EE 403/503
Introduction to Plasma Processing

Plasma States

October 12, 2011
Project
1- Boltzmann Equation
2- Plasma Modeling
3- Debye Length and Plasma Frequency
4- Plasma Probe
5- E5
6- Midterm
Boltzmann Equation

Species distribution function \( f_s(x,v,t) \)

\[
\frac{\partial}{\partial t} (n_s f_s) + \vec{v} \cdot \nabla n_s f_s + \frac{F_s}{m_s} \cdot n_s \nabla_v f_s = \sum_r C_{sr}
\]

Net rate of increase of species \( s \), as a result of collisions between particles of species \( s \) with those of species \( r \)

Number of particles \( n_s \)

Velocity

Mass of species \( s \)

External forces

Species distribution function \( f_s(x,v,t) \)

\[
C_{sr} = n_r n_s \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ f_s(v_s) f_r(v'_r) - f_s(v'_s) f_r(v_r) \right] v_{rs} b db dv_r
\]

Impact parameter

Austrian Physicist (1844-1906)
Boltzmann Equation

Maxwellian Distribution

Conservation Equations

Plasma properties

Derivation?
Modeling of Plasmas

Higher pressures

Conservation Equations

Brownian Motion

Lower pressures
Brownian Motion
Conservation Equations

To utilize the conservation equations in your application, you need:

- a set of conservation equations that are required to describes your applications

- Set up your initial condition and boundary conditions

- Provide the required thermodynamic and transport properties

Solve the conservation equations analytically or numerically
Conservation Equations

General Form

\[ \frac{\partial}{\partial t} (\rho \Phi) + \nabla \cdot (\rho \vec{u} \Phi) = \nabla \cdot (\Gamma \nabla \Phi) + S \]

- **Dependent variable**
- **Unsteady term**
- **Convection term**
- **Diffusion term**
- **Source term**

**Specific to a particular Meaning of \( \Phi \)**
Examples:

General:
\[
\frac{\partial}{\partial t} \left( \rho \Phi \right) + \nabla \cdot (\rho \vec{u} \Phi) = \nabla \cdot (\Gamma \nabla \Phi) + S
\]

\(\Phi = 1\)
\[
\frac{\partial}{\partial t} \left( \rho \right) + \nabla \cdot (\rho \vec{u}) = 0
\]

\(\Phi = \vec{u}\)
\[
\frac{\partial}{\partial t} \left( \rho \vec{u} \right) + \nabla \cdot (\rho \vec{u} \vec{u}) = \nabla \cdot (\Gamma \nabla \vec{u}) + \nabla p + S_m
\]

\(\Phi = m_l\)
\[
\frac{\partial}{\partial t} \left( \rho m_l \right) + \nabla \cdot (\rho \vec{u} m_l) = \nabla \cdot (\Gamma \nabla m_l) + R_l
\]

\(\Phi = h\)
\[
\frac{\partial}{\partial t} \left( \rho h \right) + \nabla \cdot (\rho \vec{u} h) = \nabla \cdot \left( \frac{k}{c_p} \nabla \Phi \right) + S_h
\]
Diffusivity (Page 49)

Movement of gas particles from high density to low density gas by random walk

\[ \Gamma = n v_d = -D \nabla n \]

Fick’s Law

\[ D \approx \frac{1}{3} v_c \lambda^2 = \frac{1}{3} v \lambda = \frac{1}{3} v^2 \tau \]

\[ D = 2.06 \times 10^{-12} \frac{\sqrt{mT}}{\sigma} \]

For a monolithic gas:

Cross section for binary hard-sphere collisions
2- Electrical Conductivity

Current Density \[ J = \sigma E \] [A/m²]

Electrical Conductivity \[ \sigma = \frac{e^2 n_e}{m_e v_e} \]

3- Thermal Conductivity

Heat Flux \[ q = -\kappa \nabla T \]

Thermal Conductivity \[ \kappa \approx f \bar{v} \lambda \]

Degree of Freedom \[ \kappa = \frac{25 f k}{64 \sigma} \left( \frac{\pi k T}{m} \right)^{1/2} \]

For a monolithic gas: \[ \kappa = \frac{25 f k}{64 \sigma} \left( \frac{\pi k T}{m} \right)^{1/2} = \frac{C_v \bar{v}}{3 N_{Avag} \sigma_{cross}} \]
4- Viscosity

