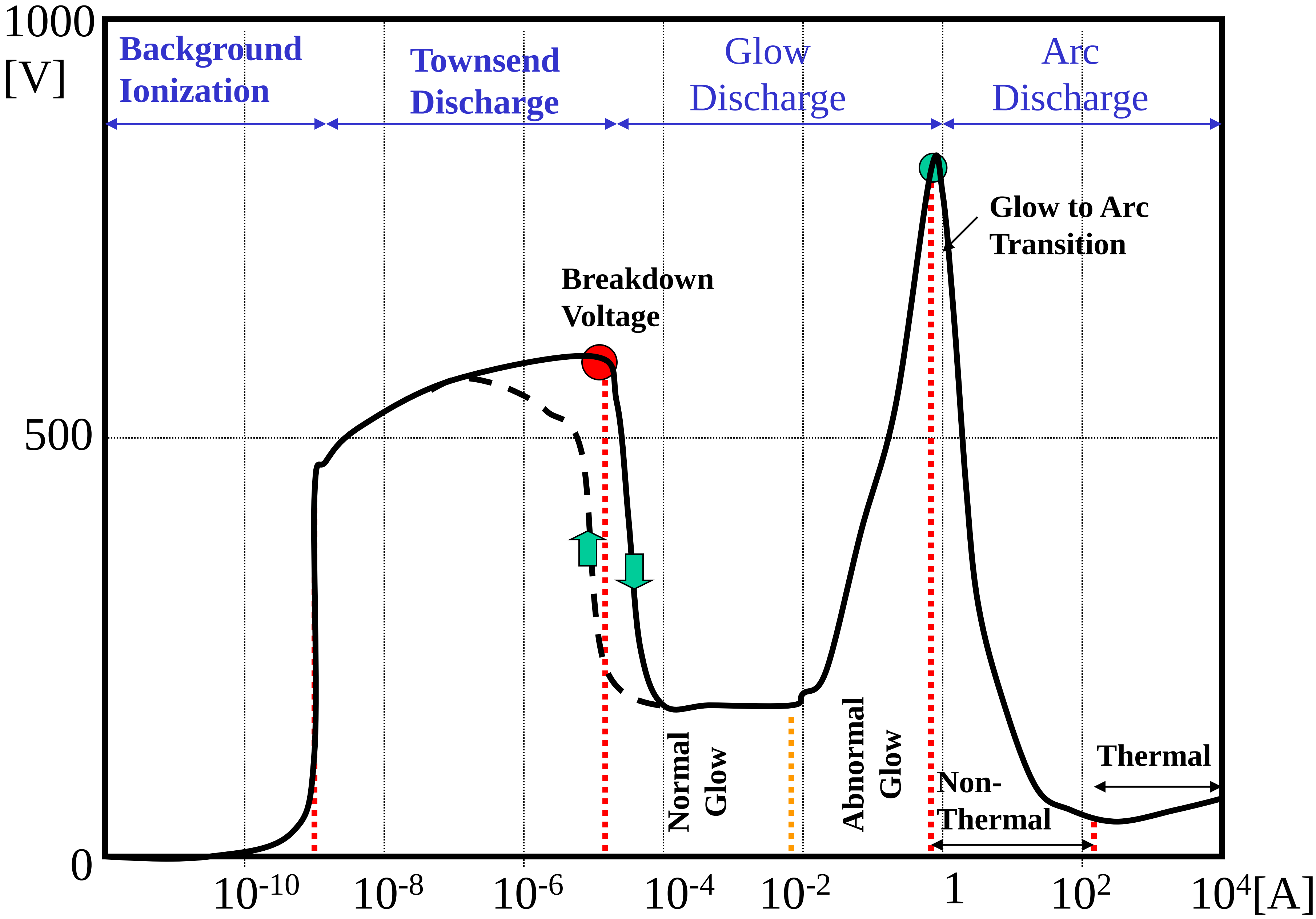
Arc Discharge

EE 403/503

Introduction to Plasma Processing

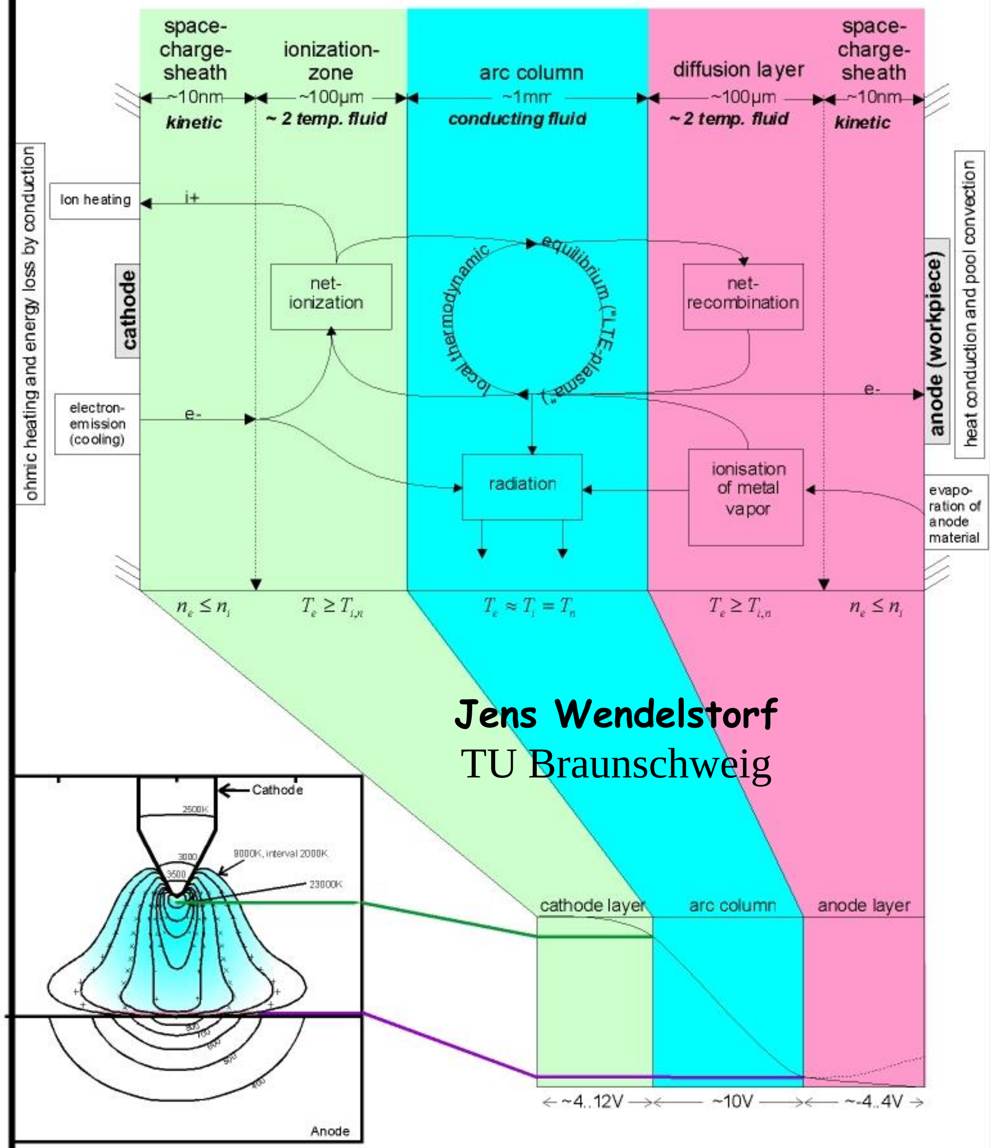
November 2, 2011

V-I Characteristic of DC Discharges



Plasma Parameters	Non-Thermal Arc	Thermal Arc
Equilibrium State	Kinetic	LTE
n_e [1/m^3]	$10^{20} < n_e < 10^{21}$	$10^{22} < n_e < 10^{23}$
Pressure [Pa]	$0.1 < p < 10^5$	$10^4 < p < 10^7$
T_e [eV]	$0.2 < T_e < 2.0$	$1.0 < T_e < 10$
T_{gas} [eV]	$0.025 < T_{gas} < 0.5$	$T_{gas} = T_e$
Current I [A]	$1 < I < 50$	$50 < I < 10^4$
E/p [V/m-Torr]	High	Low
IE [kW/cm]	IE < 1.0	IE > 1.0
Transparency	Tranparent	Opaque
Luminous Intensity	Bright	Dazzeling
Ionization fraction	Indeterminate	Saha Equation
Radiation Output	Indeterminate	LTE

Physical regions of the arc plasma:



space-charge-sheath
~10nm
kinetic

ionization-zone
~100µm
~ 2 temp. fluid

arc column
~1mm
conducting fluid

diffusion layer
~100µm
~ 2 temp. fluid

space-charge-sheath
~10nm
kinetic

ohmic heating and energy loss by conduction

Ion heating

i+

cathode

electron-emission (cooling)

e-

net-ionization

local thermodynamic equilibrium ("LTE-plasma")

net-recombination

radiation

ionisation of metal vapor

evaporation of anode material

e-

anode (workpiece)

