EE 403/503
Introduction to Plasma Processing

Plasma States

October 12, 2011

Project

Outline

1- Boltzmann Equation

2- Plasma Modeling

3- Debye Length and Plasma Frequency

4- Plasma Probe

5- E5

6- Midterm

Austrian
Physicis

(1844-1906)

Boltzmann Equation

Net rate of increase of species s, as a result of collisions between particles of species s with those of species r

Species distribution function $f_s(x,v,t)$

External forces

$$\frac{\partial}{\partial t}(n_s f_s) + \vec{v}.\nabla n_s f_s + \frac{F_s}{m_s}\cdot n_s \nabla_v f_s = \sum_r C_{sr}$$

Number of particles n_s

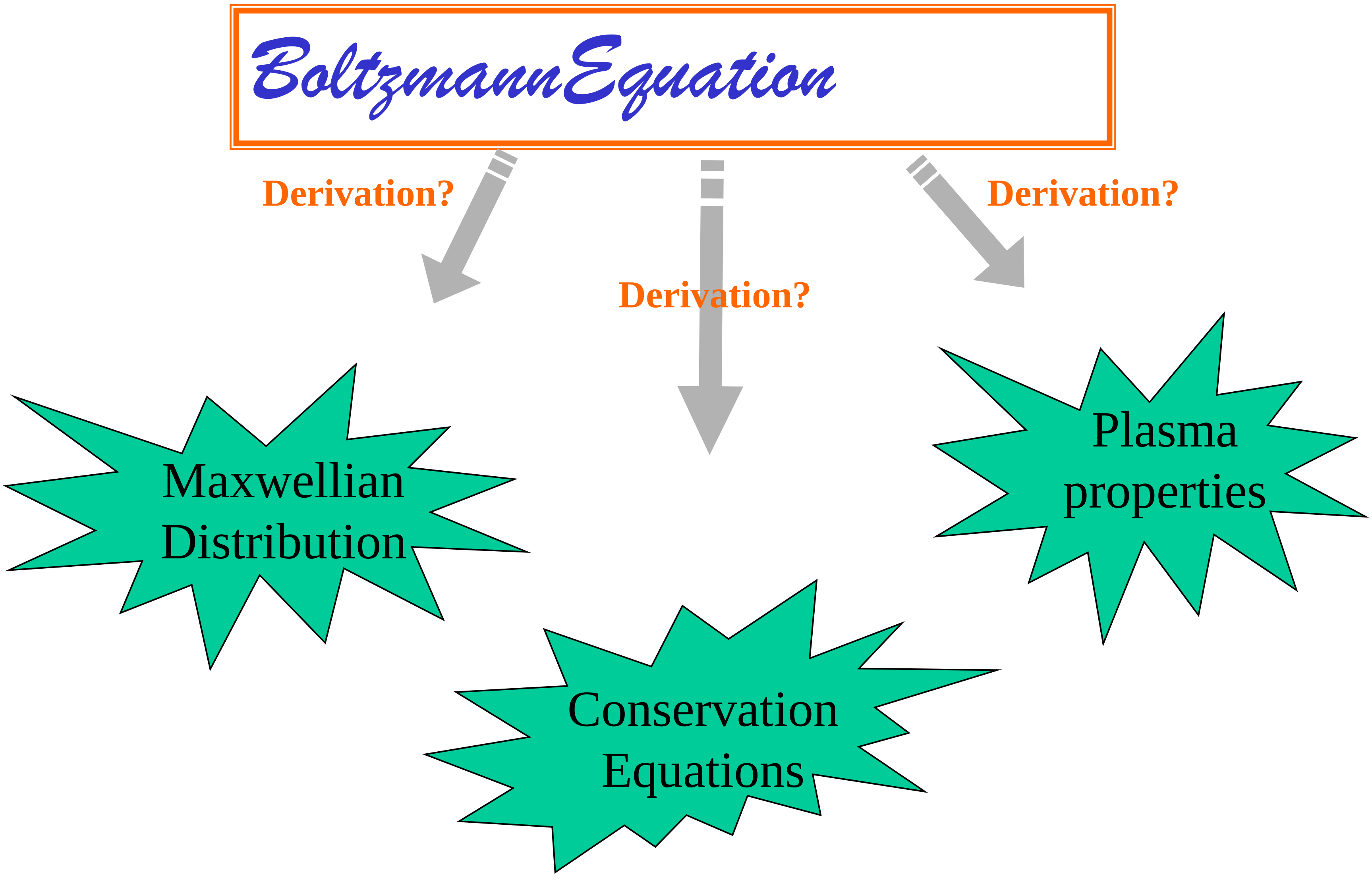
Velocity

Mass of species s

Impact parameter

after the collision

$$C_{sr} = n_r n_s \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[f_s(v_s') f_r(v_r') - f_s(v_s) f_r(v_r)\right] v_{rs}\, b\, db\, dv_r$$


BoltzmannEquation
Derivation?
Derivation?
Derivation?
Maxwellian Distribution
Conservation Equations
Plasma properties

Modeling of Plasmas

Higher pressures ⟷ Lower pressures

Conservation Equations

Brownian Motion

Brownian Motion

Conservation Equations

To utilize the conservation equations in your application, you need:

-a set of conservation equations that are required to describes your applications

-Set up your initial condition and boundary conditions

-Provide the required thermodynamic and transport properties

Solve the conservation equations analytically or numerically

Conservation Equatios

General Form

Dependent variable

Specific to a particular Meaning of Φ

$$\frac{\partial}{\partial t}(\rho\Phi) + \nabla\cdot(\rho\vec{u}\Phi) = \nabla\cdot(\Gamma\nabla\Phi) + S$$

Unsteady term

Convection term

Diffusion term

Source term

Examples:

General:

$$\frac{\partial}{\partial t}(\rho\Phi)+\nabla\cdot(\rho\vec{u}\Phi)=\nabla\cdot(\Gamma\nabla\Phi)+S$$

$\Phi = 1$

$$\frac{\partial}{\partial t}(\rho)+\nabla\cdot(\rho\vec{u})=0$$

$\Phi = \vec{u}$

$$\frac{\partial}{\partial t}(\rho\vec{u})+\nabla\cdot(\rho\vec{u}\vec{u})=\nabla\cdot(\Gamma\nabla\vec{u})+\nabla p+S_m$$

$\Phi = m_l$

$$\frac{\partial}{\partial t}(\rho m_l)+\nabla\cdot(\rho\vec{u}m_l)=\nabla\cdot(\Gamma\nabla m_l)+R_l$$

$\Phi = h$

$$\frac{\partial}{\partial t}(\rho h)+\nabla\cdot(\rho\vec{u}h)=\nabla\cdot(\frac{k}{c_p}\nabla\Phi)+S_h$$

Transport Properties

1- Diffusivity (Page 49)

Movement of gas particles from high density to low density gas by random walk

Particle Flux → Γ; Diffusion Coefficient → D; Density → n; Drift Velocity → v_d

$$\Gamma = n v_d = -D \nabla n$$

Fick's Law

Average Velocity → $\bar{v}$; Collision Frequency → v_c; Mean Free Path → λ

$$D \approx \frac{1}{3} v_c \lambda^2 = \frac{1}{3} \bar{v} \lambda = \frac{1}{3} \bar{v}^2 \tau$$

← if $\frac{\lambda}{n}\frac{\partial n}{\partial x} = \frac{\lambda}{L} << 1$

For a monolithic gas:

$$D = 2.06 \times 10^{-12} \frac{\sqrt{mT}}{\sigma}$$

σ: Cross section for binary hard-sphere collisions

2- Electrical Conductivity

Current Density → $J = \sigma E$ [A/m^2]

