes the government to withdraw safety effort, $s = 0$, causing transition to case 3B. Finally, as b increases above 2.13, both players withdraw safety efforts in case 4. Hence, although only the government suffers high unit cost b, both players suffer in case 4 which illustrates the impact of strategic interaction.

In panel (f), the scaling function F for how the company is negatively affected by the disaster causes no company safety effort in case 2 with low impact $F < 1.15$ and causes positive safety effort in case 1B with high impact $1.15 < F < 8.55$. For extremely high $F > 8.55$ (not plotted), the government is deterred from exerting safety effort, that is, case 3B.

In panel (g), the scaling function f for how the government is negatively affected by the disaster causes case 2 with impact $f > 0.69$. As f decreases below $f = 0.69$, interestingly, the fact that the government is less impacted by the disaster causes transition to case 1B where the company exerts positive safety effort. As f decreases below $f = 0.53$, more dramatically, the low impact on the government causes it to withdraw safety effort, $s = 0$, in case 3B. Finally, for $f < 0.38$, both players withdraw safety efforts in case 4.

Conclusion

A general two-stage model is developed with one company and one government. In stage 1, the government chooses a tax rate. In stage 2, the company and the government choose safety efforts, simultaneously and independently, seeking to constrain the negative consequences of a disaster. The probability of disaster is a contest between the disaster magnitude and the two players' safety efforts. As the company chooses its safety effort, it also chooses its productive effort by allocating its resources for these two purposes. High resource allocation to production gives high profit provided that the disaster does not occur. However, if the disaster does occur, high productive effort is detrimental due to the corresponding low safety effort. Conversely, too high resource allocation causes low production which may cause bankruptcy. The company strikes a balance between these two concerns. If the government chooses a too high tax rate, it increases its income, but it burdens the company excessively, making it difficult for the company to operate, and allocate effectively between production and safety. The government chooses its safety effort to strike a balance between its tax income and its cost of safety effort, taking into account the probability and magnitude of the disaster. We quantify the different extents to which the company and government are negatively affected by the disaster. The company has a unit production cost, and the unit costs of safety effort differ for the company and the government.

We first confirm four propositions presented by Hausken and Zhuang,[2] with slightly different assumptions. They assumed that the government chooses both the tax rate and the safety effort in stage 1, while the company chooses the safety effort in stage 2. They found, first, that as the safety effort of one player approaches infinity, the change in the other player's safety effort, with respect to the first player's safety effort, approaches zero. Second, an infinitely large safety effort by any player causes the disaster probability and negative impact of the disaster to decrease toward a constant. Third, the two players' safety efforts are strategic substitutes. Fourth, an increase in one player's safety effort decreases the other player's safety effort.

The analytical results of the new model in this article include the following, with various plausible assumptions: first, both the government's and the company's safety efforts decrease in the unit safety effort costs, and the company's safety effort increases in the unit production cost and in the company's resource. Second, both players' safety efforts are inverse U shaped in the disaster magnitude. Safety efforts first increase as the disaster magnitude increases to limit the increasing impact of the disaster. The two safety efforts eventually reach maximum points and thereafter decrease since the cost of providing safety against a too devastating disaster is too large and cannot be justified. Third, the company's safety effort increases, and the government's safety effort decreases, in taxation. Taxation can thus ameliorate companies' incentive to free ride on governments' provision of safety efforts. With sufficiently large production, the government prefers, and the company does not prefer, raising taxation above 0%. For the government an upper limit usually exists above which taxation cannot be profitably increased.

Whereas this article has shown that the two players' safety efforts are strategic substitutes, which we think is usually descriptive, future research should consider situations where the two players' safety efforts are partly more exhaustively strategic complements, for example, driven by one player possessing competence, knowledge, technology, and preferences not possessed by the other player. Future research should also model additional players such as the general public, subpopulations, special interest groups, environmental groups, occupational groups, insurance, and different attitudes toward risk.

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Appendix 1

Notation

A	company's unit cost of production
b	government's unit cost of safety effort
B	company's unit cost of safety effort
D	disaster magnitude
E	company's productive effort
$f(D,S,s)$	scaling function for how the government is negatively affected by disaster
$F(D,S,s)$	scaling function for how the company is negatively affected by disaster
$H(S)$	production function
$p(D,S,s)$	disaster probability
R	company's resources
s	government's safety effort
S	company's safety effort
u	government's expected profit
U	company's expected profit
τ	taxation percentage variable chosen by the government

