8 Online Supplementary Materials—Appendix

8.1 Theorem 1 in Balachandran and Schaefer (1980)

"Given Λ_i^d for $i = 1, ..., k$ as in (9), then there exists a unique equilibrium aggregate arrival rate

$$\Lambda^s = \sum_{i=1}^{k} \lambda_i^s$$

and

$$\lambda_i^s = \max(\min(\Lambda_i^d - \sum_{j=1}^{i-1} \lambda_j^s, \lambda_i^m), 0)$$

with the convention that $\sum_{j=1}^{0} \lambda_j^s = 0$."

Remark: In the above theorem, k is the number of customer classes and in our paper $k = 2$. λ_i^m is the maximum possible arrival rate for class i, which corresponds with Λ_{ig} in our paper. Λ_i^d is the desired aggregate equilibrium arrival rate by the individual from class i, which corresponds with $\widehat{\Lambda}_{ig}$ in our paper. In addition, Λ_i^d is derived by Equation (5) in Balachandran and Schaefer (1980); the individual will enter until the cost of waiting exactly offsets the reward due to service and the demand traffic is composed only by class i. λ_i^s is the stable aggregate equilibrium arrival rate for class i, which corresponds with λ_{ig} in our paper. Note that $i = 1, ..., k$ is the ordering defined in (9) and Lemma 1 in Balachandran and Schaefer (1980): If the reward net of the waiting cost of service time per value of time of class i exceeds that of class $i + 1$, then $\Lambda_i^d > \Lambda_{i+1}^d$. In our paper, when $0 < \Phi_1 < \Phi_2$, $\frac{r_g - \Phi_1 c_w/\mu}{\Phi_1 c_w} > \frac{r_g - \Phi_2 c_w/\mu}{\Phi_2 c_w}$. Thus attributes 1 and 2 in our paper correspond with classes 1 and 2 respectively in the above theorem, and in addition $\widehat{\Lambda}_{1g} > \widehat{\Lambda}_{2g}$ in our paper.

8.2 Proof to Proposition 1

The proof follows trivially from the optimization problem (5) for $\theta = b$.

8.3 Proof to Proposition 2

(i) When $\Phi_1 = 0$, applicants of attribute 1 will not experience screening or waiting, and their utilities are always positive. So potential applicants of attribute 1 will apply with probability 1. Similar analysis applies when $\Phi_2 = 0$.

(ii.a) When $\Phi_1 < \Phi_2$, recall $\widehat{\Lambda}_{1g}$ and $\widehat{\Lambda}_{2g}$ are the maximum traffic of good applicants for screening defined in Equations (10) and (11), by setting the traffic from applicants of the other attribute as zero and then solving $u_{tg}(\mathbf{\Phi}, \mathbf{p}) = 0$, $t = 1, 2$ respectively. Note that $\widehat{\Lambda}_{1g} > \widehat{\Lambda}_{2g}$ when $\Phi_1 < \Phi_2$. Using Theorem 1 in Balachandran and Schaefer (1980), there exists a unique equilibrium aggregate traffic rate for screening $\lambda_{1g} + \lambda_{2g}$, where

$$\lambda_{1g} = \max(\min(\widehat{\Lambda}_{1g}, \Phi_1\Lambda_{1g}), 0)$$

is the equilibrium aggregate arrival rate of the screened good applicants with attribute 1, and

$$\lambda_{2g} = \max(\min(\widehat{\Lambda}_{2g} - \lambda_{1g}, \Phi_2\Lambda_{2g}), 0)$$

is the equilibrium aggregate arrival rate of the screened good applicants with attribute 2.

Thus, good potential applicants' best response strategies satisfy:

$$\hat{p}_{1g} = \frac{\lambda_{1g}}{\Phi_1\Lambda_{1g}} = \frac{1}{\Phi_1\Lambda_{1g}} \max(\min(\widehat{\Lambda}_{1g}, \Phi_1\Lambda_{1g}), 0)$$

and

$$\hat{p}_{2g} = \frac{\lambda_{2g}}{\Phi_2\Lambda_{2g}} = \frac{1}{\Phi_2\Lambda_{2g}} \max(\min(\widehat{\Lambda}_{2g} - p_{1g}\Phi_1\Lambda_{1g}, \Phi_2\Lambda_{2g}), 0)$$

(ii.b) The proof is analogues to the proof to (ii.a) above.

(iii) When $\Phi_1 = \Phi_2 = \Phi$, then $\hat{p}_{1b} = \hat{p}_{2b} \doteq \hat{p}_b$, we obtain

$$\widehat{\Lambda}_{1g} = \widehat{\Lambda}_{2g} = \mu - \Phi\hat{p}_b\Lambda_b - \frac{\Phi c_w}{r_g} \doteq \widehat{\Lambda}_g.$$

and at the equilibrium, the utility for good applicants with attribute 1 is the same with the utility for good applicants with attribute 2. If $\widehat{\Lambda}_g > \Phi(\Lambda_{1g}+\Lambda_{2g}) = \Phi\Lambda_g$, then $\hat{p}_{1g} = \hat{p}_{2g} = 1$, where good applicants have positive utilities. If $\widehat{\Lambda}_g < 0$, then $\hat{p}_{1g} = \hat{p}_{2g} = 0$, where good applicants have negative utilities. If $0 \leq \widehat{\Lambda}_g \leq \Phi(\Lambda_{1g} + \Lambda_{2g}) = \Phi\Lambda_g$, then $\hat{p}_{1g} = \hat{p}_{2g} \in [0, 1]$ and satisfy $\hat{p}_{1g}\Phi\Lambda_{1g} + \hat{p}_{2g}\Phi\Lambda_{2g} = \widehat{\Lambda}_g$. Thus, we obtain that $\hat{p}_{1g} = \hat{p}_{2g} = \frac{\widehat{\Lambda}_g}{\Phi(\Lambda_{1g}+\Lambda_{2g})} = \frac{\widehat{\Lambda}_g}{\Phi\Lambda_g}$, where good applicants have zero utility.

