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DEFENDING AGAINST A STOCKPILING TERRORIST

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A government defends against a terrorist who attacks repeatedly and stockpiles its resources over time. The government defends an asset and attacks the terrorist's resources. The terrorist defends its resources and attacks the government. We find four possible equilibrium solutions: (1) the government attacks only, deterring the terrorist; (2) both players defend and attack; (3) the government defends but does not attack, and the terrorist attacks only; and (4) the terrorist attacks a passive government. Understanding which factors impact the four cases is important in order to combat terrorism. The terrorist allocates its resources over T *periods according to a geometric series with a stockpiling parameter. This article analyzes how the government and terrorist prefer low versus high stockpiling parameters and how these preferences interact with the other parameters such as the terrorist's resources and the players' asset valuations, unit defense and attack costs, and discount factors. If the terrorist's resources are small, it can be deterred in each period. If the terrorist's resources are extremely large, it allocates its resources equally across the* T *periods, whereas the government prefers a single attack. If the terrorist's resources are intermediate, the terrorist would be deterred in each period if it allocated its resources equally across the* T *periods. It thus strikes a balance where it allocates much resources to early or late periods, to facilitate attacks, and accept being deterred in the other periods. As the future becomes less important, the terrorist attacks more in early periods.*

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INTRODUCTION

Understanding and defeating terrorist threats over time is important but challenging. This article assumes that a terrorist has resources that it allocates over T time periods. It may choose a single attack, attack substantially in early periods, or stockpile for later attacks. In each time period the government defends an asset and attacks the terrorist's resources to degrade its attack capability. The terrorist defends its resources and attacks the government's asset.

The objective of the article is to understand a multiplicity of phenomena, such as how the government allocates between defending its asset and attacking the terrorist's resources, how the terrorist allocates across defense and attack and through time, how the government can deter the terrorist from attacking, how the terrorist can pacify the government, and when an interior-solution equilibrium exists where both players defend and attack. Assuming geometric resource stockpiling for the terrorist, the results depend on the terrorist's resources and the players' asset valuations, unit defense and attack costs, and discount factors.

The government is assumed to possess an asset that it defends. We assume that the terrorist takes the government's defense of this asset as given when choosing its attack strategy in each time period. Hence, in each period, we analyze a two-stage game where the government moves in the first stage and the terrorist moves in the second stage.

In earlier research, Azaiez and Bier (2007) considered the optimal resource allocation for security in reliability systems. Bier et al. (2005) and Bier and Abhichandani (2002) assumed that the defender minimizes the success probability and expected damage of an attack. Bier et al. (2005) analyzed the protection of series and parallel systems with components of different values. Levitin (2007) considered the optimal element separation and protection in complex multistate series-parallel systems and suggested an algorithm for determining the expected damage caused by a strategic attacker. Patterson and Apostolakis (2007) introduced importance measures for ranking the system elements in complex systems exposed to terrorist actions. Michaud and Apostolakis (2006) analyzed such measures of damage caused by the terror as impact on people, impact on environment, impact on public image, etc.

Bier et al. (2007) assumed that a defender allocates defense to a collection of locations, whereas an attacker chooses a location to attack. They showed that the defender allocates resources in a centralized, rather than decentralized, manner and that the optimal allocation of resources can be nonmonotonic in the value of the attacker's outside option. Furthermore, the defender prefers its defense to be public rather than secret. Also, the defender sometimes leaves a location undefended and sometimes prefers a higher vulnerability at a particular location even if a lower risk could be achieved at zero cost. Dighe et al. (2009) considered secrecy in defensive allocations as a strategy for achieving more cost-effective attacker deterrence. Zhuang and Bier (2011) modeled secrecy and deception as a signaling and resource allocation game between a government and a terrorist.

Hausken et al. (2009) considered a defender that chooses tradeoffs between investments in protection against natural disaster only, protection against terrorism only, and all-hazards protection, allowing sequential or simultaneous moves. Similarly, Zhuang and Bier (2007) studied the balance between natural disaster and terrorism, where either the defender moves first (and the attacker second) or they move simultaneously. Pinker (2007) studied the tradeoff between physical deployments of security personnel and private/public warnings in the face of a strategic attack with uncertainty in the timing and location of attacks. Levitin and Hausken (2008) considered a two-period model where the defender, moving first, distributes its resources between deploying redundant elements and protecting them from attacks. Recently, Bakshi and Gans (2010) studied a game between government, trading firms, and terrorists in a supply chain, where the government provides incentives to firms to improve security upstream in the supply chain.

For a survey of work that examines the strategic dynamics of governments vs. terrorists, see Sandler and Siqueira (2009). They surveyed advances in game-theoretic analyses of terrorism, such as proactive versus defensive countermeasures, the impact of domestic politics, the interaction between political and militant factions within terrorist groups, and fixed budgets. Further, Brown et al. (2006) considered defender–attacker–defender models. First, the defender invests in protecting the infrastructure, subject to a budget constraint. Then, a resource-constrained attack is carried out. Finally, the defender operates the residual system as best possible. Brown et al. (2006) used border control, the U.S. strategic petroleum reserve, and electric power grids as examples. Trajtenberg (2006) studied a model with a nonstrategic terrorist, targets in a given country that choose defensive measures, and a government who chooses the proactive effort level.

Some research has focused on investment substitutions across time. First, Enders and Sandler (2004) suggested that a terrorist may compile and accumulate resources during times when the government's investments are high, awaiting times when the government may relax its efforts and choose lower investments. Second, Keohane and Zeckhauser (2003) showed that "the optimal control of terror stocks will rely on both ongoing abatement and periodic cleanup" (p. 201) of "a terrorist's 'stock of terror capacity'" (p. 224). Enders and Sandler (2005) used time series to show that little has changed in overall terrorism incidents before and after 9/11. Using 9/11 as a break date, they found that logistically complex hostage-taking events have fallen as a proportion of all events, whereas logistically simple, but deadly, bombings have increased as a proportion of deadly incidents. Enders and Sandler (1993) applied data from 1968 to 1988 and found both substitutes and complements among the attack modes. Evaluating the effectiveness of six policies designed to thwart terrorism, they found that policies designed to reduce one type of attack may affect other attack modes.

Sandler and Siqueira (2006) modeled the differences between proactive and defensive policies with pseudo contest functions. They found that preemption is usually undersupplied. A country's deterrence decision involves both external benefits and costs as the terrorist threat is deflected, whereas its preemption decision typically provides external benefits when the threat is reduced for all potential targets. With damages limited to home interests, they found that a country will overdeter, whereas for globalized terror, a country will underdeter. Bandyopadhyay and Sandler (2011) considered, in a two-stage game, the interaction between preemption and defense. In the first stage, two countries decide their levels of preemption against a common threat. Preemption decreases damages at a diminishing rate. Preemption, as a public good, is subject to a free-rider problem. In the second stage, the countries decide their levels of defense against the threat adjusted by the first-stage preemption. An increase in one country's defense increases the probability of an attack against the other country. They found that high-cost defenders may rely on preemption, and too little preemption may give rise to subsequent excessive defense.

Cárceles-Poveda and Tauman (2011) studied a two-stage game. In the first stage, an endogenously determined subset of countries choose their proactive effort levels, which downgrade through a functional form the resources available to the terrorist in the second stage. In the second stage, the terrorist allocates its remaining resources to attack the countries while, at the same time, the countries choose their defensive measures.

There are significant differences between Bandyopadhyay and Sandler's (2011) paper and Cárceles-Poveda and Tauman's (2011) paper and the present article. First, we assume that both the government and the terrorist are fully strategic when allocating their resources between defense and attack. The terrorist's resources are downgraded by two fully strategic players where the government attacks and the terrorist defends its resources. In contrast, Bandyopadhyay and Sandler (2011) assumed a nonstrategic threat and Cárceles-Poveda and Tauman (2011) assumed that the resources available to the terrorist in the second stage are downgraded nonstrategically through a functional exponential form. The resources available to the terrorist in the second stage are applied in their entirety. The terrorist's strategic decision is how to allocate its downgraded resources across the countries. Second, we assume that the damage probability for the government's asset depends on the strategic decision by the government of how well to defend its asset and the strategic decision by the terrorist of how well to attack the asset using its downgraded resources, accounting for a contest intensity. In contrast, Bandyopadhyay and Sandler (2011) assumed that the terrorist's second-stage attack depends nonstrategically and functionally on the countries' first-stage preemption, and Cárceles-Poveda and Tauman (2011) assumed that the damage inflicted on country *i* is determined by a functional form that is proportional to the resources allocated by the terrorist to country *i*, proportional to the political and/or economic power of country *i*, and inverse proportional to the defense of country *i* in the second stage. Third, we consider one unitary government, which means abstracting away the collective action problem of multiple governments. In contrast, Bandyopadhyay and Sandler (2011) and Cárceles-Poveda and Tauman (2011) accounted for the collective action problem with two and multiple players, respectively. Fourth, both Bandyopadhyay and Sandler (2011) and this article determine solutions in which the government does not defend. Zhuang et al. (2010) modeled secrecy and deception as a multiperiod signaling and resource allocation and considered both long-run and short-run defenses.