Electrical Conductivity → $\sigma = \dfrac{e^2 n_e}{m_e \nu_e}$

3- Thermal Conductivity

Heat Flux → $q = -\kappa \nabla T$

Thermal Conductivity → $\kappa \approx f \bar{v} \lambda$

Degree of Freedom

Heat Capacity

For a monolithic gas: $\kappa = \dfrac{25}{64} \dfrac{fk}{\sigma} \left(\dfrac{\pi k T}{m} \right)^{1/2} = \dfrac{C_v \bar{v}}{3 N_{Avag} \sigma_{cross}}$

4- Viscosity

Force resulting from the net transport of momentum from one region to another

$$F = \eta \frac{du}{dz}$$

[N/m²]

Coefficient of Viscosity

$$\eta = \frac{1}{3} \frac{m\bar{v}}{\sigma_{cross}}$$

[Ns/m²]

Cross section for the binary hard-sphere collision

5- Mobility

Mobility

Drift Velocity

$$U_{e_d} = -\mu_e E$$

Electric Field

$$\mu_e = \frac{3}{4} \frac{e}{n\sigma} \left(\frac{2\pi}{m_e kT} \right)^{1/2}$$

Thermodynamic Properties

Link1

1- Internal Energy

2- Enthalpy

3- Entropy

4- Gibbs Free Energy

Partition Function

(Center-Piece of Statistical Mechanics)

$$Z = \sum_{s} g_s e^{-\frac{E_s}{kT}}$$

$$E_s = E_{transitional} + E_{electronic} + E_{vibrational} + E_{rotational} + E_{ionization}$$

$$\Downarrow$$

$$Z = Z_{transitional} \times Z_{electronic} \times Z_{vibrational} \times Z_{rotational} \times Z_{ionization}$$

$$Z = \sum_s g_s e^{-\frac{E_s}{kT}}$$

$$Z = Z_{transitional} + Z_{electronic} + Z_{vibrational} + Z_{rotational} + Z_{ionization}$$

$$Z_{transitional} = \int\int\int_{-\infty}^{+\infty} \frac{V}{h^3} e^{-\frac{m\vec{v}^2}{2kT}} d\vec{v} = \frac{V}{h^3}(2\pi mkT)^{3/2}$$

$$Z_{electronics} = \sum_i (2j+1) e^{-\frac{E_i}{kT}} = \sum_i 2n^2 e^{-\frac{E_i}{kT}}$$

$$Z_{rotational} = \frac{1}{2}\sum_{j=0}^{\infty}(2j+1) e^{-\left[\frac{h^2}{8\pi^2\Theta} j(j+1)\right]/kT} = \frac{8\pi^2\Theta}{2h^2} kT$$

$$Z_{oscillation} = \sum_{\nu=0}^{\infty} e^{-\frac{h\nu(\nu+1/2)}{kT}} = \frac{1}{2\sinh(h\nu/2kT)}$$

$$Z_{ionization} = e^{-\frac{\chi_i}{kT}}$$

Total partition function for a system composed of N identical, indistinguishable particles:

$$Z_{total} = \frac{Z^N}{N!}$$

$$U = \frac{kT^2}{N}\frac{\partial}{\partial t}(\ln Z_{total})$$

Internal Energy

Sum of energies of individual atoms or molecules

$$S = \frac{k}{N}\ln Z_{total} + \frac{U}{T}$$

Entropy

Definition: $dS = \frac{dQ}{T}$

$S = k \ln W$

Probability of system

$$H = U + P\frac{V}{N}$$

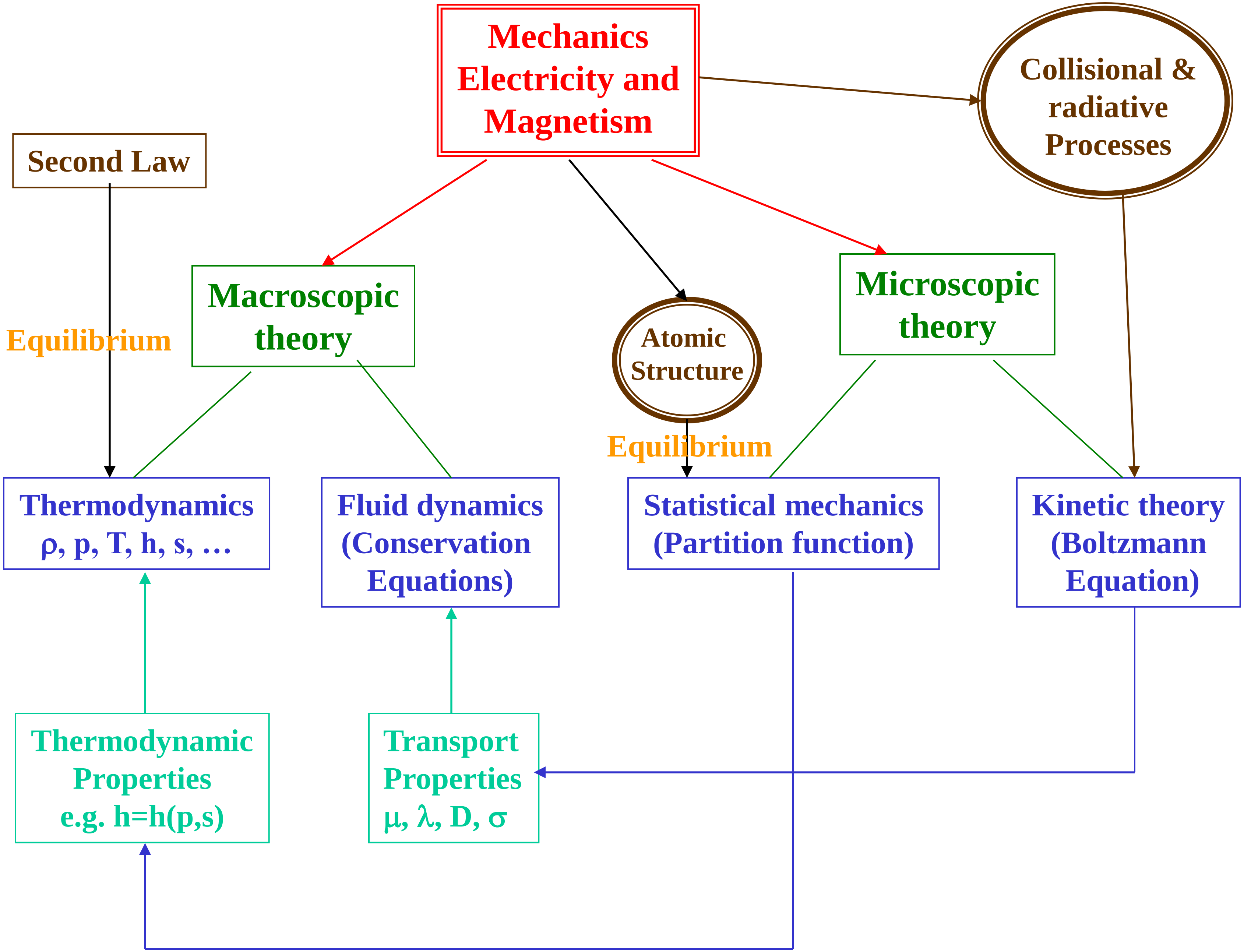
Enthalpy

Definition: H=U+pV

$$G = H - TS$$

Gibbs Free Energy

Definition: G=U-TS+pV


Mechanics
Electricity and
Magnetism
Collisional &
radiative
Processes
Second Law
Macroscopic
theory
Equilibrium
Atomic
Structure
Microscopic
theory
Equilibrium
Thermodynamics
ρ, p, T, h, s, …
Fluid dynamics
(Conservation
Equations)
Statistical mechanics
(Partition function)
Kinetic theory
(Boltzmann
Equation)
Thermodynamic
Properties
e.g. h=h(p,s)
Transport
Properties
μ, λ, D, σ

Equilibrium Concept

In a Complete Thermal Equilibrium (CTE) system the following conditions are met.

