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The impact of disaster on the strategic interaction between company and government

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ABSTRACT

A company allocates a resource between safety effort and production. The government earns taxes on production. The disaster probability is modeled as a contest between the disaster magnitude and the two players' safety efforts. The model illustrates that safety efforts are strategic substitutes and inverse U shaped in the disaster magnitude. The company's safety effort increases, and the government's safety effort decreases, in taxation. Taxation can ameliorate companies' free riding on governments' safety efforts. With sufficiently large production, the government prefers, and the company does not prefer, raising taxation above 0%. For the government, an upper limit usually exists above which taxation cannot be profitably increased. The model shows how both or no players exert safety efforts when the disaster magnitude is small and large respectively, and how they free ride on each other's safety efforts when the disaster magnitude is intermediate. The company free rides when the unit production cost is low so that the large profits outweigh the negative impact of the disaster. With endogenized taxation determined by the government, the tax rate decreases in the disaster magnitude, the unit production cost, the government's unit cost of safety effort, and how the company is negatively affected by the disaster. The tax rate increases in the company's resource and how the government is negatively affected by the disaster. The tax rate is weakly U shaped in the company's unit safety effort. The model is illustrated with numerical examples and with the oil spill disasters by BP in 2010 and by Exxon Valdez in 1989.

1. Introduction

1.1. Objectives

We develop a model assuming a contest success function between the disaster magnitude and the two players' safety efforts. By making use of credible specific functional forms, we produce exact analytical solutions for the variables, illustrated with numerical simulations. In return for the sacrifice of generality, a successful specification demonstrates that at least the minimal standard of internal consistency has been achieved. In addition, we claim the particular functional forms used here will be illuminating.[1] Using particular functional forms makes it possible to determine a range for each parameter value within which solutions to the six cases can and cannot be obtained.

1.2. Motivation

The oil spills by BP in the Gulf of Mexico between April 20 and July 15, 2010[2] and by Exxon Valdez in Alaska on March 24, 1989[3] demonstrate the challenges faced when balancing production efficiency against safety effort.[4] In this paper we seek to understand how a company strikes a balance between production and safety effort when interacting with a government who also exerts safety effort. The government earns taxes on the company's production. Both players maximize profits weighted against costly safety efforts which help mitigate the negative impact of a disaster. We focus especially on

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[1] In economics, Cobb–Douglas or CES production functions, although involving special assumptions about the functional relations between inputs and outputs, have proved to be extremely useful for advancing our understanding of productive processes and economic growth.

[2] http://en.wikipedia.org/wiki/Deepwater_Horizon_oil_spill, retrieved April 19, 2011.

[3] http://en.wikipedia.org/wiki/Exxon_Valdez_oil_spill, retrieved April 19, 2011.

[4] See http://www.infoplease.com/ipa/A0001451.html for major oil spills since 1967, and http://news.yahoo.com/s/ac/20110317/sc_ac/8079848_worst_oil_spills_in_history for the five worst oil spills in history. The gravity of disasters can be measured according to economic loss, human loss, and symbolic loss. The July 6, 1988 North Sea off Scotland Occidental Petroleum's Piper Alpha rig disaster killed 167 people, http://en.wikipedia.org/wiki/Piper_Alpha#cite_note-2. The North Sea off Norway March 27, 1980 capsize of the Alexander L. Kielland platform killed 123 people, http://en.wikipedia.org/wiki/Alexander_L._Kielland_(platform), all retrieved April 19, 2011.

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Nomenclature

R	company's resource	D	disaster magnitude
E	company's productive effort	p	disaster probability
S	company's safety effort	F	scaling function for how the company is negatively affected by disaster
s	government's safety effort	f	scaling function for how the government is negatively affected by disaster
A	company's unit cost of production	τ	taxation percentage parameter
B	company's unit cost of safety effort	u	government's expected profit
$H(S)$	production function	U	company's expected profit
h	production parameter		
b	government's unit cost of safety effort		
k	safety effort parameter		

the players' incentives to free ride on each other's safety efforts. Examples of safety efforts are to develop, implement, and ensure compliance to safety regulations. The general public's interest is represented by the government. Since preferences exist to consume oil at a low price, this requires large production which may compromise safety. The general public does not complain when disasters do not occur. But, when disasters do occur, and history tells us they do, the general public complains, people suffer, and lawsuits are filed.

Whereas production is routinely analyzed in an economic sense, analyzing a disaster economically is challenging since it may involve loss of human life, uncertain costs of future lawsuits, uncertain costs of countering future lobbying efforts by various groups affected by a disaster, and uncertain reputational ramifications after a disaster. Companies handle this challenge in many different ways. Some may ignore disasters arguing that they are unlikely, that their large profits may pay for disasters, that the government may bail them out, that they may somehow be able to handle disasters ex post, or that bankruptcy is not so bad since individual managers may seek employment elsewhere. Others may rely on safety standards imposed by laws and regulations without economic assessment. Some companies have hard working safety officials working diligently to uphold safety standards, sometimes fighting continuously with other officials seeking to boost production. Companies have different safety cultures. After a disaster within an industry, or if the CEO has a professed safety focus, or the government have imposed enhanced safety standards, larger budgets may be allocated to safety.

The precariousness, capriciousness, and sometimes irrationality present in companies' attitudes toward disasters suggest a need to analyze disasters rationally. Companies possess substantial competence to assess production rationally, and adjust production according to changes in demand, changes in other companies' production, and various market uncertainties. However, disasters need to be placed on the same footing as production. Neither the current general public nor future generations are going to accept continuous occurrence of disasters revealing that companies repeatedly fail to approach disasters professionally, fail to use common business sense when assessing disasters, and fail to assess both possible magnitudes and probabilities of disasters.

1.3. Brief literature review

From an economics perspective, the literature on industrial organization, e.g. Tirole (1988), focuses thoroughly on production but has a limited safety focus. By contrast, the literature on industrial organization from an organizational psychology perspective, e.g. Zohar (1980), focuses on safety, but deemphasizes the role of production. Hausken (2005) considers two groups striking a balance between production and contesting each other's production. Azaiez and Bier (2007) consider the optimal resource allocation for security in reliability systems.

Moreover, disaster preparedness would be a partnership game between the government and the private sector, where both players exert safety efforts. For example, Sadka (2007) provides a public economics perspective on public–private partnerships. Flinders (2005) analyzes the role of efficiency, risk, complexity, accountability, and governance in public–private partnerships. Boase (2000) discusses the idea of "governments steering and the private sector rowing," and examines various cases of public–private partnerships. Due to economic externalities, a "free-riding" problem exists on multiple players' security investment as studied by Kunreuther and Heal (2003) and Hausken (2005). Kunreuther and Useem (2010) consider strategies for reaction and response when learning from catastrophes. Kunreuther (2008) considers reducing losses from catastrophic risks through long-term insurance and mitigation.

The objective of this paper is to fill this important gap, and to place production and disaster preparedness on the same footing, weighing them against each other, assessing both rationally, and the impact on free riding and profits between a company and the government. To our best knowledge, no previous research considers the important tradeoff between production and disaster preparedness in a game between a company and the government.

Boyer and Laffont (1999) present an incomplete contract approach of a political economy of environmental policy and accident prevention using motivational stories similar to those in the present paper. They show why constitutional constraints on the instruments of environmental policy instruments can be preferable, justified by the limitations they impose on politicians' ability to distribute rents. Asche and Aven (2004) consider the business incentives for investing in safety. They observe that decision-makers do not necessarily communicate about safety in economic terms, and show how safety measures can have a value in an economic sense. England (1988) presents a profit maximization model by a firm utilizing a disaster-prone technology. Output rate and accident prevention are shown to be jointly determined by market demand, production cost and prospective accident loss data. He argues that, in the absence of government safety regulations, even a risk-neutral management is likely to choose an excessively high probability of a Bhopal-style disaster. Golbe (1986) uses data from the US airline industry and finds that the sign of the relationship between profit and safety is indeterminate and depends on risk preferences and the structure of costs and demands. Carmichael (1986) shows that for a competitive labor market with complete information, safety will be underprovided, and that in some cases government-enforced workmen's compensation can bring safety improvements. Hale (2003) considers safety management in production focusing on organizational culture and learning. Cheung and Zhuang (2012) study a regulation game and find that competition between companies lowers the incentives for a company to invest in safety effort and thus requires higher government regulation.

Zhuang and Bier (2007) and Hausken et al. (2009) abstract away from production and consider how defenders of infrastructures optimally allocate resources between terrorism, natural disaster, and all hazards. They find that when all-hazards protection is sufficiently cheaper compared with protection against natural disaster only, or terrorism only, it jointly protects against both the natural disaster and terrorism. As the cost increases, all-hazards protection is replaced with either pure natural disaster protection or pure terrorism protection. Osmundsen et al. (2010) do not consider disasters but evaluate in economic terms oil producers' incentives to assure steady supply, which means accounting for various risks that may occur. When steady supply is not assured, reputational issues emerge and contract obligations may be compromised.

2. The model

We consider two players that are representative of the company and the government. The company has a resource R (e.g. a capital good, or labor) which can be converted with unit conversion cost A into productive effort E, and with unit cost B into safety effort S, where

$$R = AE + BS,\ S \geqslant 0,\ E \geqslant 0 \quad \Longleftrightarrow \quad 0 \leqslant S = (R - AE)/B \leqslant R/B \tag{1}$$

We consider the production function $H(E) = E^h$, where h is a production parameter. $0 < h < 1$ means concave production[5], $h = 1$ means linear production, and $h > 1$ means convex production. We express $H(E)$ as $H(S)$ using (1), interpreting S as the company's strategic decision variable. Hence production decreases in safety effort. We express the disaster probability with the ratio form contest success function (Tullock, 1980; Skaperdas, 1996; Zhuang et al., 2010)

$$p(D,S,s) = \frac{D}{D+S}\frac{D}{D+s} \tag{2}$$

where D is the disaster magnitude, and s is the government's safety effort which consists in designing, implanting, and enforcing laws and procedures to ensure safety compliance, etc. The reasons why we select a disaster probability with the ratio form contest success function are: (a) it is probably the most widely accepted contest success function in the literature; (b) the ratio form function captures the essential relationship between the probability of disaster and safety efforts; that is, the probability of disaster is between 0 and 1 and increases in the disaster magnitude and decreases in both the government's and company's safety efforts; and (c) simplicity, which allows us to get nice structural analytical results. One alternative to the basic ratio form is the ratio form with a contest intensity m, $D^m/(D^m + S^m)$, which makes the analysis intractable. See Hausken and Levitin (2008) for an analysis of this function. Another alternative, though somewhat less used, is the difference form $Exp(\alpha D)/[Exp(\alpha D) + Exp(\alpha S)]$, where α is a parameter. See Hirshleifer (1989) and Hausken (2008) for a comparison of the ratio form and the difference form, and Skaperdas (1996) for axiomatic foundations. The results are qualitatively similar.