Force resulting from the net transport of momentum from one region to another

\[ F = \eta \frac{du}{dz} \quad [\text{N/m}^2] \]

Coefficient of Viscosity

\[ \eta = \frac{1}{3} \frac{mv}{\sigma_{cross}} \quad [\text{Ns/m}^2] \]

Cross section for the binary hard-sphere collision

5- Mobility

Mobility

\[ U_{e_d} = -\mu_e E \quad \text{Electric Field} \]

Drift Velocity

\[ \mu_e = \frac{3}{4} \frac{e}{n\sigma} \left( \frac{2\pi}{m_e kT} \right)^{1/2} \]
1- Internal Energy

2- Enthalpy

3- Entropy

4- Gibbs Free Energy
Partition Function

(Center-Piece of Statistical Mechanics)

\[ Z = \sum_s g_s e^{-\frac{E_s}{kT}} \]

\[ E_s = E_{\text{transitional}} + E_{\text{electronic}} + E_{\text{vibrational}} + E_{\text{rotational}} + E_{\text{ionization}} \]

\[ Z = Z_{\text{transitional}} \times Z_{\text{electronic}} \times Z_{\text{vibrational}} \times Z_{\text{rotational}} \times Z_{\text{ionization}} \]
Total partition function for a system composed of \( N \) identical, indistinguishable particles:

\[
Z_{total} = \frac{Z^N}{N!}
\]
\[ U = \frac{kT^2}{N} \frac{\partial}{\partial t} (\ln Z_{total}) \]

\[ S = \frac{k}{N} \ln Z_{total} + \frac{U}{T} \]

\[ H = U + P \frac{V}{N} \]

\[ G = H - TS \]

Internal Energy
Sum of energies of individual atoms or molecules

Entropy
Definition: \[ dS = \frac{dQ}{T} \]
\[ S = k \ln W \]
Probability of system

Enthalpy
Definition: \[ H=U+pV \]

Gibbs Free Energy
Definition: \[ G=U-TS+pV \]
In a Complete Thermal Equilibrium (CTE) system the following conditions are met.

1- Radiation emitted by the system follow Black Body Radiation
2- All species have Maxwellian distribution
3- Kinetic equilibrium must exist, ie., $T_e = T_h$ (E/P is small, T is high)
4- Collision process establish excitation and ionization equilibrium
5- Spatial variations are small

Local Thermodynamic Equilibrium (LTE) means that conditions 2-5 are met.
Partial Local Thermodynamic Equilibrium (PLTE) means that one condition is only partially validated (e.g. excitation may not exactly follow the Boltzmann distribution).
Characteristics of Plasma

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Debye Length, $\lambda_D$

1- Charge Neutrality

2- Plasma Sheaths

3- Shielded Coulomb Potential

Plasma Frequency, $\nu_p$ or $\omega_p$

Plasma Response Time
Two fundamental Plasma Parameters

Debye Length

\[ \lambda_D = 69 \times \sqrt{\frac{T_e}{n_e}} \] [m]

Plasma Frequency

\[ \nu_p = \frac{\omega_p}{2\pi} = 8.97 \times \sqrt{n_e} \] [sec\(^{-1}\)]
Debye Length

\[ \lambda_D = 69 \times \sqrt{\frac{T_e}{n_e}} \quad [\text{m}] \]

Plasma Frequency

\[ \nu_p = 8.97 \times \sqrt{n_e} \quad \text{Hz} \]
\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]

\[ \frac{dE_y}{dy} = \frac{ne}{\varepsilon_0} \]

\[ E_y = 0 \]

\[ E_y = \frac{ne}{\varepsilon_0} y \]

\[ E_y = \frac{ne}{\varepsilon_0} y_0 \]
The work necessary to move each electron an additional distance \( d \) is:

\[
W = \int_0^d e \left( \frac{ne}{\varepsilon_0} y_0 \right) dy_0 = \frac{ne^2 d^2}{\varepsilon_0}
\]

Mean Thermal Energy

\[
\frac{kT_e}{2} = \frac{ne^2 \lambda_D^2}{\varepsilon_0} \frac{\lambda_D^2}{2}
\]

\[
\lambda_D = \left( \frac{\varepsilon_0 kT_e}{ne^2} \right)^{1/2} = 69 \left( \frac{T_e}{n} \right)^{1/2} \quad [\text{m}]
\]