1- Radiation emitted by the system follow Black Body Radiation

2-All species have Maxwellian distribution

3- Kinetic equilibrium must exist, ie., $T_e = T_h$ (E/P is small, T is high)

4- Collision process establish excitation and ionization equilibrium

5- Spatial variations are small

Local Thermodynamic Equilibrium (LTE) means that conditions 2-5 are met.

Partial Local Thermodynamic Equilibrium (PLTE) means that one condition is only partially validated (e.g. excitation may not exactly follow the Boltzmann distribution).

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Characteristics of Plasma

October 12, 2011

Debye Length, λ_D Link

1- Charge Neutrality

2- Plasma Sheaths

3- Shielded Coulomb Potential

Plasma Frequency, ν_p or ω_P

Plasma Response Time

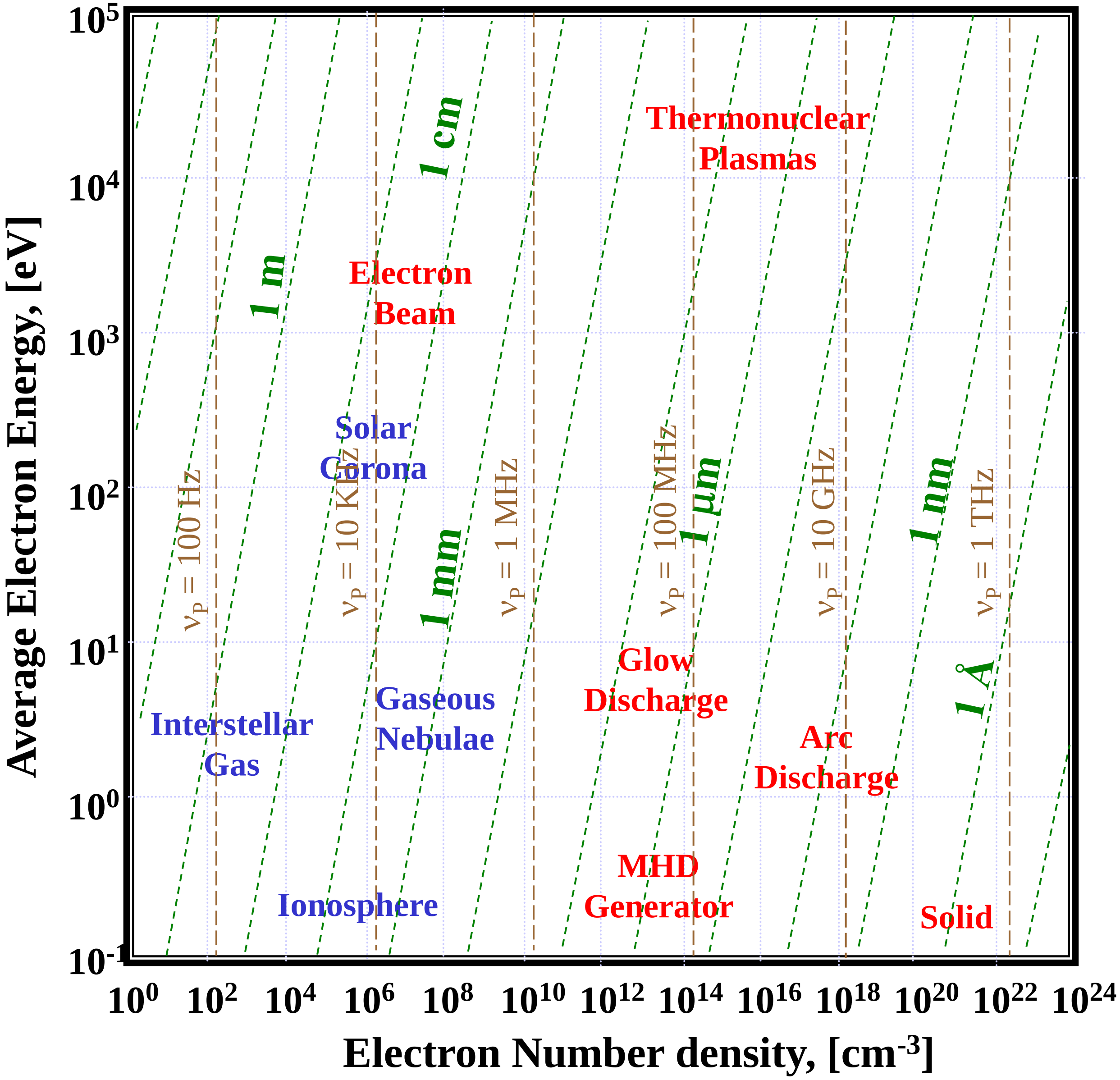
Two fundamental Plasma Parameters

Debye Length

$$\lambda_D = 69 \times \sqrt{\frac{T_e}{n_e}} \qquad [\mathrm{m}]$$

Plasma Frequency

$$\nu_p = \frac{\omega_p}{2\pi} = 8.97 \times \sqrt{n_e} \qquad [\mathrm{sec}^{-1}]$$



