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A COMPARISON OF SOLUTION STRATEGIES FOR SIMULATION-BASED MULTIDISCIPLINARY DESIGN OPTIMIZATION

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Abstract

The design of multidisciplinary systems (such as aircraft, automobiles, and others) often requires an iterative cycle that includes a design initialization, a system analysis, a sensitivity analysis, and design optimization. This design cycle is standard in the field of Multidisciplinary Design Optimization (MDO) and has often been referred to in literature as the "Multiple-Discipline-Feasible" (MDF) approach. The name stems from the fact that complete multidisciplinary feasibility is maintained each and every design cycle. The drawback of MDF is that it can be a timely and a costly procedure. Numerous researchers have developed alternate means for posing and subsequently solving the multidisciplinary design problem. One such solution procedure has been referred to both as "Simultaneous Analysis and Design" (SAND) and "All-at-Once" (AAO), and treats the entire multidisciplinary design cycle as one large optimization problem. Another alternate solution procedure has been referred to as "Individual-Discipline-Feasible" (IDF); this procedure exhibits characteristics which lie in between the two extremes exemplified by MDF and AAO. IDF assures that each individual discipline is feasible on every design cycle, while driving the entire system (all disciplines) towards multidisciplinary feasibility. The present work will present a rigorous numerical comparison of these solution strategies over a wide variety of problem sizes and complexities. The multidisciplinary design test problems that are used for these comparisons are generated by a robust simulation tool called CASCADE.

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Background and Motivation

Concurrent engineering is a systematic approach to the integrated, simultaneous design of products and their related processes. Many of the recently developed capabilities to address concurrent engineering have stemmed from the emerging area of Multidisciplinary Design Optimization, or MDO. The origins of MDO can be traced back to the early 1980's, where Sobieski[14] used a linear decomposition approach to subdivide the design of a large engineering system into a grouping of related and more manageable subsystems. However, such a decomposition often results in a grouping of subsystems which cannot be placed into a definitive top-down hierarchy. The resultant decomposition grouping is typically hybrid-hierarchic in nature as shown in the example decomposition of Figure 1.

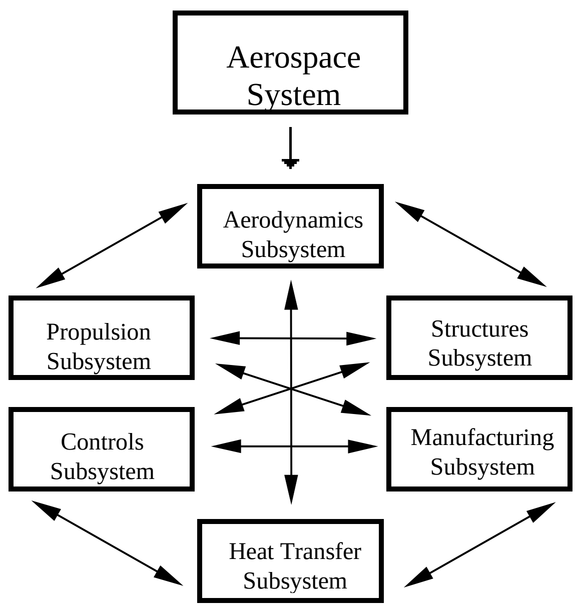

Figure 1: Hybrid-hierarchic decomposition

This inherent lack of hierarchy requires that the system analysis (associated with the overall design cycle) be initialized to some set of values, and iteratively converged thereafter. Subsequent to attaining a converged analysis solution, a sensitivity analysis is performed. The sensitivity analysis can be a numerical procedure such as finite differencing or the Global Sensitivity Equation (GSE)[15,4] method. The sensitivity analysis is required for the optimization of the overall design. The optimization step itself will typically cause certain optimization variables to change, which then necessitates the re-convergence of the system analysis. Hence, the entire design cycle repeats itself until a converged solution is attained. A summary of such non-hierarchic design synthesis is illustrated in Figure 2.

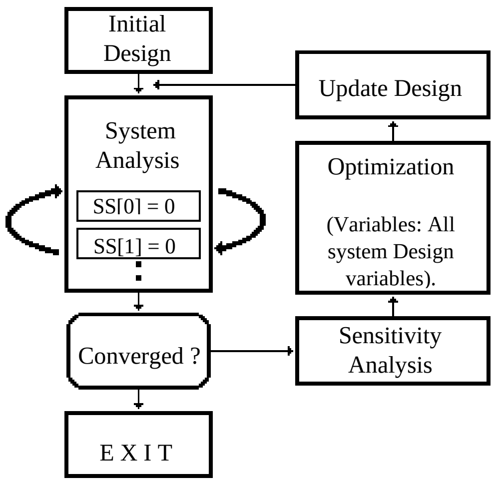

Figure 2: Non-hierarchic design synthesis - The "Multiple-Discipline-Feasible" (MDF) strategy

The design cycle that has just been described has been referred to in literature as the "Multiple-Discipline-Feasible", or MDF approach[1,5,6]. It has been demonstrated more than any other approach on non-hierarchic multidisciplinary examples[1]. The advantages of the MDF approach include its commonality to most MDO researchers, and its optimization problem, which treats only design variables (and **not** behavior variables) as optimization variables. The primary disadvantage of the MDF approach is that it is potentially very time and cost consuming. At each optimization iteration, complete multidisciplinary feasibility is enforced[6]. At each design cycle, a great deal of time may be inefficiently spent while fully re-converging the system analysis portion of a design that is still very far from its optimal solution.