Appendix 2

Proof of Propositions 1–3

Proof of Proposition 1

Assuming $s=0$ implies $\partial s/\partial b=0$; so we only consider $s>0$. Differentiating the first equation in equation (5) gives

$$\begin{aligned}
&\frac{\partial}{\partial b}\left(f(D,S,s)\frac{\partial p(D,S,s)}{\partial s}\right) = \frac{\partial}{\partial b}\left(-b-\frac{\partial f(D,S,s)}{\partial s}p(D,S,s)\right)\\
&\Rightarrow \frac{\partial f(D,S,s)}{\partial s}\frac{\partial s}{\partial b}\frac{\partial p(D,S,s)}{\partial s}+f(D,S,s)\frac{\partial^2 p(D,S,s)}{\partial s^2}\frac{\partial s}{\partial b}\\
&= -1-\frac{\partial^2 f(D,S,s)}{\partial s^2}\frac{\partial s}{\partial b}p(D,S,s)-\frac{\partial f(D,S,s)}{\partial s}\frac{\partial p(D,S,s)}{\partial s}\frac{\partial s}{\partial b}\\
&\Rightarrow \frac{\partial s}{\partial b} = -\frac{1}{2\frac{\partial f(D,S,s)}{\partial s}\frac{\partial p(D,S,s)}{\partial s}+f(D,S,s)\frac{\partial^2 p(D,S,s)}{\partial s^2}+\frac{\partial^2 f(D,S,s)}{\partial s^2}p(D,S,s)}\leqslant 0
\end{aligned} \tag{11}$$

The three last inequalities in Proposition 1 follow from equation (1) by taking the total derivative with respect to B, A, and R, respectively. QED.

Proof of Proposition 2

When $S = 0$, $dS/dD = d^2S/dD^2 = 0$ is always zero so we only consider $S > 0$. From the third equation in equation (5), we have

$$\begin{aligned}
&F\frac{\partial p(D,S,s)}{\partial S} = (1-\tau)\frac{\partial H(S)}{\partial S}\\
&\Rightarrow \frac{d}{dD}\left(F\frac{\partial p(D,S,s)}{\partial S}\right) = \frac{d}{dD}\left((1-\tau)\frac{\partial H(S)}{\partial S}\right) = 0\\
&\Rightarrow 0 = \frac{d}{dD}\left(\frac{\partial p(D,S,s)}{\partial S}\right) = \frac{\partial^2 p(D,S,s)}{\partial S\partial D}\\
&+ \frac{\partial^2 p(D,S,s)}{\partial S^2}\frac{dS}{dD} \Rightarrow \frac{dS}{dD} = -\frac{\frac{\partial^2 p(D,S,s)}{\partial S\partial D}}{\frac{\partial^2 p(D,S,s)}{\partial S^2}}
\end{aligned} \tag{12}$$

Since we assume $\partial^2 p(D,S,s)/\partial S^2 \geqslant 0$, we have $dS/dD \geqslant 0$ if and only if $\partial^2 p(D,S,s)/\partial S\partial D \leqslant 0$.

From equation (12), we have

$$\begin{aligned}
\frac{d^2S}{dD^2} &= \frac{d}{dD}\left(-\frac{\frac{\partial^2 p(D,S,s)}{\partial S\partial D}}{\frac{\partial^2 p(D,S,s)}{\partial S^2}}\right)\\
&= \frac{-\left(\frac{\partial^3 p(D,S,s)}{\partial S\partial D^2}\right)\left(\frac{\partial^2 p(D,S,s)}{\partial S^2}\right) + \left(\frac{\partial^2 p(D,S,s)}{\partial S\partial D}\right)\left(\frac{\partial^3 p(D,S,s)}{\partial S^2\partial D}\right)}{\left(\frac{\partial^2 p(D,S,s)}{\partial S^2}\right)^2}
\end{aligned} \tag{13}$$

So, we have $d^2S/dD^2 \leqslant 0$ if and only if equation (8) is satisfied. The proof for $d^2S/dD^2 \leqslant 0$ is analogous using the first equation in equation (5), and we require $\partial^2 p(D,S,s)/\partial s^2 \geqslant 0$ and $\partial^2 p(D,S,s)/\partial s\partial D \leqslant 0$. For the second part, when D goes to infinity, we must have $dS/dD \leqslant 0$ and $ds/dD \leqslant 0$. Since s and S must be non-negative, the limits exist. QED.

Proof of Proposition 3

Assuming $s = 0$ implies $\partial s/\partial\tau = 0$; so we only consider $s > 0$. Differentiating the first equation in equation (5) gives

$$\begin{aligned}
&\frac{\partial}{\partial\tau}\left(f(D,S,s)\frac{\partial p(D,S,s)}{\partial s}\right) = \frac{\partial}{\partial\tau}\left(-b - \frac{\partial f(D,S,s)}{\partial s}p(D,S,s)\right)\\
&\Rightarrow \frac{\partial f(D,S,s)}{\partial s}\frac{\partial s}{\partial\tau}\frac{\partial p(D,S,s)}{\partial s} + f(D,S,s)\frac{\partial^2 p(D,S,s)}{\partial s^2}\frac{\partial s}{\partial\tau}\\
&= -\frac{\partial^2 f(D,S,s)}{\partial s^2}\frac{\partial s}{\partial\tau}p(D,S,s) - \frac{\partial f(D,S,s)}{\partial s}\frac{\partial p(D,S,s)}{\partial s}\frac{\partial s}{\partial\tau}\\
&\Rightarrow \frac{\partial s}{\partial\tau} = 0
\end{aligned} \tag{14}$$