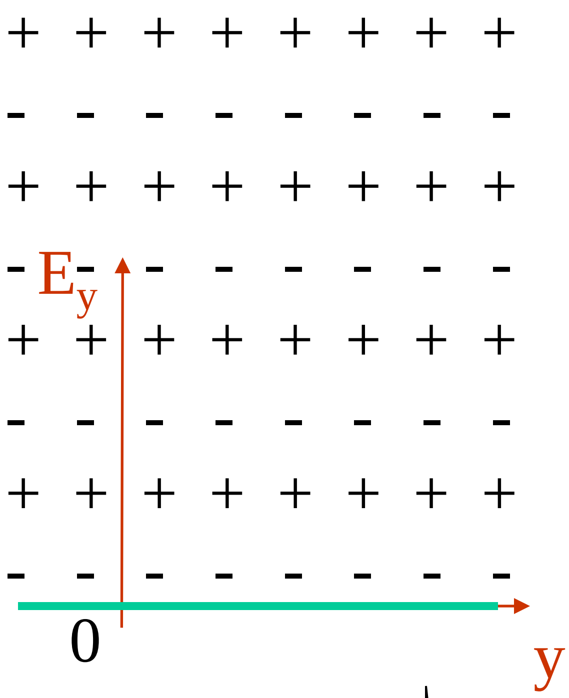
Debye Length

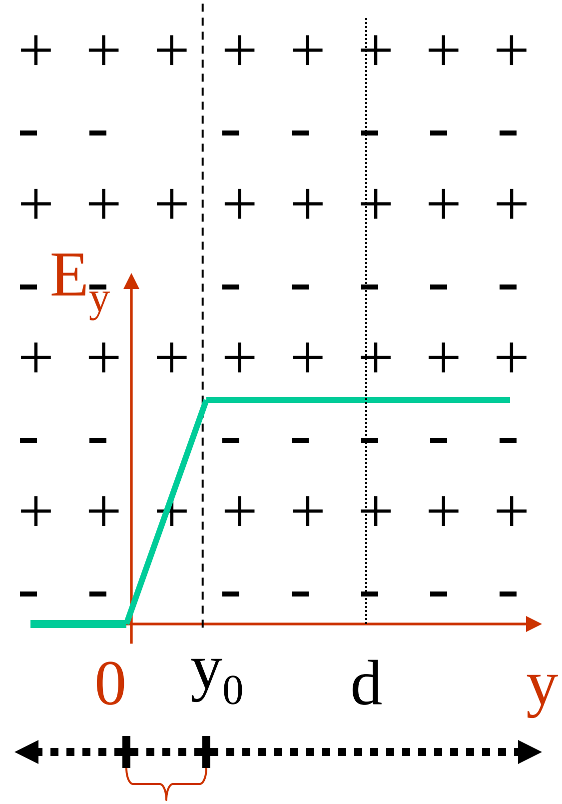
$$\lambda_D = 69 \times \sqrt{\frac{T_e}{n_e}} \quad [\text{m}]$$

Plasma Frequency

$$\nu_p = 8.97 \times \sqrt{n_e} \quad \text{Hz}$$

Charge Neutrality





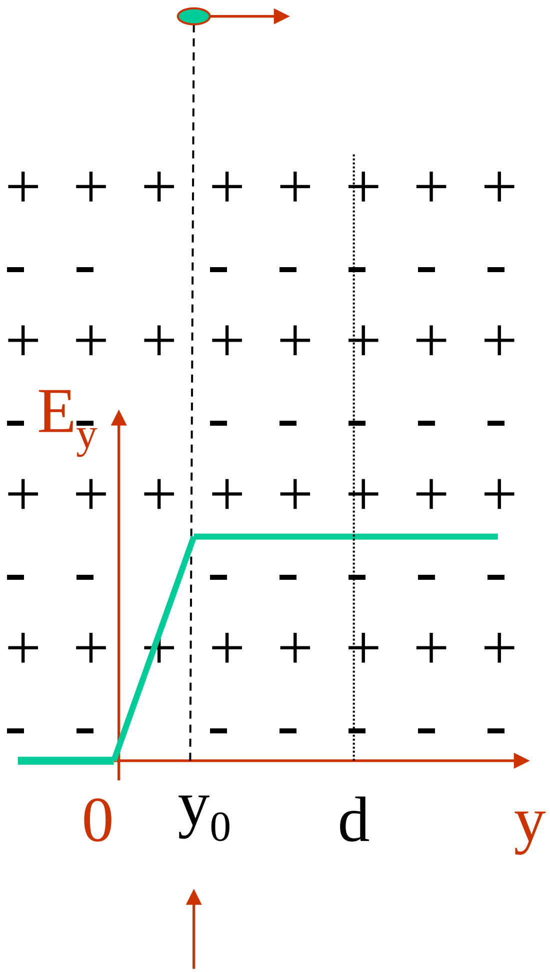
$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\frac{dE_y}{dy} = \frac{ne}{\varepsilon_0}$$

$$E_y = 0$$

$$E_y = \frac{ne}{\varepsilon_0} y$$

$$E_y = \frac{ne}{\varepsilon_0} y_0$$



$$E_y = \frac{ne}{\varepsilon_0} y_0$$

The work necessary to move each electron an additional distance d is:

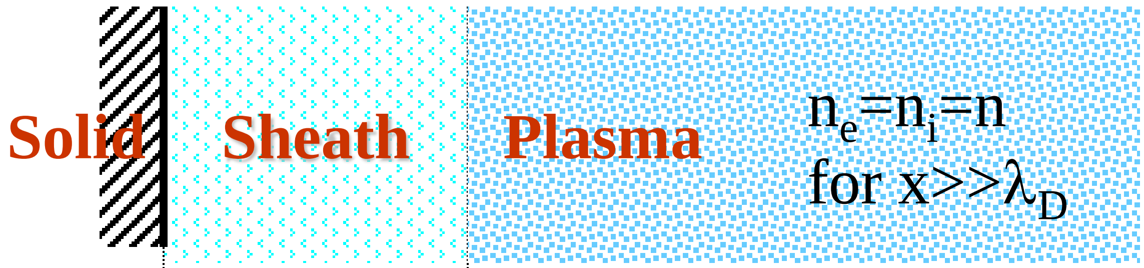
$$W = \int_0^d e\left(\frac{ne}{\varepsilon_0} y_0\right) dy_0 = \frac{ne^2}{\varepsilon_0}\frac{d^2}{2}$$

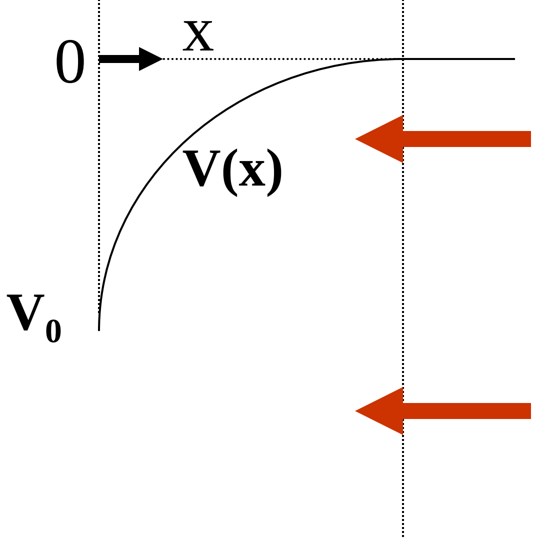
Mean Thermal Energy

$$\frac{kT_e}{2} = \frac{ne^2}{\varepsilon_0}\frac{\lambda_D^2}{2}$$

$$\lambda_D = \left(\frac{\varepsilon_0 k T_e}{ne^2}\right)^{1/2} = 69\left(\frac{T_e}{n}\right)^{1/2} \quad \text{[m]}$$

Plasma Sheaths





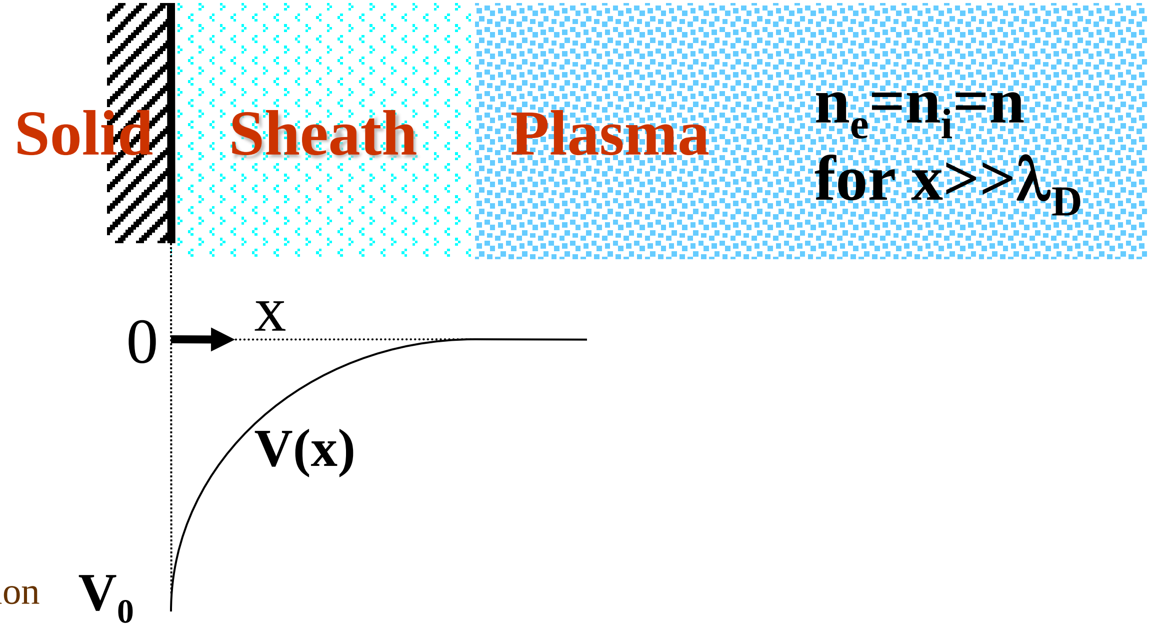
$$\Gamma_e = n \int_{v_x > v_{x0}} v_x f_e(v_x) dv_x = \frac{n\overline{v_e}}{4} e^{\frac{eV_0}{kT}}$$