Raczynski (2004) simulated the dynamic interactions between terror and antiterror groups. Feichtinger and Novak (2008) used differential game theory to study the intertemporal strategic interactions of Western governments and terror organizations. They also illustrated long-run persistent oscillations. Berman and Gavious (2007) studied a leader–follower game in which the state provides counterterrorism support across multiple metropolitan areas to minimize losses, and the terrorist attacks one of the metropolitan areas to maximize his utility. Berrebi and Lakdawalla (2007) considered for 1949–2004 how terrorists sought targets in Israel, responding to costs and benefits, and found that long periods without an attack signaled lower risk for most localities but higher risk for important areas. Barros et al. (2006) applied parametric and semiparametric hazard model specifications to study durations between Euskadi Ta Askatasuna's (ETA, a Spain-based terrorist group) terrorist attacks, which seem to increase in the summer and decrease with respect to, for example, deterrence and political variables. Udwadia et al. (2006) considered the dynamic behavior of terrorists, those susceptible to terrorist and pacifist propaganda, military/police intervention to reduce the terrorist population, and nonviolent, persuasive intervention to influence those susceptible to becoming pacifists. Hausken (2008) considered a terrorist that defends an asset that grows from the first to the second period. The terrorist seeks to eliminate the asset optimally across the two periods. Telesca and Lovallo (2006) found that a terror event is not independent from the time elapsed since the previous event, except for severe attacks, which approach a Poisson process. This latter finding suggests that attack and defense decisions are not unit-periodic in nature but that there are linkages through time. One objective of the current article is to understand more thoroughly the nature of such linkages through time, affected by changes in resources, unit costs of defense and attack, etc.

Our article builds upon and extends earlier research mentioned above. On the one hand, we enrich the one-period model by allowing both the government and terrorist to both defend and attack. The government defends itself and at the same time attacks the terrorist's resources. Analogously, the terrorist defends its resources and at the same time uses its surviving resources to attack the government. On the other hand, we repeat the one-period model T times to understand how long the terrorist can be deterred. A resourceful terrorist is obviously harder to deter than a less resourceful terrorist, but it is not obvious how a government should allocate its resources to defending itself and attacking the terrorist and, analogously, how the terrorist should allocate its resources to defend itself and attack the government. This article seeks to determine the key factors that impact such resource allocation problems, which are important for governments, policy makers, and even terrorists to understand.

In an earlier paper, Hausken and Zhuang (2011b) assumed fixed government resources so that the government has only one free decision variable. Similarly, Hausken and Zhuang (2011a) assumed that the government only defends and does not have the option to attack the terrorist's resources. Furthermore, Hausken and Zhuang (2011a) only studied a two-period game. By contrast, this article allows both attacks and defenses for both government and the terrorist in a T-period game, and the defender's two decision variables are free because there is no fixed resource. Such extensions provide more general insights and modeling framework to the attacker–defender game literature.

The following section presents the model. The next section solves the two-stage game, illustrated in the subsequent section. Then we analyze and illustrate the T-period game, and the final secction concludes the article.

THE MODEL

Notation

- A_t Terrorist's attack effort attacking the asset in period t
- a_t Government's attack effort attacking the terrorist's resources R_t in period t
- B_t Terrorist's unit attack cost in period t
- b_t Government's unit defense cost in period t

D_t Terrorist's defense effort protecting its resources in period t
d_t Government's defense effort protecting the asset in period t
P_t Probability of asset destruction in period t
R Terrorist's total resources in the T-period game
R_t Terrorist's resources in period t
r_t Government's resources in period t
T Number of time periods
t Time period, $t = 1, \ldots, T$
U Terrorist's accumulative discounted utility
U_t Terrorist's expected utility in period t
u Government's accumulative discounted utility
u_t Government's expected utility in period t
V_t Terrorist's asset valuation
v_t Government's asset valuation
W Terrorist's stockpiling parameter
Δ Terrorist's discount factor
δ Government's discount factor

Assumptions

We assume that the terrorist has resources R_t that are transformed at unit cost $B_t \geq 0$ into attack A_t of an asset[1] controlled by the government and transformed at unit cost 1 into defense D_t against the government's attack. The unit cost 1 is a benchmark against which B_t is compared. The government attacks the terrorist's resources with a_t at unit cost 1. We model the probability of survival of the terrorist's resources with the common ratio form (Skaperdas 1996; Tullock 1980) contest success function; that is,

$$Q_t(a_t, D_t) = \begin{cases} 1 & \text{if } a_t = D_t = 0 \\ \dfrac{D_t}{D_t + a_t} & \text{otherwise} \end{cases} \tag{1}$$

where $\partial Q_t / \partial D_t \geq 0$ and $\partial Q_t / \partial a_t \leq 0$. When $a_t = 0$, the government does not attack, so there is no need for the terrorist to defend, which gives $D_t = 0$. We define this event such that the terrorist invests an arbitrarily small but positive amount D_t that allows it to retain its entire resources so

[1]For analytical tractability one asset is considered. The model also applies for collections of assets interpreted as a joint asset. One example of a collection of assets is the four targets of the 9/11 attack; that is, the World Trade Center's North and South Towers, the Pentagon, and the White House (which was not hit). Focusing on one asset means that we do not analyze how the government and terrorist substitute resources across assets. See Enders and Sandler (2004), Hausken (2006), Bier et al. (2007, 2008), and Hao et al. (2009) for when a government allocates defense to a collection of locations and a terrorist chooses a location to attack.

that $Q_t = 1$. The terrorist's original resources in each period is R_t, but it decreases to $Q_t R_t$ due to the government's attack. The terrorist's resource allocation equation can thus be expressed as:

$$Q_t(a_t, D_t)R_t = B_t A_t + D_t \tag{2}$$

Equation (2) means that the terrorist strikes a balance between attacking the asset and defending its resources. The government defends its asset with $d_t \geq 0$ at unit cost $b_t \geq 0$. The government unit attack cost 1 is a benchmark against which b_t is compared. For the probability of asset destruction in period t, we consider the following form of the contest success function:

$$P_t(d_t, A_t) = \begin{cases} 0 & \text{if } A_t = d_t = 0 \\ \dfrac{A_t}{A_t + d_t} & \text{otherwise} \end{cases} \tag{3}$$

where $\partial P_t/\partial d_t \leq 0$ and $\partial P_t/\partial A_t \geq 0$. When $A_t = 0$, the terrorist does not attack, so there is no need for the government to defend, which gives $d_t = 0$. We define this event such that the government invests an arbitrarily small but positive amount d_t that allows it to retain its entire asset so that the probability of asset destruction equals zero.

The probability that the asset is not destroyed is $1 - P_t(d_t, A_t)$, which the government maximizes, accounting for the asset valuation v_t, asset defense expenditures $b_t d_t$, and resource attack expenditures a_t. Conversely, the terrorist maximizes the probability $P_t(d_t, A_t)$ of asset destruction, striking a balance between defending its resources R_t and attacking the asset valued at V_t. The government's and terrorist's expected utilities in period t are

$$\begin{aligned} u_t(d_t, a_t, A_t) &= [1 - P_t(d_t, A_t)]v_t - b_t d_t - a_t \\ U_t(d_t, a_t, A_t) &= P_t(d_t, A_t)V_t \end{aligned} \tag{4}$$

This means that the government's resources are a variable determined by $r_{\mathrm{t}} = b_t d_t + a_t$, which is realistic because the government has the capacity needed to compile the resources needed for optimizing behavior; for example, transfer funds from other government branches. The terrorist's resources compilation mechanism is different. Its focus is narrower, it cannot easily transfer funds from other branches, and it cannot easily take up loans. Its resources are determined by incoming funds, which can be one-shot or increase or decrease over time. We thus assume that the terrorist's resources are fixed to R_t in period t, which allows us to analyze how the terrorist stockpiles resources over time. Inserting (2) and (3) into (4) gives:

$$u_t(d_t, a_t, D_t, A_t) = \begin{cases} v_t - b_t d_t - a_t & \text{if } A_t = 0 \\ \dfrac{d_t}{R_t/B_t + d_t} v_t - b_t d_t & \text{if } A_t > 0, a_t = 0 \\ \dfrac{d_t}{\left(\left[\frac{D_t}{D_t + a_t} R_t - D_t\right] \Big/ B_t\right) + d_t} v_t - b_t d_t - a_t & \text{otherwise} \end{cases}$$

$$U_t(d_t, a_t, D_t, A_t) = \begin{cases} 0 & \text{if } A_t = 0 \\ \dfrac{R_t/B_t}{R_t/B_t + d_t} V_t & \text{if } A_t > 0, a_t = 0 \\ \dfrac{\left(\left[\frac{D_t}{D_t + a_t} R_t - D_t\right] \Big/ B_t\right)}{\left(\left[\frac{D_t}{D_t + a_t} R_t - D_t\right] \Big/ B_t\right) + d_t} V_t & \text{otherwise} \end{cases} \tag{5}$$

The government's two free-choice variables are d_t and a_t. The terrorist's one free-choice variable is D_t, where A_t follows from (2). We assume common knowledge so that both players know all parameters and the game structure.

Problem Formulation

To determine the subgame perfect Nash equilibrium, we assume that the government chooses d_t and a_t simultaneously in the first stage. The terrorist observes d_t and a_t and chooses A_t in the second stage. The game is solved with backward induction.

Definition 1. *A strategy pair* (a_t, d_t, D_t) *is a subgame perfect Nash equilibrium if and only if*

$$D_t = D_t(a_t, d_t) = \arg\max_{D_t \geq 0} U_t(a_t, d_t, D_t) \tag{6}$$

and

$$(a_t, d_t) = \arg\max_{a_t \geq 0, d_t \geq 0} u_t(a_t, d_t, D_t(a_t, d_t)) \tag{7}$$

SOLVING THE TWO-STAGE GAME

We solve the game with backward induction, starting with the second stage. The terrorist's first-order condition in the second stage implies:

$$\frac{\partial U_t(d_t, a_t, D_t, A_t)}{\partial D_t} = 0 \Rightarrow D_t = \begin{cases} \sqrt{R_t a_t} - a_t & \text{iff } a_t < R_t \\ 0 & \text{iff } a_t \geq R_t \end{cases} \tag{8}$$

Although we allow the terrorist to choose $D_t \in [0, R_t]$, (8) implies that when the government attacks too severely in the first stage, $a_t \geq R_t$, the terrorist chooses $D_t = 0$, which implies $A_t = 0$ according to (2).