Whereas S and s are intentionally chosen strategic decision variables, the disaster magnitude is a variable chosen by nature. In the simulations we treat D as an exogenously determined parameter. Various methods are used to measure disaster magnitude. Hurricanes are measured in categories 1, 2, 3, 4, 5, etc. Earthquakes are measured on the Richter's scale. Storms are measured by wind speed. Floods are measured by how much the water rises. Explosions are measured in the amount of energy released. The disaster probability increases in the disaster magnitude and decreases in both safety efforts. If one player is negligent, the other player must exert substantial safety effort to decrease $p(D,S,s)$. If both players are negligent, $p(D,S,s)$ is large. Eq. (2) consists of one term $D/(D + S)$ which expresses the disaster probability caused by the company being negligible, multiplied by one term $D/(D + s)$ which expresses the disaster probability caused by the government being negligible. The disaster probability is the product of these two probabilities.

The company's profit is

$$\begin{aligned} U &= (1-\tau)H(S) - Fp(D,S,s) \\ &= (1-\tau)\left(\frac{R-BS}{A}\right)^h - F\frac{D}{D+S}\frac{D}{D+s} \end{aligned} \tag{3}$$

where τ, $0 \leqslant \tau \leqslant 1$, is a taxation percentage parameter, and thus the company keeps the fraction $1 - \tau$ after having paid taxes. The parameter F scales the extent to which the company is negatively affected by the disaster.

The government's profit is

$$\begin{aligned} u &= \tau H(S) - fp(D,S,s) - bs^k \\ &= \tau\left(\frac{R-BS}{A}\right)^h - f\frac{D}{D+S}\frac{D}{D+s} - bs^k \end{aligned} \tag{4}$$

where k is a parameter and f scales the extent to which the government is negatively affected by the disaster, and b is the government's unit safety effort. $0 < k < 1$ means concavely increasing safety cost, $k = 1$ means linearly increasing safety cost, and $k > 1$ means convexly increasing safety cost.

The probability of a disaster, which in (2) is a function of the three strategic variables D, S, s, should be assessed in the context of uncertainty, which is the main source of difficulty in estimating the risk of a disaster. First, the disaster magnitude D is uncertain, and the consequences of D are often difficult to estimate. Second, the two players are affected by uncertainty in the parameters; e.g. the unit safety efforts B and b, and furthermore uncertainty in the parameters A, F, f, τ. Thus uncertainty in D, combined with uncertainty in other parameters, are compounded causing an uncertain environment within which the two players make their strategic choices. In this paper we account for such uncertainty in the sensitivity analysis in Section 4 where we systematically vary each parameter value relative to a baseline, which provides some insights on the impact of these uncertainties. For example, we observe that both government and company safety efforts are inverse U-shaped in the disaster magnitude D, which means these efforts could first increase then decrease in D. In other words, uncertainty in D significantly impacts the players' strategic choices. The nature of the simultaneous-move game is such that uncertainty is present in the sense that we do not assume that the players observe each other's safety effort (and thus do not observe the exact costs of prevention and remediation). Instead, those safety efforts are determined simultaneously in a Nash equilibrium. This paper does not model incomplete information which we suggest is suitable for future research; e.g. where parameter values are drawn from probability distributions thus using expected values (Wang and Zhuang, 2011), or drawn in a range without knowledge of the distribution thus using robust optimization (Nikoofal and Zhuang, 2012).

3. Analyzing the model

The government has one free choice variable s. The company has one free choice variable S. We analyze the game for simultaneous moves. The government and company choose their strategies simultaneously and independently.

[5] The common concave production functions are a good approximation in highly developed economies, for example when the ratio of capital to labor is large (Skiba, 1978).

Table 1
Solution to subgame perfect Nash equilibrium when $h = k = 1$.

Cases	Case 1A	Case 1B	Case 2	Case 3A	Case 3B	Case 4
Conditions	$D \leqslant Min\left\{\delta - \frac{R}{B}, \left(\frac{\chi^2 D^2}{\delta}\right)^{\frac{1}{3}} - \frac{R}{B}\right\}$	$\left(\frac{\chi^2 D^2}{\delta}\right)^{\frac{1}{3}} - \frac{R}{B} < D < Min\left\{\frac{\chi^2}{\delta}, \frac{\delta^2}{\chi}\right\}$	$\frac{\chi^2}{\delta} \leqslant D < \delta$	$\delta - \frac{R}{B} < D < \sqrt{\chi D} - \frac{R}{B}$	$Max\left\{\sqrt{\chi D} - \frac{R}{B}, \frac{\delta^2}{\chi}\right\} \leqslant D < \chi$	$D \geqslant Max\{\chi, \delta\}$
Scenarios for safety effort	Active company and active government	Active company and active government	Inactive company and active government	Active company and inactive government	Active company and inactive government	Inactive company and inactive government
$S=$	R/B	$\left(\left(\frac{AFAFb}{(1-\tau)B(1-\tau)Bf}\right)^{\frac{1}{3}} - D^{\frac{1}{3}}\right)D^{\frac{2}{3}}$	0	R/B	$\left(\sqrt{\frac{AF}{(1-\tau)B}} - \sqrt{D}\right)\sqrt{D}$	0
$s=$	$D\left(\sqrt{\frac{f}{b(D+R/B)}} - 1\right)$	$\left(\frac{\left(\frac{f}{b}\right)^{\frac{2}{3}}}{\left(\frac{AF}{(1-\tau)B}\right)^{\frac{1}{3}}} - D^{\frac{1}{3}}\right)D^{\frac{2}{3}}$	$\left(\sqrt{\frac{f}{b}} - \sqrt{D}\right)\sqrt{D}$	0	0	0
$p=$	$\frac{D}{D+\frac{R}{S}}\frac{1}{\sqrt{\frac{f}{b(D+R/B)}}}$	$\left(\frac{D(1-\tau)B}{AF}\right)^{\frac{1}{3}}\left(\frac{Db}{f}\right)^{\frac{1}{3}}$	$\left(\frac{bD}{f}\right)^{\frac{1}{2}}$	$\frac{D}{D+R/B}$	$\left(\frac{D(1-\tau)B}{AF}\right)^{\frac{1}{2}}$	1
$U=$	$\frac{-bD\left(2\sqrt{f/b}-\sqrt{D+R/B}\right)}{\sqrt{D+R/B}}$	$\frac{(1-\tau)(R+BD)}{A} - \frac{2FD^{\frac{2}{3}}}{(AFf)^{\frac{1}{3}}}((1-\tau)Bb)^{\frac{1}{3}}$	$\frac{(1-\tau)R}{A} - F\sqrt{Db/f}$	$\frac{-fD}{D+R/B}$	$\frac{(1-\tau)(R+BD)}{A} - 2F\sqrt{D(1-\tau)B/(AF)}$	$(1-\tau)\frac{R}{A} - F$
$u=$	$\frac{-FD}{\sqrt{f/b}\sqrt{D+R/B}}$	$\frac{\tau(R+BD)}{A} - \frac{\frac{\tau}{1-\tau}FD^{\frac{2}{3}}}{\left(\frac{AF f}{(1-\tau)Bb}\right)^{\frac{1}{3}}} - 2b\frac{\left(\frac{f}{b}\right)^{\frac{2}{3}}D^{\frac{2}{3}}}{\left(\frac{AF}{(1-\tau)B}\right)^{\frac{1}{3}}} + bD$	$\frac{\tau R}{A} + bD - 2b\sqrt{\frac{f}{b}D}$	$\frac{-FD}{D+R/B}$	$\frac{\tau(R+BD)}{A} - \left(\frac{\tau F}{1-\tau} + f\right)\frac{\sqrt{D}}{\sqrt{\frac{AF}{(1-\tau)B}}}$	$\tau\frac{R}{A} - f$

Definition 1. A strategy pair (s, S) is a *Nash equilibrium* if and only if

$$S = S(s) = \arg\max_{S\geqslant 0} U(s,S), \quad s = \arg\max_{s\geqslant 0} u(s,S) \tag{5}$$

Appendix A: Supplementary material solves the model generating solutions within the six cases shown in Table 1 where $h = k = 1$. We define $\chi = \frac{AF}{B(1-\tau)}$ and $\delta = \frac{f}{b}$ which imply $\frac{A^2F^2b}{(1-\tau)^2B^2f} = \frac{\chi^2}{\delta}$ and $\frac{(1-\tau)f^2B}{b^2AF} = \frac{\delta^2}{\chi}$. We hereafter mainly confine attention to the common linear production function (Hirshleifer, 1995), and provide one-way sensitivity analysis around $h = k = 1$. We consider the linear function because: (a) it captures the essential increasing relationship between input and output in a production function; and (b) simplicity, which allows us to get nice structural analytical results.

The conditions in Table 1 for cases 1A and 3A are extremely unusual, causing $S = R/B$ and company bankruptcy through for example very large tax rate τ or very low resource R. Case 1A is empty when at least one of the arguments in the Min function is negative.

The parameter χ expresses how the company is negatively affected by the disaster multiplied with its unit cost of production, divided by the percentage of the untaxed production, and divided by its unit cost of safety effort.[6] In particular, high taxation τ causes high χ which does not benefit the company. The parameter δ expresses how the government is negatively affected by the disaster divided by its unit cost of safety effort.[7] If the unit costs A, B, b are fixed, the company prefers low χ and the government prefers low δ. This is reflected in Table 1 in that $\chi \leqslant \delta$ excludes case 3B where the company exerts safety effort, and $\chi \geqslant \delta$ excludes case 2 where the government exerts safety effort. $\chi = \delta$ implies case 1B when $D < \delta$, cases 2 and 3B are impossible, and case 4 occurs when $D \geqslant \delta$.

Case 1B with both government and company safety efforts occurs when the disaster is small. As the disaster increases in magnitude from a small level, a transition to either cases 2 or 3B occurs, where either the government or the company exerts safety effort, but not both. Eventually, as the disaster magnitude increases above that handled by either cases 2 or 3B, a transition to case 4 occurs with neither government nor company safety efforts.