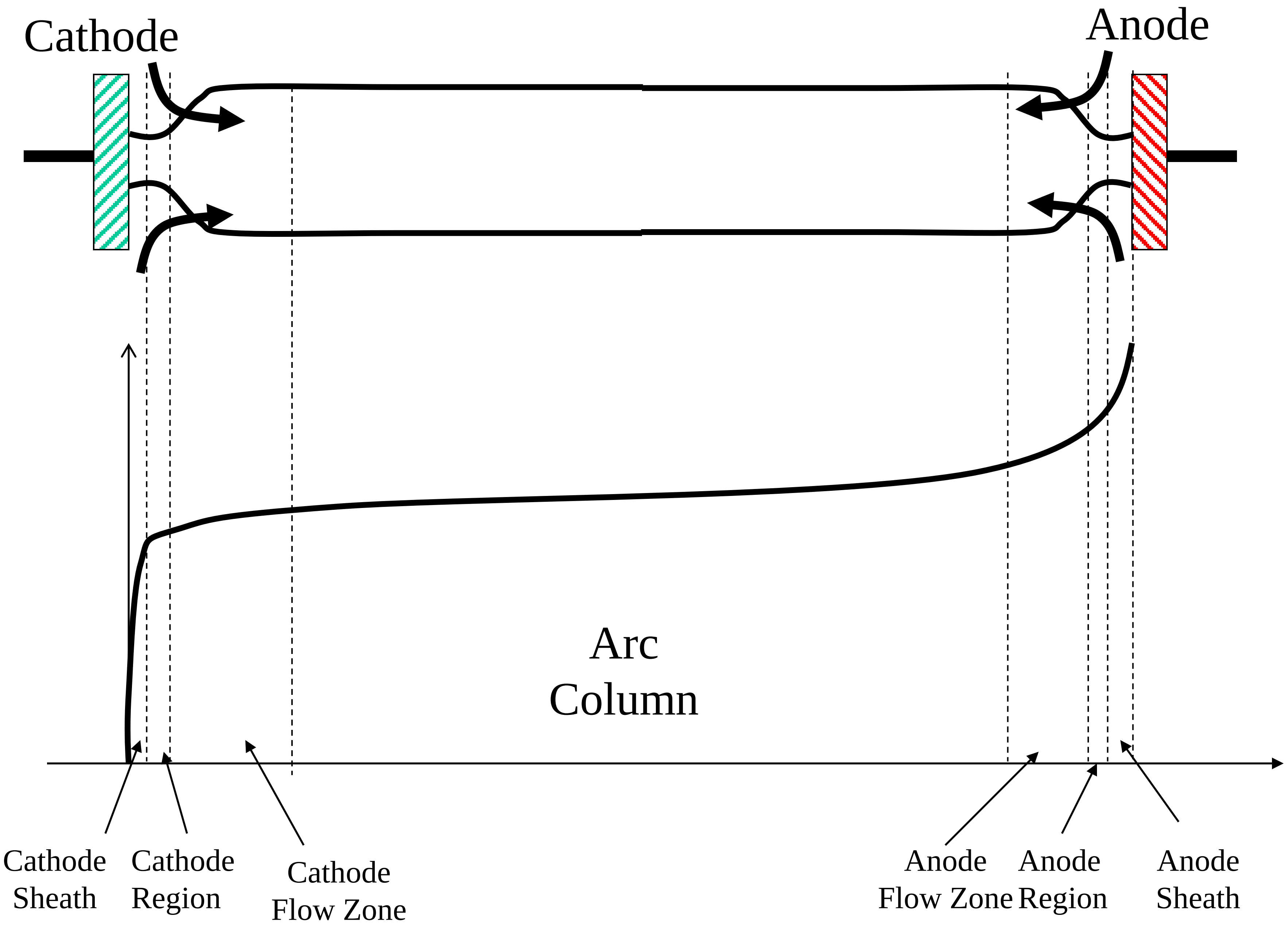
heat conduction and pool convection

$n_e \le n_i$ | $T_e \ge T_{i,n}$ | $T_e \approx T_i = T_n$ | $T_e \ge T_{i,n}$ | $n_e \le n_i$

cathode layer | arc column | anode layer

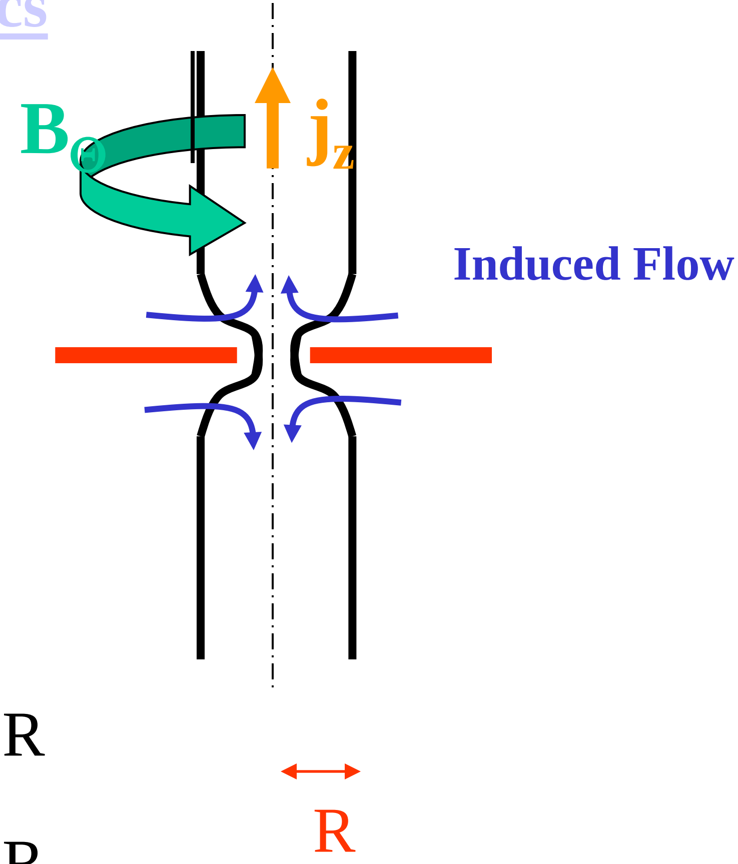
~4..12V | ~10V | ~-4..4V

Jens Wendelstorf
TU Braunschweig


Cathode
Anode
Arc
Column
Cathode
Sheath
Cathode
Region
Cathode
Flow Zone
Anode
Flow Zone
Anode
Region
Anode
Sheath