8.4 Proof to Proposition 3

According to Proposition 1, when the screening probability is larger than or equal to s_b, none of bad potential applicants submit applications. As a result, the approver will never get a strictly better payoff by performing a screening probability strictly larger than s_b. Recall that we assume in Section 2.1 that when the approver is indifferent between different levels of screening probabilities, she will choose the lowest level. Therefore, here the optimal non-discriminatory screening probability must be either s_b, or some value in the interval $[0, s_b)$.

1. When $\Phi = s_b$, then $\hat{p}_{1b} = \hat{p}_{2b} = 0$, there are no bad applicants. Then the traffic for screening is composed only by good applicants. Using Proposition 2(iii) and Equations

(10-11), we obtain $\widehat{\Lambda}_g = \widehat{\Lambda}_{1g} = \widehat{\Lambda}_{2g} = \mu - \frac{s_b c_w}{r_g}$, and the demand rate of the screened good applicants is $\max(0, \min(\widehat{\Lambda}_g, s_b\Lambda_g))$. Divided by $\Phi = s_b$, the demand rate of the good applicants is $\max(0, \min(\frac{\widehat{\Lambda}_g}{s_b}, \Lambda_g))$. Then the approver's objective value is:

$$R \max(0, \min(\frac{\mu}{s_b} - \frac{c_w}{r_g}, \Lambda_g)).$$

2. When $\Phi \in [0, s_b)$, then $\hat{p}_{1b} = \hat{p}_{2b} = 1$, all bad potential applicants are submitting. Using Proposition 2(iii) and Equations (10-11), we obtain $\widehat{\Lambda}_g = \mu - \Phi\Lambda_b - \frac{\Phi c_w}{r_g}$. Then the demand rate of the screened good applicants is $\max(0, \min(\widehat{\Lambda}_g, \Phi\Lambda_g))$. Divided by Φ, the demand rate of the good applicants is $\max(0, \min(\frac{\widehat{\Lambda}_g}{\Phi}, \Lambda_g))$.[8] The optimal strategy is to solve:

$$\max_{\Phi \in [0, s_b)} R \max(0, \min(\frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{r_g}, \Lambda_g)) - C(1 - \Phi)\Lambda_b$$

Now we rewrite the nondiscriminatory optimization problem. Let

$$\begin{aligned} J_1(\Phi) &= R \max(0, \min(\frac{\mu}{\Phi} - \frac{c_w}{r_g}, \Lambda_g)) \\ J_2(\Phi) &= R \max(0, \min(\frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{r_g}, \Lambda_g)) + C\Lambda_b\Phi - C\Lambda_b \end{aligned}$$

and

$$J(\Phi) = \begin{cases} J_1(\Phi), & \Phi = s_b \\ J_2(\Phi), & 0 \leq \Phi < s_b \end{cases}$$

Thus, the optimal non-discriminatory strategy is to solve

$$\max_{0 \leq \Phi \leq s_b} J(\Phi)$$

Note that for a given value of $\Phi \in [0, 1]$, we have

$$J_1(\Phi) \geq J_2(\Phi) \tag{19}$$

Then let $J^* = \max_{0 \leq \Phi \leq s_b} J(\Phi)$, $J_1^* = J_1(s_b)$, and $J_2^* = \sup_{0 \leq \Phi < s_b} J_2(\Phi)$. (We use "sup" because $J_2(\Phi)$ might not obtain its maximum in $[0, s_b)$.) We rewrite $J_2(\Phi)$ as a piece-wise linear function:

$$J_2(\Phi) = \begin{cases} R\Lambda_g + C\Lambda_b\Phi - C\Lambda_b, & \Phi \leq \mu/(\Lambda_g + \Lambda_b + \frac{c_w}{r_g}) & \text{(i)} \\ \frac{R\mu}{\Phi} - R\Lambda_b - R\frac{c_w}{r_g} + C\Lambda_b\Phi - C\Lambda_b, & \mu/(\Lambda_g + \Lambda_b + \frac{c_w}{r_g}) < \Phi \leq \mu/(\Lambda_b + \frac{c_w}{r_g}) & \text{(ii)} \\ C\Lambda_b\Phi - C\Lambda_b, & \Phi > \mu/(\Lambda_b + \frac{c_w}{r_g}) & \text{(iii)} \end{cases}$$

Table 5: Additional Notations

Notation	Explanation
x_1	$\mu/(\Lambda_g + \Lambda_b + \frac{c_w}{r_g})$
x_2	$\mu/(\Lambda_b + \frac{c_w}{r_g})$
x_3	$\mu/(\Lambda_g + \frac{c_w}{r_g})$
x_4	$\mu\frac{r_g}{c_w}$
x_5	$\sqrt{\frac{R\mu}{C\Lambda_b}}$
x_6	$1 - \frac{R\Lambda_g}{C\Lambda_b}$
x_7	$\frac{R\mu}{R\Lambda_g + C\Lambda_b(x_1-1)+R\frac{cw}{rg}}$

Table 5 provides notations $x_i, i = 1, ..., 7$ that are used in this proof. Most of them represent the intersection points.

