E 4

EE403/503 Introduction to Plasma Processing

Examples solutions
Test 1: Boltzmann Distribution

Example 1. **For an atmospheric pressure hydrogen plasma having a temperature of 10,000K determined the following:**

- **Number density of atoms, ions and electrons**
- **Maximum number densities of atoms, ions and electrons that can contribute to excitation of hydrogen atoms from ground state**
- **Density distribution of energy states of hydrogen atoms**
- **Intensity [J/cm^3] of hydrogen spectral line at 1215.72Å (first excited to ground state, g_u=8, Z_H=2, A_{ul}=4.699×10^8/s)**

Solution:

Part I

Step 1:

$$\frac{n_e n_i}{n_a} = \frac{(2\pi m_e kT)^{3/2}}{h^3} \frac{2Z_i}{Z_a} e^{\frac{-e(E_i - \Delta E_i)}{kT}} = f(T)$$

$$\left\{ \begin{array}{l} \Delta E_i = 6.69\times 10^{-9} n_e^{\ 1/3} \\ Z_i = 2 \ \ \textit{for hydrogen ion} \\ Z_a(T, \Delta E) \ \ \textit{from table} \end{array} \right.$$

Step 2: $n_e = n_i$ *singly ionized plasma*

Step 3: $P = (n_e + n_i + n_a)kT$

$n_e = n_i$ → $\frac{n_e^2}{n_a} = f(T)$ & $P = (2n_e + n_a)kT$

$$n_e^2 + 2 \times f(T) \times n_e - \frac{P \times f(T)}{kT} = 0$$

Initial value $n_e = 10^{20} (m^{-3})$, *find* ΔE_i & Z_a

⇒ *find f(T)*

⇒ *find a reasonable value for* n_e *from the polynomial*

⇒ *go for next iteration*

Preparation:

$$f(T) = \frac{[2\pi(9.1\times10^{-31})(1.38\times10^{-23})(1000)]^{3/2}}{(6.6\times10^{-34})} \times \frac{2\times2}{Z_a} \exp\left(\frac{-(1.6\times10^{-19})(13.595 - \Delta E_i)}{1.38\times10^{-23}\times10000}\right)$$

$$f(T) = \frac{2.409\times10^{27}}{Z_a} \exp(-15.7623 + 1.1594\times\Delta E_i)$$

$$n_e = -f(T) + \sqrt{f(T)^2 + \frac{P \times f(T)}{kT}}$$

First iteration:

$$n_e = 10^{20} \Rightarrow \begin{cases} \Delta E = 0.0323 \\ Z_a = 2.000 \end{cases} \Rightarrow \begin{cases} f(T) = 3.1019\times10^{21} \\ n_e = 4.4722\times10^{22} \end{cases}$$

Second iteration:

$$n_e = 4.4722\times10^{22} \Rightarrow \begin{cases} \Delta E = 0.2470 \\ Z_a = 2.000 \end{cases} \Rightarrow \begin{cases} f(T) = 2.2893\times10^{20} \\ n_e = 1.2738\times10^{22} \end{cases}$$

Third iteration:

$$n_e = 1.2738\times10^{22} \Rightarrow \begin{cases} \Delta E = 0.1625 \\ Z_a = 2.000 \end{cases} \Rightarrow \begin{cases} f(T) = 2.0757\times10^{20} \\ n_e = 1.2140\times10^{22} \end{cases}$$

Forth iteration:

$$n_e = 1.2140\times10^{22} \Rightarrow \begin{cases} \Delta E = 0.1600 \\ Z_a = 2.000 \end{cases} \Rightarrow \begin{cases} f(T) = 2.0695\times10^{20} \\ n_e = 1.2182\times10^{22} \end{cases}$$

Thus, $n_e = 1.2182\times10^{22}$, $n_i = 1.2182\times10^{22}$, $n_a = \dfrac{n_e^{\,2}}{f(T)} = 7.1709\times10^{23}\,(m^{-3})$