Inserting (8) into (5) to yield the government's first stage utility:

$$u_t(d_t, a_t) = \begin{cases} \dfrac{d_t v_t}{\left([\sqrt{R_t} - \sqrt{a_t}]^2/B_t\right) + d_t} - b_t d_t - a_t & \text{iff } a_t < R_t \\ v_t - b_t d_t - a_t & \text{iff } a_t \geq R_t \end{cases} \tag{9}$$

There are five possible optimal solutions, referred to as cases 1–5:

Case 1: $a_t \geq R_t$. Then the terrorist is fully deterred with optimal variables $a_t = R_t$, $d_t = 0$, and government utility $u_t(d_t, a_t) = v_t - R_t \equiv u_{t1}$, which is nonnegative when $v_t \geq R_t$.

Case 2: $0 < a_t < R_t$, $d_t > 0$. Then we can solve the two first-order conditions simultaneously:

$$\begin{aligned} \frac{\partial u_t(d_t, a_t)}{\partial d_t} &= \frac{([\sqrt{R_t} - \sqrt{a_t}]^2/B_t)v_t}{([\sqrt{R_t} - \sqrt{a_t}]^2/B_t + d_t)^2} - b_t = 0 \\ \frac{\partial u_t(d_t, a_t)}{\partial a_t} &= \frac{d_t([\sqrt{R_t} - \sqrt{a_t}]/B_t)v_t}{\sqrt{a_t}([\sqrt{R_t} - \sqrt{a_t}]^2/B_t + d_t)^2} - 1 = 0 \end{aligned} \tag{10}$$

and obtain

$$\begin{aligned} \sqrt{a_t} &= \frac{(v_t - b_t\sqrt{R_t})}{B_t - b_t} => a_t = \frac{(v_t - b_t\sqrt{R_t})^2}{(B_t - b_t)^2}, \\ d_t &= \frac{(v_t - b_t\sqrt{R_t})(\sqrt{R_t}B_t - v_t)}{b_t(B_t - b_t)^2} \end{aligned} \tag{11}$$

This is possible if and only if:

$$0 < a_t = \frac{(v_t - b_t\sqrt{R_t})^2}{(B_t - b_t)^2} < R_t, d_t = \frac{(v_t - b_t\sqrt{R_t})(\sqrt{R_t}B_t - v_t)}{b_t(B_t - b_t)^2} > 0, \tag{12}$$

which implies:

$$0 < \frac{(v_t - b_t\sqrt{R_t})^2}{(B_t - b_t)^2} < R_t, \left(\frac{v_t}{B_t}\right)^2 < R_t < \left(\frac{v_t}{b_t}\right)^2 \tag{13}$$

and the government's utility is

$$u_t(d_t, a_t) = \frac{\left(\frac{v_t}{b_t} - \sqrt{R_t}\right)(B_t - \sqrt{R_t})}{b_t(B_t - b_t)} \equiv u_{t2} \tag{14}$$

Case 3: $a_t = 0$, $d_t > 0$. Then we solve the first-order condition:

$$\frac{\partial u_t(d_t, a_t)}{\partial d_t} = \frac{([\sqrt{R_t} - \sqrt{a_t}]^2/B_t)v_t}{([\sqrt{R_t} - \sqrt{a_t}]^2/B_t + d_t)^2} - b_t = 0, \tag{15}$$

which gives

$$a_t = 0, d_t = \sqrt{\frac{R_t v_t}{B_t b_t}} - \frac{R_t}{B_t}, \tag{16}$$

which is possible if and only if:

$$d_t = \sqrt{\frac{R_t v_t}{B_t b_t}} - \frac{R_t}{B_t} > 0 \Leftrightarrow R_t < \frac{B_t v_t}{b_t}, \tag{17}$$

which gives the utility:

$$\begin{aligned} u_t(d_t, a_t) &= \frac{\left(\sqrt{\frac{R_t v_t}{B_t b_t}} - \frac{R_t}{B_t}\right) v_t}{\frac{R_t}{B_t} + \sqrt{\frac{R_t v_t}{B_t b_t}} - \frac{R_t}{B_t}} - b_t \left(\sqrt{\frac{R_t v_t}{B_t b_t}} - \frac{R_t}{B_t}\right) \\ &= \left(\sqrt{v_t} - \sqrt{\frac{b_t R_t}{B_t}}\right)^2 \equiv u_{t3} \end{aligned} \tag{18}$$

where (17) ensures that the expression being squared is nonnegative.

Case 4: $a_t = 0$, $d_t = 0$, which gives $u_t(d_t, a_t) = 0 \equiv u_{t4}$.

Case 5: $0 < a_t < R_t$, $d_t = 0$. Then we solve the first-order condition:

$$\frac{\partial u_t(d_t, a_t)}{\partial a_t} = \frac{d_t([\sqrt{R_t} - \sqrt{a_t}]/B_t)v_t}{\sqrt{a_t}([\sqrt{R_t} - \sqrt{a_t}]^2/B_t + d_t)^2} - 1 = -1 = 0, \tag{19}$$

which is impossible.

In summary, and after some calculations, the four possible cases are summarized in Table 1.

Comparing u_{t1} and u_{t3} when these are nonnegative gives:

$$u_{t1} \geq u_{t3} \Leftrightarrow v_t - R_t \geq \left(\sqrt{v_t} - \sqrt{\frac{b_t R_t}{B_t}}\right)^2 \Leftrightarrow R_t \leq v_t \frac{4b_t/B_t}{(1 + b_t/B_t)^2} \tag{20}$$

Property 1. When $B_t/b_t \leq 1$, the government prefers to attack the terrorist's resources ($a_t = R_t$ and $d_t = 0$), rather than passively defending its asset ($a_t = 0$ and $d_t > 0$).

Proof. The rightmost inequality in (20) must be less than v_t, because $v_t \frac{4b_t/B_t}{(1+b_t/B_t)^2} \leq v_t \Leftrightarrow 0 \leq (1 - b_t/B_t)^2$, which is always satisfied. Hence, u_{t1} always weakly dominates u_{t3} when $B_t/b_t \leq 1$, which follows from $v_t \geq R_t$ and (17).

Therefore, when $B_t/b_t \leq 1$, we only need to compare u_{t1}, u_{t2}, and u_{t4} (Appendix A). When $B_t/b_t > 1$ we get three cases. First, when $v_t < R_t b_t/B_t$, neither cases 1 nor 3 apply, and we only need to compare u_{t2} and u_{t4} (Appendix B). Second, when $R_t b_t/B_t \leq v_t < R_t$, case 3 applies, and case 1 does not apply, so we only need to compare u_{t2}, u_{t3}, and u_{t4} (Appendix C). Third, when $v_t \geq R_t$, both cases 1 and 3 apply. Equation (20) can be rewritten as $u_{t1} \geq u_{t3} \Leftrightarrow R_t \frac{(1+b_t/B_t)^2}{4b_t/B_t} \leq v_t$. Hence, when $v_t \geq R_t \frac{(1+b_t/B_t)^2}{4b_t/B_t} \geq R_t$, we only need to compare u_{t1}, u_{t2}, and u_{t4} (Appendix A). When $v_t \leq R_t \frac{(1+b_t/B_t)^2}{4b_t/B_t}$, we only need to compare u_{t2}, u_{t3}, and u_{t4} (Appendix C). ■

ILLUSTRATING THE TWO-STAGE GAME

We first consider the special cases $B_t = b_t$, which excludes case 2 according to (13). The government benefits from a terrorist with little resources in case 1 and suffers from a resourceful terrorist in case 4. The government's utility is continuous through $v_t = R_t$, whereas the terrorist's utility increases discontinuously when R_t increases above v_t and the government abruptly ceases to deter. Figure 1 plots a_t, d_t, A_t, D_t, u_t, and U_t as functions of R_t when $B_t = b_t = V_t = v_t = 1$. When the government is superior, expressed with $v_t \geq R_t$, the government deters the terrorist (case 1), where a_t increases and u_t decreases in R_t. Case 3 is excluded because the government prefers to deter the terrorist rather than jointly protect and attack. When the terrorist is

Table 1. Four Possible Cases of Subgame Perfect Nash Equilibrium

Cases	Case 1 (u_{t1})	Case 2 (u_{t2})	Case 3 (u_{t3})	Case 4 (u_{t4})
Government scenarios	Government attacks only, deterring the terrorist	Government defends and attacks	Government defends but does not attack	Government neither defends nor attacks
Terrorist scenarios	Terrorist is deterred	Terrorist defends and attacks	Terrorist attacks only	Terrorist attacks only
a_t	R_t	$\frac{(v_t-b_t\sqrt{R_t})^2}{(B_t-b_t)^2}$	0	0
d_t	0	$\frac{(v_t-b_t\sqrt{R_t})(\sqrt{R_t}B_t-v_t)}{b_t(B_t-b_t)^2}$	$\sqrt{\frac{R_t v_t}{B_t b_t}} - \frac{R_t}{B_t}$	0
A_t	0	$(\frac{\sqrt{R_t}(v_t-b_t\sqrt{R_t})}{B_t(B_t-b_t)} - \frac{(v_t-b_t\sqrt{R_t})^2}{B_t(B_t-b_t)^2})(\frac{B_t\sqrt{R}-v_t}{v_t-b_t\sqrt{R_t}})$	R_t/B_t	R_t/B_t
D_t	0	$\frac{\sqrt{R_t}(v_t-b_t\sqrt{R_t})}{(B_t-b_t)} - \frac{(v_t-b_t\sqrt{R_t})^2}{(B_t-b_t)^2}$	0	0
u_t	$v_t - R_t$	$\frac{(\frac{v_t}{b_t}-\sqrt{R_t})(B_t-\sqrt{R_t})}{b_t(B_t-b_t)}$	$(\sqrt{v_t} - \sqrt{\frac{b_t R_t}{B_t}})^2$	0
U_t	0	$\frac{V_t(\frac{\sqrt{R_t}(v_t-b_t\sqrt{R_t})}{(B_t-b_t)} - \frac{(v_t-b_t\sqrt{R_t})^2}{(B_t-b_t)^2})(\frac{(B_t-b_t)\sqrt{R_t}}{(v_t-b_t\sqrt{R_t})} - 1)}{[\frac{(B_t-b_t)\sqrt{R_t}}{(v_t-b_t\sqrt{R_t})} - 1](\frac{\sqrt{R_t}(v_t-b_t\sqrt{R_t})}{(B_t-b_t)} - \frac{(v_t-b_t\sqrt{R_t})^2}{(B_t-b_t)^2}) + \frac{B_t(v_t-b_t\sqrt{R_t})(\sqrt{R_t}B_t-v_t)}{b_t(B_t-b_t)^2}}$	$V_t\sqrt{\frac{R_t b_t}{v_t B_t}}$	V_t