We first calculate for J_1^* using the intersection points specified in Table 5:

$$J_1^* = J_1(s_b) = \begin{cases} R\Lambda_g, & s_b \le x_3 \\ R(\frac{\mu}{s_b} - \frac{c_w}{r_g}), & x_3 < s_b \le x_4 \\ 0, & s_b > x_4 \end{cases} \tag{20}$$

Now we solve for J^*. Note that the first and the third pieces of $J_2(\Phi)$ are both non-decreasing in Φ. The second piece is more complex. It decreases first and then increases. Let x_5 be its minimum point. Solving the first order condition of the second piece of $J_2(\Phi)$: $\frac{R\mu}{\Phi} + C\Lambda_b\Phi - R\Lambda_b - R\frac{c_w}{r_g} - C\Lambda_b$, we obtain $x_5 = \sqrt{\frac{R\mu}{C\Lambda_b}}$. Thus we have the following three cases.

1. $x_5 \le x_1$. Then the second piece of $J_2(\Phi)$ is non-decreasing in Φ, too. Thus,

$$J_2^* = J_2(s_b)$$

Due to Inequality (19), we obtain

$$J_2(s_b) \le J_1(s_b)$$

In addition, from the result (20), we have

$$J_2^* \le J_1^*$$

Thus, in this case,

$$J^* = J_1^* = \begin{cases} R\Lambda_g, & s_b \le x_3 \\ R(\frac{\mu}{s_b} - \frac{c_w}{r_g}), & x_3 < s_b \le x_4 \\ 0, & s_b > x_4 \end{cases} \tag{21}$$

and $\Phi^* = s_b$.

[8]Note that the screening probability might be zero here. In that case, if $\widehat{\Lambda}_g$ is positive, then the demand rate is Λ_g; else if $\widehat{\Lambda}_g$ is negative, the demand rate is zero.

2. $s_b \leq x_1 < x_5$. Then $[0, s_b)$ is the first piece of $J_2(\Phi)$. This piece is non-decreasing in Φ. Thus we have

$$J_2^* = J_2(s_b)$$

Due to Inequality (19), we obtain

$$J_2(s_b) \leq J_1(s_b)$$

From the result (20), we have

$$J_2^* \leq J_1^*$$

Note that $s_b \leq x_1 < x_3 < x_4$. Due to (21), we have

$$J^* = J_1^* = R\Lambda_g \text{ and } \Phi^* = s_b.$$

3. $x_5 > x_1$ and $s_b > x_1$. Then $J_2(\Phi)$ is non-decreasing in $[0, x_1]$. If $s_b \leq x_5$, $J_2(\Phi)$ is decreasing in $[x_1, s_b)$. If $s_b > x_5$, $J_2(\Phi)$ is non-increasing in $[x_1, x_5]$ and non-decreasing in $[x_5, s_b)$. Then,

$$J_2^* = \max(J_2(x_1), J_2(s_b))$$

Since $J_2(s_b) \leq J_1(s_b) = J_1^*$, we only need to compare the values of $J_2(x_1)$ and J_1^*. Note that $J_1^* \geq 0$. So we only need to compare the values of $J_2(x_1)$ and J_1^* when $J_2(x_1) \geq 0$. The sufficient and necessary condition for $J_2(x_1) \geq 0$ is

$$x_1 \geq 1 - \frac{R\Lambda_g}{C\Lambda_b} \doteq x_6 \text{ (because } J_2(x_1) \geq 0 \Longleftrightarrow R\Lambda_g + C\Lambda_b(x_1 - 1) \geq 0) \tag{22}$$

Next we analyze the following three cases according to the values of J_1^*.

(a) $s_b \leq x_3$. Then $J_1^* = R\Lambda_g$. Since $x_1 < s_b \leq 1$,

$$J_2(x_1) = R\Lambda_g + C\Lambda_b(x_1 - 1) < R\Lambda_g = J_1^*$$

Thus,

$$J^* = J_1^* = R\Lambda_g \text{ and } \Phi^* = s_b.$$

(b) $s_b > x_4$. Then $J_1^* = 0$. Due to Condition (22), we obtain:

$$J^* = \begin{cases} J_1^* = 0, & x_1 \leq x_6, \\ J_2(x_1), & x_1 > x_6, \end{cases}$$

and

$$\Phi^* = \begin{cases} s_b, & x_1 \leq x_6, \\ x_1, & x_1 > x_6. \end{cases}$$

(c) $x_3 < s_b \le x_4$. Then $J_1^* = R(\frac{\mu}{s_b} - \frac{c_w}{r_g}) \ge 0$. When $J_2(x_1) \ge 0$ (or equivalently, $x_1 \ge x_6$),

$$J_2(x_1) > J_1^* \Longleftrightarrow R\Lambda_g + C\Lambda_b(x_1 - 1) > R(\frac{\mu}{s_b} - \frac{c_w}{r_g})$$
$$\Longleftrightarrow s_b > \frac{R\mu}{R\Lambda_g + C\Lambda_b(x_1 - 1) + R\frac{c_w}{r_g}} \doteq x_7.$$

Note that when $J_2(x_1) \ge 0$ (or equivalently, $x_1 \ge x_6$), $x_7 \in (x_3, x_4]$, which is because

$$x_7 = \frac{R\mu}{J_2(x_1) + R\frac{c_w}{r_g}} \le \mu\frac{c_w}{r_g} = x_4$$

and

$$x_1 < 1 \text{ (because } 1 > s_b > x_1)$$
$$\Longrightarrow \frac{R\mu}{R\Lambda_g + C\Lambda_b(x_1 - 1) + R\frac{c_w}{r_g}} > \frac{R\mu}{R\Lambda_g + R\frac{c_w}{r_g}} = \frac{\mu}{\Lambda_g + \frac{c_w}{r_g}}$$
$$\Longrightarrow x_7 > x_3.$$