Proposition 1. *(a) The six cases in Table 1 are both collectively exhaustive and mutually exclusive. (b) Assume parameter values excluding the unlikely cases 1A and 3A. $\chi \leqslant \delta$ implies cases 1B, 2, 4 (Supplementary Fig. S1). $\chi \geqslant \delta$ implies cases 1B, 3B, 4 (Supplementary Fig. S2). Safety efforts s and S depend only on χ and δ.*

Proof. Please see Appendix B: Supplementary material. □

Excluding the company's safety effort in case 3B when $\frac{AF}{B(1-\tau)} \leqslant \frac{f}{b}$ occurs when the unit production cost is low (so the company really needs to focus on production), when the company can keep much of its production through paying limited taxes (so τ is low), when the company's unit safety effort cost is larger than the government's unit safety effort cost (so safety effort is more cost efficiently exerted by the government than by the company), and when the company is less negatively affected than the government by the disaster (so the company can get away with not exerting safety effort). Conversely, excluding the government's safety effort

[6] High χ means that the company is negatively affected by the disaster and/or has a high unit cost of production, and/or has a low percentage of the untaxed production and/or low unit cost of safety effort. Low χ means that the company is less negatively affected by the disaster and/or has a low unit cost of production, and/or high unit cost of safety effort.

[7] High δ means that the government is negatively affected by the disaster and/or has a low unit cost of safety effort. Low δ means that the government is less negatively affected by the disaster and/or has a high unit cost of safety effort.

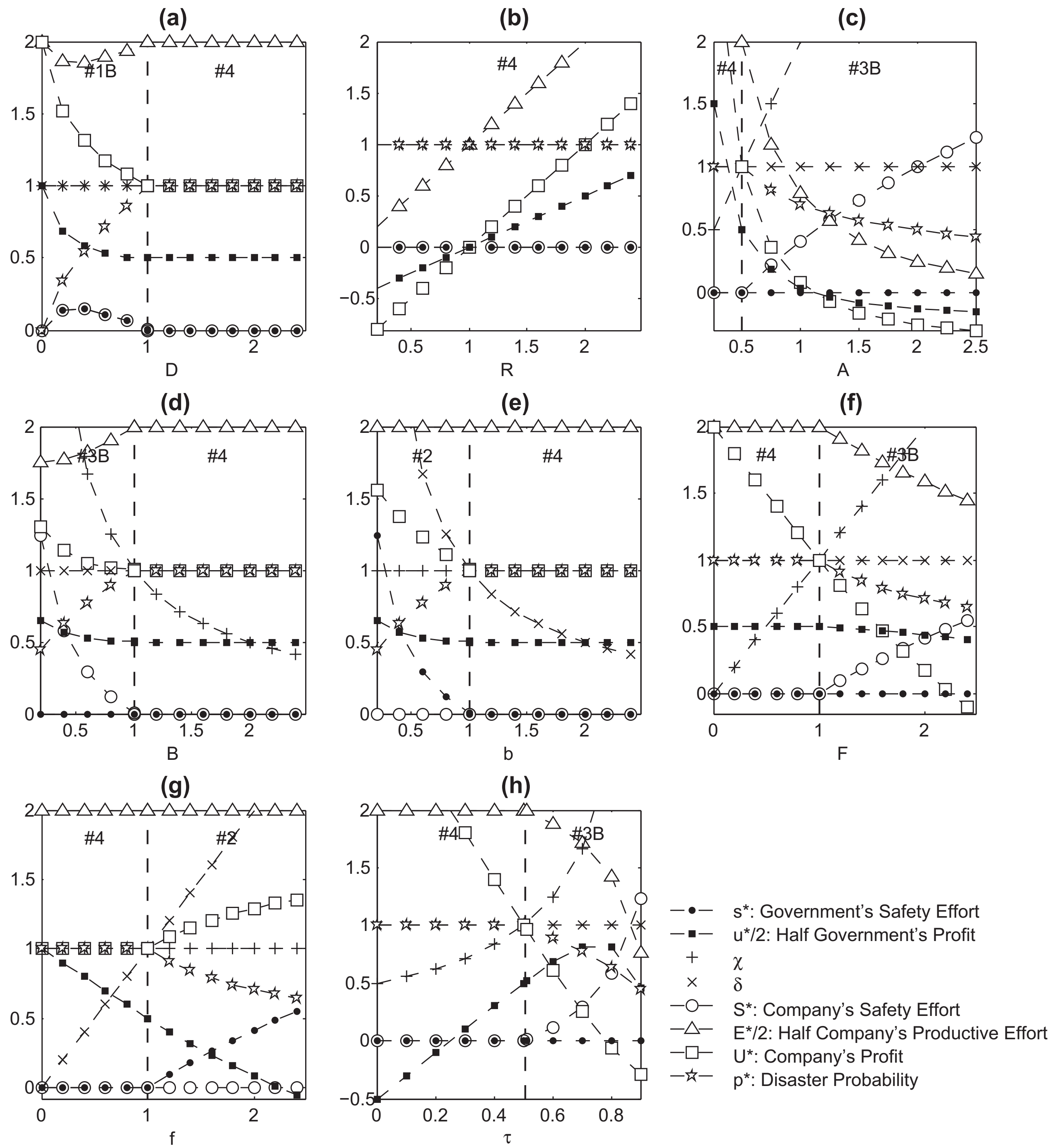

Fig. 1. Equilibrium behavior as a function of $D, R, A, B, b, F, f, \tau$ with baseline values $R = 2$, $\chi = \delta = D = B = b = F = f = h = k = 1$, and $A = \tau = 0.5$.

in case 2 when $\frac{AF}{B(1-\tau)} \geqslant \frac{f}{b}$ occurs when these four conditions are reversed. In particular, imposing taxes is one method by which a government can transfer safety effort from itself to the company.[8]

[8] See Sunley et al. (2002, Table 2, pp. 15–16) for income tax rates, production sharing, royalties, etc. Sunley et al. (2002, p. 14) write that "It is typical to have at least 50–60 percent of profit oil going to the state, but in some countries a higher share applies." The Norwegian petroleum industry is taxed $\tau = 0.78$, the US petroleum industry is taxed $\tau = 0.48$ (http://www.ipaa.org/news/docs/Tax_Issue_Talking_Points_02-2011.pdf, retrieved May 1, 2011), and various companies seek tax shelters such as e.g. Bermuda.

Proposition 1 implies that the conditions in Table 1 are the conditions for each of the six cases to be optimal. Thus, we are able to summarize the solution in Table 1 below, where an active player refers to a player exerting safety effort.

Proposition 2. *Assume parameter values excluding the unlikely cases 1A and 3A. Safety efforts are inverse U shaped in the disaster magnitude within cases 1B and 3B for the company and within cases 1B and 2 for the government. The disaster probability increases concavely in the disaster magnitude reaching 1 at the transition to case 4. Both profits are convex in the disaster magnitude. The* *company's profit decreases convexly in the disaster magnitude for case 2. The company's safety effort increases convexly in taxation. The government's safety effort and the disaster probability decrease convexly in taxation. Within cases 2 and 4 the company's profit decreases linearly in taxation while the government's profit increases linearly in taxation. Within cases 1B and 3B the company's profit is convex in taxation.*

Proof. See Table 3 in Appendix C: Supplementary material, and p is continuous through cases 1B, 2, and 3B. □

Proposition 2 shows how safety efforts are large for intermediate disaster magnitudes and low when the disaster is negligible or overwhelming. Profits are convexly impacted by the disaster. Increasing taxation causes the company to exert convexly increasing safety effort, while the government exerts convexly decreasing safety effort, and the disaster probability decreases convexly. Within cases 2 and 4 the company's profit decreases linearly in taxation while the government's profit increases linearly in taxation. Within cases 1B and 3B the results are usually similar as we proceed to illustrate in the next section, though theoretically the interaction between the parameters for the profits is more complicated as shown in Table 1. Table 3 in Appendix C and the next section show the dependence on the other parameters R, A, B, b, F, f.

4. Illustrating the model

Fig. 1 assumes the baseline $R = 2$, $D = B = b = F = f = h = k = 1$, and $A = \tau = 0.5$, which gives $S = s = U = u = 0$, $p = 1$. Division with 2, i.e. $u^*/2$ and $E^*/2$, is done for scaling purposes. Technically this solution is case 4 where no players invest, profits are zero, and the disaster is guaranteed. Arbitrarily small changes in parameter values can give either case 3B, case 2, or case 1B. Panel a gives direct transition from cases 1B to 4 as D increases through $D = 1$, excluding cases 2 and 3B. Both the government's and the company's safety efforts are inverse U shaped which is common for such phenomena (Hausken et al., 2009; Hausken and Zhuang, 2011a,b,c). A small disaster can be ignored, while defending against an overwhelming disaster has negligible impact and is not worthwhile. The production $E = (R - BS)/A$ is U shaped within case 1B. A small D guarantees case 1B. A large D guarantees case 4.

Panel b gives case 4 since $D = \chi = \delta = 1$. The profits increase in R with no safety efforts. Panel c gives transition from cases 4 to 3B as A increases through $A = 0.5$, causing $\chi > \delta$. The company's high unit production cost $A > 0.5$ interestingly causes it to be the sole provider of safety effort, where S increases in A, causing the disaster probability p to decrease in A. One policy recommendation is that when unit production costs are low, one should be especially concerned about preventing companies from shifting their focus from safety to production. Conversely, panel d gives transition from cases 3B to 4 as B increases through $B = 1$, which in turn causes $\chi < \delta$. The company's low unit safety effort cost $B < 1$ causes it to be the sole provider of safety effort, and decreasing B causes lower disaster probability p. Analogously panel e gives transition from cases 2 to 4 as b increases through $b = 1$, which in turn causes $\chi < \delta$. Here $b < 1$ causes the government to be the sole provider of safety effort, and p increases in b. Panel f gives transition from cases 4 to 3B as F increases through $F = 1$ causing $\chi > \delta$. Larger F means that the company is more negatively affected by the disaster inducing it to provide sole safety effort to decrease p, and the government to free ride on safety effort. Analogously panel g gives transition from cases 4 to 2 as f increases through $f = 1$ causing $\chi > \delta$. As f increases above 1, the government is so negatively affected by the disaster that it incurs substantial safety effort to decrease p, despite earning negative profit, and enabling the company to free ride on the government's safety effort, while earning positive profit. Panel h gives transition from cases 4 to 3B as τ increases through $\tau = 0.5$ causing $\chi > \delta$, where δ is independent of τ and χ increases in τ towards infinity. Increasing taxation for the company is similar to increasing unit production cost A and thus a similar transition is observed in panel c, inducing the company to exert increasing safety effort and decreasing productive effort. Within case 3B the government's profit is inverse U shaped, the company's profit is U shaped, and the disaster probability p decreases in τ.