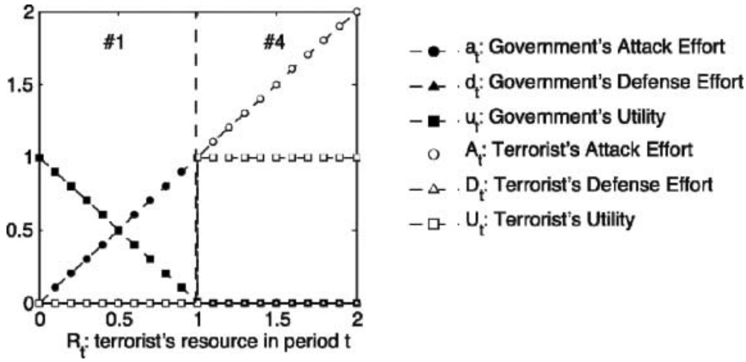

Figure 1. a_t**,** d_t**,** A_t**,** D_t**,** u_t**, and** U_t **as functions of** R_t **when** $B_t = b_t = V_t = v_t = 1$**.**

superior, expressed as $v_t \leq R_t$, the government neither defends nor attacks (case 4).

The direct transition from cases 1 to 4, in the sense of no intermediate steps through cases 2 and/or 3, also occurs when $B_t < b_t$. A strong terrorist (with low unit attack cost) tends to induce an all-or-nothing governmental response. Either the government deters the terrorist or it gives up. The transition between cases 1 and 4 is smooth in the sense that the government's utility decreases gradually to zero as the terrorist's resources increase to the transition point between cases 1 and 4.

A weakened terrorist, with a higher unit attack cost than $B_t = 1$, allows cases 2 and 3 to occur, which are not all-or-nothing governmental responses. Figure 2 plots a_t, d_t, A_t, D_t, u_t, and U_t as functions of R_t when $b_t = 1$ and $B_t = 2$. All panels give case 1, which deters the terrorist when R_t is low, and case 4, where the government gives up, when R_t is large. Because the terrorist is disadvantaged with the large $B_t = 2$, there is no direct transition from cases 1 to 4. Instead, there is transition through case 3 only when $V_t = v_t$ is very small or very large (Figures 2a, 2b, 2f–2i), through cases 2 and 3 when $V_t = v_t$ is small or large (Figures 2c and 2e), and through case 2 only when $V_t = v_t$ is intermediate ($V_t = v_t = 2$). The extreme value $V_t = v_t$ allows case 3 where the government defends only and the terrorist attacks only. The intermediate value $V_t = v_t = 2$ is needed to allow case 2 where both players prefer, and accept, that both players both defend and attack.

SENSITIVITY ANALYSIS ON $\mathbf{B_t}$ *AND* $\mathbf{b_t}$

This section studies the sensitivity when the terrorist's unit attack cost B_t and/or the government's unit defense cost b_t vary. Such changes may

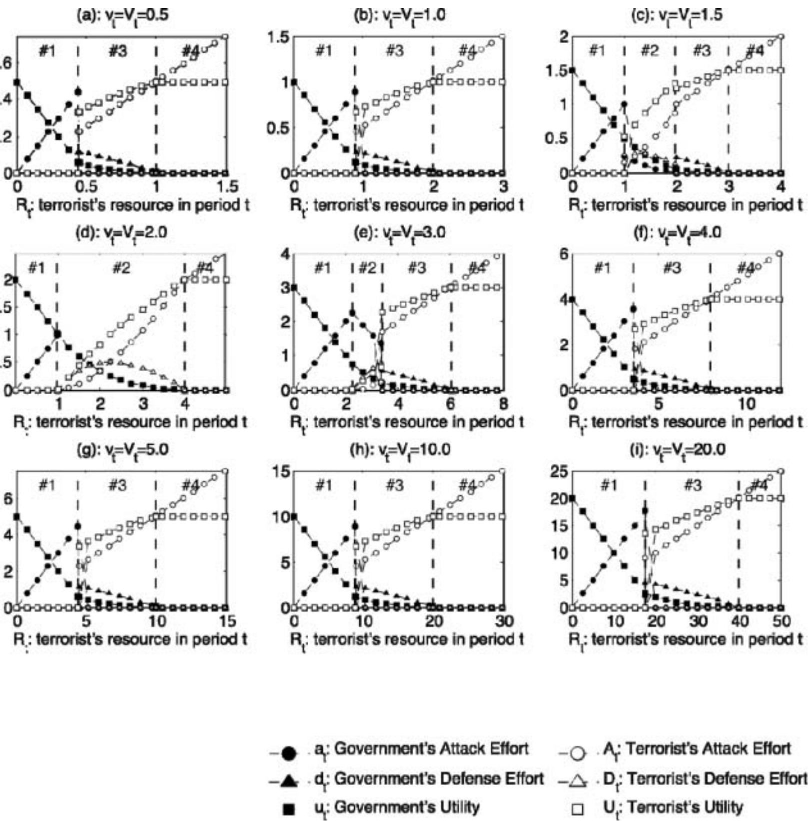

Figure 2. a_t, d_t, A_t, D_t, u_t, **and** U_t **as functions of** R_t **when** $b_t = 1$ **and** $B_t = 2$.

occur for a variety of reasons. One reason is that the terrorist and the government learn as time progresses. Learning may cause B_t and b_t to decrease over time. They become better suited to choose the right tasks to execute, carry out these tasks more efficiently, and utilize their equipment more proficiently. B_t and b_t may also increase over time; for example, due to fatigue or equipment being subject to wear and tear, requiring more frequent maintenance.

Sensitivity of the Terrorist's Unit Attack Cost B_t

Figure 3 plots a_t, d_t, A_t, D_t, u_t, and U_t as functions of B_t when $b_t = 1$ for various R_t and $V_t = v_t$.

Figure 3a ($R_t = 0.5$, $V_t = v_t = 1$) shows case 1 to the government's advantage when $B_t < 5.83$ and case 3 when $B_t > 5.83$. A large attacker unit

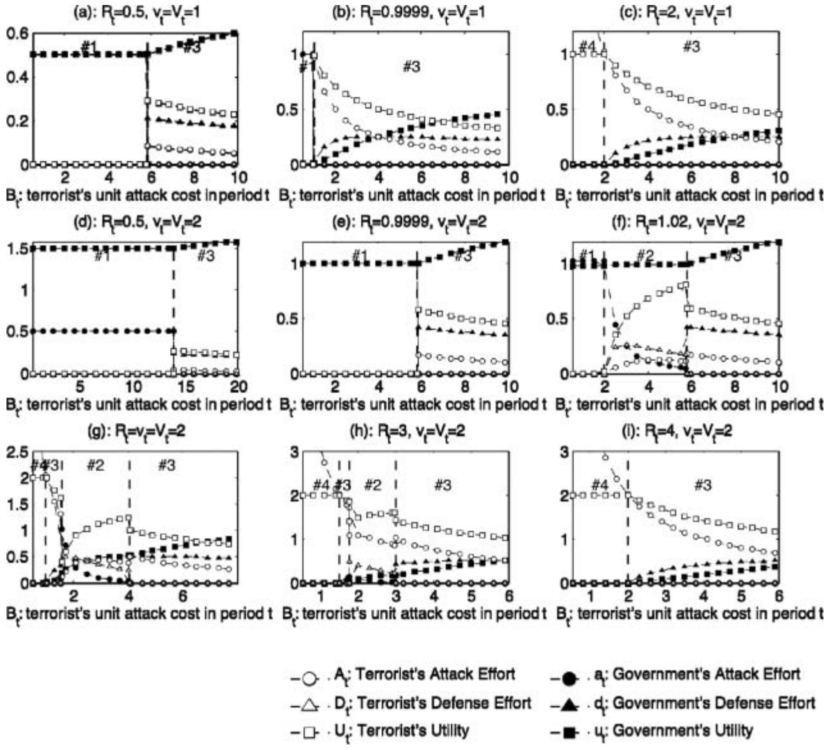

Figure 3. a_t, d_t, A_t, D_t, u_t, **and** U_t **as functions of** B_t **when** $b_t = 1$ **for various** R_t **and** $V_t = v_t$.

attack cost when the terrorist has little resources causes the government to allow the terrorist to attack because protection can easily and cost-efficiently be furnished against the modest attack. Increasing R_t causes the range for case 1 to shrink. Figure 3b ($R_t = 0.9999$, $V_t = v_t = 1$) shows case 1 when $B_t < 1.02$. Increasing R_t to $R_t = 1$ causes case 1 to be replaced with case 4 in accordance with Figure 1 when $B_t = 1$. Increasing R_t above 1 causes case 4 to expand to the terrorist's advantage. Figure 3c ($R_t = 2$, $V_t = v_t = 1$) shows case 4 when $B_t < 2$ and case 3 when $B_t > 2$.

The absence of case 2 in Figures 3a–3c is in accordance with Figure 2b where $V_t = v_t = 1$. To introduce case 2, Figures 3d–3f assume $V_t = v_t = 2$ as in Figure 2d. Figure 3d ($R_t = 0.5$, $V_t = v_t = 2$) shows case 1 to the government's advantage when $B_t < 13.93$ and case 3 when $B_t > 13.93$. Increasing R_t again causes the range for case 1 to shrink. Figure 3e ($R_t = 0.9999$, $V_t = v_t = 2$) shows case 1 when $B_t < 5.83$.