Thus we have

$$J^* = \begin{cases} R(\frac{\mu}{s_b} - \frac{c_w}{r_g}), & \begin{array}{l} x_1 \le x_6, \\ \text{or } x_1 > x_6 \text{ and } x_3 < s_b \le x_7 \end{array} \\ J_2(x_1), & x_1 > x_6 \text{ and } x_7 < s_b \le x_4 \end{cases}$$

and

$$\Phi^* = \begin{cases} s_b, & \begin{array}{l} x_1 \le x_6, \\ \text{or } x_1 > x_6 \text{ and } x_3 < s_b \le x_7 \end{array} \\ x_1, & x_1 > x_6 \text{ and } x_7 < s_b \le x_4 \end{cases}$$

Summarizing all the above results, we find that if and only if $x_7 < s_b$ and $\min(1, x_5) > x_1 > x_6$, $J^* = J_2(x_1)$and $\Phi^* = x_1$; otherwise, $J^* = J_1^*$ as defined by Equation (20), and $\Phi^* = s_b$.

8.5 Proof to Proposition 4

The non-discriminatory optimization problem is identical to the discriminatory optimization problem plus an additional constraint: $\Phi_1 = \Phi_2 = \Phi$.

8.6 Proof to Proposition 5

The proof follows trivially from the optimization problem (13) for $\theta = b$.

8.7 Proof to Proposition 6

The proof is exactly the same as the proof to Proposition 2 based on the updated expressions of $\widehat{\Lambda}_{1g}$ and $\widehat{\Lambda}_{2g}$ in Equations (15-16).

8.8 Proof to Proposition 7

According to Proposition 5, when the screening probability is larger than or equal to s'_b, none of bad potential applicants submit applications. In addition, the submission probability of good potential applicants decreases as the screening probability increases. Thus as a result, the approver will never get a strictly better payoff by performing a screening probability strictly larger than s'_b. Recall that we assume that when the approver is indifferent between different levels of screening probabilities, she will choose the lowest level. Therefore, the optimal non-discriminatory screening probability must be either s'_b, or some value in the interval $[0, s'_b)$. Note that the approver's objective function under the nondiscriminatory policy is

$$(1 - e_g\Phi)\lambda_g R - (1 - (1 - e_b)\Phi)\lambda_b C$$

1. When $\Phi = s'_b \leq 1$, then $\hat{p}_{1b} = \hat{p}_{2b} = 0$, there are no bad applicants. Then the traffic for screening is composed only of good applicants. Using Proposition 6(iii) and Equations (15-16), we obtain $\widehat{\Lambda}_g = \widehat{\Lambda}_{1g} = \widehat{\Lambda}_{2g} = \mu - \frac{s'_b c_w}{(1-e_g s'_b)r_g}$, and the demand rate of the screened good applicants is $\max(0, \min(\widehat{\Lambda}_g, s'_b\Lambda_g))$. Divided by $\Phi = s'_b$, the demand rate of the good applicants is $\max(0, \min(\frac{\widehat{\Lambda}_g}{s'_b}, \Lambda_g))$. Then the approver's objective value is:

$$(1 - e_g s'_b) \max(0, \min(\frac{\mu}{s'_b} - \frac{c_w}{(1 - e_g s'_b)r_g}, \Lambda_g))R.$$

2. When $\Phi \in [0, s'_b)$, then $\hat{p}_{1b} = \hat{p}_{2b} = 1$, all bad potential applicants are submitting their applications. Using Proposition 2(iii) and Equations (10-11), we obtain $\widehat{\Lambda}_g = \mu - \Phi\Lambda_b - \frac{\Phi c_w}{(1-e_g\Phi)r_g}$. Then the demand rate of the screened good applicants is $\max(0, \min(\widehat{\Lambda}_g, \Phi\Lambda_g))$. Divided by Φ, the demand rate of the good applicants is $\max(0, \min(\frac{\widehat{\Lambda}_g}{\Phi}, \Lambda_g))$.[9] The optimal strategy is to solve:

$$\max_{\Phi\in[0,s'_b)} (1 - e_g\Phi) \max(0, \min(\frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{(1 - e_g\Phi)r_g}, \Lambda_g))R - (1 - (1 - e_b)\Phi)\Lambda_b C$$

[9]Note that the screening probability might be zero here. In that case, $\widehat{\Lambda}_g = \mu > 0$, then the demand rate is Λ_g.

Now we rewrite the nondiscriminatory optimization problem. Let

$$J_1(\Phi) = (1 - e_g\Phi)R\max(0, \min(\frac{\mu}{\Phi} - \frac{c_w}{(1 - e_g\Phi)r_g}, \Lambda_g))$$
$$J_2(\Phi) = (1 - e_g\Phi)R\max(0, \min(\frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{(1 - e_g\Phi)r_g}, \Lambda_g)) + (1 - e_b)C\Lambda_b\Phi - C\Lambda_b$$

and

$$J(\Phi) = \begin{cases} J_1(\Phi), & \Phi = s'_b \\ J_2(\Phi), & 0 \le \Phi < s'_b \end{cases}$$