Arc Dynamics

$$\underbrace{\rho\frac{d\vec{v}}{dt}}_{=0 \text{ for steady state}} = -\nabla p + \vec{j}\times\vec{B}$$



$$\vec{B} = (0, B_\Theta(r), 0)$$

$$\vec{j} = (0,0,j_z(r)) \qquad r<R$$

$$\vec{j} = 0 \qquad r>R$$

$$\boxed{-j_z(r)B_\Theta(r) = \frac{dp}{dr}} \Rightarrow \boxed{\Delta p(r) = \int_r^R j_z B_\Theta(r)\,dr} \quad r<R$$

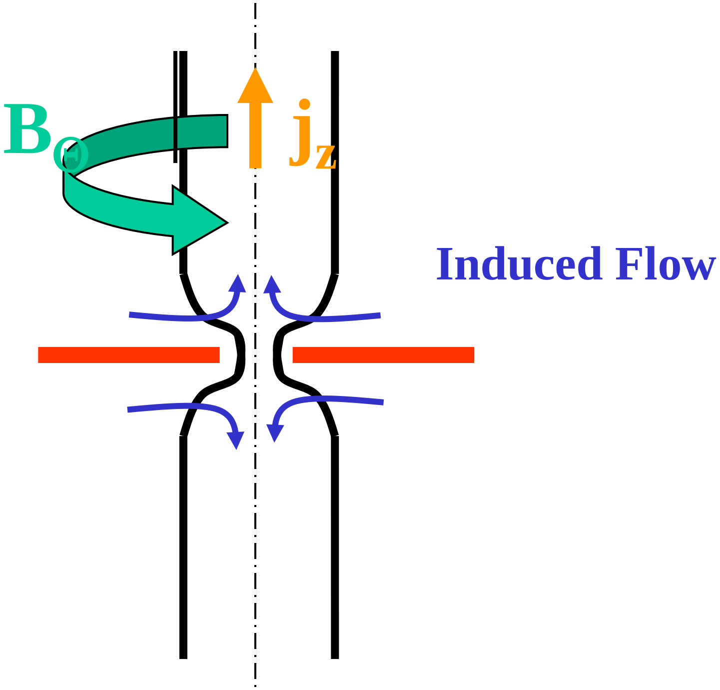
$$\boxed{\Delta p(r) = \int_r^R j_z B_\Theta(r)dr}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

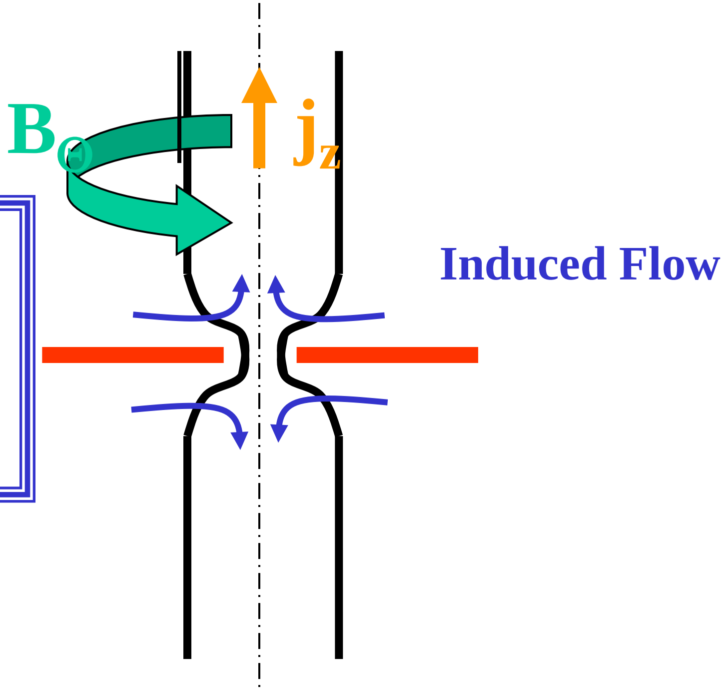
$$\frac{1}{r}\frac{d}{dr}(rB(r)) = \mu_0 \vec{j}$$

$$\boxed{B(r) = \mu_0 \frac{1}{r}\int_0^r jrdr}$$

$$\boxed{\Delta p = \mu_0 \int_r^R j(r)\left(\frac{1}{r}\int_0^r j(r)rdr\right)dr}$$



$$\boxed{\Delta p = \mu_0 \int_r^R j(r)\left(\frac{1}{r}\int_0^r j(r)\,r\,dr\right)dr}$$