$$\frac{1}{2} m_e v_{x0}^2 \equiv -eV_0$$

$$\Gamma_i = \frac{n\overline{v_i}}{4}$$

$$\Gamma_e = \Gamma_i \Rightarrow$$

$$-V_0 = \frac{kT}{e} \ln \frac{\overline{v_e}}{\overline{v_i}} = \frac{kT}{e} \ln \left(\frac{m_i}{m_e} \right)^{1/2}$$



Poisson Equation

$$\frac{d^2V}{dx^2}=-\frac{\rho(x)}{\varepsilon_0}=-\frac{1}{\varepsilon_0}(n_i(x)-n_e(x))=-\frac{e}{\varepsilon_0}(n_{i_0}e^{-\frac{eV(x)}{kT}}-n_{e_0}e^{\frac{eV(x)}{kT}})$$

$$n_{i_0}=n_{e0}=n$$

Taylor Expansion:

$$\frac{d^2V}{dx^2}=-\frac{ne}{\varepsilon_0}(1-\frac{eV}{kT}-1-\frac{eV}{kT}+\ldots)\cong\frac{2ne^2}{\varepsilon_0 kT}V=\frac{2}{\lambda_D^2}V$$

Approximate Solution

$$V(x)=V_0e^{-\frac{\sqrt{2}x}{\lambda_D}}$$

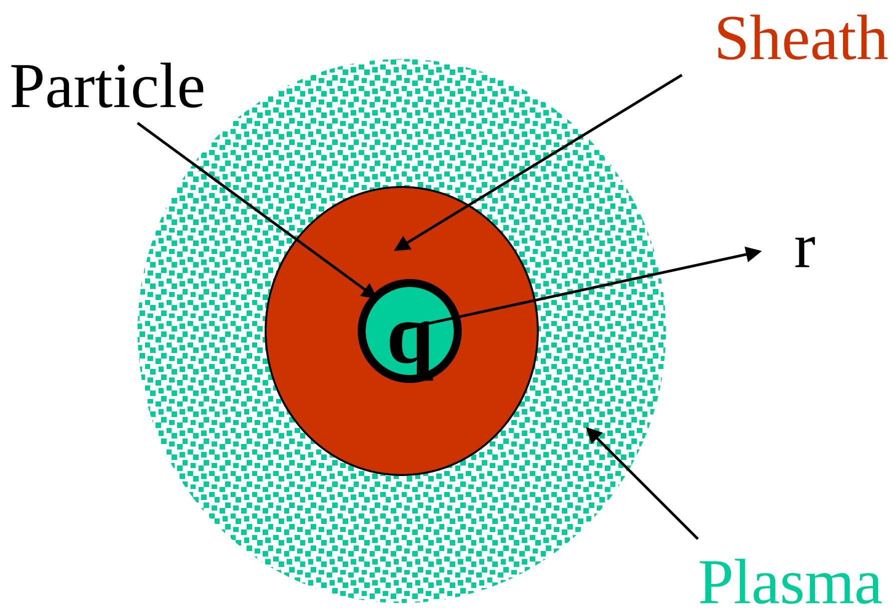
$$\lambda_D=69\left(\frac{T_e}{n}\right)^{1/2}$$

Shielded Coulomb Potential

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

Poisson Equation

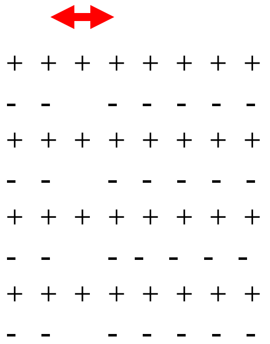
$$\nabla^2 V = -\frac{e}{\varepsilon_0}\left[n_i(r) - n_e(r)\right]$$



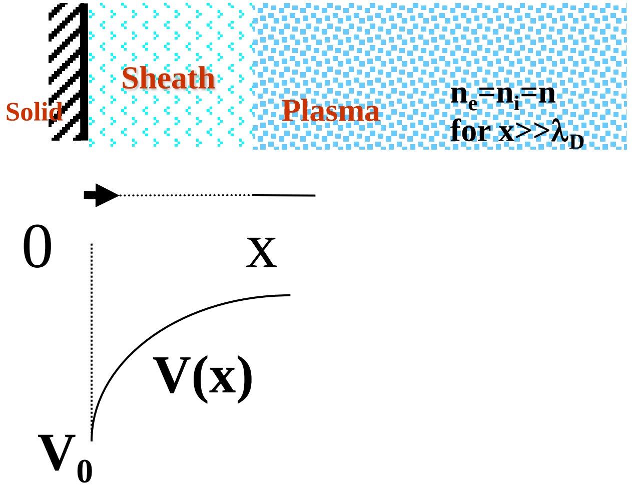
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dx}\right) = -\frac{ne}{\varepsilon_0}\left(e^{-\frac{eV(r)}{kT}} - e^{\frac{eV(r)}{kT}}\right)$$

$$\boxed{V(r) = \frac{qe^{-\sqrt{2}r/\lambda_D}}{4\pi\varepsilon_0 r}}$$

Summary

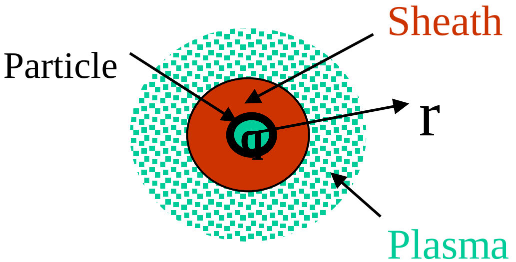


$$\lambda_D = 69\left(\frac{T_e}{n_e}\right)^{1/2}$$



$$V(x) = V_0 e^{-\frac{\sqrt{2}x}{\lambda_D}}$$

$$x = \lambda_D \Rightarrow V(\lambda_D) = 0.24 \times V_0$$



$$V(r) = \frac{qe^{-\sqrt{2}r/\lambda_D}}{4\pi\varepsilon_0 r}$$

$$r = \lambda_D \Rightarrow V(\lambda_D) = \frac{q}{4\pi\varepsilon_0\lambda_D} \times 0.24$$