Part II

First excited energy state, $n = 2$, $E_n = 13.5995(1-\dfrac{1}{n^2})$

$$\Rightarrow E_n = 10.1994eV$$

$$E_n = 10.1994eV \times 1.6\times10^{-19} = 1.632\times10^{-18}\,J$$

Atom and Ion have almost the same mass and need twice the energy to transfer enough energy in inelastic collision for excitation

$$E = \frac{1}{2} m V^2 = 2 \times 1.632 \times 10^{-18}$$

$$m_{H\&N^+} = 1.66 \times 10^{-27}$$

$$V = \sqrt{\frac{4 \times 1.632 \times 10^{-18}}{1.66 \times 10^{-27}}} = 6.27 \times 10^4 \, m/s$$

For electrons,

$$E = \frac{1}{2} m V^2 = 1.632 \times 10^{-18}$$

$$V = \sqrt{\frac{2 \times 1.632 \times 10^{-18}}{9.108 \times 10^{-31}}} = 1.89 \times 10^6 \, m/s$$

Refer to Example E3

$$\begin{cases} \dfrac{N_V}{N_a} = 1 + \dfrac{2}{\sqrt{\pi}} U e^{-U^2} - erf(U) \\ U = \sqrt{\dfrac{m V^2}{2kT}} \end{cases}$$

Atoms & Ions

$$U = \sqrt{\frac{(1.66 \times 10^{-27})(6.27 \times 10^4)^2}{2(1.38 \times 10^{-23})(10000)}} = 4.8626$$

Electron

$$U = \sqrt{\frac{(9.108 \times 10^{-31})(1.89 \times 10^6)^2}{2(1.38 \times 10^{-23})(10000)}} = 3.4334$$

For atoms & Ions

$$\frac{N_V}{N_a} = 1 + \frac{2}{\sqrt{\pi}}(4.8626)e^{-(4.8626)^2} - erf(4.8626)$$

$$\Rightarrow \quad \frac{N_V}{(7.17\times 10^{23})} = 1.0424 - erf(4.8626)$$

$$\Rightarrow \quad N_V = 7.17\times 10^{23}\times(1.0424-1)$$

$$\Rightarrow \quad N_V = 3.04\times 10^{22}$$

For electron

$$\frac{N_V}{(1.21\times 10^{22})} = 1 + \frac{2}{\sqrt{\pi}}(3.4334)e^{-(3.4334)^2} - erf(3.4334)$$

$$\frac{N_V}{(1.21\times 10^{22})} = 0.125 \qquad \Rightarrow \qquad N_V = 1.89\times 10^{20}$$

Part III

$$n_i = n_a\frac{g_i}{Z_a}\exp(\frac{E_i}{kT})$$

$$n_a = 7.1709\times 10^{23}$$

$$g_i = 2i^2$$

Z_a almost equal 2 at T=10000K

$$n_i = 7.1709\times10^{23}\times\frac{2i^2}{2}\exp(\frac{-13.5995(1-\frac{1}{i^2})\times1.6\times10^{-19}}{1.38\times10^{-23}\times10000})$$

$$n_i = 7.1709\times10^{23}\times i^2\exp(-15.768(1-\frac{1}{i^2}))$$

$i = 1$	$n_1 = 7.1709\times10^{23}$
$i = 2$	$n_2 = 2.1\times10^{19}$
$i = 3$	$n_3 = 5.3\times10^{18}$
$i = 4$	etcs

Part IV

$$I = \frac{1}{4\pi}n_i\times\frac{hc}{\lambda_{2\to1}}\times A_{2\to1}$$

$$I = \frac{1}{4\pi}(2.1\times10^{19})\times\frac{6.6\times10^{-34}\times3\times10^{8}}{1215.67\times10^{-8}}\times4.699\times10^{8}$$

$$I = 1.284\times10^{7}J/m^{3}$$