To test the sensitivity around $h = k = 1$, Fig. 2 assumes the baseline $R = 2$, $B = b = F = f = 1$, and $A = \tau = 0.5$, but sets $D = 0.5$ which gives the middle position within case 1B for the interior solution $(S > 0, s > 0)$ in Fig. 1 panel a. In the left panel where $k = 1$, increasing h gives increasing government safety effort s^* and decreasing company safety effort S^*. The explanation, since $E^* > 1$ in (3), is that increasing h gives increased production which the company

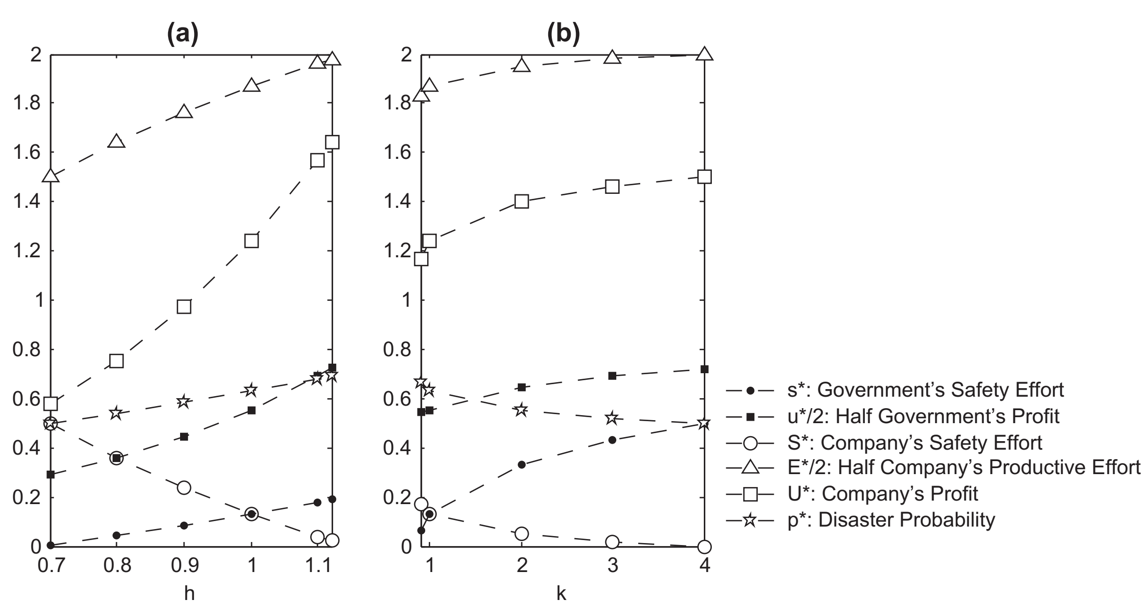

Fig. 2. Equilibrium behavior as a function of h when $k = 1$, and as a function of k when $h = 1$, when $R = 2$, $\chi = \delta = B = b = F = f = 1$, $A = \tau = 0.5$, and D = 0.5.

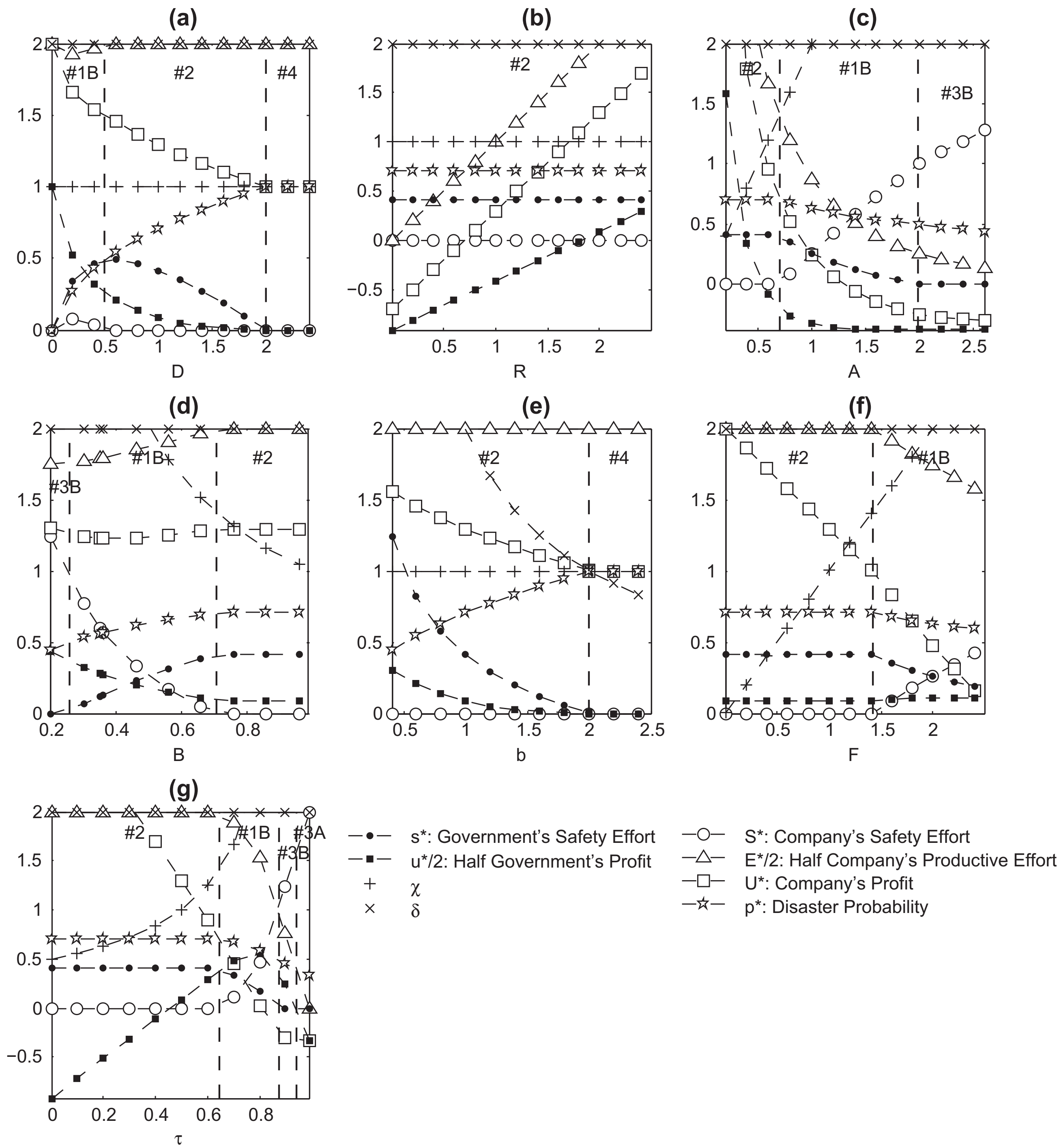

Fig. 3. Equilibrium behavior as a function of D, R, A, B, b, F, τ, with baseline values $R = 2$, $D = B = b = F = h = k = 1$, $f = 2$, and $A = \tau = 0.5$.

exploits by decreasing safety effort, analogously to decreasing unit production cost also causing decreasing company safety effort. In these situations the government steps in with increased safety effort. Both players, but especially the company, earn higher profits due to higher h. Similarly, in the right panel where $h = 1$, increasing k also gives increasing s^* and decreasing S^*, but the explanation is different. Since $s^* < 1$, an increasing exponent k in (4) gives decreased government cost of safety effort from which both players benefit. Again the company free rides on the government's safety effort.

Fig. 3 assumes $R = 2$, $D = B = b = F = 1$, $f = 2$, and $A = \tau = 0.5$. Since $\chi < \delta$, panel a gives cases 1B, 2, 4, excluding case 3B where the company exerts safety effort. Again both the government's and the company's safety efforts are inverse U shaped. The large $\delta = 2$ means that government is negatively affected by the disaster so s is large, extends over cases 1B and 2, and decreases to 0 at the transition to case 4. The government's profit thus decreases strongly from $u = 1$ when $D = 0$ to $u = 0$ when $D \geqslant 2$. The low $\chi = 1$ means that the company is less negatively affected by the disaster so S is small, extends only over case 1B, and decreases to 0 at the transition to case 2. The small s implies large production $E = (R - BS)/A$ which is U shaped within case 1B, and $E = 1$ when $D \geqslant 0.5$. The company's profit decreases more moderately from $U = 2$ when $D = 0$ to $U = 1$ when $D \geqslant 2$.