Increasing R_t above 1 causes the emergence of case 2. Figure 3f ($R_t = 1.02$, $V_t = v_t = 2$) shows case 2 when $1.99 < B_t < 5.78$ where the terrorist enjoys a substantial utility increase compared with $R_t = 0.9999$ because the government no longer deters the terrorist. The government nevertheless enjoys a utility close to 1.

Increasing R_t above 1.9 causes alternation of cases 2 and 3, and increasing R_t above 2 causes case 1 to be replaced with case 4 in accordance with Figure 1 when $B_t = 1$. Figure 3g ($R_t = V_t = v_t = 2$) shows case 4 when $B_t < 1$, case 3 when $1 < B_t < 1.54$ or $B_t > 4.05$, and case 2 when $1.54 < B_t < 4.05$.

Increasing R_t further above 2 causes case 4 to expand upwards and the upper case 3 to expand downwards. Figure 3h ($R_t = 3$, $V_t = v_t = 2$) shows case 4 when $B_t < 1.5$, case 3 when $1.5 < B_t < 1.77$ or $B_t > 2.99$, and case 2 when $1.77 < B_t < 2.99$.

Increasing R_t above 4 causes the disappearance of case 2. (We have case 2 in a small range when $R_t = 3.99$, which is not shown.) Figure 3i ($R_t = 4$, $V_t = v_t = 2$) shows case 4 when $B_t < 2$ and case 3 when $B_t > 2$. A low attacker unit attack cost when the terrorist has many resources causes the government to give up. As B_t increases, the government starts to defend itself. Decreased B_t—for example, as a consequence of terrorist learning—enables the terrorist to enjoy a switch from case 3 to case 4.

Sensitivity of the Government's Unit Defense Cost b_t

Figure 4 plots a_t, d_t, A_t, D_t, u_t, and U_t as functions of b_t when $B_t = 1$ for various R_t and $V_t = v_t$. Figure 4a ($R_t = 0.5$, $V_t = v_t = 1$) shows case 1 when

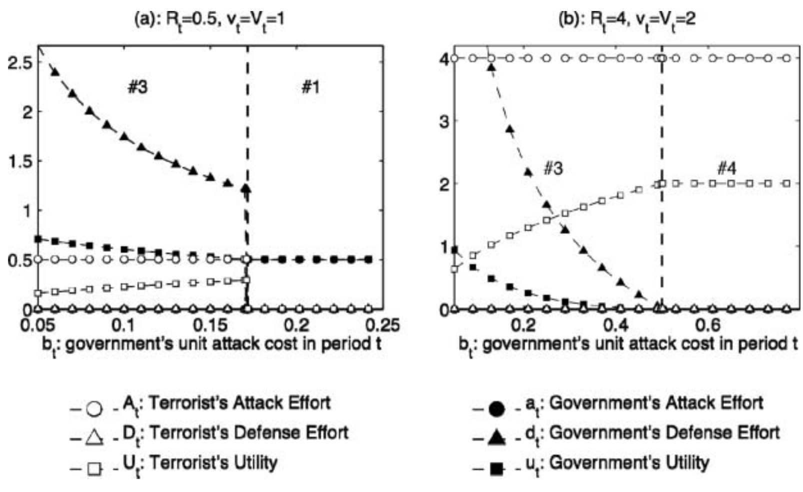

Figure 4. a_t, d_t, A_t, D_t, u_t**, and** U_t **as functions of** b_t **when** $B_t = 1$ **for various** R_t **and** $V_t = v_t$**.**

$b_t > 0.17$ and case 3 when $b_t < 0.17$. Observe the left–right reversal of cases 1 and 3 compared with Figure 3a. Regardless of how large b_t is, the government can deter the terrorist by attacking and, thus, enjoy case 1. As b_t decreases below 0.17, the government can exploit the low unit defense cost to defend, enjoying a larger utility than by pure deterrence and, thus, allow the terrorist to attack. Figure 4b ($R_t = 4$, $V_t = v_t = 2$) shows case 3 when $b_t < 0.5$ and case 4 when $b_t > 0.5$. Observe the left–right reversal of cases 3 and 4 compared with Figure 3i. A resourceful terrorist and a high governmental unit defense cost cause the government to neither defend nor attack. Decreased b_t, which may occur, for example, as a consequence of government learning, enables the government to enjoy a switch from case 4 to case 3.

Sensitivity of $B_t = b_t$

When $b_t = B_t$, the parameter values $\mathrm{R_t} = 0.5$ and $\mathrm{V_t} = \mathrm{v_t} = 1$ cause case 1 ($\mathrm{A_t} = \mathrm{D_t} = \mathrm{U_t} = \mathrm{d_t} = 0$, $\mathrm{a_t} = \mathrm{u_t} = 0.5$), and $\mathrm{R_t} = 4$ and $\mathrm{V_t} = \mathrm{v_t} = 2$ cause case 4 ($\mathrm{D_t} = \mathrm{a_t} = \mathrm{d_t} = \mathrm{u_t} = 0$, $\mathrm{A_t} = 4/\mathrm{B_t}$, $\mathrm{U_t} = 2$). The equal unit defense costs cause a situation of either case 1 where the terrorist is deterred or case 4 where the government gives up. Equal unit defense costs fluctuating, for example, downwards may arise; for example, when the government and terrorist learn at the same rate.

THE T *PERIOD GAME*

Theoretical Development

The two-stage game is played T times, referred to as *periods*, where T can be arbitrarily large. We assume that the time between periods is sufficiently longer than the time between stages so that each two-stage game can be solved with backward induction for each period. This means that the players are myopic and boundedly rational in the sense that they only consider one two-stage game in each period. The terrorist allocates resources R over T time periods, facing a complex decision-making procedure through time. Making R_t a free-choice variable to be optimized in each time period is quite a challenge both for the terrorist and the researcher. We thus simplify the decision-making procedure by assuming that the terrorist allocates R_1 to the first period and changes its allocation according to the geometric progression:

$$R_t = W R_{t-1}, \quad \text{for } 1 < \mathrm{t} \leq T, \tag{21}$$

such that

$$R = \sum_{t=1}^{T} R_t = R_1 \frac{W^T - 1}{W - 1}. \tag{22}$$

This simplification enables the terrorist to focus on the one parameter W, which is determined before the first period. This makes the problem tractable and allows for sufficiently different attack scenarios as W changes from zero to infinity. The parameter W determines the strategy of resource allocation through the T periods: $W > 1$ corresponds to increasing the resource allocation. $W = \infty$ corresponds to allocating the entire resources in the last time period $R_T = R$, which is also analyzed as the two-stage game, for the last period. $W < 1$ corresponds to decreasing the resource allocation. $W = 1$ corresponds to even resource distribution across the K attacks. Then the analysis in each time period is equivalent to the analysis of the two-stage game, replacing R with R/T. Finally, $W = 0$ corresponds to allocating the entire resources in the first time period $R_1 = R$, which is also analyzed as the two-stage game, for the first period. We thus get:

$$R_1 = \begin{cases} R\dfrac{W - 1}{W^T - 1} & \textit{when } W \neq 1 \\ \dfrac{R}{T} & \textit{when } W = 1 \end{cases}, \quad R_t = W^{t-1} R_1. \tag{23}$$

The government uses intelligence and spies to compile complete information about R_t allocated to each attack. Hence, all parameters are common knowledge. The contest in each time period is such that the government may either keep its asset, lose its asset, keep a fraction of its asset, or keep its asset with a specified probability. In each subsequent time period, the contest starts anew over an asset with a specified value; that is, valued as v_t by the government and valued as V_t by the terrorist.

We define the players' accumulative discounted utilities over the T time periods as:

$$u = \begin{cases} \sum_{t=1}^{T} \delta^{t-1} u_t & \textit{when } 0 < \delta \leq 1 \\ u_1 & \textit{when } \delta = 0 \end{cases},$$

$$U = \begin{cases} \sum_{t=1}^{T} \Delta^{t-1} U_t & \textit{when } 0 < \Delta \leq 1 \\ U_1 & \textit{when } \Delta = 0 \end{cases} \tag{24}$$

where δ and Δ are time discount factors. If the government's asset is destroyed or partly destroyed in one period, it is refurnished from an outside source prior to the subsequent period. The terrorist chooses W before the first period, has a time horizon of T periods, and uses (24) to determine its accumulative discounted utility over the T periods. Analogously, the government uses (24) to determine its accumulative discounted utility over the T periods.

Illustrative Examples

Figure 5 plots the equilibrium dynamics as a function of time period t for various W and R when R_t follows the geometric progression, assuming $\delta = \Delta = b_t = B_t = V_t = v_t = 1$ and $T = 10$. With these parameter values, Figure 2b implies that only cases 1 and 3 are possible. When $W = 1$, the terrorist's resources are spread equally through the $T = 10$ periods, which gives $R_t = 5/10 = 0.5$ in Figure 5b and $R_t = 15/10 = 1.5$ in Figure 5e. This means that the terrorist receives incoming resources R_t in equal amounts in each time period and decides to allocate this entire portion R_t in each period, without planning for the future or somehow assessing whether $W \neq 1$ may be appropriate.