Thus the optimal non-discriminatory strategy is to solve

$$\max_{0\le\Phi\le1} J(\Phi)$$

Note that for a given value of $\Phi \in [0, 1]$, we have

$$J_1(\Phi) \ge J_2(\Phi) \tag{23}$$

Before analyzing the structure of the objective functions, we first show properties of some useful functions. We denote $A \triangleq \mu e_g > 0$, $B \triangleq \frac{c_w}{r_g} > 0$, $f_+(y) \triangleq \frac{(y+A+B)+\sqrt{(y+A+B)^2-4Ay}}{2e_g y}$, and $f_-(y) \triangleq \frac{(y+A+B)-\sqrt{(y+A+B)^2-4Ay}}{2e_g y}$. Then when $y > 0$,

$$\begin{aligned} \frac{df_+(y)}{dy} &= \frac{1}{2e_g}[-\frac{A+B}{y^2} + \frac{1}{y^2}(\frac{(y+A+B)-2A}{\sqrt{(y+A+B)^2-4Ay}}y - \sqrt{(y+A+B)^2-4Ay})] \\ &= \frac{1}{2e_g y^2}[-A-B+\frac{(y+A+B)y-2Ay-(y+A+B)^2+4Ay}{\sqrt{(y+A+B)^2-4Ay}}] \\ &= \frac{1}{2e_g y^2}[-A-B+\frac{(A-B)y-(A+B)^2}{\sqrt{(y+A+B)^2-4Ay}}] \end{aligned}$$

$$\begin{aligned} \frac{df_+(y)}{dy} &< 0 \Leftrightarrow -A-B+\frac{(A-B)y-(A+B)^2}{\sqrt{(y+A+B)^2-4Ay}} < 0 \\ &\Leftrightarrow (A-B)y-(A+B)^2 < (A+B)\sqrt{(y+A+B)^2-4Ay} \end{aligned}$$

If $(A-B)y-(A+B)^2 < 0$, the above inequality obviously holds. Else if $(A-B)y-(A+B)^2 \ge 0$,

$$\begin{aligned} (A-B)y-(A+B)^2 &< (A+B)\sqrt{(y+A+B)^2-4Ay} \\ &\Leftrightarrow ((A-B)y-(A+B)^2)^2 < (A+B)^2((y+A+B)^2-4Ay) \\ &\Leftrightarrow -4ABy^2 < 0 \text{ (which obviously holds)} \end{aligned}$$

Thus $\frac{df_+(y)}{dy} < 0$ holds. Similarly, we can prove $\frac{df_-(y)}{dy} < 0$ holds.

$$\begin{aligned}
\lim_{y\to+\infty} f_+(y) &= \frac{1}{2e_g}\lim_{y\to+\infty}\frac{(y+A+B)+\sqrt{(y+A+B)^2-4Ay}}{y} \\
&= \frac{1}{2e_g}+\frac{1}{2e_g}\lim_{y\to+\infty}\frac{\sqrt{(y+A+B)^2-4Ay}}{y}=\frac{1}{e_g}
\end{aligned}$$

and

$$\begin{aligned}
\lim_{y\to 0^+} f_-(y) &= \frac{1}{2e_g}\lim_{y\to 0^+}\frac{(y+A+B)-\sqrt{(y+A+B)^2-4Ay}}{y} \\
&= \frac{1}{2e_g}\lim_{y\to 0^+}\Big(1-\frac{1}{2}\frac{2(y+A+B)-4A}{\sqrt{(y+A+B)^2-4Ay}}\Big) \text{ (Using l'Hospital's rule)} \\
&= \frac{A}{A+B}\cdot\frac{1}{e_g}
\end{aligned}$$

We obtain:

$$f_+(y) > \frac{1}{e_g} \geq 1 \text{ and } f_-(y) < \frac{A}{A+B}\cdot\frac{1}{e_g}, \text{ when } y>0$$

Summarizing the above results, we have the following lemma:

Lemma 1 *For* $f_+(y) \triangleq \frac{(y+A+B)+\sqrt{(y+A+B)^2-4Ay}}{2e_g y}$, *and* $f_-(y) \triangleq \frac{(y+A+B)-\sqrt{(y+A+B)^2-4Ay}}{2e_g y}$, *where* $A>0$ *and* $B>0$, $y>0$,

1. *Both* $f_+(y)$ *and* $f_-(y)$ *are strictly decreasing in* y.
2. $f_+(y) > \frac{1}{e_g} \geq 1$.
3. $0 < f_-(y) < \frac{A}{A+B}\cdot\frac{1}{e_g}$.

Note that

$$\begin{aligned}
\frac{\mu}{\Phi}-\Lambda_b-\frac{c_w}{(1-e_g\Phi)r_g} < \Lambda_g &\Longleftrightarrow \frac{\mu}{\Phi}-\frac{c_w}{(1-e_g\Phi)r_g} < \Lambda_b+\Lambda_g \\
&\Longleftrightarrow \mu(1-e_g\Phi)-\frac{c_w}{r_g}\Phi < (1-e_g\Phi)\Phi(\Lambda_b+\Lambda_g) \\
&\Longleftrightarrow (\Lambda_b+\Lambda_g)e_g\Phi^2-(\Lambda_b+\Lambda_g+\mu e_g+\frac{c_w}{r_g})\Phi+\mu < 0
\end{aligned}$$

The roots of the above inequality are

$$\frac{(\Lambda_b+\Lambda_g+\mu e_g+\frac{c_w}{r_g})\pm\sqrt{(\Lambda_b+\Lambda_g+\mu e_g+\frac{c_w}{r_g})^2-4\mu(\Lambda_b+\Lambda_g)e_g}}{2(\Lambda_b+\Lambda_g)e_g} = f_\pm(\Lambda_b+\Lambda_g).$$

Due to Lemma 1 and because $\Phi \in [0,1]$, we obtain that

$$\begin{cases} \frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{(1-e_g\Phi)r_g} < \Lambda_g, & f_-(\Lambda_b+\Lambda_g) < \Phi \le 1 \\ \frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{(1-e_g\Phi)r_g} \ge \Lambda_g, & 0 \le \Phi \le f_-(\Lambda_b+\Lambda_g) \end{cases}$$