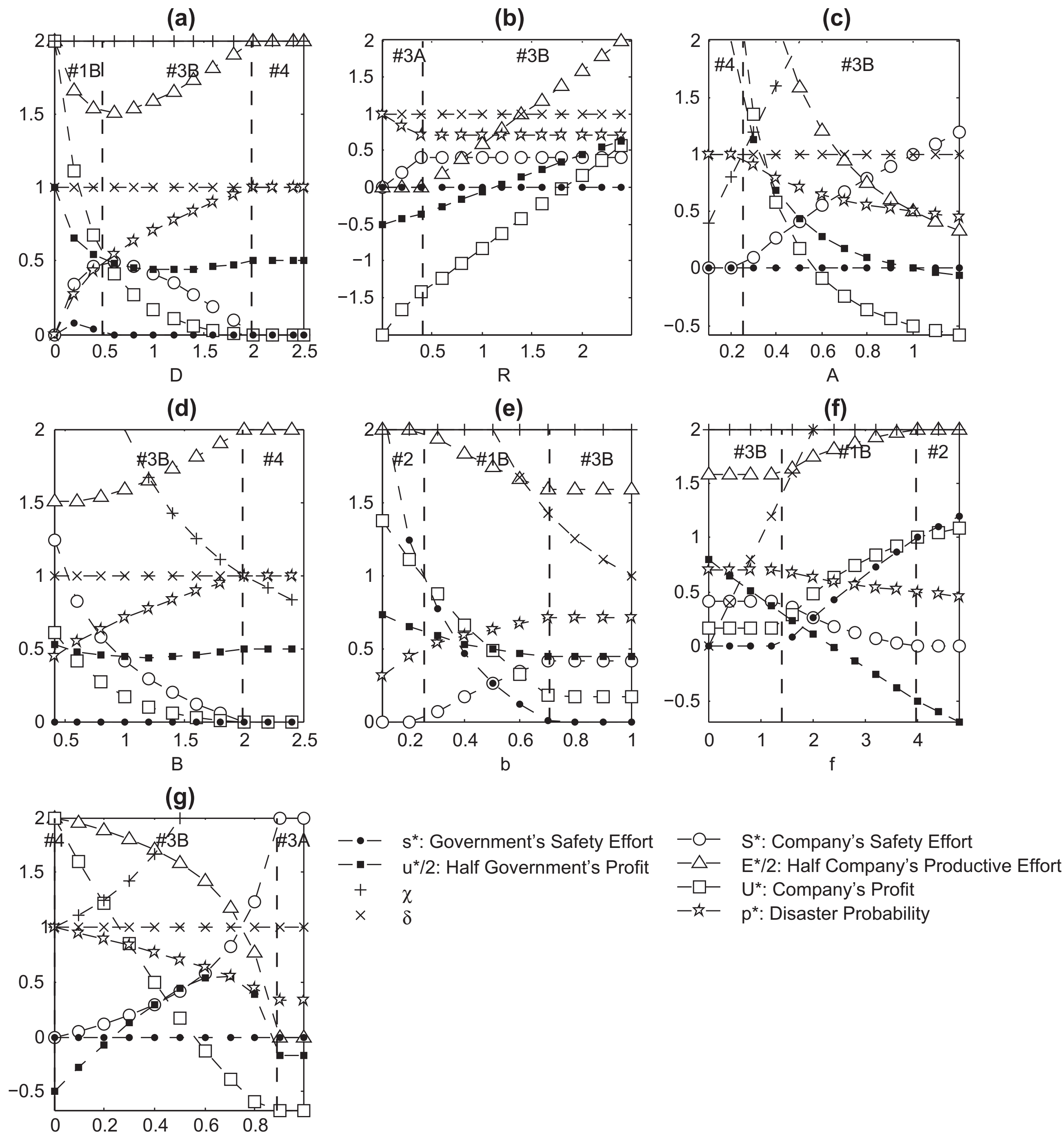

Fig. 4. Equilibrium behavior as a function of D, R, A, B, b, f, τ with baseline values $R = 2$, $D = B = b = f = h = k = 1$, $F = 2$, and $A = \tau = 0.5$.

Panel b gives case 2 since $\chi < \delta$. The government is more negatively affected by the disaster, exerts positive safety effort, and earns lower profit than the company. Panel c gives transitions from cases 2 to 1B and from cases 1B to 3B as A increases through $A = 0.7$ and $A = 2.0$, respectively. The first crossing according to Table 1 follows since $\chi^2/\delta = D = 1$ implies $A = \sqrt{2}/2 \approx 0.7$. The second crossing follows since $\delta^2/\chi = D = 1$ implies $A = 2$. For low unit production cost the government solely provides safety effort (case 2). The company receives substantial profit from production and discounts the relative impact of the disaster. In contrast, the government is substantially impacted by the disaster ($f = 2$) and has a higher incentive to exert safety effort, although also enjoying the high profits. A possible policy implication is that the government should look for cases where firms earn substantial profits, and assess whether to exert safety effort. For intermediate unit production cost both players exert safety efforts (case 1B). The company earns lower benefit of production and assesses the relative impact of the disaster more seriously, thus increasing S as A increases. This in turn enables the government to free ride on the company's safety effort. Also influenced by the lower profits, the government thus decreases s as A increases. For very high unit production cost the company becomes the sole provider of safety effort (case 3B), S continuing to increase in A. This situation is quite unfortunate. The benefit of production is so low that the government continues to free ride on the company's safety effort, and cannot justify exerting its own safety effort. The company weighs the low but positive benefit of production against the negative impact of the disaster, as expressed with the two terms for U in (3). The large A implies that the first term is low even when S is low, so the company increases S to ensure that the second negative term does not become too large. The company earns negative profit when $A > 1.2$. Panel d gives transitions from cases 3B to 1B and from cases 1B to 2 as B increases through $B = 0.25$ and $B = 0.7$, respectively. The sequence is opposite that of panel c. The first crossing follows since $\delta^2/\chi = D = 1$ implies $B = 0.25$. The second crossing follows since $\chi^2/\delta = D = 1$ implies $B = \sqrt{1/2} \approx 0.7$. With low unit cost B of safety effort the company exerts it solely while the government free rides (case 3B). For intermediate B both players exert safety effort (case 2). For large B the company withdraws safety effort focusing solely on production, trusting the government to exert safety effort which it does since it would otherwise be not provided and since it benefits from the production (case 2). This case may occur when the company finds it difficult, or lacks the expertise, knowhow, or equipment, to incur safety effort. Governments need to be observant of this case and step in with safety effort. Panel e gives transition from cases 2 to 4 as b increases through $b = 2$, analogously to Fig. 1, but now f is twice as large so the transition occurs when b is twice as large. The government is willing to incur safety effort until its unit cost is twice as large, but not when $b > 2$. Panel f gives transition from cases 2 to 1B as F increases through $F = 1.4$. This follows since $\chi^2/\delta = D = 1$ implies $F = \sqrt{2} \approx 1.41$. When F is low, the company is little affected by the disaster and free rides on the government's safety effort. When F is large, the company joins with safety effort. The large $f = 2$ causes the government to incur safety effort for all F, in contrast to Fig. 1 where it does not incur safety effort for all F. Plotting as a function of f disables $f = 2$ causing the same plot as in Fig. 1 which is omitted. In panel g the large $\delta = f/b = 2$ induces the government to exert safety effort when τ is small, as in panel c when A is small. This removes case 4 present in Fig. 1. Case 2 occurs when $0 \leqslant \tau < 0.65$. As τ increases and production becomes less profitable for the company, it joins exerting safety effort causing case 1B when $0.65 < \tau < 0.87$. With excessive taxation $0.87 < \tau \leqslant 0.94$, as in panel c when $A > 4$, case 3B occurs where only the tax burdened company exerts safety effort. More extremely, case 3A occurs when $0.94 < \tau \leqslant 1$ causing company safety effort $S = R/B = 2$ and bankruptcy profit $U = -0.33$. The disaster probability p decreases in taxation τ, and in unit production cost A in panel c, consistently with Table 1.

Fig. 4 assumes $D = R = B = b = f = 1$, $F = 2$, and $A = \tau = 0.5$. Since $\chi > \delta$, panel a gives cases 1B, 3B, 4, excluding case 2 where the company exerts safety effort. The small $\delta = 1$ which benefits the government causes a small inverse U shaped s for the government within case 1B and a large inverse U shaped S for the company within cases 1B and 3B. Hence production is U shaped and substantially lower within cases 1B and 3B. The company's profit decreases strongly from $U = 2$ when $D = 0$ to $U = 0$ when $D \geqslant 2$. The government's profit decreases within case 1B reaching a minimum when $D = 1$ within case 3B. The government benefits from the company's increasing production as D increases from 1 through case 3B, reaching $u/2 = 0.5$ when $D \geqslant 2$. Panel b gives case 3B when $R > 0.42$ since $\chi > \delta$, and case 3A with $S = R/B$ when $R < 0.42$. The company earns negative profit when $R < 1.1$. Panel c gives transition from cases 4 to 3B as A increases through $A = 0.25$. Since the company is more negatively affected by the disaster, the transition to case 3B occurs for a unit production cost A which is half of that in Fig. 1. Panel d gives transition from cases 3B to 4 as B increases through $B = 2$. The company is willing to be the sole provider of safety effort for twice the unit cost of safety effort compared with Fig. 1.

Panel e gives transitions from cases 2 to 1B and from cases 1B to 3B as b increases through $b = 0.25$ and $b = 0.7$, respectively. The crossing points follow from solving $\chi^2/\delta = D = 1$ and $\delta^2/\chi = D = 1$ respectively with respect to b. The panel has some qualitative similarities with panel c in Fig. 3. Increasing the government's unit safety effort cost b when $F = 2$ has qualitative similarities with increasing the company's unit production cost A when $f = 2$. Plotting as a function of F disables $F = 2$ causing the same plot as in Fig. 1 which is omitted. Panel f gives transitions from cases 3B to 1B and from cases 1B to 2 as f increases through $f = 1.4$ and $f = 4$. The crossing points follow from solving $\delta^2/\chi = D = 1$ and $\chi^2/\delta = D = 1$ respectively with respect to f. When f is below 1.4, the government is so little affected by the disaster that it does not exert safety effort, while the company exerts safety effort (case 3B). When f is intermediate, $1.4 < f < 4$, both players incur safety effort. As f increases above 1.4, the government provides safety effort solely (case 2). The large $F = 2$ prevents case 4 which occurs in Fig. 1. The transition to the government providing safety effort occurs for a large $f = 4$, in contrast to $f = 1$ in Fig. 1, since $F = 2$ is so large. In panel g the large $F = 2$ induces the company to exert more substantial safety effort than in Figs. 1 and 3 when τ increases causing case 3B when $0 < \tau \leqslant 0.89$, as in panel c when $A > 0.5$, and case 3A with $S = R/B$ when $0.89 < \tau \leqslant 1$. Only at the extreme limit with no taxation, $\tau = 0$, does the company focus exclusively on production with no safety effort causing case 4.

5. Two disaster examples

Let us link the model to two disasters to visualize a way of thinking. The first is the April 20–July 15, 2010 Gulf of Mexico oil spill disaster involving British Petroleum, estimated to cost \$100 billion (Becker and Posner, 2010).[9] BP's 2009 profits were \$14 billion.[10] The second is the March 24, 1989 Exxon Valdez oil spill disaster in Prince William Sound, Alaska, estimated to cost \$7 billion. Exxon's 1989 and 1990 profits were 3.8 billion and \$5 billion, respectively.[11]

BP's CEO Tony Hayward claimed that the Gulf disaster had "a one in a million" chance of happening. Becker and Posner (2010) argue that if the claim is correct, and if the disaster costs \$100 billion, then the discounted value (at a 5% interest rate) of this expected cost over time would be approximately 20 times \$100 billion divided by one million, i.e. \$2 million, which is modest for any large business. However, if the annual probability is more like 1/10,000, the expected discounted cost would be about \$200 million, a more considerable sum. Becker and Posner (2010) correctly acknowledge that estimating low probability costly events is difficult.

Whereas the previous section considered the baseline $D = R = B = b = F = f = h = k = 1$ and $A = \tau = 0.5$ causing $S = s = U = u = 0$, $p = 1$, this section considers parameter values causing lower disaster probability within case 1B, for low probability costly events. First we set $D = 10^{-3}$ causing $p = 10^{-2}$ when $B = b = F = f = 1$ and $A = \tau = 0.5$. This is still a large disaster probability, so let us determine which parameter values decrease p further. Assume unit costs $B = b = 1$ and $A = \tau = 0.5$, and that the government is more affected by the disaster than the company. One example is $F = 10$ and $f = 100$ to reflect a company less concerned about a disaster if it does not affect its profit. This gives $\chi = \frac{AF}{B(1-\tau)} = 10$, and $\delta = \frac{f}{b} = 100$. Inserting into Table 1 gives $S = (1 - D^{1/3})D^{2/3}$,

[9] This includes the \$20 billion fund established to compensate Gulf Coast residents, http://money.cnn.com/2011/02/19/smallbusiness/bp_claims_final_protocol/index.htm, retrieved April 20, 2011.