For $j = \dfrac{I}{\pi R^2}$

$$\Delta p = \mu_0 j^2 \frac{1}{4}(R^2 - r^2)$$

$$\Delta p = \frac{\mu_0}{4\pi} I \times j\left(1 - \frac{r^2}{R^2}\right)$$

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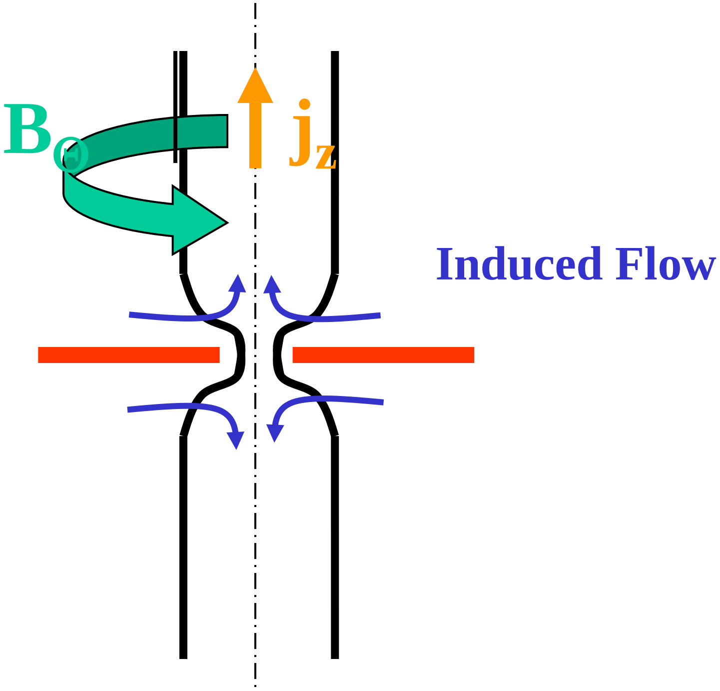


Bernoulli's equation

$$\frac{1}{2}\rho V_z^2 + p_z = const$$

$$\frac{1}{2}\rho \nabla (V_z^2) = -\nabla p_z$$

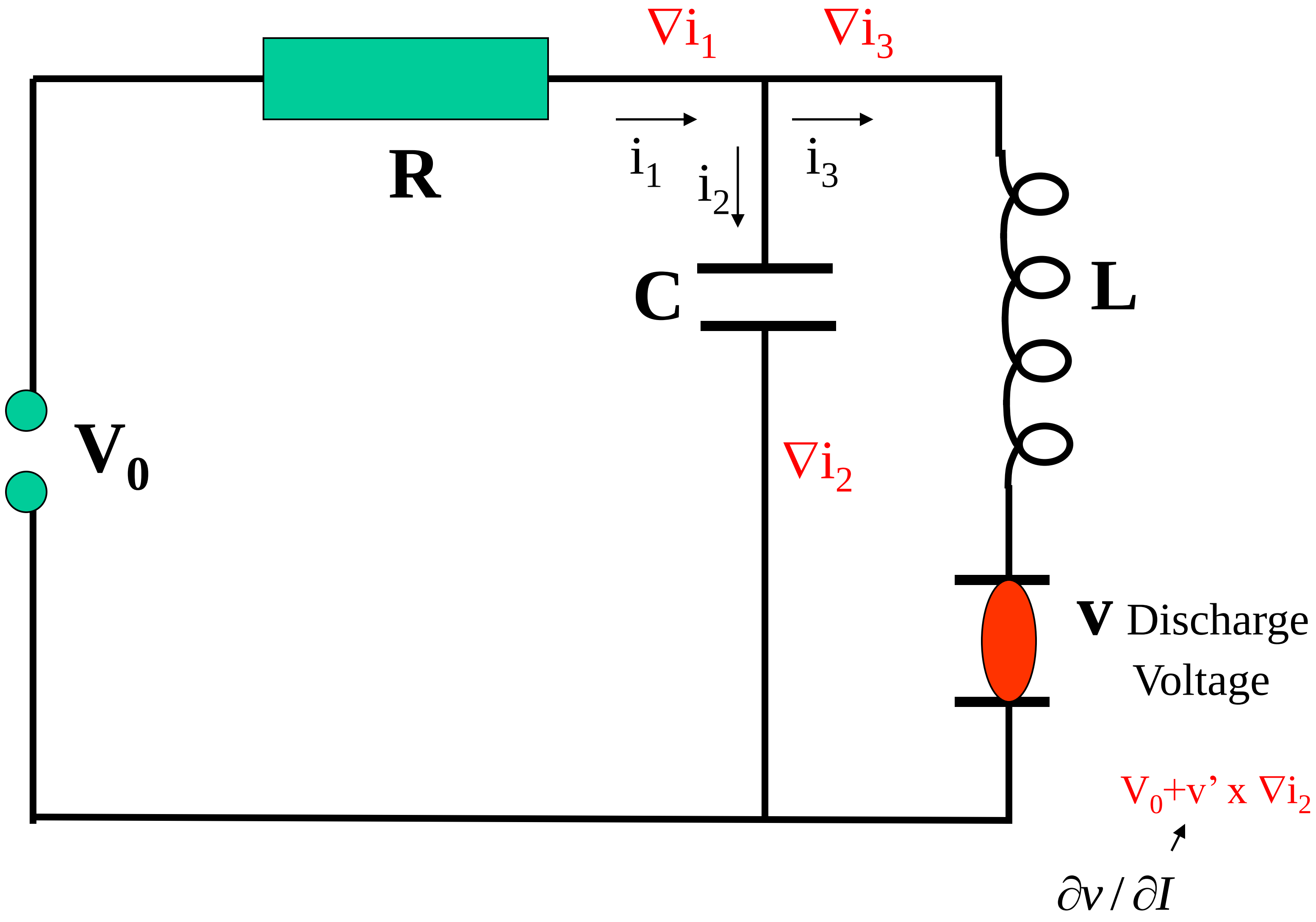
$$p_{\max} = \frac{1}{2}\rho V_{z,\max}^2 = \frac{\mu_0}{4\pi} I \times j$$