\( E_y = \frac{ne}{\varepsilon_0} y_0 \)
Plasma Sheaths

Solid Sheath Plasma

\( n_e = n_i = n \)

for \( x >> \lambda_D \)

\[ \Gamma_e = n \int v_x f_e(v_x) dv_x = \frac{n V_e}{4} e^{\frac{e V_0}{kT}} \]

\[ \Gamma_i = \frac{n V_i}{4} \]

\[ \frac{1}{2} m_e v_{x0}^2 = -e V_0 \]

\[ -V_0 = \frac{kT}{e} \ln \frac{v_e}{v_i} = \frac{kT}{e} \ln \left( \frac{m_i}{m_e} \right)^{1/2} \]
Solid Sheath Plasma

\[ n_e = n_i = n \text{ for } x >> \lambda_D \]

Poisson Equation

\[
\frac{d^2 V}{dx^2} = - \frac{\rho(x)}{\varepsilon_0} = - \frac{1}{\varepsilon_0} \left( n_i(x) - n_e(x) \right) = - \frac{e}{\varepsilon_0} \left( n_{i_0} - n_{e_0} \right) eV(x) = - \frac{e}{kT} \left( n_{i_0} eV(x) - n_{e_0} eV(x) \right)
\]

Taylor Expansion:

\[
\frac{d^2 V}{dx^2} = - \frac{ne}{\varepsilon_0} \left( 1 - \frac{eV}{kT} - 1 - \frac{eV}{kT} + \ldots \right) \approx \frac{2ne^2}{\varepsilon_0 kT} V = \frac{2}{\lambda_D^2} V
\]

Approximate Solution

\[
V(x) = V_0 e^{-\frac{\sqrt{2x}}{\lambda_D}}
\]

\[ \lambda_D = 69 \left( \frac{T_e}{n} \right)^{1/2} \]
\[ \nabla^2 V = -\frac{\rho}{\varepsilon_0} \]

Poisson Equation

\[ \nabla^2 V = -\frac{e}{\varepsilon_0} \left[ n_i(r) - n_e(r) \right] \]

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dV}{dx} \right) = -\frac{ne}{\varepsilon_0} \left( e^{-\frac{eV(r)}{kT}} - e^{\frac{eV(r)}{kT}} \right) \]

\[ V(r) = \frac{qe^{-\sqrt{2r}/\lambda_D}}{4\pi\varepsilon_0 r} \]
\[ \lambda_D = 69 \left( \frac{T_e}{n_e} \right)^{1/2} \]

\[ V(x) = V_0 e^{-\frac{\sqrt{2x}}{\lambda_D}} \]

\[ x = \lambda_D \Rightarrow V(\lambda_D) = 0.24 \times V_0 \]

\[ V(r) = \frac{qe^{-\sqrt{2r}/\lambda_D}}{4\pi\varepsilon_0 r} \]

\[ r = \lambda_D \Rightarrow V(\lambda_D) = \frac{q}{4\pi\varepsilon_0 \lambda_D} \times 0.24 \]
Plasma Frequency

\[
n\Delta x = (n + \delta n) \left\{ [x_1 + \xi(x_1)] - [x + \xi(x)] \right\}
\]

\[
\approx (n + \delta n) \left\{ x_1 + \xi(x) + \Delta x \frac{d\xi}{dx} \right\} - [x + \xi(x)]
\]

\[
= (n + \delta n) \left( 1 + \frac{d\xi}{dx} \right) \Delta x
\]

\[
\delta n = \frac{n}{1 + d\xi / dx} - n = \frac{-nd\xi / dx}{1 + d\xi / dx} \approx -n \frac{d\xi}{dx}
\]

Assuming \( d\xi / dx \ll 1 \)
\[ \delta n \cong -n \frac{d\xi}{dx} \]

\[ \frac{dE_x}{dx} = -\frac{e\delta n}{\varepsilon_0} = \frac{ne}{\varepsilon_0} \frac{d\xi}{dx} \]

\[ E_x(x) = \frac{ne}{\varepsilon_0} \xi(x) \]

\[ m_e \frac{d^2\xi}{dt^2} = -eE_x(x + \xi) \cong -eE_x(x) = -\frac{ne^2}{\varepsilon_0} \xi \]