[10] http://www.huffingtonpost.com/2010/05/27/bps-profits-far-out-weigh_n_591992.html, retrieved April 14, 2011, retrieved April 14, 2011.

[11] http://www.grist.org/article/2010-05-26-will-bp-take-responsibility-or-squeeze-this-disaster-for-profits, retrieved April 14, 2011.

$s = (10 - D^{1/3})D^{2/3}$, $p = D^{2/3}/10$. We have $S < s$ for all $D > 0$. We must have $D < 1$ to have case 1B otherwise S will be negative (which gives case 2 where the company does not invest). For example, let $D = 10^{-3}$, which gives $S = 0.009$, $s = 0.099$, $p = 10^{-3}$, $u = R - 0.028$, $U = R - 0.019$. This gives case 1B where the company invests less than the government in safety effort. The disaster probability $p = 10^{-3}$ is 10^3 times larger than Tony Hayward's suggestion, and gives a discounted cost of about \$2 million $\times 10^3$ = \$2 billion. Such a cost gets the attention of any company, but is only 14% of BP's 2009 profits, and 53% of Exxon's 1989 profits.

This paper's model allows estimating the impact of changes in parameter values. Let us consider a few such changes, keeping $D = 10^{-3}$. Using the data from Wikipedia (2012a,b), the spill volume was 260–750 thousand barrels for the Exxon Valdez disaster but 4.9 million barrels for the Gulf of Mexico disaster, which is roughly 1:10. We thus assume that F was small for the Exxon Valdez disaster, and in the order of 10 times larger for the Gulf disaster due to various factors including increased environmental awareness. To reflect this, assume $F = 100$ caused by laws and regulations inducing companies to be more concerned about disasters. Table 1 implies $S = s = (10^{2/3} - D^{1/3})D^{2/3} \approx 0.045$, $p = D^{2/3}/10^{4/3} = 10^{-10/3} \approx 4.6 \times 10^{-4}$. That is, the company increases its safety effort five times, the government cuts its safety effort in half (free rider effect), and the disaster probability is less than half, with a discounted cost of less than \$1 billion if the disaster strikes.

Table 2 shows the variables and the approximate discounted cost C in million dollars for various parameter values. The first two rows show the examples discussed above. Thereafter follow six rows with small changes in A, $B = b$, and $F = f$, respectively. The next five rows show examples where the company withdraws safety effort, $S = 0$. This occurs when the unit production cost is low, the company's unit safety effort cost is high, the government's unit safety effort cost is low (so the company can free ride on the government's safety effort), the company is minusculely impacted by the disaster, or the government is strongly impacted by the disaster. Low tax rate $\tau = 0$ is not sufficient to cause $S = 0$, so the subsequent row assumes $\tau = 0$ causing $S = 0.0053$.

Conversely, in the six rows thereafter the government withdraws safety effort, $s = 0$. This occurs when the unit production cost is high (so the government can free ride on the company's safety effort), the company's unit safety effort cost is low, the government's unit safety effort cost is high, the company is strongly impacted by the disaster, the government is weakly impacted by the disaster, or the tax rate is large. Thereafter follow 12 examples where both players withdraw safety effort, $S = s = 0$, guaranteeing $p = 1$. This occurs when at least two parameters take extreme values; i.e., (1) when the unit production cost is low and the government's unit safety effort cost is high, (2) when the unit production cost is low and the government is minusculely impacted by the disaster, (3) when both unit safety effort costs are high, (4) when the company's unit safety effort cost is high and the government is minusculely impacted by the disaster, (5) when the government's unit safety effort cost is high and the company is minusculely impacted by the disaster, (6) when both players are minusculely impacted by the disaster. The tax rate constrained to $0 \leqslant \tau \leqslant 1$ is less flexible than the other parameters to adjust to $S = s = 0$, so we present the corresponding six examples where $\tau = 0.9$.

The two rows thereafter show examples where the disaster probability is $p = 10^{-6}$, as proposed by BP's CEO, which occurs

Table 2
Examples of parameter values when $D = 10^{-3}$.

	S	s	E	p	u	U	C
$B = b = F = f = 1$, $A = \tau = 0.5$	0.009	0.009	$2R - 0.018$	10^{-2}	-0.028	-0.019	2×10^4
$B = b = 1$, $F = 10$, $f = 10^2$, $A = \tau = 0.5$	0.009	0.099	$2R$-0.018	10^{-3}	$R - 0.028$	$R - 0.019$	2000
$A = 0.25$, $B = b = 1$, $F = 10$, $f = 10^2$, $\tau = 0.5$	0.0053	0.125	$4R - 0.021$	1.3×10^{-3}	$2R - 0.262$	$2R - 0.02$	2520
$A = 1$, $B = b = 1$, $F = 10$, $f = 10^2$, $\tau = 0.5$	0.0149	0.078	$R - 0.0149$	7.9×10^{-4}	$R/2 - 0.165$	$0.5R - 0.015$	1587
$B = b = 0.5$, $F = 10$, $f = 10^2$, $A = \tau = 0.5$	0.0116	0.125	$2R - 0.012$	6.3×10^{-4}	$R - 0.131$	$R - 0.01$	1260
$B = b = 2$, $F = 10$, $f = 10^2$, $A = \tau = 0.5$	0.0069	0.078	$2R - 0.028$	1.6×10^{-3}	$R - 0.329$	$R - 0.03$	3175
$B = b = 1$, $F = f = 10^2$, $A = \tau = 0.5$	0.0454	0.045	$2R - 0.091$	4.6×10^{-4}	$R - 0.137$	$R - 0.09$	928
$B = b = 1$, $F = f = 10^3$, $A = \tau = 0.5$	0.099	0.099	$2R - 0.198$	10^{-4}	$R - 0.298$	$R - 0.20$	200
$A = 5 \times 10^{-2.5}$, $B = b = 1$, $F = 10$, $f = 10^2$, $\tau = 0.5$	0	0.315	$2 \times 10^{1.5}R$	3.2×10^{-3}	$10^{1.5}R - 0.63$	$10^{1.5}R - 0.03$	6325
$B = 10^{1.5}$, $b = 1$, $F = 10$, $f = 10^2$, $A = \tau = 0.5$	0	0.315	$2R$	3.2×10^{-3}	$R - 0.632$	$R - 0.03$	6325
$B = 1$, $b = 10^{-3}$, $F = 10$, $f = 10^2$, $A = \tau = 0.5$	0	9.999	$2R$	10^{-4}	$R - 0.02$	$R - 10^{-3}$	200
$B = b = 1$, $F = 10^{-0.5}$, $f = 10^2$, $A = \tau = 0.5$	0	0.315	$2R$	3.2×10^{-3}	$R - 0.63$	$R - 0.032$	6325
$B = b = 1$, $F = 1$, $f = 10^3$, $A = \tau = 0.5$	0	0.999	$2R$	10^{-3}	$R - 2.00$	$R - 10^{-3}$	2000
$B = b = 1$, $F = 10$, $f = 10^2$, $A = 0.5$, $\tau =$	0.0053	0.1250	$2(R - 0.0106)$	0.0013	-0.251	$2R - 0.0232$	2519.8
$A = 5 \times 10^5$, $B = b = 1$, $F = 10$, $f = 10^2$, $\tau = 0.5$	99.999	0	$2(10^{-6}R - 10^{-4})$	10^{-5}	$10^{-6}R - 10^{-3}$	$10^{-6}R - 2 \times 10^{-4}$	20
$B = 10^{-6}$, $b = 1$, $F = 10$, $A = 10^2$, $A = \tau = 0.5$	99.999	0	$2(R - 10^{-4})$	10^{-5}	$R - 10^{-3}$	$R - 2 \times 10^{-4}$	20
$B = 1$, $b = 10^3$, $F = 10$, $f = 10^2$, $A = \tau = 0.5$	0.099	0	$2(R - 0.099)$	10^{-2}	$R - 1.10$	$R - 0.199$	2×10^4
$B = b = 1$, $F = 10^7$, $f = 10^2$, $A = \tau = 0.5$	99.999	0	$2(R - 99.999)$	10^{-5}	$R - 10^2$	$R - 200.00$	20
$B = b = 1$, $F = 10$, $f = 0.1$, $A = \tau = 0.5$	0.099	0	$2(R - 0.099)$	10^{-2}	$R - 0.1$	$R - 0.199$	2×10^4
$A = 0.5$, $B = b = 1$, $F = 10$, $f = 10^2$, $\tau = 0.9999995$	99.999	0	$2(R - 199.998)$	10^{-5}	$R - 199.9989$	$R - 2 \times 10^{-4}$	20
$A = 5 \times 10^{-5}$, $B = 1$, $b = 10^5$, $F = 10$, $f = 10^2$, $\tau = 0.5$	0	0	2×10^4R	1	$10^4R - 10^2$	$10^4R - 10$	2×10^6
$A = 5 \times 10^{-5}$, $B = 1$, $b = 1$, $F = 10$, $f = 10^{-3}$, $\tau = 0.5$	0	0	2×10^4R	1	$10^4R - 10^{-3}$	$10^4R - 10$	2×10^6
$B = 10^4$, $b = 10^5$, $F = 10$, $f = 10^2$, $A = \tau = 0.5$	0	0	$2R$	1	$R - 10^2$	$R - 10$	2×10^6
$B = 10^4$, $b = 1$, $F = 10$, $f = 10^{-3}$, $A = \tau = 0.5$	0	0	$2R$	1	$R - 10^{-3}$	$R - 10$	2×10^6
$B = 1$, $b = 10^5$, $F = 10^{-3}$, $f = 10^2$, $A = \tau = 0.5$	0	0	$2R$	1	$R - 10^2$	$R - 10^{-3}$	2×10^6
$B = 1$, $b = 1$, $F = f = 10^{-3}$, $A = \tau = 0.5$	0	0	$2R$	1	$R - 10^{-3}$	$R - 10^{-3}$	2×10^6
$A = 10^{-5}$, $B = 1$, $b = 10^5$, $F = 10$, $f = 10^2$, $\tau = 0.9$	0	0	10^5R	1	$9 \times 10^4R - 100$	$10^4R - 10$	2×10^6
$A = 10^{-5}$, $B = b = 1$, $F = 10$, $f = 10^{-3}$, $\tau = 0.9$	0	0	10^5R	1	$9 \times 10^4R - 10^{-3}$	$10^4R - 10$	2×10^6
$A = 0.5$, $B = 5 \times 10^4$, $b = 10^5$, $F = 10$, $f = 10^2$, $\tau = 0.9$	0	0	$2R$	1	$1.8R - 10^2$	$0.2R - 10$	2×10^6
$A = 0.5$, $B = 5 \times 10^4$, $b = 1$, $F = 10$, $f = 10^{-3}$, $\tau = 0.9$	0	0	$2R$	1	$1.8R - 10^{-3}$	$0.2R - 10$	2×10^6
$A = 0.5$, $B = b = 1$, $F = 2 \times 10^{-4}$, $f = 10^{-3}$, $\tau = 0.9$	0	0	$2R$	1	$1.8R - 10^{-3}$	$0.2R - 2 \times 10^{-4}$	2×10^6
$A = 0.5$, $B = 1$, $b = 10^5$, $F = 2 \times 10^{-4}$, $f = 10^2$, $\tau = 0.9$	0	0	$2R$	1	$1.8R - 10^2$	$0.2R - 2 \times 10^{-4}$	2×10^6
$B = b = 10^{-4.5}$, $F = 10$, $f = 10^2$, $A = \tau = 0.5$	0.3152	3.161	$2R - 2 \times 10^{-5}$	10^{-6}	$R - 2 \times 10^{-4}$	$R - 2 \times 10^{-5}$	2
$B = b = 1$, $F = f = 10^6$, $A = \tau = 0.5$	0.999	0.999	$2R - 1.998$	10^{-6}	$R - 2.998$	$R - 1.999$	2
$A = 5 \times 10^{-7.75}$, $B = b = 10^{-4.5}$, $F = f = 10^6$, $\tau = 0.5$	0	5623	$2 \times 10^{6.75}R$	1.8×10^{-7}	$10^{6.75}R - 0.36$	$10^{6.75}R - 0.18$	0.356
$A = 5 \times 10^5$, $B = b = 10^{-4.5}$, $F = f = 10^6$, $\tau = 0.5$	3.16×10^5	0.315	$2(10^{-6}R - 10^{-5})$	10^{-11}	$10^{-6}R - 3 \times 10^{-5}$	$10^{-6}R - 2 \times 10^{-5}$	2×10^{-5}