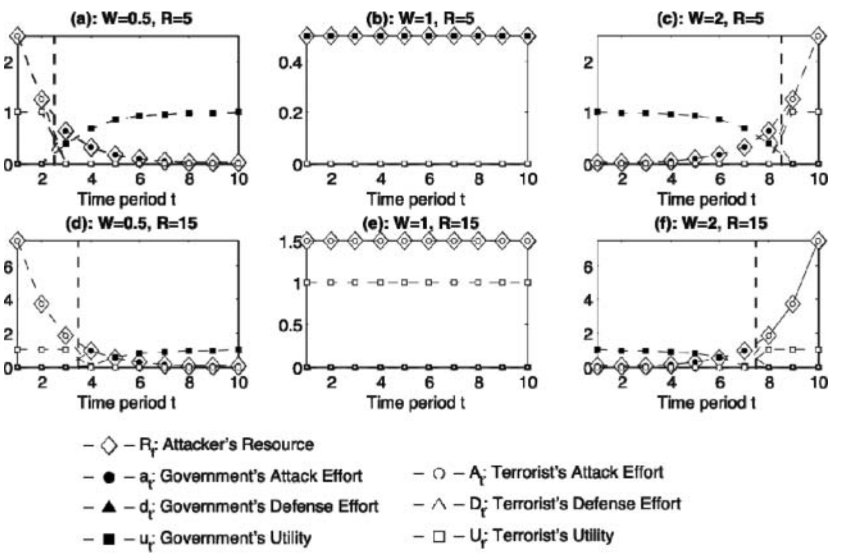

Figure 5. Equilibrium dynamics as a function of t for various W and R, $b_t = B_t = V_t = v_t = 1, T = 10$.

Figure 5b gives case 1 and is the only panel among the six panels where the government successfully deters the terrorist throughout the 10 periods. For each period $t = 1, \ldots, 10$, we have $a_t = R_t = u_t = 0.5$ and $d_t = A_t = D_t = U_t = 0$. The accumulative utilities are $u = 10 \times 0.5 = 5$ and $U = 0$. Figure 5e gives case 4 and is the only panel among the six panels where the government neither defends nor attacks throughout the 10 periods, while the terrorist attacks throughout the 10 periods. For each period $t = 1, \ldots, 10$, we have $R_t = A_t = 1.5$, $U_t = 1$, and $a_t = d_t = D_t = u_t = 0$. The accumulative utilities are $u = 0$ and $U = 10 \times 1 = 10$.

When $W = 0.5$, R_t decreases geometrically through the 10 periods. In Figure 5a, case 4 arises when $t \leq 2$, transitioning to case 1 when $t > 2$, with accumulative utilities $u = 6.75$ and $U = 2.00$. Therefore, both players prefer $W = 0.5$ rather than $W = 1$. In Figure 5d, where the terrorist is more resourceful, the transition from cases 3 to 1 occurs when $t = 3$, with $u = 5.14$ and $U = 3.00$ because the attacker is more resourceful. Intuitively, the higher R is, the longer the terrorist enjoys case 4 before switching to case 1. Thus, when $R = 15$ the government prefers $W = 0.5$ rather than $W = 1$, whereas the terrorist prefers the reverse.

Analogously, in Figures 5c and 5f where $W = 2$, R_t increases geometrically with transitioning from cases 1 to 3 in periods 9 and 8, respectively. The higher R is, the sooner the terrorist enjoys the switch from cases 1 to 3. When the Ws are chosen such that $W = 0.5$ is the inverse of $W = 2$, the utilities of Figure 5a always equal the utilities of Figure 5c, where Figure 5c is the time-reversed version of Figure 5a. Analogously, the utilities of Figure 5d always equal the utilities of Figure 5f. The terrorist prefers Figure 5d and 5f over Figure 5a and 5c because it has more resources, whereas the government prefers Figures 5a and 5c over Figures 5d and 5f by contrast.

Figure 6 plots the equilibrium dynamics as a function of time period t for various W and R when R_t follows the geometric progression, assuming $b_t = 1$, $V_t = v_t = B_t = 2$, and $T = 10$, which should be compared with Figure 2d. When $W = 1$ in Figure 6, Figure 6b is the only panel where case 1 occurs throughout the 10 periods ($u = 15$, $U = 0$), and Figure 6e is the only panel where case 2 occurs throughout the 10 periods ($u = 6.01$, $U = 4.49$). When $W = 0.5$ and $R = 5$ (Figure 6a), case 2 occurs when $t \leq 2$, case 1 occurs when $t > 2$, and case 4 does not arise (because the attacker is not sufficiently advantaged when $t = 0$ and R is as low as 5), with $u = 15.71$ and $U = 1.40$. When $W = 0.5$ and $R = 15$ (Figure 6d), case 4 occurs when $t \leq 1$, case 2 occurs when $1 < t \leq 3$, and case 1 occurs when $t > 3$, with $u = 12.54$ and $U = 4.61$. Therefore, both players prefer $W = 0.5$ rather than $W = 1$ when $R = 5$ or $R = 15$. Again, Figures 6c and 6f are the time-reversed versions to Figures 6a and 6d, respectively. The

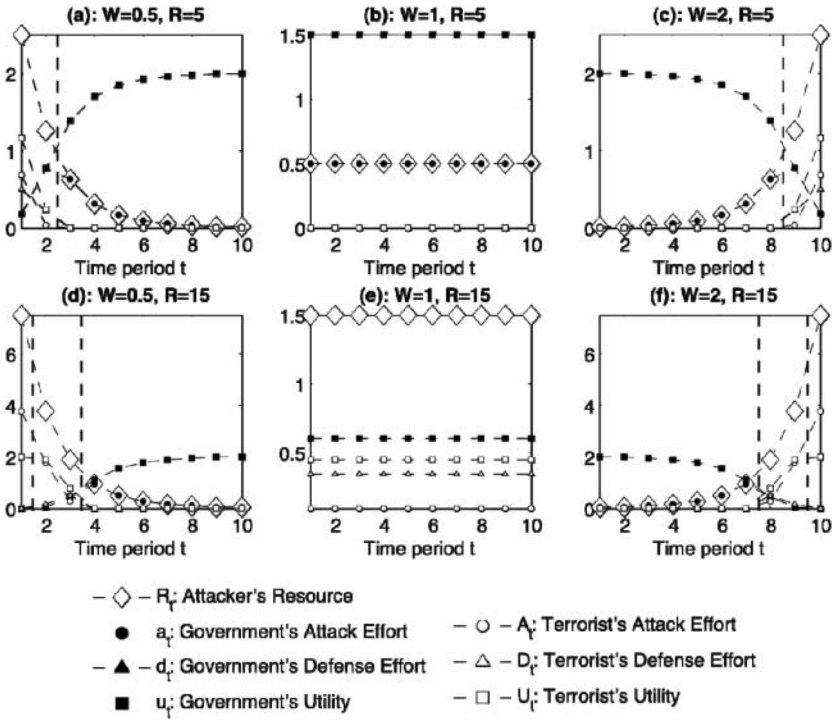

Figure 6. Equilibrium dynamics as a function of t for various W and R, $b_t = 1$, $V_t = v_t = B_t = 2$, $T = 10$.

terrorist prefers Figures 6d and 6f over Figures 6a and 6c because it has more resources, whereas the government prefers Figures 6a and 6c over Figures 6d and 6f by contrast.

According to (24), the government's accumulative utility u increases in δ, the terrorist's accumulative utility U increases in Δ, and, of course, U increases in R.

Figure 7 plots the accumulative utilities u and U as functions of W when $b_t = B_t = V_t = v_t = 1$, $T = 10$, for various R and $\delta = \Delta$. When $\delta = 1$ the government's highest preference is that case 1 arises in each period, which is possible when $R < 1$, as seen in Figure 1, and then W is irrelevant. The government's second highest preference is that case 4 arises once and that case 1 arises in the remaining $T - 1$ periods. That is possible when $1 < R$, and then the government prefers the single attack $W = 0$ or $W = \infty$. In contrast, as R increases above 1, the terrorist prefers to spread its attack over several periods and earns $U_t = V_t$ in each period of its attacks. For example, in the upper right panel in Figure 7 where $R = 5$ and $\delta = \Delta = 1$, the terrorist earns its maximum $U = 2$ from attacking in periods 1 and

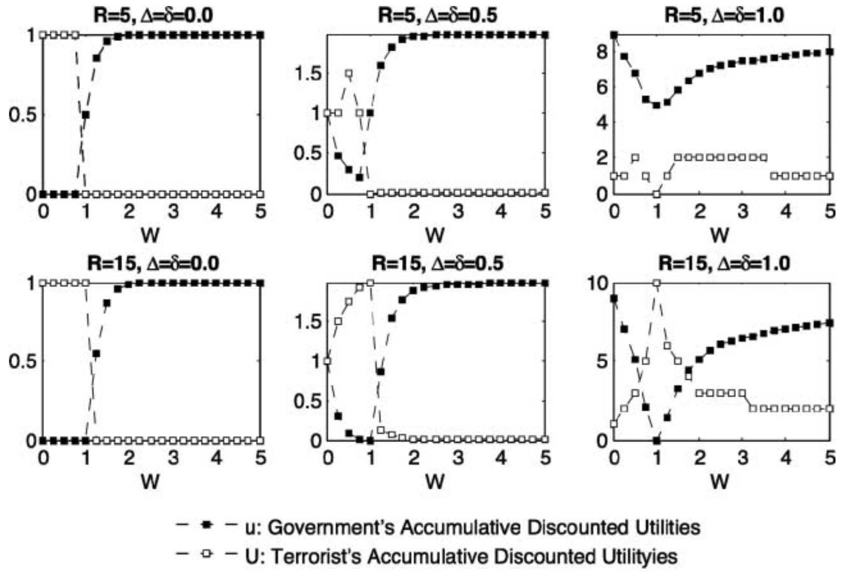

Figure 7. The accumulative utilities *u* and *U* as functions of *W* for various *R* and $\delta = \Delta$ when $b_t = B_t = V_t = v_t = 1$, $T = 10$.

2 when $0.28 < W < 0.75$ and in periods 9 and 10 when $1.34 < W < 3.62$ (two ranges exist because of the time-reversed logic when $\delta = \Delta = 1$) and is deterred in all 10 periods when $0.83 < W < 1.20$.[2] When $R = 5$, the terrorist prefers W to be intermediately small (or intermediately large) to be able to attack in two periods. When W is too low (or too high), a large attack occurs in only one period. When W is too close to 1, the terrorist spreads its resources too equally across all periods and is deterred in each period.