Similarly,

$$\frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{(1-e_g\Phi)r_g} < 0 \Longleftrightarrow \Lambda_b e_g \Phi^2 - (\Lambda_b + \mu e_g + \frac{c_w}{r_g})\Phi + \mu < 0$$

The roots of the above inequality are

$$\frac{(\Lambda_b + \mu e_g + \frac{c_w}{r_g}) \pm \sqrt{(\Lambda_b + \mu e_g + \frac{c_w}{r_g})^2 - 4\mu\Lambda_b e_g}}{2\Lambda_b e_g} = f_\pm(\Lambda_b).$$

Due to Lemma 1 and because $\Phi \in [0,1]$, we obtain that

$$\begin{cases} \frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{(1-e_g\Phi)r_g} < 0, & f_-(\Lambda_b) < \Phi \le 1 \\ \frac{\mu}{\Phi} - \Lambda_b - \frac{c_w}{(1-e_g\Phi)r_g} \ge 0, & 0 \le \Phi \le f_-(\Lambda_b) \end{cases}$$

Now we are able to rewrite $J_2(\Phi)$ when $\Phi \in [0,1]$ as:

$$J_2(\Phi) = \begin{cases} [(1-e_b)C\Lambda_b - e_g R\Lambda_g]\Phi + R\Lambda_g - C\Lambda_b, & 0 \le \Phi \le f_-(\Lambda_b+\Lambda_g) \\ \frac{\mu R}{\Phi} + [e_g R\Lambda_b + (1-e_b)C\Lambda_b]\Phi - (\frac{c_w}{r_g}R + e_g R\mu + R\Lambda_b + C\Lambda_b), & f_-(\Lambda_b+\Lambda_g) < \Phi \le f_-(\Lambda_b) \\ (1-e_b)C\Lambda_b\Phi - C\Lambda_b & f_-(\Lambda_b) < \Phi \le 1 \end{cases}$$

Obviously, when $0 \le \Phi \le f_-(\Lambda_b+\Lambda_g)$

$$\begin{aligned} &\max_{0\le\Phi\le f_-(\Lambda_b+\Lambda_g)} J_2(\Phi) \\ = &\begin{cases} J_2(0) = R\Lambda_g - C\Lambda_b, & (1-e_b)C\Lambda_b \le e_g R\Lambda_g \\ J_2(f_-(\Lambda_b+\Lambda_g)) = [(1-e_b)C\Lambda_b - e_g R\Lambda_g]f_-(\Lambda_b+\Lambda_g) + R\Lambda_g - C\Lambda_b, & (1-e_b)C\Lambda_b > e_g R\Lambda_g \end{cases} \end{aligned}$$

When $f_-(\Lambda_b+\Lambda_g) \le \Phi \le f_-(\Lambda_b)$, $\frac{d^2 J_2(\Phi)}{d\Phi^2} = \frac{2\mu R}{\Phi^3} > 0$, which implies $J_2(\Phi)$ is strictly convex in $[f_-(\Lambda_b+\Lambda_g), f_-(\Lambda_b)]$. Therefore

$$\max_{f_-(\Lambda_b+\Lambda_g)\le\Phi\le f_-(\Lambda_b)} J_2(\Phi) = \max\{J_2(f_-(\Lambda_b+\Lambda_g)), J_2(f_-(\Lambda_b))\}.$$

When $f_-(\Lambda_b) \le \Phi \le 1$, $J_2(\Phi)$ is increasing in Φ. So $\max_{f_-(\Lambda_b)\le\Phi\le 1} J_2(\Phi) = J_2(1)$, from which we can see $f_-(\Lambda_b)$ is dominated by 1.

In summary, the optimal screening probability can only take a value among $\{0, f_-(\Lambda_b+\Lambda_g), 1\}$.

Now we analyze $J_1(s'_b)$ when $s'_b \le 1$.

Since

$$\frac{\mu}{s_b'} - \frac{c_w}{(1-e_g s_b')r_g} < \Lambda_g \Longleftrightarrow \Lambda_g e_g (s_b')^2 - (\Lambda_g + \mu e_g + \frac{c_w}{r_g})s_b' + \mu < 0$$

Taking s_b' as the independent variable, the roots of the above inequality are

$$\frac{(\Lambda_g + \mu e_g + \frac{c_w}{r_g}) \pm \sqrt{(\Lambda_g + \mu e_g + \frac{c_w}{r_g})^2 - 4\mu\Lambda_g e_g}}{2\Lambda_b e_g} = f_\pm(\Lambda_g).$$

Due to Lemma 1 and the fact $s_b' \leq 1$, we obtain that when $f_-(\Lambda_g) < s_b' \leq 1$, $\frac{\mu}{s_b'} - \frac{c_w}{(1-e_g s_b')r_g} < \Lambda_g$; when $0 \leq s_b' \leq f_-(\Lambda_g)$, $\frac{\mu}{s_b'} - \frac{c_w}{(1-e_g s_b')r_g} \geq \Lambda_g$. In addition, we have

$$\begin{aligned} \frac{\mu}{s_b'} - \frac{c_w}{(1-e_g s_b')r_g} \quad &< \quad 0 \Longleftrightarrow (1-e_g s_b')\mu - \frac{c_w s_b'}{r_g} < 0 \\ &\Longleftrightarrow \quad s_b' > \frac{\mu}{\mu e_g + \frac{c_w}{r_g}} \end{aligned}$$

Thus we rewrite $J_1(s_b')$ when $s_b' \leq 1$ as:

$$J_1(s_b) = \begin{cases} (1-e_g s_b')R\Lambda_g, & 0 \leq s_b' \leq f_-(\Lambda_g) \\ \frac{\mu}{s_b'}R - (\mu e_g + \frac{c_w}{r_g})R, & f_-(\Lambda_g) < s_b' \leq \frac{\mu}{\mu e_g + \frac{c_w}{r_g}} \\ 0 & \frac{\mu}{\mu e_g + \frac{c_w}{r_g}} < s_b' \leq 1 \end{cases}$$

Next we discuss $J^* = \max_{0 \leq \Phi \leq 1} J(\Phi)$.