Total differentiation and inserting equation (14) gives

$$\begin{aligned}
\frac{ds}{d\tau} &= \frac{\partial s}{\partial\tau} + \frac{\partial s}{\partial S}\frac{\partial S}{\partial\tau} = \frac{\partial s}{\partial S}\frac{\partial S}{\partial\tau},\\
\frac{dS}{d\tau} &= \frac{\partial S}{\partial\tau} + \frac{\partial S}{\partial s}\frac{\partial s}{\partial\tau} = \frac{\partial S}{\partial\tau}
\end{aligned} \tag{15}$$

Assuming $S = 0$ implies $\partial S/\partial\tau = 0$ so we only consider $S > 0$. Differentiating the third equation in equation (5) gives

$$\begin{aligned}
&\frac{\partial}{\partial\tau}\left((1-\tau)\frac{\partial H(S)}{\partial S} - \frac{\partial F(D,S,s)}{\partial S}p(D,S,s)\right) = \frac{\partial}{\partial\tau}\left(F(D,S,s)\frac{\partial p(D,S,s)}{\partial S}\right)\\
&\Rightarrow (1-\tau)\frac{\partial^2 H(S)}{\partial S^2}\frac{\partial S}{\partial\tau} - \frac{\partial H(S)}{\partial S} - \frac{\partial^2 F(D,S,s)}{\partial S^2}\frac{\partial S}{\partial\tau}p(D,S,s)\\
&- \frac{\partial F(D,S,s)}{\partial S}\frac{\partial p(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau} = \frac{\partial F(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}\frac{\partial p(D,S,s)}{\partial S}\\
&+ F(D,S,s)\frac{\partial^2 p(D,S,s)}{\partial S^2}\frac{\partial S}{\partial\tau} \Rightarrow \frac{\partial S}{\partial\tau} =\\
&\frac{\frac{\partial H(S)}{\partial S}}{\left[(1-\tau)\frac{\partial^2 H(S)}{\partial S^2} - \frac{\partial^2 F(D,S,s)}{\partial S^2}p(D,S,s) - F(D,S,s)\frac{\partial^2 p(D,S,s)}{\partial S^2} - 2\frac{\partial F(D,S,s)}{\partial S}\frac{\partial p(D,S,s)}{\partial S}\right]}
\end{aligned} \tag{16}$$

From the assumptions and $\partial H/\partial S < 0$ assumed after equation (1), it follows that $\partial S/\partial\tau > 0$ which is inserted into equation (15) to give $dS/d\tau = \partial S/\partial\tau > 0$. Differentiating equation (3) gives

$$\begin{aligned}
\frac{du(S,s,\tau)}{d\tau} &= H(S) + \tau\frac{\partial H}{\partial S}\frac{\partial S}{\partial\tau} - f(D,S,s)\\
&\times\left[\frac{\partial p(D,S,s)}{\partial s}\frac{\partial s}{\partial\tau} + \frac{\partial p(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}\right]\\
&- p(D,S,s)\left[\frac{\partial f(D,S,s)}{\partial s}\frac{\partial s}{\partial\tau} + \frac{\partial f(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}\right] - b\frac{\partial s}{\partial\tau}\\
&= H(S) + \tau\frac{\partial H}{\partial S}\frac{\partial S}{\partial\tau} - f(D,S,s)\frac{\partial p(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}\\
&- p(D,S,s)\frac{\partial f(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}
\end{aligned} \tag{17}$$

Differentiating equation (2) gives

$$\begin{aligned}
\frac{dU(S,s,\tau)}{d\tau} &= -H(S) + (1-\tau)\frac{\partial H}{\partial S}\frac{\partial S}{\partial\tau}\\
&- F(D,S,s)\left[\frac{\partial p(D,S,s)}{\partial s}\frac{\partial s}{\partial\tau} + \frac{\partial p(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}\right]\\
&- p(D,S,s)\left[\frac{\partial F(D,S,s)}{\partial s}\frac{\partial s}{\partial\tau} + \frac{\partial F(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}\right]\\
&= -H(S) + (1-\tau)\frac{\partial H}{\partial S}\frac{\partial S}{\partial\tau} - F(D,S,s)\frac{\partial p(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}\\
&- p(D,S,s)\frac{\partial F(D,S,s)}{\partial S}\frac{\partial S}{\partial\tau}
\end{aligned} \tag{18}$$

Applying $dS(s)/ds \leqslant 0$ and the assumptions in Proposition 5, inserting into equation (15), and applying $\partial S/\partial\tau > 0$ gives $ds/d\tau \leqslant 0$. QED.