More recently, researchers have focused on alternate methods for posing and solving the multidisciplinary design problem. An approach has been developed which treats the entire multidisciplinary design cycle seen in Figure 2 as a single large optimization problem. This is accomplished by converting the system analysis equations into equality constraints, and by treating both system design variables and subsystem outputs (behavior variables) as optimization variables. Such an approach has been referred to in literature as both "Simultaneous Analysis and Design" (SAND)[1,5,6,8] and "All-at-Once" (AAO)[1,5,6]. The primary advantage of AAO is the elimination of an iterative design cycle for attaining an optimal design through the outright elimination of costly iterative analysis evaluations. One possible disadvantage of AAO is that a much more complicated optimization problem results. More optimization variables and more equality constraints are present in the AAO formulation. These variables and equations stem from the addition of the system analysis equations to the optimization problem statement. A second disadvantage is that disciplinary feasibility is only attained at a relative or at an absolute extremum. This reduces the possibility of attaining a valid design solution if the optimizer is unsuccessful in attaining the global optimum solution. A generalized summary of the AAO strategy is seen in Figure 3. Notice that the "Residual Evaluator" has replaced the iterative System Analysis seen in Figure 2. In the Residual Evaluator, the analysis equality constraints are posed.

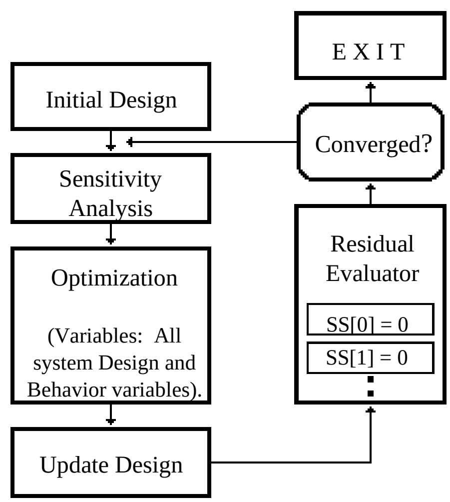

Figure 3: The "All-at-Once" (AAO) strategy

Another alternative to the classical MDF approach exhibits characteristics which lie between the extremes of MDF and AAO. Recall that MDF requires full disciplinary feasibility at each and every optimization iteration, while AAO only enforces disciplinary feasibility at the final solution (a local or global optimum), if attained. An intermediate approach is called "Individual-Discipline-Feasible" (IDF)[1,5,6,9]. With IDF, each individual discipline (or subsystem) is independently feasible at every optimization iteration. The optimizer eventually drives all of the individual disciplines towards multidisciplinary feasibility by controlling the interdisciplinary data[6]. In this formulation, all coupling variables (behavior variables that are required inputs to other subsystems) are promoted to being optimization variables. This takes place by temporarily substituting a replacement "surrogate" variable for each coupling variable in the optimization problem statement. Auxiliary equality constraints are added to the problem formulation to ensure that each and every behavior variable is equal to its corresponding surrogate variable, at optimization convergence. These constraints may be thought of as "equilibrium" constraints. A generalized summary of the IDF strategy is seen in Figure 4. Notice that the "Analysis Solver" has replaced the iterative System Analysis seen in Figure 2. In the Analysis Solver, both the single analysis solution (non-iterative) and the equilibrium constraint formulation take place.

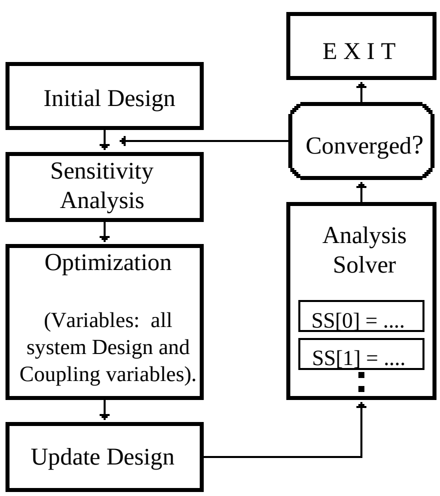

Figure 4: The "Individual-Discipline-Feasible" (IDF) strategy

With a fundamental understanding of each of the three solution strategies, the authors' means for obtaining a wide variety of coupled multidisciplinary test systems can now be discussed. For this task, a robust simulation tool called CASCADE[10,11] has been used.