Case i: when $s_b' > 1$ (or equivalently, $e_b > \frac{c_b}{c_b + r_b}$), all bad potential applicants are submitting their applications ($\hat{p}_b = 1$), then $J^* = \max_{0 \leq \Phi \leq 1} J_2(\Phi)$:

1. $f_-(\Lambda_b + \Lambda_g) > 1$. Then $0 \leq \Phi < f_-(\Lambda_b + \Lambda_g)$ always holds. So from the above analysis, we obtain

$$\begin{aligned} J_2^* \quad &= \quad \max\{J_2(0), J_2(1)\} \\ &= \quad \max\{R\Lambda_g - C\Lambda_b, (1-e_g)R\Lambda_g - e_b C\Lambda_b\}. \end{aligned}$$

   Therefore if $\frac{R\Lambda_g}{C\Lambda_b} \geq \frac{1-e_b}{e_g}$, then $\Phi^* = 0$; else if $\frac{R\Lambda_g}{C\Lambda_b} < \frac{1-e_b}{e_g}$, then $\Phi^* = 1$.

2. $f_-(\Lambda_b + \Lambda_g) \leq 1 \leq f_-(\Lambda_b)$. Then from the above analysis, we obtain

$$\begin{aligned} J_2^* \quad &= \quad \max\{J_2(0), J_2(f_-(\Lambda_b + \Lambda_g)), J_2(1)\} \\ &= \quad \max\{R\Lambda_g - C\Lambda_b, [(1-e_b)C\Lambda_b - e_g R\Lambda_g]f_-(\Lambda_b + \Lambda_g) + R\Lambda_g - C\Lambda_b, \\ &\qquad (1-e_g)(\mu - \Lambda_b)R - e_b C\Lambda_b - \frac{c_w}{r_g}R\} \end{aligned}$$

Therefore if $\frac{R\Lambda_g}{C\Lambda_b} \geq \frac{1-e_b}{e_g}$, then $J_2^* = \max\{J_2(0), J_2(1)\}$. In this case if $[\Lambda_g - (1-e_g)(\mu - \Lambda_b) + \frac{c_w}{r_g}]R \geq (1-e_b)C\Lambda_b$, $\Phi^* = 0$; if $[\Lambda_g - (1-e_g)(\mu-\Lambda_b) + \frac{c_w}{r_g}]R < (1-e_b)C\Lambda_b$, $\Phi^* = 1$.

If $\frac{R\Lambda_g}{C\Lambda_b} < \frac{1-e_b}{e_g}$, then $J_2^* = \max\{J_2(f_-(\Lambda_b+\Lambda_g)), J_2(1)\}$. In this case if $f_-(\Lambda_b+\Lambda_g) \geq \frac{[(1-e_g)(\mu-\Lambda_b)-\Lambda_g-\frac{c_w}{r_g}]R+(1-e_b)C\Lambda_b}{(1-e_b)C\Lambda_b-e_gR\Lambda_g}$, $\Phi^* = f_-(\Lambda_b+\Lambda_g)$; if $f_-(\Lambda_b+\Lambda_g) < \frac{[(1-e_g)(\mu-\Lambda_b)-\Lambda_g-\frac{c_w}{r_g}]R+(1-e_b)C\Lambda_b}{(1-e_b)C\Lambda_b-e_gR\Lambda_g}$, $\Phi^* = 1$.

3. $f_-(\Lambda_b+\Lambda_g) < f_-(\Lambda_b) < 1$. Then from the above analysis, we obtain

$$\begin{aligned} J_2^* &= \max\{J_2(0), J_2(f_-(\Lambda_b+\Lambda_g)), J_2(1)\} \\ &= \max\{R\Lambda_g - C\Lambda_b, [(1-e_b)C\Lambda_b - e_gR\Lambda_g]f_-(\Lambda_b+\Lambda_g) + R\Lambda_g - C\Lambda_b, -e_bC\Lambda_b\} \end{aligned}$$

If $\frac{R\Lambda_g}{C\Lambda_b} \geq \frac{1-e_b}{e_g}$, then $J_2^* = \max\{J_2(0), J_2(1)\}$. In this case since $\frac{R\Lambda_g}{C\Lambda_b} \geq \frac{1-e_b}{e_g} \geq 1-e_b$, $J_2(0) \geq J_2(1)$ and then $\Phi^* = 0$.

If $\frac{R\Lambda_g}{C\Lambda_b} < \frac{1-e_b}{e_g}$, then $J_2^* = \max\{J_2(f_-(\Lambda_b+\Lambda_g)), J_2(1)\}$. In this case if $f_-(\Lambda_b+\Lambda_g) \geq \frac{(1-e_b)C\Lambda_b - R\Lambda_g}{(1-e_b)C\Lambda_b-e_gR\Lambda_g}$, $\Phi^* = f_-(\Lambda_b+\Lambda_g)$; else if $f_-(\Lambda_b+\Lambda_g) < \frac{(1-e_b)C\Lambda_b-R\Lambda_g}{(1-e_b)C\Lambda_b-e_gR\Lambda_g}$, $\Phi^* = 1$.