Example:

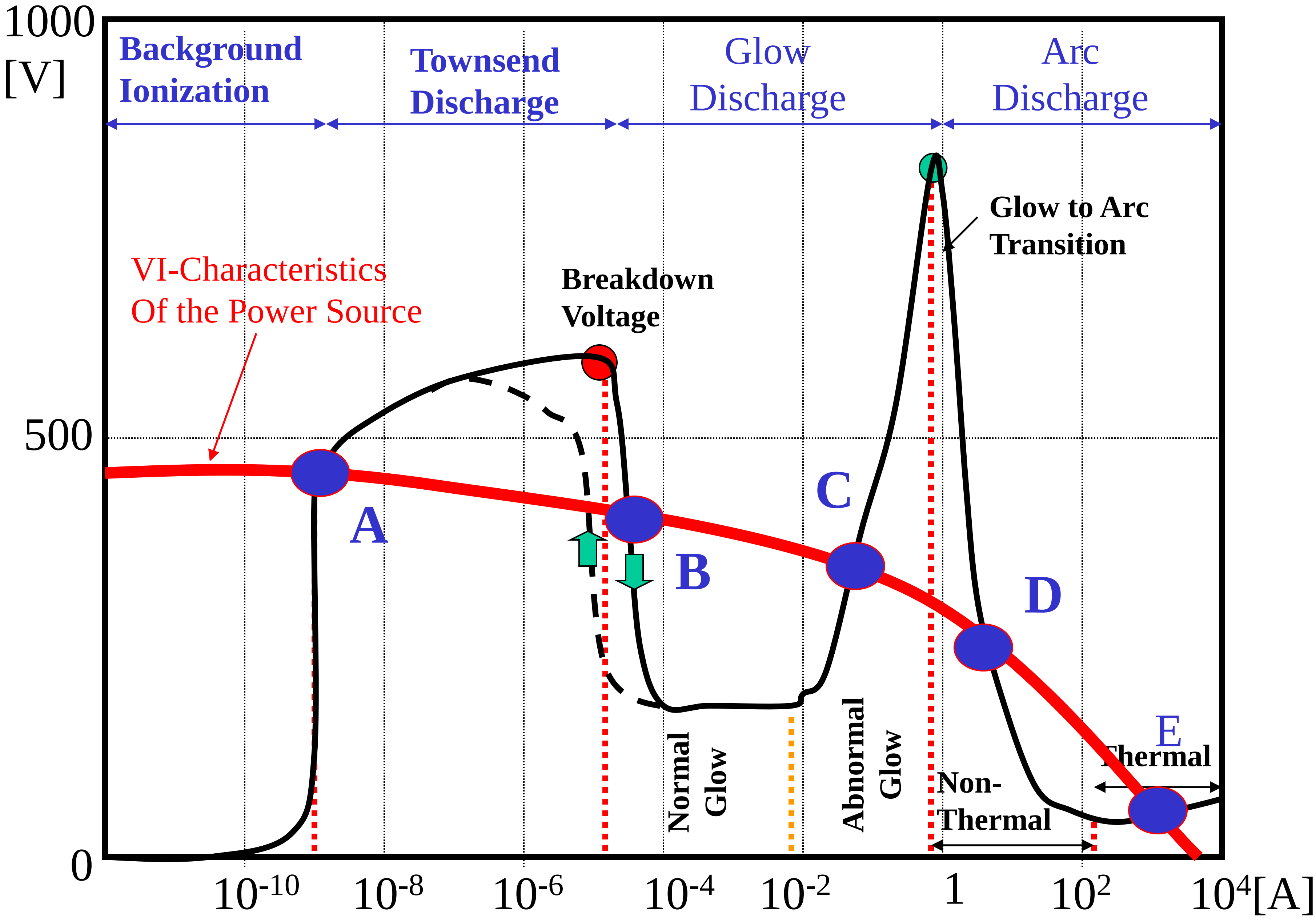
$I = 200$ A
$\rho = 0.015$ Kg/m^3
$j = 4.5 \times 10^7$ A/m^2