Simulation of coupled systems

A number of previous research efforts involving the comparison of the MDF, IDF, and AAO solution strategies have been limited to a classical two subsystem multidisciplinary system, such as an aero-elastic example. It is desired to assess the utility of these solution strategies over a wide variety of system sizes and subsystem-level coupling densities. It is only after such testing is completed that heuristic rules can be developed which will govern the appropriateness of a given solution strategy for a given set of coupled system characteristics.

The rigorous testing of these solution strategies requires the use of a wide variety of stable test systems. For this reason, the CASCADE simulator has been used for the generation of analytical test systems. CASCADE is an acronym which stands for "Complex Application Simulator for the Creation of Analytical Design Equations". A thorough description of CASCADE can be found in past literature. A brief overview will be presented here for completeness.

CASCADE is a computer tool that generates a coupled system that consists of analytical equations of user-specified size. CASCADE has the capability of generating equations which represent both a coupled system analysis and an associated coupled optimization problem. The optimization portion consists of an objective function, inequality constraint functions, and side constraints. The intended use of such systems is for the advancement of any research work that involves the use of simulated coupled system behavior. CASCADE-generated test systems have been used for Multidisciplinary Design Optimization research involving sequencing[12], convergence[12], and coupling suspension[7] strategies, response surface methods[3], parallel processing with Java[2], distributed computing with PVM[10,11], design space visualization via graph morphing[17], multidisciplinary data fusion, and others. CASCADE can be accessed on the World Wide Web at the MDO Test Suite[13].

For clarity, a sample CASCADE system is presented as follows (Figure 5).

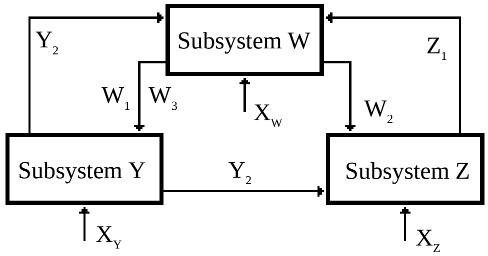

Figure 5: Decomposed three-subsystem coupled system

Figure 5 consists of a multidisciplinary system that has been decomposed into three inter-related subsystems. Each subsystem has its own set of independent design variables as input (the "X's"), as well as dependent behavior variables - outputs from other subsystems - also serving as input (the "W's", "Y's", and "Z's"). The CASCADE-generated analysis equations that exhibit the coupled behavior illustrated in Figure 5 might appear as in Equation [1]:

$$W_1 = 0.22X_W^1 + 0.05Y_2^3 - 0.46Z_1^2 + 0.73(X_W Y_2)^1$$
$$W_2 = -0.96Y_2^2 + 0.56Z_1^3 - 0.03(X_W Y_2)^2$$
$$W_3 = 0.36X_W^2 + 0.93(Z_1 Y_2)^2$$
$$Y_1 = 0.08X_Y^3 - 0.05W_1^1 + 0.11W_3^1 - 0.09(X_Y W_1)^3$$
$$Y_2 = 0.59W_3^1 + 0.41W_1^2 + 0.99(X_Y W_3)^3$$
$$Z_1 = -0.43X_Z^2 - 0.88W_2^2 + 0.25(W_2 X_Z)^2$$

[1]

Note that each behavior variable output equation is a polynomial function of both single-coupling terms and double-coupling (interaction) terms. The CASCADE-generated optimization problem is a function of the same design and behavior variables that are found in the analysis equations, and might appear as in Equations [2] and [3]:

Minimize:

$$F = 0.04X_W^2 + 0.96X_Y^3 + 0.15X_Z^1 - 0.26W_1^2 + 0.44W_2^1 + 0.57W_3^3 - 0.07Y_1^1 + 0.68Y_2^2 - 0.02Z_1^3$$

[2]

Subject to:

$$g_W = -578.9 + 0.36Y_2^3 + 0.55\,X_W^1 + 0.09(X_W Z_1)^3 \le 0$$
$$g_Y = -226.7 + 0.26X_Y^3 + 0.51\,W_1^2 + 0.53(X_Y W_3)^1 \le 0$$
$$g_Z = -1095.1 + 0.33\,Y_2^2 + 0.47(X_Z W_2)^1 \le 0$$
$$-9999.0 \le X_W, X_Y, X_Z \le +9999.0$$

[3]

Note again the presence of both single-coupling and interaction terms in the objective and inequality constraint functions. CASCADE generates the inequality constraints around the converged system analysis in such away that the initial design point is a feasible design point. Once generated, the analysis equations, objective function and constraint functions are written to three separate output files, each of which is a compilable ANSI C-based header file. The utility of these functions is at the discretion of the user. For the present research, the functions are used to numerically compare and contrast the MDF, IDF, and AAO solution strategies. The next section will present an example CASCADE system, and a corresponding problem statement for each of the three solution strategies.

Integration of CASCADE with the multidisciplinary solution strategies

To further illustrate the nature of the present research, the problem statements for each of the three solution strategies will be presented, corresponding to Figure 5 and Equations [1] through [3].