Harmonic motion

\[ \omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m_e}} = 56.36 \sqrt{n_e} \]
n_e < n_{critical}

n_e

n_e > n_{critical}

\nu_{radiation} > \nu_P

\omega_P = 56.36 \sqrt{n_e} \iff n_{critical} = 0.0124 \cdot \nu_{radiation}^2
When we place a probe in plasmas, a “sheath” is around the probe formed. The potential drop across the sheath is needed to measure plasma potential.
Ion saturation current

Electron saturation current

$V_{\text{Floating}}$

$V_{pl} = V_{pr}$
**Assumptions:**

- Plasma is not moving
- Probe is not emitting
- Sheath thickness is much smaller than the probe dimensions so that the sheath may be treated as having plane symmetry
- If B is not equal zero, the orientation of the probe will effect the measurements.
\[ I_{\text{Probe}} = I_e^* - I_i^* \exp \left( \frac{-e(V_{pr} - V_{pl})}{kT} \right) \]

\[ V_{pr} > V_{pl} \]

\[ I_e^* = Ae \frac{nv_e}{4} \]

\[ I_i^* = Ae \frac{nv_i}{4} \]

\[ V_{pr} < V_{pl} \]

\[ I_{\text{Probe}} = I_e^* \exp \left( \frac{-e(V_{pl} - V_{pr})}{kT} \right) - I_i^* \]
\[ V_{pr} < V_{pl} \quad I_{Probe} = I_e^* \exp \left( \frac{-e(V_{pl} - V_{pr})}{kT} \right) - I_i^* \quad I_e^* = A e \frac{n v_e}{4} \]

\[ V_{pr} > V_{pl} \quad I_{Probe} = I_e^* - I_i^* \exp \left( \frac{-e(V_{pr} - V_{pl})}{kT} \right) \quad I_i^* = A e \frac{n v_i}{4} \]

\[ I_i^* \] can be measured directly by applying large negative bias voltages to the probe.

The electron temperature can be determined by fitting a straight line on semi-log graph of \( \ln(I_{Probe} + I_i^*) \) vs \( V_{Pr} \), for values in the transition regime near \( \Phi_f \) or from

\[ \frac{kT}{e} = I_i \left( \frac{dV_{Pr}}{dI} \right)_{I=0} \]

\( n_e \) is calculated from

\[ I_i^* = A^* \frac{env_i}{4} \]
Example

A potential probe with a surface area of $10^{-8}$ m$^2$ is inserted in an atmospheric pressure hydrogen plasma at 10,000 K (see example 4 & 5), Draw the ideal I-V characteristic of the probe.
Midterm
Requirements:

- High sensitivity
- High frequency response for fast events
- Minimal disturbance of the plasma

\[ V_{\text{ind}} = kN \frac{d\Phi}{dt} \]

\[ \Phi = \int_{A} B dA \]
\[ V_{ind} = kN \frac{d\Phi}{dt} \]

\[ \Phi = \int_A B dA \]

Assuming the coil diameter is very small so that \( B(x,y) \) is constant:

\[ \Phi = B(t)A \]

\[ V_{ind} = kN \frac{\partial B(t)}{\partial t} \]

When coil is oriented with magnetic field:

\[ B(t) = \frac{1}{kAN} \int_0^t V_{ind} dt \]
\[ V_{\text{ind}} = IR + \frac{1}{C} \int_{0}^{t} Idt \]

IR >> V_C implies: \( V_{\text{ind}} = IR \)

\[ \int_{0}^{t} V_{\text{ind}} dt = R \int_{0}^{t} Idt = RCV_C(t) \]

\[ B(t) = \frac{RCV_C(t)}{kNA} \]

\[ B(t) = \frac{1}{kAN} \int_{0}^{t} V_{\text{ind}} dt \]
A magnetic probe often used in pulsed discharges and provide the following information

1- Symmetry and detection of instabilities

2- Map of magnetic field B

3- Current density distribution: $\nabla \times B = \mu_0 (j + \varepsilon_0 \frac{\partial \vec{E}}{\partial t})$

4- Electrical conductivity

$$\vec{j} = \sigma (\vec{E} + \vec{V} \times \vec{B})$$

Note that the electrical conductivity is a function of orientation with respect to $\vec{B}$