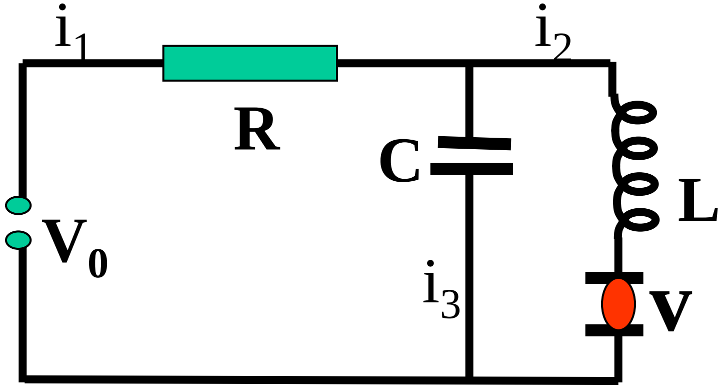
$V = 350$ m/s

Electrical Stability Criteria



V-I Characteristic of DC Discharges





Current at operating point

$$i_1 = i_2 + i_3$$

$$i_2 = i_1$$

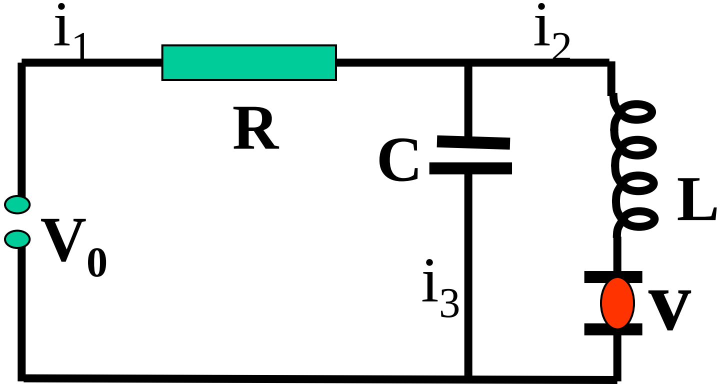
$$i_3 = 0$$

$$i_1 = i_0 + \Delta i_1$$

$$i_2 = i_0 + \Delta i_2$$

$$i_3 = \Delta i_3$$

$$\boxed{i_1 = i_2 + i_3 \Rightarrow \Delta i_1 = \Delta i_2 + i_3}$$



$$\begin{cases}(i_0+\Delta i_1)R+L\frac{d}{dt}(i_0+\Delta i_2)+v_0+v^{'}\Delta i_2=V_0\\ i_0R+v_0=V_0\end{cases}$$

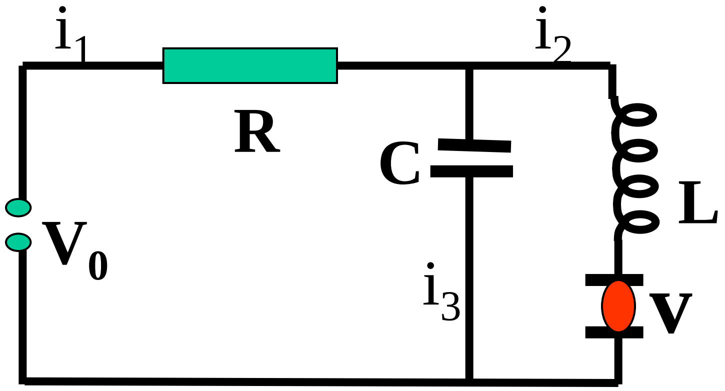
$$\boxed{\Delta i_1R+L\frac{d}{dt}(\Delta i_2)+v'\Delta i_2=0}$$

$$\Delta i_1 R + L\frac{d}{dt}(\Delta i_2) + v'\Delta i_2 = 0$$

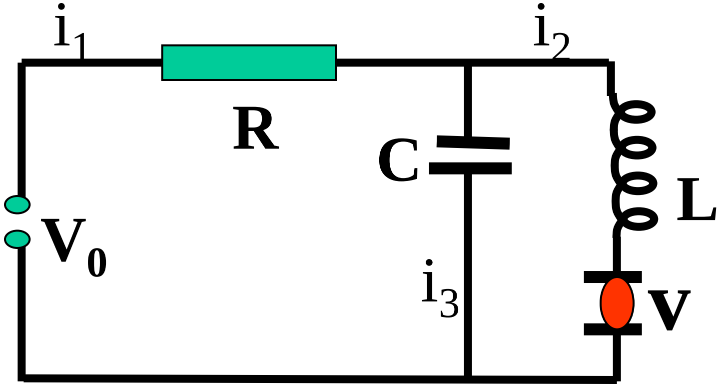
$$\Delta i_1 = \Delta i_2 + i_3$$

$$i_3 = C\frac{dv_c}{dt}$$

$$v_c = L\frac{d\Delta i_2}{dt} + v_0 + v'\Delta i_2$$



$$\frac{d^2\Delta i_2}{dt^2} + \left(\frac{v'}{L} + \frac{1}{RC}\right)\frac{d\Delta i_2}{dt} + \frac{1}{LC}\left(\frac{v'}{R} + 1\right)\Delta i_2 = 0$$



Linear, homogeneous second-order differential equation

$$\boxed{\frac{d^2\Delta i_2}{dt^2}+\underbrace{\left(\frac{v'}{L}+\frac{1}{RC}\right)}_{\mathbf{a}}\frac{d\Delta i_2}{dt}+\underbrace{\frac{1}{LC}\left(\frac{v'}{R}+1\right)}_{\mathbf{b}}\Delta i_2=0}$$