MDF

- Optimization variables: X_W, X_Y, and X_Z
- Analysis: Equation [1]. Full iterative re - convergence every MDO cycle.
- Optimization: Equations [2] and [3].
- Comment: The problem is solved as posed in equations [1] through [3] and in a cyclic manner similar to that seen in Figure 2.

IDF

- Optimization variables: X_W, X_Y, X_Z, and surrogate variables X_{W1}, X_{W2}, X_{W3}, X_{Y2}, and X_{Z1}
- Analysis: A single non-iterative "solution" of Equation [1] on each cycle, modified as follows. Note the presence of replacement "surrogate" variables on the right hand side of the equations:

$$W_1 = 0.22X_W^1 + 0.05X_{Y2}^3 - 0.46X_{Z1}^2 + 0.73(X_W X_{Y2})^1$$
$$W_2 = -0.96X_{Y2}^2 + 0.56X_{Z1}^3 - 0.03(X_W X_{Y2})^2$$
$$W_3 = 0.36X_W^2 + 0.93(X_{Z1} X_{Y2})^2$$
$$Y_1 = 0.08X_Y^3 - 0.05X_{W1}^1 + 0.11X_{W3}^1 - 0.09(X_Y X_{W1})^3$$
$$Y_2 = 0.59X_{W3}^1 + 0.41X_{W1}^2 + 0.99(X_Y X_{W3})^3$$
$$Z_1 = -0.43X_Z^2 - 0.88X_{W2}^2 + 0.25(X_{W2} X_Z)^2$$

- Optimization:

Minimize:
$F = 0.04X_W^2 + 0.96X_Y^3 + 0.15X_Z^1 - 0.26X_{W1}^2 + 0.44X_{W2}^1 + 0.57X_{W3}^3 - 0.07X_{Y1}^1 + 0.68X_{Y2}^2 - 0.02X_{Z1}^3$

Subject to:
$g_W = -578.9 + 0.36X_{Y2}^3 + 0.55X_W^1 + 0.09(X_W X_{Z1})^3 \leq 0$
$g_Y = -226.7 + 0.26X_Y^3 + 0.51X_{W1}^2 + 0.53(X_Y X_{W3})^1 \leq 0$
$g_Z = -1095.1 + 0.33X_{Y2}^2 + 0.47(X_Z X_{W2})^1 \leq 0$
$0 = X_{W1} - W_1$
$0 = X_{W2} - W_2$
$0 = X_{W3} - W_3$
$0 = X_{Y2} - Y_2$
$0 = X_{Z1} - Z_1$
$-9999. \leq X_W, X_Y, X_Z, X_{W1}, X_{W2}, X_{W3}, X_{Y2}, X_{Z1} \leq 9999.$

- Comment: Note that X_{Y1} is not an optimization variable, since it is not used as input by any subsystem in the analysis equations.

AAO

- Optimization variables: X_W, X_Y, X_Z, W_1, W_2, W_3, Y_1, Y_2, and Z_1
- Analysis: None. ("Analysis" is included in the optimization problem).
- Optimization:

Minimize:
$F = 0.04X_W^2 + 0.96X_Y^3 + 0.15X_Z^1 - 0.26W_1^2 + 0.44W_2^1 + 0.57W_3^3 - 0.07Y_1^1 + 0.68Y_2^2 - 0.02Z_1^3$

Subject to:
$g_W = -578.9 + 0.36Y_2^3 + 0.55\, X_W^1 + 0.09(X_W Z_1)^3 \leq 0$
$g_Y = -226.7 + 0.26X_Y^3 + 0.51\, W_1^2 + 0.53(X_Y W_3)^1 \leq 0$
$g_Z = -1095.1 + 0.15Y_1^1 + 0.33\, Y_2^2 + 0.47(X_Z W_2)^1 \leq 0$
$0 = 0.22X_W^1 + 0.05Y_2^3 - 0.46Z_1^2 + 0.73(X_W Y_2)^1 - W_1$
$0 = -0.96Y_2^2 + 0.56Z_1^3 - 0.03(X_W Y_2)^2 - W_2$
$0 = 0.36X_W^2 + 0.93(Z_1 Y_2)^2 - W_3$
$0 = 0.08X_Y^3 - 0.05W_1^1 + 0.11W_3^1 - 0.09(X_Y W_1)^3 - Y_1$
$0 = 0.59W_3^1 + 0.41W_1^2 + 0.99(X_Y W_3)^3 - Y_2$
$0 = -0.43X_Z^2 - 0.88W_2^2 + 0.25(W_2 X_Z)^2 - Z_1$
$-9999. \leq X_W, X_Y, X_Z, W_1, W_2, W_3, Y_1, Y_2, Z_1 \leq 9999.$

- Comment: All design and behavior variables are controlled by the optimizer.

With a general understanding of the three different means for posing the MDO test systems that are generated by CASCADE, the results of several numerical comparisons are seen in the next section.