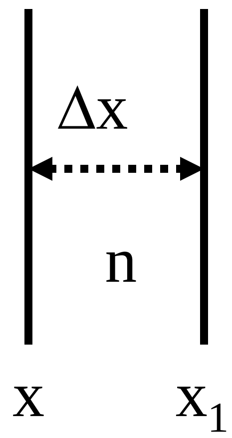
Plasma Frequency

$$n\Delta x = (n+\delta n)\{[x_1+\xi(x_1)] - [x+\xi(x)]\}$$

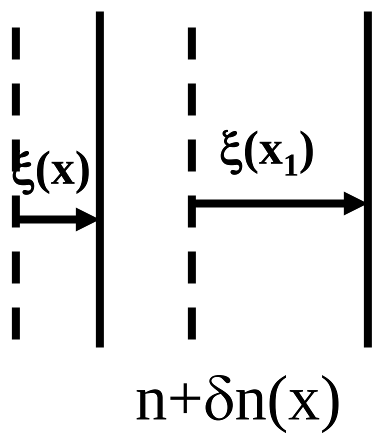
$$\cong (n+\delta n)\left\{\left[x_1+\xi(x)+\Delta x\frac{d\xi}{dx}\right] - [x+\xi(x)]\right\}$$

$$=(n+\delta n)\left(1+\frac{d\xi}{dx}\right)\Delta x$$

before



after

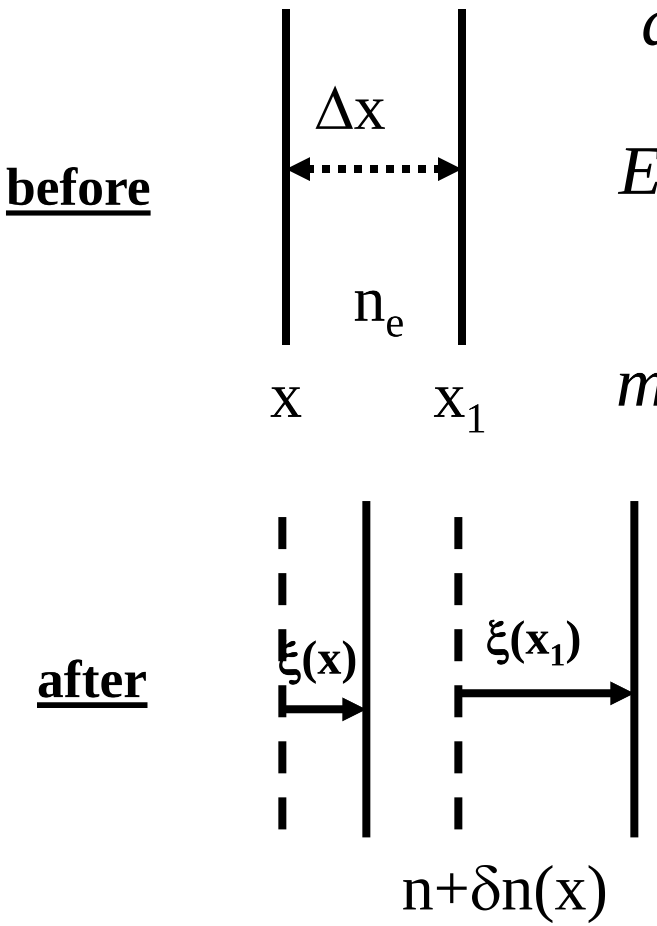


$$\delta n = \frac{n}{1+d\xi/dx} - n = \frac{-n\,d\xi/dx}{1+d\xi/dx} \cong -n\frac{d\xi}{dx}$$

Assuming $d\xi/dx \ll 1$

$$\delta n \cong -n\frac{d\xi}{dx}$$

$$\frac{dE_x}{dx} = -\frac{e\delta n}{\varepsilon_0} = \frac{ne}{\varepsilon_0}\frac{d\xi}{dx}$$

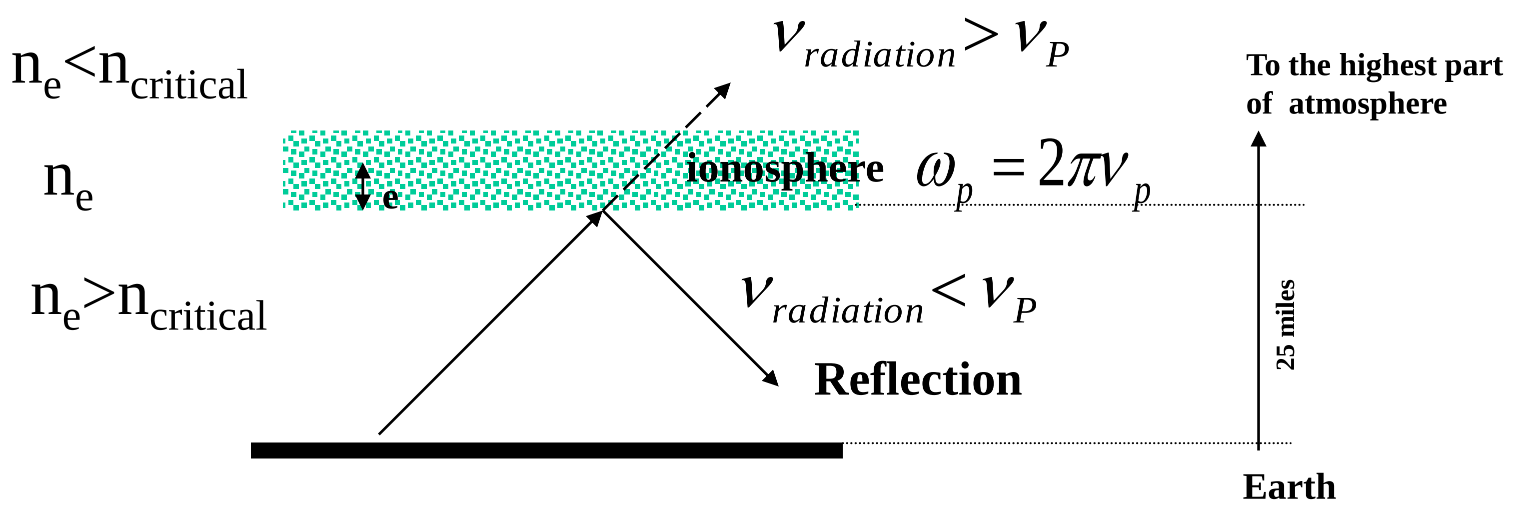


$$E_x(x) = \frac{ne}{\varepsilon_0}\xi(x)$$

$$m_e\frac{d^2\xi}{dt^2} = -eE_x(x+\xi) \cong -eE_x(x) = -\frac{ne^2}{\varepsilon_0}\xi$$