Solution:

$$\Delta i_2 = ce^{\lambda t} \Rightarrow \quad \lambda^2+a\lambda+b=0 \Rightarrow \lambda = -\frac{a}{2} \pm \sqrt{\frac{a^2}{4}-b}$$

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$$\frac{a^2}{4} - b > 0$$

Overcritical Damping

$$\frac{a^2}{4} - b = 0$$

critical damping

$$\frac{a^2}{4} - b < 0 \Rightarrow \lambda_{1,2} = -\frac{a}{2} \pm j\sqrt{b - \frac{a^2}{4}}$$

underdamping (oscillation)

$$\Delta i_2 = e^{-\frac{a}{2}t}\left[C_1 \sin\sqrt{b - \frac{a^2}{4}}\,t + C_2 \cos\sqrt{b - \frac{a^2}{4}}\,t\right]$$

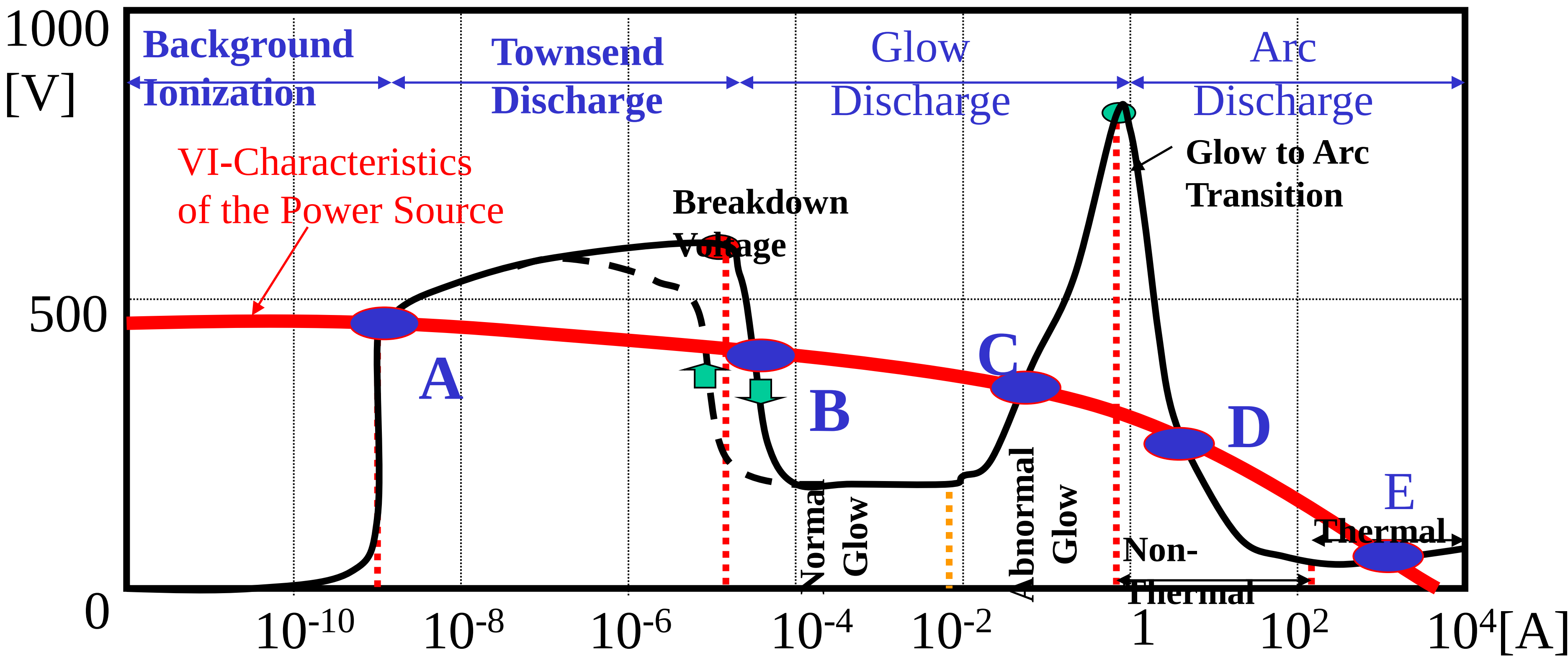
$$\Delta i_2 = e^{-\frac{a}{2}t}\left[C_1 \sin\sqrt{b-\frac{a^2}{4}}t + C_2 \cos\sqrt{b-\frac{a^2}{4}}t\right]$$

Stable solution when $\begin{matrix}\Delta i_2 \to 0 \\ t \to \infty\end{matrix}$ $\left\{\begin{matrix} a > 0 \Rightarrow \frac{v'}{L} + \frac{1}{RC} > 0 \\ b > 0 \Rightarrow \frac{v'}{R} + 1 > 0 \end{matrix}\right.$

1- The criterion that R<L/(|v’|C) is usually met

2- v’>0 every intercept of the load line with the arc characteristic leads to a stable point.

3- v’<0 stable point when R>|v’| -> Load line must intercept the characteristic line from above



1- v’>0   every intercept of the load line with the arc characteristic leads to a stable point. A & C

2- v’<0 stable point when R>|v’| -> Load line must intercept the characteristic line from above B & D are not; E is

(The criterion that R<L/(|v’|C) is usually met)