Results

CASCADE has been used to generate a total of five simulations of coupled multidisciplinary systems. These five test systems vary in size and coupling complexity. The ANSI-C translated version of Automated Design Synthesis (ADS)[16] has been used as the optimization software for these MDO test systems. The strategy - optimizer combination that has been used for the acquisition of all the results in this section is *Sequential Linear Programming - Method of Feasible Directions*. Internal finite difference methods have been used to attain gradient information, within ADS. All trial executions were performed on a *SUN Ultra 1 Creator 3D* workstation, under comparable network conditions. The primary characteristics of the five test systems are summarized in Table 1. (Recall that the number of *coupling variables* for each solution strategy corresponds to the number of behavior variables which are required as input by at least one other subsystem).

Test system #1 has 3 subsystems (W, Y, and Z), which have 2, 1, and 3 behavior variables per subsystem, 3, 2, and 1 design variables per subsystem, and 1, 3, and 2 inequality constraints per subsystem, respectively. The initial value of the objective function is -678.71. Figure 6 provides a detailed illustration of the coupling nature of the first test system. Figure 7 compares the Test system #1 objective function histories for all three solution strategies. Table 2 summarizes the "best" results attained (where "best" implies *lowest* objective function) for the first test system, for each of the three solution strategies, after numerous trial executions. All solution strategies achieve approximately the same optimal design point, with the MDF strategy attaining the lowest objective function value of -1711.79.

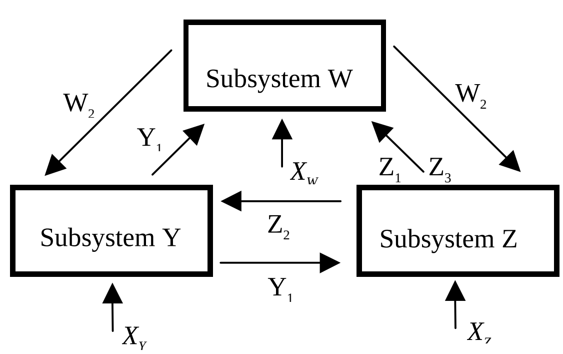

Figure 6: Schematic of Test System #1

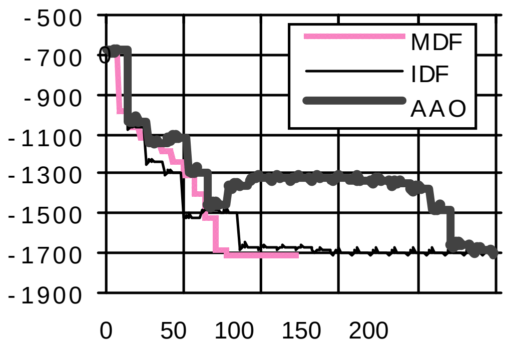

**Figure 7: Test system #1 -
Objective function value vs. evaluation number**

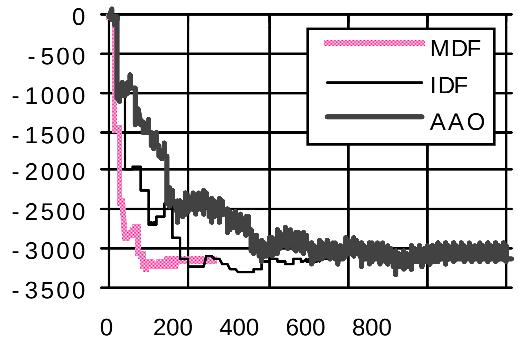

**Figure 9: Test system #2 -
Objective function value vs. evaluation number**

Test system #2 has 5 subsystems (U, V, W, Y, and Z), which have 1, 2, 1, 3, and 2 behavior variables per subsystem, 3, 2, 1, 2, and 3 design variables per subsystem, and 4, 0, 1, 2, and 3 inequality constraints per subsystem, respectively. The initial value of the objective function is -43.91. Figure 8 provides a detailed illustration of the coupling nature of the second test system. Figure 9 compares the Test system #2 objective function histories for all three solution strategies. Table 3 summarizes the "best" results attained for the second test system, for each of the three solution strategies, after numerous trial executions. Again, all solution strategies achieve approximately the same optimal design point. Here, the MDF strategy attains the lowest objective function value of -3150.08.

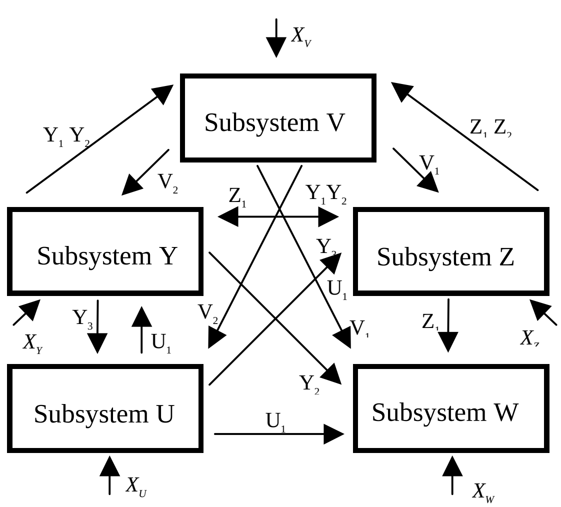

Figure 8: Schematic of Test System #2

Test system #3 has 10 subsystems, a total of 20 behavior variables, 40 design variables, and 20 inequality constraints. (For the final three test systems, detailed coupling illustrations and objective function histories are omitted for brevity.) The initial value of the objective function is 1622.74. Table 4 summarizes the "best" results attained for the third test system, for each of the three solution strategies, after numerous trial executions. Once again, all solution strategies achieve approximately the same optimal design point. The MDF strategy attains the lowest objective function value of -11859.7.