As R increases above 10, the terrorist can guarantee its preferred case 4 in each period by choosing $0.92 < W < 1.09$. This is illustrated in the lower right panel in Figure 7 where $R = 15$ and $\delta = \Delta = 1$ and where the players have exactly opposite preferences. The government prefers $W = 0$ or $W = \infty$ and prefers to avoid W close to 1, whereas the terrorist prefers $0.92 < W < 1.09$ and prefers to avoid $W = 0$ or $W = \infty$.

As the discount factors decrease below 1, the time-reversed logic for the utilities no longer applies, and the players strike balances between their preferences for W and time discounting. These preferences are also opposite. When $\delta = \Delta = 0$, only the first period matters and hence the government prefers $W = \infty$, which gives a single attack in the last period $T = 10$, whereas the terrorist prefers $W = 0$, which gives a single attack

[2]The terrorist earns $U = 1$ from attacking only in period 1 when $W < 0.28$ and $0.75 < W < 0.83$ and attacking only in period 10 when $1.20 < W < 1.34$ and $W > 3.62$.

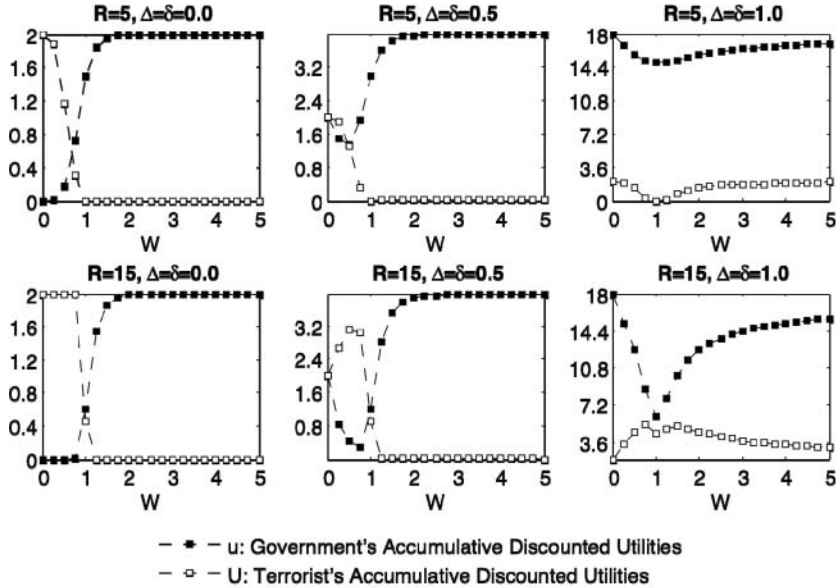

Figure 8. The accumulative utilities *u* and *U* as functions of *W* for various *R* and $\delta = \Delta$ when $b_t = 1, B_t = V_t = v_t = 2, T = 10$.

in the first period. That is, when the government becomes more myopic, it prefers that the terrorist stockpile rather than use its resources now. More generally, the government enjoys case 1 regardless of δ and W when $R < 1$ and prefers $W = \infty$ when $\delta < 1$. The terrorist always prefers $W \leq 1$ when $\Delta < 1$ because the time-reversed logic for the utilities does not apply. The range for the terrorist's resources where the terrorist enjoys case 4 are the same regardless of Δ, but $\Delta < 1$ causes discounted utility U as illustrated when $\delta = \Delta = 0$ in Figure 7.

Figure 8 plots u and U as functions of W when $b_t = 1$, $B_t = V_t = v_t = 2$, $T = 10$, for various R and $\delta = \Delta$. The terrorist's unit attack cost B_t is twice as high as in Figure 7. As in Figure 7, the government prefers the single attack $W = 0$ or $W = \infty$ when $\delta = 1$ and prefers $W = \infty$ when $d < 1$. As seen in Figure 2d, when choosing $W = 1$, the terrorist suffers case 1 in all periods when $R/T = R_t < 1$, gets case 2 in all periods when $1 < R/T = R_t < 4$, and enjoys case 4 in all periods when $R/T = R_t > 4$. Hence, the terrorist chooses the single attack $W = 0$ when inferior with $R = 5$ and $\Delta = 1$ (upper right panel) and chooses $W = 0.74$ when more resourceful with $R = 15$ and $\Delta = 1$ (lower right panel). $W = 0.74$ in Figure 8 (lower right panel) corresponds to the terrorist enjoying case 4 in period 1, gets case 2 in periods 2–5, and suffers case 1 in periods 6–10.[3] One crucial characteristic of Figure 8 is the dependence on case 2. As R_t decreases through the 10 periods, the terrorist transitions from case 4 through case 2 to case 1. In contrast to Figure 7, the range for the terrorist's resources where the terrorist experiences the various cases indeed depends on the discount factor Δ. For example, when $R = 15$, the terrorist chooses $W = 0.74$ when $\Delta = 1$ and chooses $W = 0.60$ when $\Delta = 0.5$. That is, a lower discount factor causes the terrorist to attack more thoroughly in the early periods. The terrorist strikes a balance between stockpiling and time discounting. It stockpiles less when the future becomes less important.

CONCLUSION

In this article we analyze a model where a government defends against a terrorist who attacks repeatedly and stockpiles its resources over time. In each time period the government defends an asset and attacks the terrorist's resources. The terrorist defends its resources and uses the surviving portion of its resources to attack the government.

For a one-period game we demonstrate four possible equilibrium solutions: the government attacks only, which deters the terrorist; both players defend and attack; the government defends but does not attack and the terrorist attacks only; the terrorist attacks only and the government neither defends nor attacks. We show how the four cases depend on the terrorist's resources and the players' asset valuations and unit defense and attack costs. Understanding which factors impact the four cases is important in order to combat terrorism.

Considering a T-period game, we analyze how the terrorist stockpiles resources. To our knowledge, this article is the first to study resource stockpiling in the literature of attacker–defender games. To handle the many ways in which the terrorist can stockpile within a time horizon of T periods, we assume that before the game starts the terrorist determines how much of its resources to use in the first period and its geometric stockpiling parameter. A stockpiling parameter equal to 1 means equal resource allocation in each period; a parameter larger (smaller) than 1 means increasing (decreasing) resource allocation; a parameter equal to zero means a single attack in the first period; and a parameter equal to infinity means a single attack in the last period. Both players have time discount parameters that determine their accumulative discounted utilities over the T periods.

[3]This follows because inserting $W = 0.74$, $R = 15$, and $T = 10$ into (21)–(23) implies $R_1 = 4.10$, $R_2 = 3.04$, $R_3 = 2.25$, $R_4 = 1.66$, $R_5 = 1.23$, $R_6 = 0.91$, $R_7 = 0.67$, $R_8 = 0.50$, $R_9 = 0.37$, $R_{10} = 0.27$, and the cases are determined from Figure 2d.

The article analyzes how the government and terrorist prefer low versus high stockpiling parameters and how these preferences interact with the other parameters such as the terrorist's resources and the players' asset valuations, unit defense and attack costs, and discount factors. If the terrorist's resources are small, it can be deterred in each period by the government's attack. If the terrorist's resources are extremely large, it allocates its resources equally across the T periods for maximum impact, whereas the government prefers a single attack. If the terrorist's resources are intermediate, it would be deterred in each period if it allocated its resources equally across the T periods. It thus strikes a balance where it allocates many resources to early or late periods to facilitate attacks and accepts being deterred in the other periods. As the future becomes less important, the terrorist attacks more in early periods.

One shortcoming of this article is that we make explicit assumptions about functional forms, such as how the terrorist's resources are reduced by the government's attack, the probability of asset destruction dependent on the players' defense and attack, and the utilities. By making use of credible specific functional forms we produce exact analytical solutions for the variables, illustrated with numerical simulations. In return for the sacrifice of generality, a successful specification demonstrates internal consistency and equilibrium solutions. In addition, we claim that the particular functional forms used here are illuminating. (In economics, Cobb-Douglas and Constant Elasticity of Substitution (CES) production functions, with special assumptions about the functional relations between inputs and outputs, have proved useful for understanding production and economic growth.) Using particular functional forms makes it possible to determine ranges of parameter values within which the four possible equilibrium solutions arise.

A second shortcoming is the assumption of complete information, which provides a benchmark and is common game theoretically. In reality, gathering information is arguably one of the most important and difficult tasks facing the government in counterterrorism. Furthermore, it is difficult for the terrorist to know how well an asset is protected. Future research can allow a variety of terrorist characteristics to be incompletely known, such as its resources, unit attack and defense costs, asset valuations, and time discount factor. Future research can also consider an infinitely repeated game.

A further possible avenue of future research is to merge the model in this article, in which the terrorist strikes a balance between attacking and protecting its resources, with Morselli et al.'s (2007) model, in which the terrorist strikes a balance between secrecy and efficiency. That is, the terrorist chooses some value for its efficiency parameter. The higher this value is, the more likely the attack will be successful but also the more likely the government will be able to thwart the attack. The terrorist needs to find the right balance of secrecy and efficiency to carry out its attack.