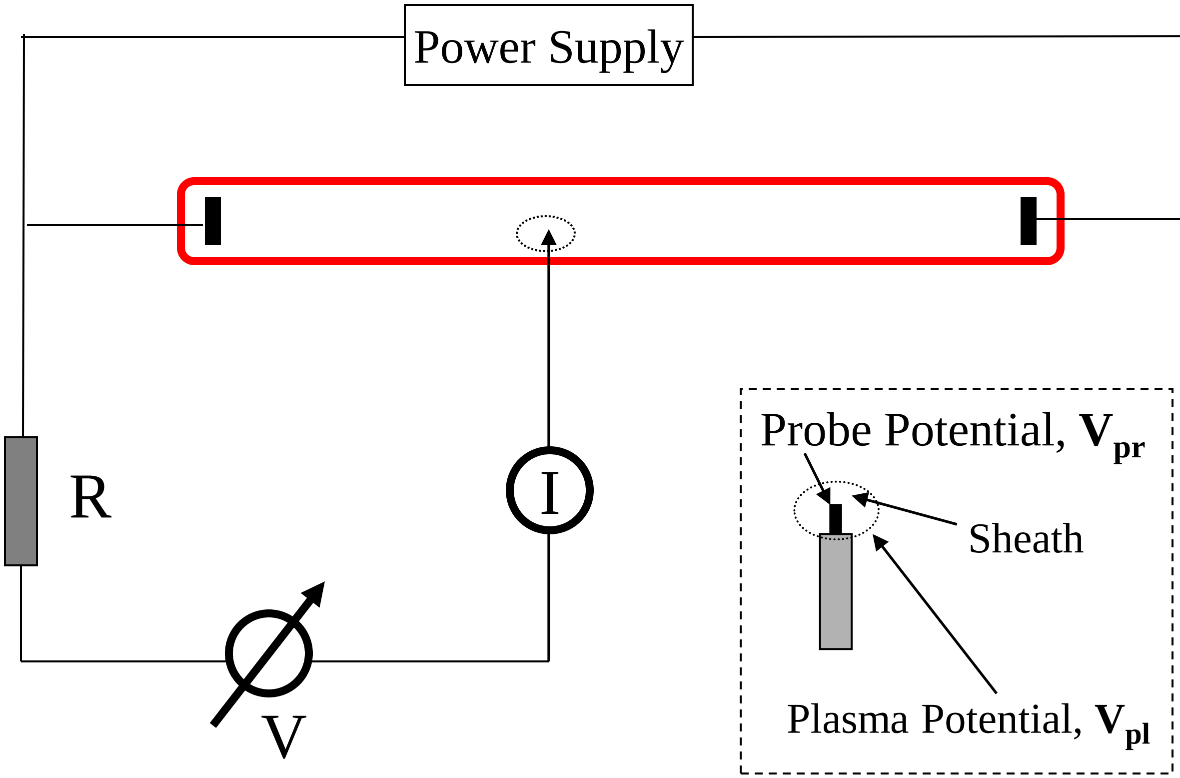
Harmonic motion

$$\omega_P = \sqrt{\frac{ne^2}{\varepsilon_0 m_e}} = 56.36\sqrt{n_e}$$

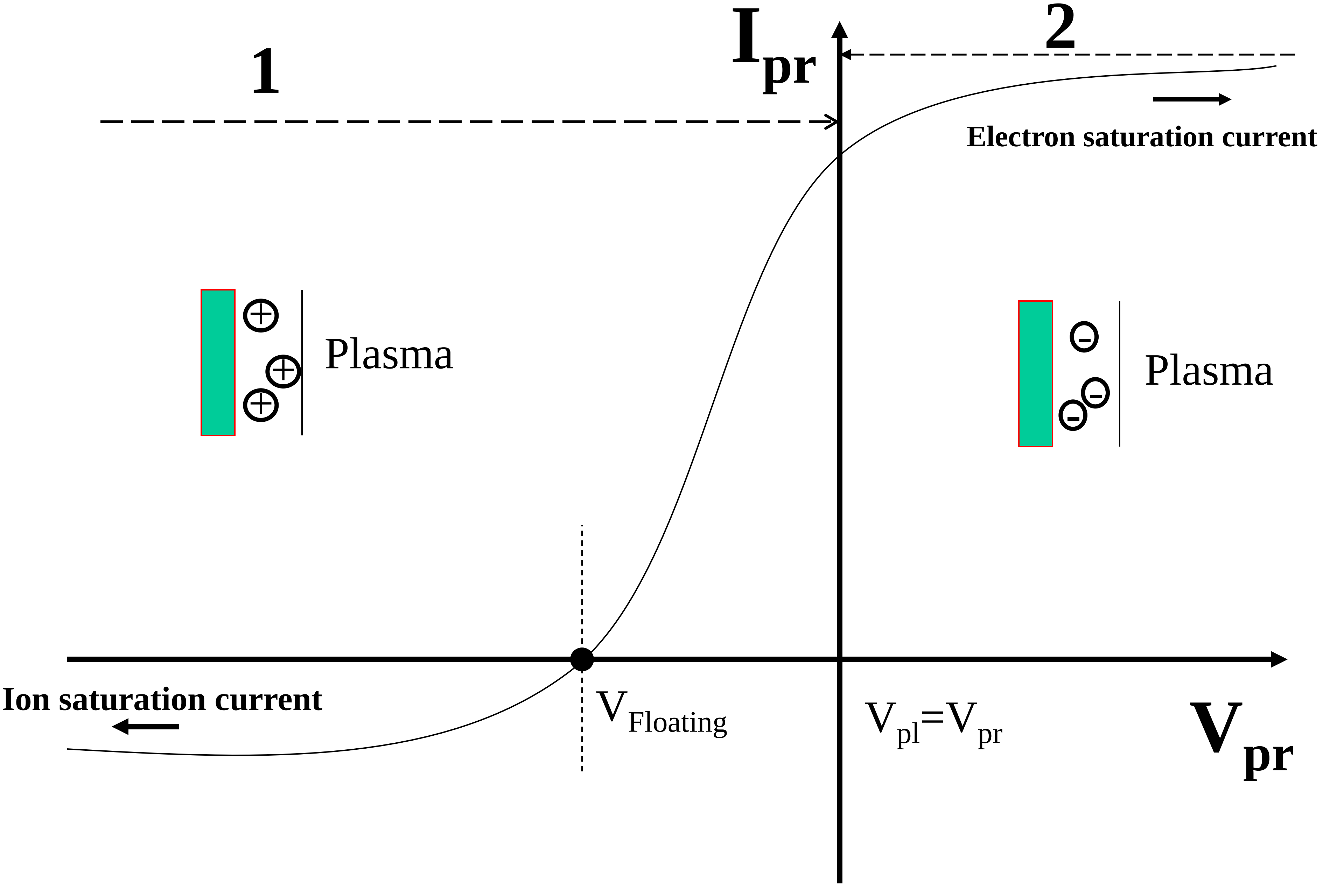


$$\omega_P = 56.36\sqrt{n_e} \Leftrightarrow n_{critical} = 0.0124 \cdot \nu^2_{radiation}$$

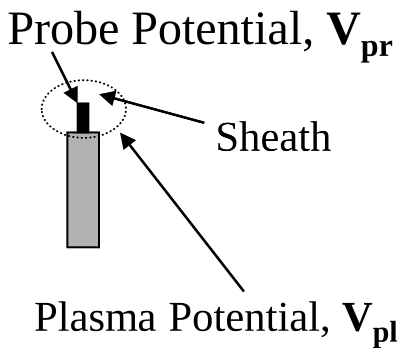
Potential Probe



When we place a probe in plasmas, a "sheath" is around the probe formed. The potential drop across the sheath is needed to measure plasma potential.



I_{pr}
1
2
Electron saturation current
Plasma
Plasma
Ion saturation current
$V_{Floating}$
$V_{pl}=V_{pr}$
V_{pr}



Assumptions:

-Plasma is not moving

-Probe is not emitting

-Sheath thickness is much smaller than the probe dimensions so that the sheath may be treated as having plane symmetry

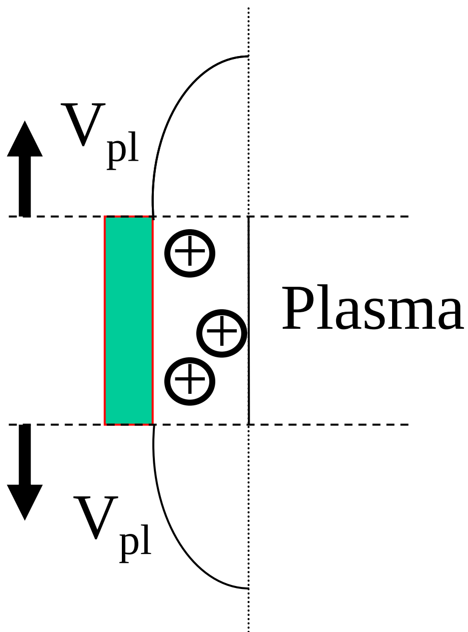
-If B is not equal zero, the orientation of the probe will effect the measurements.