Case ii: when $s_b' \leq 1$ (or equivalently, $e_b \leq \frac{c_b}{c_b+r_b}$), $J^* = \max\{J_1(s_b'), \max_{0\leq\Phi\leq s_b} J_2(\Phi)\}$. Note that $\forall s_b' \in (0,1]$, we have $J_2(s_b') < J_1(s_b')$ (due to the inequality (23)). Then we obtain:

$$J^*(\Phi) = \begin{cases} \max\{J_2(0), J_1(s_b')\} & 0 < s_b' \leq f_-(\Lambda_b+\Lambda_g) \\ \max\{J_2(0), J_2(f_-(\Lambda_b+\Lambda_g)), J_1(s_b')\}, & f_-(\Lambda_b+\Lambda_g) < s_b' \leq 1 \end{cases}$$

Case ii has the following subcases.

1. If $0 < s_b' \leq f_-(\Lambda_b+\Lambda_g)(< f_-(\Lambda_g))$, $J_2(0) = R\Lambda_g - C\Lambda_b$ and $J_1(s_b') = (1-e_gs_b')R\Lambda_g$. Then when $\frac{R\Lambda_g}{C\Lambda_b} \geq \frac{1}{e_gs_b'}$, $\Phi^* = 0$; when $\frac{R\Lambda_g}{C\Lambda_b} < \frac{1}{e_gs_b'}$, $\Phi^* = s_b'$.

2. If $f_-(\Lambda_b+\Lambda_g) < s_b' \leq 1$, $J^*(\Phi) = \max\{J_2(0), J_2(f_-(\Lambda_b+\Lambda_g)), J_1(s_b')\}$.

2.1. $\frac{R\Lambda_g}{C\Lambda_b} \geq \frac{1-e_b}{e_g}$, then $J_2(0) \geq J_2(f_-(\Lambda_b+\Lambda_g))$ and thus $J^*(\Phi) = \max\{J_2(0), J_1(s_b')\}$

2.1.1. $s_b' \leq f_-(\Lambda_g)$, $J_1(s_b') = (1-e_gs_b')R\Lambda_g$, then when $\frac{R\Lambda_g}{C\Lambda_b} \geq \frac{1}{e_gs_b'}$, $\Phi^* = 0$; when $\frac{R\Lambda_g}{C\Lambda_b} < \frac{1}{e_gs_b'}$, $\Phi^* = s_b'$.

2.1.2. $f_-(\Lambda_g) < s_b' \leq \frac{\mu}{\mu e_g + \frac{c_w}{r_g}}$, $J_1(s_b') = \frac{\mu}{s_b'}R - (\mu e_g + \frac{c_w}{r_g})R$, then when $(\Lambda_g - \frac{\mu}{s_b'} + \mu e_g + \frac{c_w}{r_g})R \geq C\Lambda_b$, $\Phi^* = 0$; when $(\Lambda_g - \frac{\mu}{s_b'} + \mu e_g + \frac{c_w}{r_g})R < C\Lambda_b$, $\Phi^* = s_b'$.

2.1.3 $\frac{\mu}{\mu e_g + \frac{c_w}{r_g}} < s_b' \leq 1$, $J_1(s_b') = 0$, then when $\frac{R\Lambda_g}{C\Lambda_b} \geq 1$, $\Phi^* = 0$; when $\frac{R\Lambda_g}{C\Lambda_b} < 1$, $\Phi^* = s_b'$.

2.2. $\frac{R\Lambda_g}{C\Lambda_b} < \frac{1-e_b}{e_g}$, then $J_2(0) < J_2(f_-(\Lambda_b+\Lambda_g))$ and thus $J^*(\Phi) = \max\{J_2(f_-(\Lambda_b+\Lambda_g)), J_1(s_b')\}$.

2.2.1. $s_b' \leq f_-(\Lambda_g)$, $J_1(s_b') = (1-e_gs_b')R\Lambda_g$. Since $f_-(\Lambda_b+\Lambda_g) < s_b' \leq 1 \leq \frac{C\Lambda_b - e_gs_b'R\Lambda_g}{(1-e_b)C\Lambda_b-e_gR\Lambda_g}$, we obtain $J_2(f_-(\Lambda_b+\Lambda_g)) < J_1(s_b')$ and thus $\Phi^* = s_b'$.

2.2.2. $f_-(\Lambda_g) < s_b' \le \frac{\mu}{\mu e_g + \frac{c_w}{r_g}}$, $J_1(s_b) = \frac{\mu}{s_b'}R - (\mu e_g + \frac{c_w}{r_g})R$, then when $f_-(\Lambda_b + \Lambda_g) \ge \frac{C\Lambda_b + (\frac{\mu}{s_b'} - \mu e_g - \frac{c_w}{r_g} - \Lambda_g)R}{(1-e_b)C\Lambda_b - e_g R\Lambda_g}$, $\Phi^* = f_-(\Lambda_b + \Lambda_g)$; when $f_-(\Lambda_b + \Lambda_g) < \frac{C\Lambda_b + (\frac{\mu}{s_b'} - \mu e_g - \frac{c_w}{r_g} - \Lambda_g)R}{(1-e_b)C\Lambda_b - e_g R\Lambda_g}$, $\Phi^* = s_b'$.

2.2.3 $\frac{\mu}{\mu e_g + \frac{c_w}{r_g}} < s_b' \le 1$, $J_1(s_b') = 0$, then when $f_-(\Lambda_b + \Lambda_g) \ge \frac{C\Lambda_b - R\Lambda_g}{(1-e_b)C\Lambda_b - e_g R\Lambda_g}$, $\Phi^* = f_-(\Lambda_b + \Lambda_g)$; when $f_-(\Lambda_b + \Lambda_g) < \frac{C\Lambda_b - R\Lambda_g}{(1-e_b)C\Lambda_b - e_g R\Lambda_g}$, $\Phi^* = s_b'$.