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APPENDIX A: COMPARING $\mathbf{u}_{t1}$*,* $\mathbf{u}_{t2}$*, AND* $\mathbf{u}_{t4}$

Appendix A.1: When (13) Holds (Case 2 Is Feasible)

Comparing u_{t1}, u_{t2}, and u_{t4} is sufficient when $B_t/b_t \leq 1$ (Property 1) or when $B_t/b_t > 1$ and $v_t \geq R_t \frac{(1+b_t/B_t)^2}{4b_t/B_t} \geq R_t$. Note that

$$u_{t1} > u_{t4} \Leftrightarrow v_t - R_t > 0 \Leftrightarrow v_t > R_t \tag{A1}$$

and

$$
\begin{aligned}
& u_{t1} > u_{t2} \\
& \Leftrightarrow v_t - R_t > \frac{\left(\dfrac{v_t}{b_t} - \sqrt{R_t}\right)(B_t - \sqrt{R_t})}{b_t(B_t - b_t)} \\
& \Leftrightarrow \begin{cases} v_t > \dfrac{b_t\sqrt{R_t}\left(\sqrt{R_t}b_tB_t + \sqrt{R_t} - \sqrt{R_t}b_t^2 - B_t\right)}{\left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)} \\ \qquad \text{if } \left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)(B_t - b_t) > 0 \\ v_t < \dfrac{b_t\sqrt{R_t}\left(\sqrt{R_t}b_tB_t + \sqrt{R_t} - \sqrt{R_t}b_t^2 - B_t\right)}{\left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)} \\ \qquad \text{if } \left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)(B_t - b_t) < 0 \end{cases}
\end{aligned} \tag{A2}
$$

$$
\begin{aligned}
& u_{t4} > u_{t2} \\
& \Leftrightarrow 0 > \frac{\left(\frac{v_t}{b_t} - \sqrt{R_t}\right)(B_t - \sqrt{R_t})}{b_t(B_t - b_t)} \\
& \Leftrightarrow \begin{cases} v_t < b_t\sqrt{R_t} & \text{if } (B_t - \sqrt{R_t})(B_t - b_t) > 0 \\ v_t > b_t\sqrt{R_t} & \text{if } (B_t - \sqrt{R_t})(B_t - b_t) < 0 \end{cases}
\end{aligned} \tag{A3}
$$

Therefore, u_{t1} is highest if and only if

$$
\begin{cases} v_t > \max\left\{R_t, \dfrac{b_t\sqrt{R_t}(\sqrt{R_t}b_tB_t + \sqrt{R_t} - \sqrt{R_t}b_t^2 - B_t)}{(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t})}\right\} \\ \qquad \text{if}(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t})(B_t - b_t) > 0 \\ R_t < v_t < \dfrac{b_t\sqrt{R_t}\left(\sqrt{R_t}b_tB_t + \sqrt{R_t} - \sqrt{R_t}b_t^2 - B_t\right)}{\left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)} \\ \qquad \text{if}(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t})(B_t - b_t) < 0 \end{cases} \tag{A4}
$$

And u_{t2} is highest if and only if

$$\begin{cases} b_t\sqrt{R_t} < v_t < \dfrac{b_t\sqrt{R_t}\left(\sqrt{R_t}b_tB_t + \sqrt{R_t} - \sqrt{R_t}b_t^2 - B_t\right)}{\left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)} \\ \quad \text{if}(B_t - \sqrt{R_t})(B_t - b_t) > 0 \text{ and } \left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)(B_t - b_t) > 0 \\ v_t > \max\left\{b_t\sqrt{R_t}, \dfrac{b_t\sqrt{R_t}\left(\sqrt{R_t}b_tB_t + \sqrt{R_t} - \sqrt{R_t}b_t^2 - B_t\right)}{\left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)}\right\} \\ \quad \text{if}(B_t - \sqrt{R_t})(B_t - b_t) > 0 \text{ and } \left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)(B_t - b_t) < 0 \\ v_t < \min\left\{b_t\sqrt{R_t}, \dfrac{b_t\sqrt{R_t}\left(\sqrt{R_t}b_tB_t + \sqrt{R_t} - \sqrt{R_t}b_t^2 - B_t\right)}{\left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)}\right\} \\ \quad \text{if}(B_t - \sqrt{R_t})(B_t - b_t) < 0 \text{ and } \left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)(B_t - b_t) > 0 \\ \dfrac{b_t\sqrt{R_t}\left(\sqrt{R_t}b_tB_t + \sqrt{R_t} - \sqrt{R_t}b_t^2 - B_t\right)}{\left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)} < v_t < b_t\sqrt{R_t} \\ \quad \text{if}(B_t - \sqrt{R_t})(B_t - b_t) < 0 \text{ and } \left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)(B_t - b_t) < 0 \end{cases} \tag{A5}$$

And u_{t4} is highest if and only if:

$$\begin{cases} v_t < \min\{R_t, b_t\sqrt{R_t}\} & \text{if } (B_t - \sqrt{R_t})(B_t - b_t) > 0 \\ R_t > v_t > b_t\sqrt{R_t} & \text{if } (B_t - \sqrt{R_t})(B_t - b_t) < 0 \end{cases} \tag{A6}$$

Lemma 1. *The parameter combination*

$$\begin{aligned} &(B_t - \sqrt{R_t})(B_t - b_t) < 0 \quad \text{and} \\ &\left(b_t^2B_t - b_t^3 - B_t + \sqrt{R_t}\right)(B_t - b_t) < 0 \end{aligned} \tag{A7}$$

is impossible.

Proof. Assuming $B_t > b_t$ implies $B_t < \sqrt{R_t} < 0$ and $b_t^2(B_t - b_t) - B_t + \sqrt{R_t} < 0$, which is impossible. Assuming $B_t < b_t$ implies $B_t > \sqrt{R_t}$ and $b_t^2(B_t - b_t) - B_t + \sqrt{R_t} > 0$, which is impossible. Assuming $B_t = b_t$ causes (A7) not to be satisfied. ■

The government is advantaged in case 1, which is possible when the government's valuation v_t is above a minimum. In contrast, the terrorist is advantaged in case 4, which is possible when the government's valuation v_t is below a maximum. Case 2 gives an interior solution where both players defend and attack.

Appendix A.2: When (13) Does Not Hold (Case 2 Is Not Feasible)

Comparing u_{t1} and u_{t4} is sufficient when $B_t/b_t \leq 1$ (Property 1) or when $B_t/b_t > 1$ and $v_t \geq R_t \frac{(1+b_t/B_t)^2}{4b_t/B_t} \geq R_t$. Note that

$$u_{t1} > u_{t4} \Leftrightarrow v_t - R_t > 0 \Leftrightarrow v_t > R_t \tag{A9}$$

Therefore, we have case 1 if (A9) holds and case 4 if (A9) does not hold.

APPENDIX B: COMPARING $\mathbf{u_{t2}}$ AND $\mathbf{u_{t4}}$

Comparing u_{t2} and u_{t4} is sufficient when $B_t/b_t > 1$ and $v_t < R_t b_t/B_t$.
When (13) holds (case 2 is feasible), we have

$$\begin{aligned} u_{t4} > u_{t2} &\Leftrightarrow 0 > \frac{\left(\frac{v_t}{b_t} - \sqrt{R_t}\right)(B_t - \sqrt{R_t})}{b_t(B_t - b_t)} \\ &\Leftrightarrow \begin{cases} v_t < b_t\sqrt{R_t} & \text{if } (B_t - \sqrt{R_t})(B_t - b_t) > 0 \\ v_t > b_t\sqrt{R_t} & \text{if } (B_t - \sqrt{R_t})(B_t - b_t) < 0 \end{cases} \end{aligned} \tag{B1}$$

When (13) does not hold (case 2 is feasible), we have case 4.

APPENDIX C: COMPARING $\mathbf{u_{t2}}$, $\mathbf{u_{t3}}$, AND $\mathbf{u_{t4}}$

Comparing u_{t2}, u_{t3}, and u_{t4} is sufficient when $\{B_t/b_t > 1,\ R_t b_t/B_t \leq v_t \leq R_t\}$ or when $\{B_t/b_t > 1,\ v_t > R_t,\ R_t \frac{(1+b_t/B_t)^2}{4b_t/B_t} \geq v_t\}$. These inequalities imply $u_{t3} = (\sqrt{v_t} - \sqrt{\frac{b_t R_t}{B_t}})^2 > 0$ and therefore we only compare u_{t2} (case 2) and u_{t3} (case 3) because u_{t4} is strictly dominated. We thus get

$$u_{t3} > u_{t2} \Leftrightarrow \left(\sqrt{v_t} - \sqrt{\frac{b_t R_t}{B_t}}\right)^2 > \frac{\left(\frac{v_t}{b_t} - \sqrt{R_t}\right)(B_t - \sqrt{R_t})}{b_t(B_t - b_t)} \tag{C1}$$

where the left-hand side is strictly positive.

Note that we also need (13) to hold in order to get case 2. When (13) does not hold, we have case 3.

In case 3, the government defends but does not attack, and the terrorist attacks only; the functional forms are $a_t = 0$, $d_t = \sqrt{\frac{R_t v_t}{B_t b_t}} - \frac{R_t}{B_t}$, $A_t = R_t/B_t$, $D_t = 0$, $u_t = (\sqrt{v_t} - \sqrt{\frac{b_t R_t}{B_t}})^2$, and $U_t = V_t\sqrt{\frac{R_t b_t}{v_t B_t}}$.

BIOGRAPHICAL SKETCHES

Kjell Hausken has been Professor of Economics and Societal Safety at the University of Stavanger, Norway, since 1999. His research fields are strategic interaction, risk analysis, public choice, conflict, game theory, terrorism, information security, and economic risk management. He holds a Ph.D. from the University of Chicago (1990–1994), was a postdoc at the Max Planck Institute (Cologne) 1995–1998, and a visiting scholar at Yale University 1989–1990. He has published 140 articles, is on the editorial board for *Theory and Decision* and *Defence and Peace Economics*, has refereed for 50 journals, and advises and has advised five Ph.D. students.

Jun Zhuang has been an assistant professor of industrial and systems engineering at the University at Buffalo, the State University of New York (SUNY–Buffalo), since he obtained his Ph.D. in industrial engineering in 2008 from the University of Wisconsin–Madison. Dr. Zhuang has an M.S. in agricultural economics in 2004 from the University of Kentucky and a bachelor's degree in industrial engineering in 2002 from Southeast University, China. Dr. Zhuang's long-term research goal is to integrate operations research and game theory to better mitigate, prepare for, respond to, and recover from both natural and man-made hazards. Other areas of interest include health care, transportation, logistics and supply chain management, and sustainability. Dr. Zhuang has published more than 20 peer-reviewed journal articles. He is on the editorial board of *Decision Analysis*, is the coeditor of *Decision Analysis Today*, and has reviewed proposals for NSF, book chapters for Springer, and articles for more than 40 academic journals and conferences.