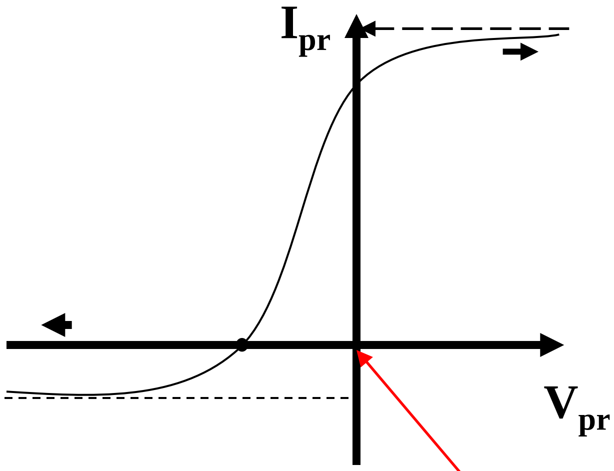
$$I_{\mathrm{Probe}} = I_e^* - I_i^* \exp\left(\frac{-e(V_{pr} - V_{pl})}{kT}\right)$$

$$I_e^* = Ae\frac{n\overline{v_e}}{4}$$

$$I_i^* = Ae\frac{n\overline{v_i}}{4}$$

$$V_{pr} > V_{Pl}$$





$$V_{pr} = V_{Pl}$$

$$V_{pr} < V_{Pl}$$

$$I_{\mathrm{Probe}} = I_e^* \exp\left(\frac{-e(V_{pl} - V_{pr})}{kT}\right) - I_i^*$$

$V_{pr} < V_{Pl}$ $$I_{\Pr obe} = I_e^* \exp\left(\frac{-e(V_{pl} - V_{pr})}{kT}\right) - I_i^* \qquad I_e^* = Ae\frac{n\overline{v_e}}{4}$$

$V_{pr} > V_{Pl}$ $$I_{\Pr obe} = I_e^* - I_i^* \exp\left(\frac{-e(V_{pr} - V_{pl})}{kT}\right) \qquad I_i^* = Ae\frac{n\overline{v_i}}{4}$$

I_i^* can be measured directly by applying large negative bias voltages to the probe

The electron temperature can be determined by fitting a straight line on semi-log graph of ln(I_{Probe}+I_i^*) vs V_{Pr}, for values in the transition regime near Φ_f or from

$$\frac{kT}{e} = I_i\left(\frac{dV_{\Pr}}{dI}\right)_{I=0}$$

n_e is calculated from $$I_i^* = A*\frac{en\overline{v_i}}{4}$$

Example

A potial probe with a surface area of 10^{-8} m^{-2} is inserted in an atmospheric pressure hydrogen plasma at 10,000 K (see example 4 & 5), Draw the ideal I-V characteristic of the probe.

E5

Midterm

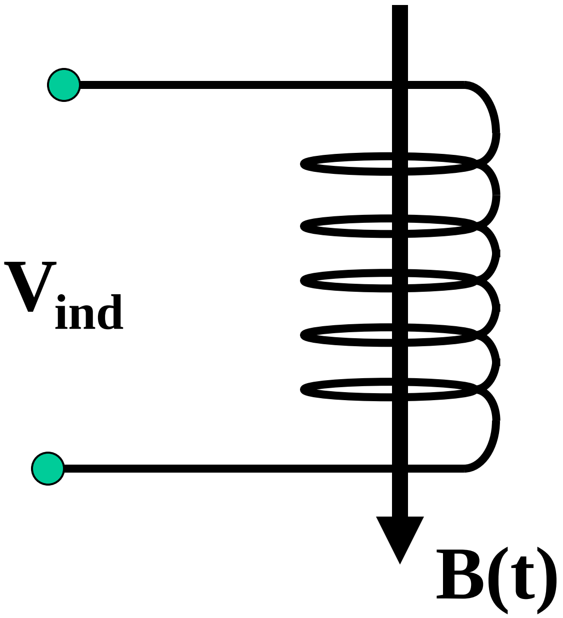
Magnetic Probes

Requirements:

-High sensitivity
-High frequency response for fast events
-Minimal disturbance of the plasma

$$V_{ind} = kN\frac{d\Phi}{dt}$$

$$\Phi = \int_A B\,dA$$



$$V_{ind} = kN\frac{d\Phi}{dt}$$

$$\Phi = \int_A B dA$$

Assuming the coil diameter is very small so that B(x,y) is constant:

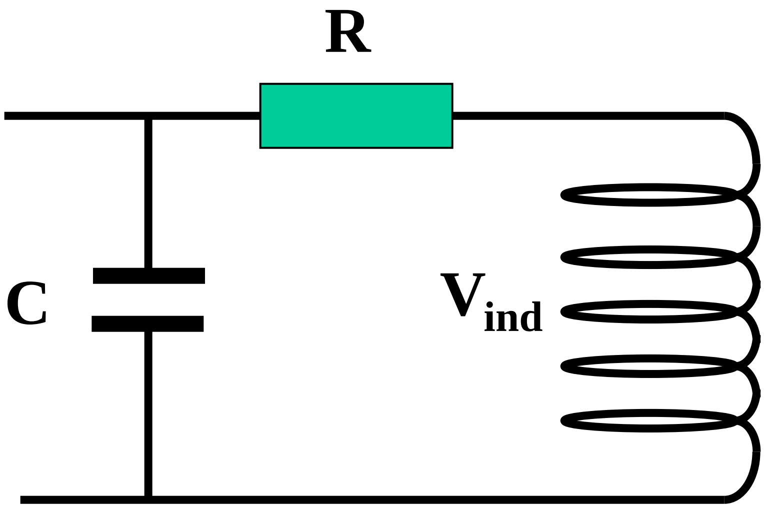
$$\Phi = B(t)A$$

$$V_{ind} = kN\frac{\partial B(t)}{\partial t}$$

When coil is oriented with magnetic field:

$$B(t) = \frac{1}{kAN}\int_0^t V_{ind} dt$$

$$V_{ind} = IR + \frac{1}{C}\int_0^t I\,dt$$



IR >> V_C implies: $V_{ind}=IR$

$$V_C(t) = \frac{1}{C}\int_0^t I\,dt$$

$$\int_0^t V_{ind}\,dt = R\int_0^t I\,dt = RCV_C(t)$$

$$B(t) = \frac{1}{kAN}\int_0^t V_{ind}\,dt$$

$$B(t) = \frac{RCV_C(t)}{kNA}$$

A magnetic probe often used in pulsed discharges and provide the following information

1- Symmetry and detection of instabilities

2- Map of magnetic field B

3- Current density distribution: $\nabla \times B = \mu_0 (j + \varepsilon_0 \frac{\partial \vec{E}}{dt})$

4- Electrical conductivity $\vec{j} = \overline{\overline{\sigma}} (\vec{E} + \vec{V} \times \vec{B})$

Note that the electrical conductivity is a function of orientation with respect to B