Test system #4 has 15 subsystems, a total of 45 behavior variables, 45 design variables, and 90 inequality constraints. The initial value of the objective function is -820.18. Table 5 summarizes the "best" results attained for the fourth test system, for each of the three solution strategies, after numerous trial executions. In this case, all three solution strategies do not arrive at "equivalent" solutions. The MDF strategy attains the lowest objective function value of -10118.8, which is approximately 10% lower than the next best solution attained, by the IDF strategy.

Finally, test system #5 has 20 subsystems, a total of 100 behavior variables, 40 design variables, and only 3 inequality constraints. The initial value of the objective function is 197.40. Table 6 summarizes the "best" results attained for the fifth test system, for each of the three solution strategies, after numerous trial executions. Here again, there is a distinct difference between the final solutions reached by each of the three solution strategies. MDF again achieves the lowest objective function value of -12012.6, considerably lower than the next best solution attained, by the AAO strategy.

Test system number	Number of subsystems	Number of behavior variables	Number of design variables	Number of inequality constraints	Number of coupling variables	Initial objective function
1	3	6	6	6	5	-678.71
2	5	9	11	10	8	-43.91
3	10	20	40	20	12	1622.74
4	15	45	45	90	43	-820.18
5	20	100	40	3	92	197.40

Table 1: Generalized summary of the five test systems

Strategy	Total Optimization variables	Final objective function	Active side constraints	Active inequality constraints	Number of analysis evaluations	Number of objective function evaluations	Total execution time (sec.)
MDF	6	-1711.79	3	1	4260	120	0.486
IDF	11	-1706.45	3	1	3612	601	1.916
AAO	12	-1705.15	3	1	0	469	2.029

Table 2: Solution summary for Test system #1

Strategy	Total Optimization variables	Final objective function	Active side constraints	Active inequality constraints	Number of analysis evaluations	Number of objective function evaluations	Total execution time (sec.)
MDF	11	-3150.08	4	1	17037	265	1.094
IDF	19	-3144.08	4	1	5238	581	7.245
AAO	20	-3124.61	4	1	0	1009	9.146

Table 3: Solution summary for Test system #2

Strategy	Total Optimization variables	Final objective function	Active side constraints	Active inequality constraints	Number of analysis evaluations	Number of objective function evaluations	Total execution time (sec.)
MDF	40	-11859.7	15	4	255940	1887	22.927
IDF	52	-11407.5	13	6	35020	1750	27.023
AAO	60	-11316.8	13	6	0	2990	64.201

Table 4: Solution summary for Test system #3

Strategy	Total Optimization variables	Final objective function	Active side constraints	Active inequality constraints	Number of analysis evaluations	Number of objective function evaluations	Total execution time (sec.)
MDF	45	-10118.8	13	22	523845	1473	57.854
IDF	88	-8882.84	11	10	188325	4184	307.943
AAO	90	-8483.45	11	12	0	1821	260.995

Table 5: Solution summary for Test system #4

Strategy	Total Optimization variables	Final objective function	Active side constraints	Active inequality constraints	Number of analysis evaluations	Number of objective function evaluations	Total execution time (sec.)
MDF	40	-12012.6	26	2	1255300	1559	98.179
IDF	132	-10253.4	13	2	572100	5720	1892.150
AAO	140	-10980.8	14	2	0	10999	3098.899

Table 6: Solution summary for Test system #5

The final four Figures relate to the results of Test system #5 (Table 6), and provide a visual interpretation of general trends that are evident in a majority of the results that have been presented in this work. Figure 10 is a plot of final objective function vs. solution strategy; Figure 11 is a plot of total analysis evaluations vs. solution strategy; Figure 12 is a plot of total objective function evaluations vs. solution strategy, and Figure 13 is a plot of total execution time vs. solution strategy.