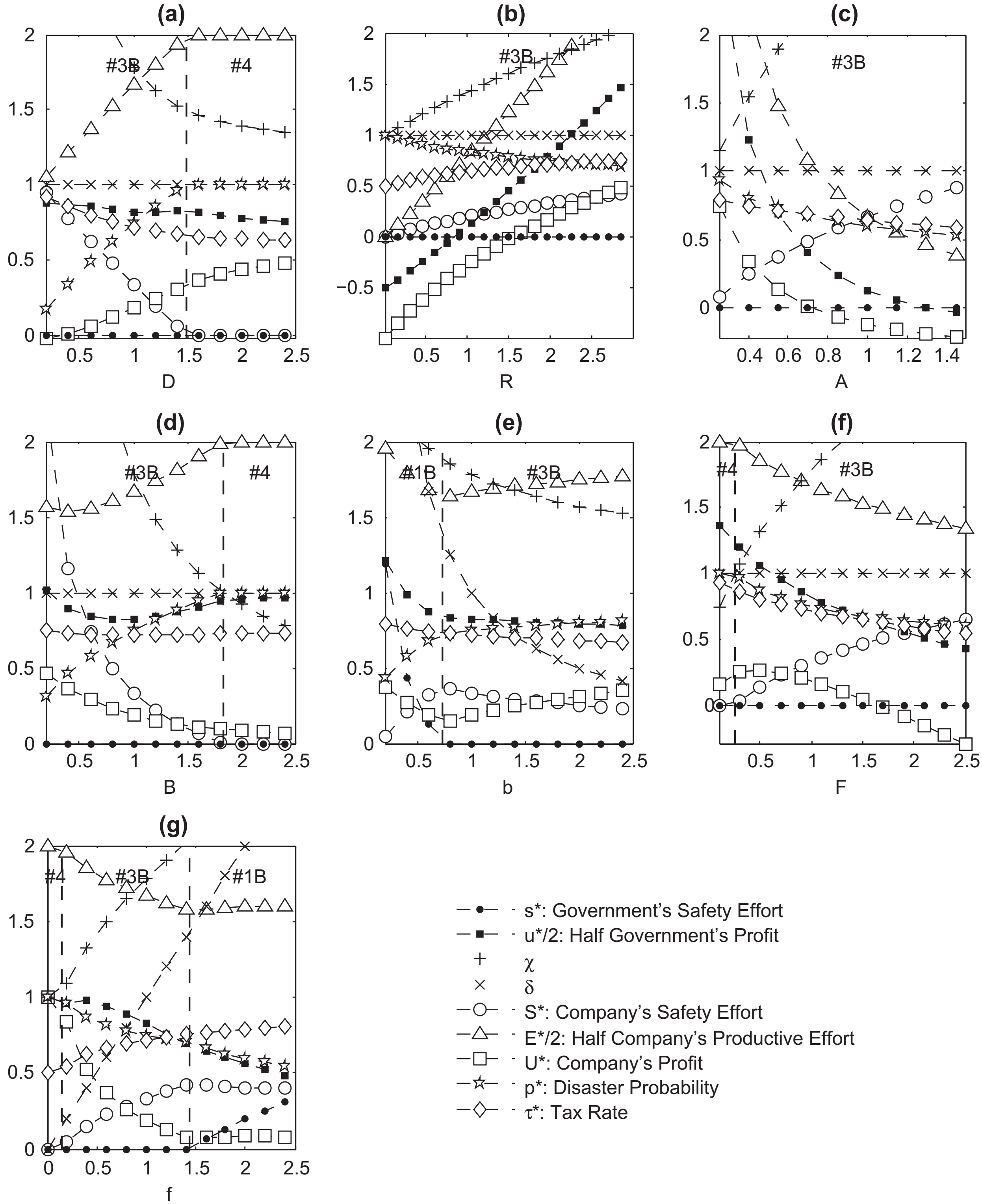

Fig. 5. Replication of Fig. 1 with optimal τ.

when both players' unit safety effort costs are low, or both players are strongly impacted by the disaster. The second last row shows an interesting benchmark where both players have very low unit safety effort costs, and are strongly affected by the disaster. However, the unit production cost is so extremely low that the company withdraws safety effort, ignores the disaster, and relies entirely on the government to exert extremely high safety effort, which cause low disaster probability and very high profits. Conversely, in the last row both players have very low unit safety effort costs, and are strongly affected by the disaster, but the unit production cost is so high that the company exerts very high safety effort, very low production effort, and the disaster probability and profits are very low.

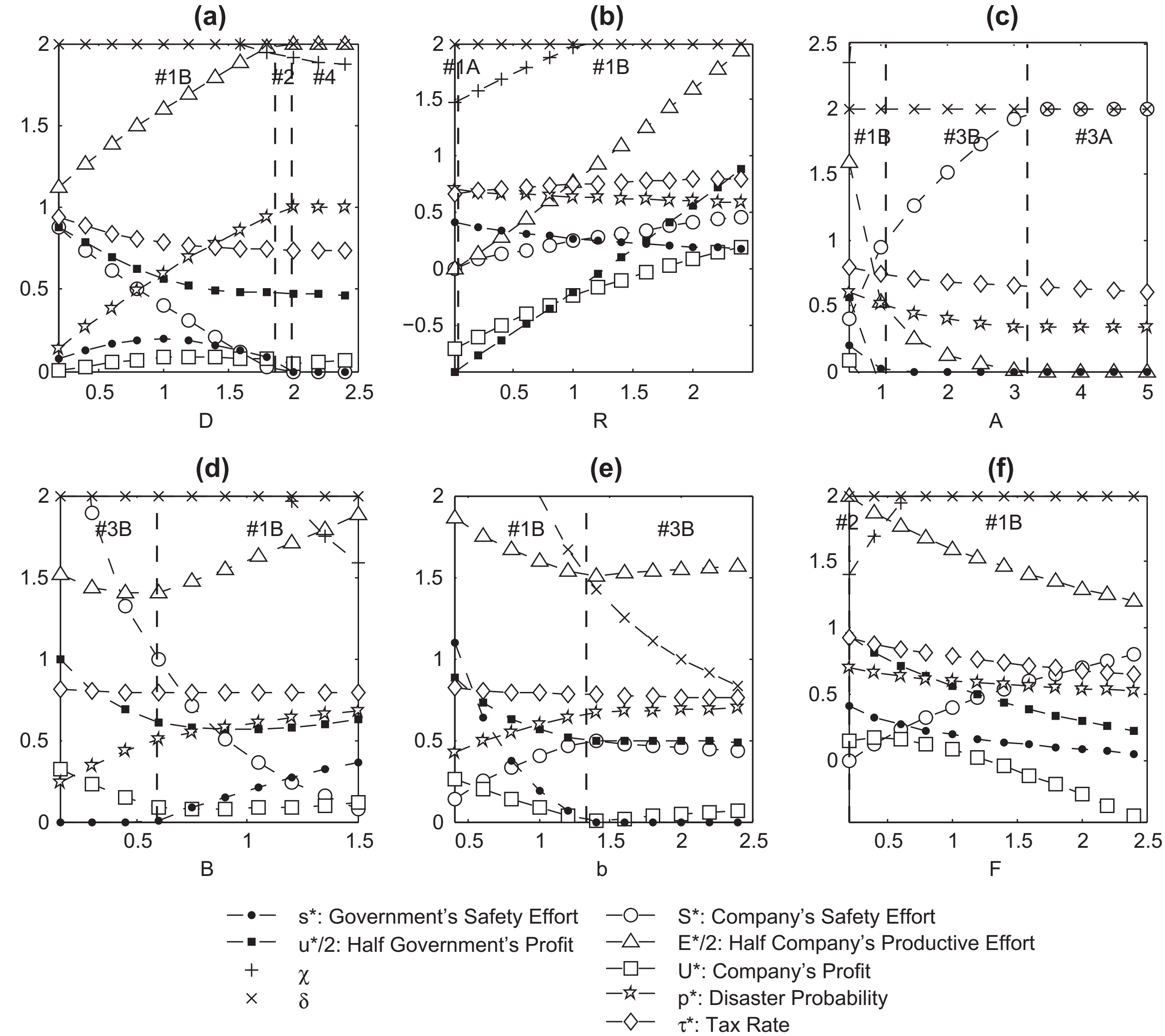

Fig. 6. Replication of Fig. 3 with optimal τ.