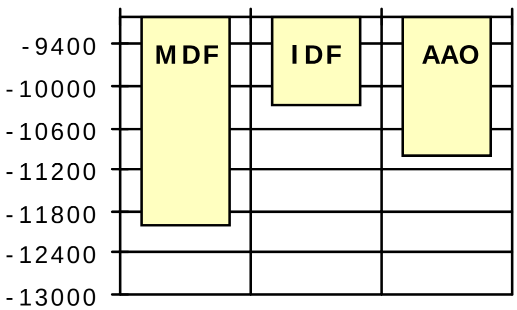

Figure 10: Test system #5 - final Objective function

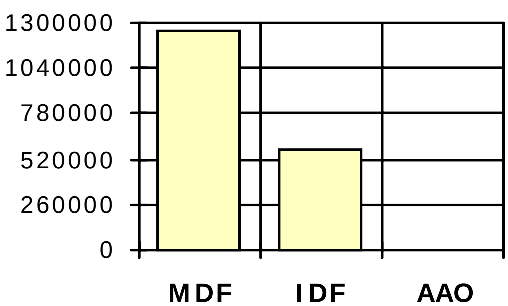

Figure 11: Test system #5 - analysis evaluations

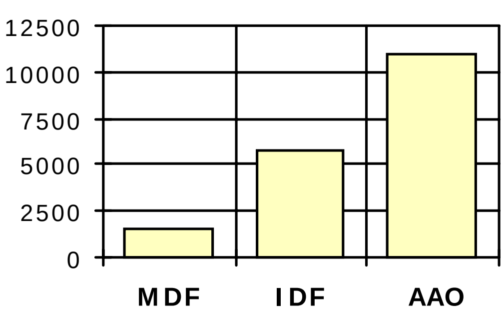

Figure 12: Test system #5 - objective function evals.

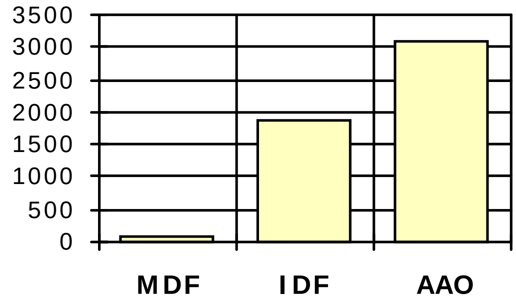

Figure 13: Test system #5 - execution time (seconds)

The next section will take a closer look at the results obtained from the five test systems, and will make some heuristic observations regarding the utility of each of the solution strategies.

Discussion

The first major point to be made regarding the results is that by using the three solution strategies to pose the same problem, a design manager will arrive at three different problem statements, each with its own "problem dynamic". The solution path for each problem will likely be totally unique, and highly dependent on the initial design point. Hence, having three separate means for posing and solving an MDO problem can only be advantageous to a design manager. This was clearly found to be the case in the present work. The authors would typically perform several trial executions on a given test problem using a given solution strategy, and attain what appeared to be the "global" optimum solution. Thereafter, numerous trial executions of the same test problem with a different solution strategy would often result in an improved solution, hence revealing the initial "global" optimum solution as being just a good local optimum solution.

The "problem dynamic" that is mentioned in the previous paragraph results from the nature of each solution strategy; the means with which the multidisciplinary design problem is posed for a given strategy. Problems which contain a large system analysis portion (i.e. problems which have a large number of behavior variables) will result in a large iterative system analysis for the MDF strategy. The same scenario will result in no system analysis at all for AAO, but will instead result in a large number of non-linear inequality constraints in the optimization. This same scenario results in an intermediate state for IDF - an equilibrium equality constraint and a single analysis evaluation are required for each behavior variable that promotes coupling.

A large factor that is affected by this topic of "problem dynamic" is the use of move limits within the optimizer. CASCADE has been known to generate treacherous design spaces that are cluttered with a multitude of local minima, any of which may contain the initial design point. Often times, very large move limits are required to retreat such a scenario. Because MDF (and to a lesser degree, IDF) has a distinct analysis step, large move limits must be handled with caution. If a design manager allows the design variables to change by a large degree on a given optimization iteration, the ensuing system analysis may see large changes in the behavior variable magnitudes. This may cause the initial design on the subsequent optimization cycle to be infeasible, from which the optimizer might not be able to recover, depending on the degree of infeasibility. The authors' trial executions of the five test systems solidified these statements.

The second major area of discussion involves the "goodness" of the results attained. Thus far, the only figure of merit that has been mentioned is the final value of the objective function. Figures 7 and 9 plot the objective function histories of the first and second test systems, respectively. In both of these test systems, the MDF strategy, which has the fewest optimization variables, attains the lowest final objective function, and reaches its final solution the most quickly. The AAO strategy, which has the largest number of optimization variables, tends to lag towards its final solution much more slowly. Not surprisingly, the IDF strategy tends to follow a path which is intermediate to the extremes of the MDF and AAO strategies.

For all of the five test systems, the MDF solution strategy attains the lowest objective function -- but at what expense? Included in Tables 2 through 6 are columns which report the analysis evaluations, objective function evaluations, and total clock time required to attain the final objective function, for each test problem and solution strategy. The general trend is that MDF is the most reliable solution strategy for attaining the lowest objective function, but at a huge cost in terms of analysis evaluations. For example, for test system #5, the MDF solution strategy attained the lowest objective function value by approximately 10%, but required over 1250000 analysis evaluations to attain its global optimum solution. Refer to Table 6 and Figures 10 and 11. The IDF solution strategy showed the least improvement in objective function value, but required less than half as many analysis evaluations as MDF. The final solution attained by the AAO solution strategy was slightly superior to that attained by IDF, and AAO required *zero* iterative analysis evaluations. When one thinks of a system analysis for a large scale design, one envisions a series of complicated Finite Element Method (FEM) and/or Computational Fluid Dynamic (CFD) matrix algebra computations, which are typically very timely and costly. Hence, eliminating the need to iteratively converge around the system analysis modules could be extremely beneficial.