6. The government chooses the tax rate

In this section we consider a sequential game where the government chooses the optimal tax rate in period 1 and both players choose optimal safety efforts s and S simultaneously and independently in period 2. Solving with backward induction, we first solve period 2 which gives the first period profit u in Table 1 which is differentiated with respect to τ to yield the first order condition

$$\frac{\partial u}{\partial \tau} = -\frac{bBD^{2/3}\left(2f(\tau-1)\left(\frac{f}{b}\right)^{2/3}\left(\frac{AF}{B}\right)^{2/3} - F(\tau-3)\left(\frac{AfF}{bB}\right)^{2/3}\right) - 3fF(1-\tau)^{5/3}(BD+R)}{3AfF(1-\tau)^{5/3}} = 0 \qquad (6)$$

which is a fifth order equation with analytical solution. Instead of the exogenously given $\tau = 0.5$, Figs. 5–7 replicate Figs. 1,3,4 with the optimal tax rate τ. In Fig. 5 panel a, the optimal tax rate τ decreases from $\tau = 0.92$ when $D = 0$ to $\tau = 0.67$ when $D = 1.4$ causing transition from cases 3B to 4, and to $\tau = 0.63$ when $D = 2.4$. The high taxation for low D suppresses the company's productivity substantially causing low profit, and increases the government's profit. This causes case 3B with no government safety effort, in contrast to case 1B and joint safety effort in Fig. 1. As D increases above $D = 1.4$, no safety effort is worthwhile. In panel b the tax rate increases from $\tau = 0.5$ when $R = 0$ to $\tau = 0.7$ when $R = 2.5$. The larger tax rate causes case 3B and positive company safety in contrast to case 4 in Fig. 1. The company earns negative profit when $R < 1.5$. In panel c the tax rate decreases from $\tau = 0.78$ when $A = 0.25$ to $\tau = 0.6$ when $A = 1.45$. The larger tax rate for low A causes case 3B rather than case 4 as in Fig. 1. The company earns negative profit when the unit production cost $A > 0.7$, in contrast to $A > 1.2$ in Fig. 1. The company suffers from the high taxation. In panel d the tax rate is marginally U shaped with $\tau = 0.75$ when $B = 0.2$, and $\tau = 0.73$ when $B = 2.4$. The higher tax rate causes transition from cases 3B to 4 when $B = 1.8$, in contrast to $B = 1$ in Fig. 1. In panel e the tax rate decreases from $\tau = 0.78$ when $b = 0.2$ to $\tau = 0.67$ when $b = 2.4$. This causes switch from cases 1B to 3B when $b = 0.7$. The higher tax rate causes case 3B and company safety effort rather than case 4 as in Fig. 1 when b is large, and causes case 1B and joint safety effort by both players rather than case 2 (no company safety effort) as in Fig. 1 when b is low. In panel f the tax rate decreases from $\tau = 0.93$ when $F = 0.1$ to $\tau = 0.67$ when $F = 2.4$. The high tax rate expands the range of case 3B down to $F = 0.2$. In panel g the tax rate increases from $\tau = 0.5$ when $f = 0$ to $\tau = 0.8$ when $f = 2.4$. The high tax rate removes most of case 4, causes case 3B when $0.15 < f < 1.42$, and case 1B when $f > 1.42$.

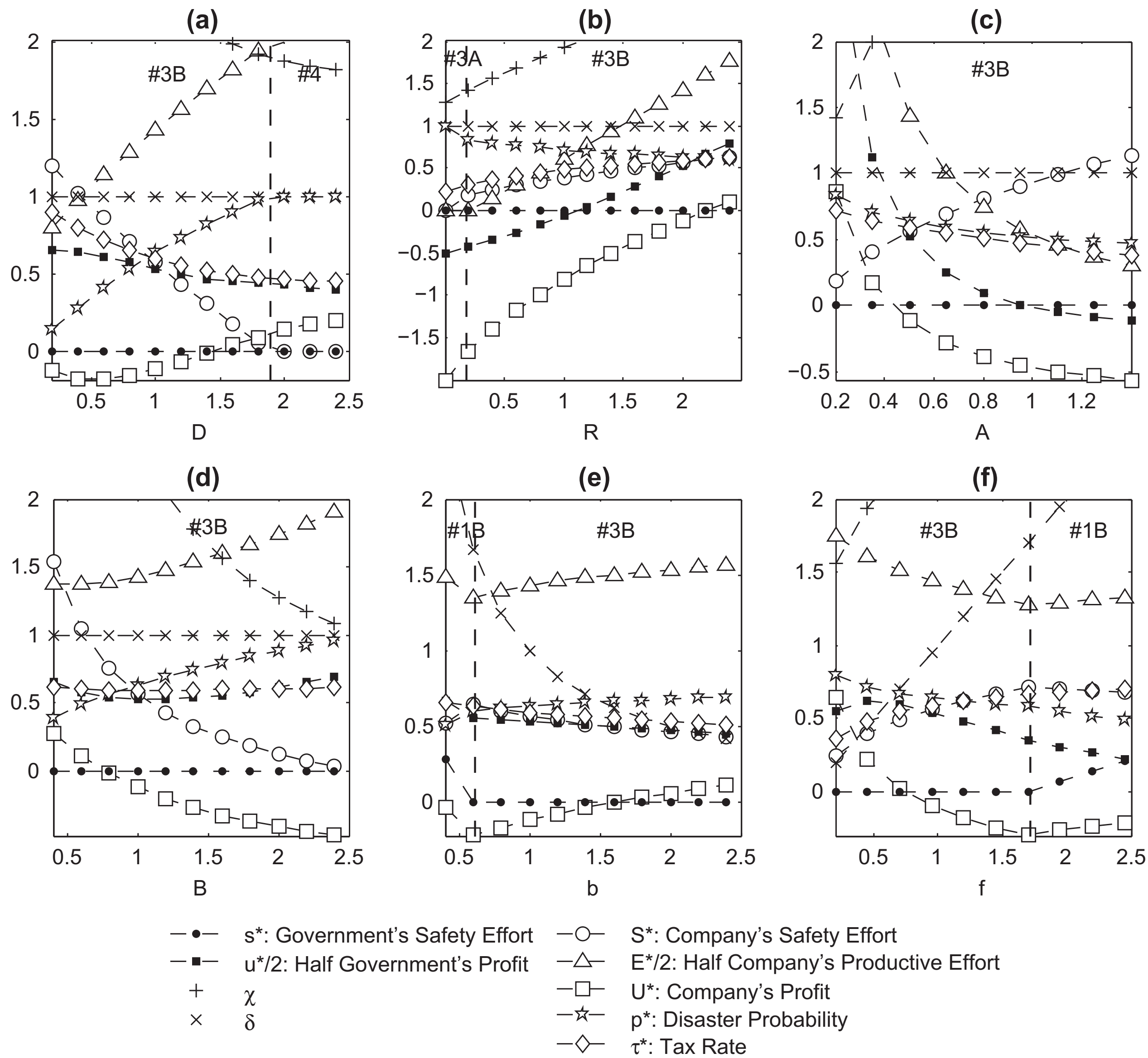

Fig. 7. Replication of Fig. 4 with optimal τ.

Case 2, where only the government exerts safety effort, no longer occurs because of the large tax rate. The company earns positive but very low profit when f is large. Overall, compared with Fig. 1, case 2 is removed since the large tax rate induces the company to exert safety effort, case 4 is less common and case 3B is more common since the company is more willing to exert safety effort.

Fig. 6 doubles f, how the government is negatively affected by disaster, to $f = 2$. Compared with Fig. 5, the main difference is a replacement of case 3B with case 1B as the government becomes more willing to exert safety effort. Furthermore, the tax rate is generally larger since the government needs more money to furnish safety effort. Compared with Fig. 3, the main difference is that case 2 is virtually removed.

Fig. 7 doubles F, how the company is negatively affected by disaster, to $F = 2$. Compared with Fig. 5, the main difference is an expansion of case 3B where the company becomes more willing to exert safety effort. Furthermore, the tax rate is generally lower to enable the company to allocate more resources to safety effort. Compared with Fig. 4, we also observe an expansion of case 3B. Summing up, first, the optimal tax rate decreases in D, A, F, and b, where D affects both players adversely, A affects production adversely, F affects the company adversely (the company exerts safety effort and pays less taxes), and b affects the government's safety effort adversely (the government exerts lower or no safety effort as b increases, and thus needs less taxes). Second, the optimal tax rate increases in R and f, where R affects the company positively (enabling the government to exploit), f affects the government adversely (the government needs taxes to exert larger safety effort). Third, the optimal tax rate is weakly U shaped in B. The government taxes a company advantaged with a low unit safety effort, and also taxes a company with a high unit safety effort to encourage safety effort through denying the company to keep a large proportion of its production.

7. Conclusion

Some policy implications are that the company's and the government's profits do not necessarily change in the same direction. Second, the disaster magnitude significantly impacts the equilibrium solution. However, in practice, this parameter may be highly uncertain. Thus, the perception on risk is critical. Third, one reason the company does not exert sufficient safety efforts is that it has to balance production and safety. The company may be more willing to invest in safety if the government could somehow decrease the production unit cost, decrease the unit cost of safety effort, increase the company's resource, or increase how negatively affected the company is by the disaster.

In the analysis, the two players' safety efforts are strategic substitutes which we think is most realistic. Future research may analyze special scenarios for example where the government cannot access the company's location, equipment, or competence, or the company does not have the government's abilities, so that the two players' safety efforts to some extent are strategic complements.

When taxation is endogenized as a free government choice variable, the tax rate decreases in the disaster magnitude, the unit production cost, how the company is negatively affected by the disaster, and the government's unit cost of safety effort, increases in the company's resource and how the government is negatively affected by the disaster, and is weakly U shaped in the company's unit safety effort.

Future research can also model the general public as a third player, and various special interest groups as further players. Such players' safety efforts may consist in making everyone aware of the dangers of various companies' operations, in the form of writing, public demonstrations, etc. In this paper we have assumed that the preferences of such further players are largely aligned with the government's preferences. This holds true in democracies where governments are elected by the public to represent their interests. Furthermore, for example regarding oil production, both prefer safe production and low oil price. However, the preferences of various environmental organizations, various professional groups (fishermen, farmers, etc.), sometimes differ from the government's preferences, which can be modeled in future research. The disaster can also be modeled as an adaptive adversary which means analyzing the strategic interaction between a company and the government when facing a strategic threat.

Finally, future research can generalize the production function to production functions with multiple inputs (e.g. oil and gas) (e.g., Cobb–Douglas or CES production functions, Leontief production function). Such generalization could allow more realism (depending on the modeled context) and thus improve results.

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Appendices A, B, and C. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.ejor.2012.09.047.

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