For all five of the test systems, MDF required the fewest objective function evaluations within ADS, (usually) followed by IDF, and then AAO. (Refer to Figure 12, which plots test system #5 data). This makes sense, in that more function evaluations are required for the problem statement which has the greater number of total optimization variables. Total execution times for the five test systems tended to follow the same trend - MDF attained its solution the fastest, followed by IDF and AAO. (Refer to Figure 13, which plots test system #5 data). For the MDF solution strategy (FULL iterative system analysis), total execution times for the five test problems were approximately 0.5, 1.1, 22.9, 57.9, and 98.2 seconds, respectively, where the corresponding number of optimization variables for each test problem is 6, 11, 40, 45, and 40, respectively. For the AAO solution strategy (NO system analysis), total execution times were approximately 2.0, 9.1, 64.2, 261.0, and 3098.9 seconds for the five test systems, corresponding to 12, 20, 60, 90, and 140 optimization variables, respectively. While the system analysis simulations have *some* impact on the total execution time, it is clear from the above data that an increase in the number of total optimization variables will have a profound, non-linear effect on the execution time spent within the optimizer.

One must keep in mind that these execution times are clock times required for multidisciplinary design *simulations*. To attain an understanding of the total time required to attain a solution for a real-life system (of the same coupling nature that is being simulated by CASCADE), one would have to factor in the relative times required for analysis and objective function calculations for the real-life system itself. Analysis calculations are likely to be the largest factor in terms of total execution time, by a considerable margin, which is not being reflected in the CASCADE test problem simulations.

The third and final area of discussion presents some general closing observations. It is the authors' theory that for any given MDO problem at some initial starting point, an improved local (or hopefully global) optimum solution can be found, regardless of the solution strategy that is applied. The real issue is the "ease" with which the best found solution is attained, with each of the strategies. To this end, MDF consistently arrived at the "best found" design point on one of the first few trial executions, and with minor modifications of the default optimizer settings, or no modifications at all. The IDF and AAO solution strategies were quite the contrary - many more trial executions were required to attain the corresponding "best found" solution for each of these strategies. Moreover, substantial modifications to the default optimizer settings were required. Some of the default optimizer parameters which were commonly adjusted include: relative move limits, relative and absolute finite difference step sizes, the value of the penalty parameter for equality constraints (IDF and AAO only), maximum iterations and convergence criteria at both the optimizer and strategy levels, and the binary switch for variable scaling. Variable scaling worked consistently well for the MDF solution strategy, whose optimization variables consist solely of design variables. Often times, the IDF and AAO strategies required that variable scaling not be used, likely because these strategies have both design and behavior variables acting as optimization variables and numerous additional equality constraints resulting from the analysis equations.

Conclusions

This research effort has presented a numerical comparison of the MDF, IDF, and AAO solution strategies across simulated multidisciplinary design systems of varying size and complexity. The multidisciplinary test systems have been generated by CASCADE, a robust simulation tool. From the results that have been attained, some general conclusions can be drawn. Given the ability to pose a multidisciplinary design problem three different ways, a design manager has three distinct means for solving the design problem, which should only be to his or her advantage. Each of the three solution strategies exemplifies a unique problem dynamic, greatly dependent on the initial design point, and in most cases, on the settings of the optimizer. Numerous figures of merit must be considered when choosing an appropriate solution strategy. The MDF strategy has been found to arrive at the "best found" solution for all of the test problems in this effort, but always at a high cost in terms of analysis evaluations. AAO usually attains final solutions that are vastly improved (over the initial design) without any form of costly iterative system analysis, but at the expense of a complex design space that is cluttered with non-linear equality constraints. IDF has shown itself to be a good trade-off between the extremes of MDF and AAO; fewer analysis evaluations than MDF, and (equilibrium) equality constraints that are simpler, and usually fewer in quantity than those of AAO, depending on the coupling nature of the system. The reader must realize that the results that have been presented represent MDO problem *simulations*, and must be interpreted as such. In a real-world MDO problem, total solution times for the MDF strategy would likely have been the longest by a considerable margin (due to the many costly iterative analysis evaluations that it requires), followed by IDF (fewer, non-iterative analysis evaluations) and AAO (no iterative analysis evaluations). Finally, the authors reason that an improved local (or hopefully global) optimum solution can be found, regardless of the solution strategy used. The ultimate goal of the design manager is to utilize the solution strategy which will achieve the "best found" design with the least difficulty. To this end, the MDF strategy was found to constantly achieve its "best found" solution with minimal modification of optimizer settings, and with the smallest number of trial executions.

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