TEST - II SOLUTION

Problem #1: $\vec{E} = -\hat{r} + \hat{\phi}$ at Point $P = (3, \pi/2, \pi)$

now:- $\vec{E} = -\underbrace{(\hat{R}\sin\theta + \hat{\theta}\cos\theta)}_{\hat{r}} + \phi + 0\cdot\hat{z}$

$= -\hat{R}\sin\theta + \hat{\phi} - \hat{\theta}\cos\theta$ / at $(3, \pi/2, \pi)$

$\therefore \vec{E}_{(3,\pi/2,\pi)} = -\hat{R}(\sin(\frac{\pi}{2})) + \hat{\phi} - \hat{\theta}\cos(\pi/2)$

$\boxed{= -\hat{R} + \hat{\phi}}$ solution.

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2. Given $V = xy$. & $\vec{A} = \hat{x} + \hat{y}$

$\nabla V = \frac{\partial(xy)}{\partial x}\hat{x} + \frac{\partial(xy)}{\partial y}\hat{y}$

$= y\hat{x} + x\hat{y}$

& $\hat{a}_l = \frac{\hat{x}+\hat{y}}{\sqrt{1^2+1^2}} = \frac{\hat{x}+\hat{y}}{\sqrt{2}}$

$\therefore \frac{dV}{dl} = \nabla V \cdot \hat{a}_l$

$= (y\hat{x} + x\hat{y})\cdot\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right)$

$= \frac{y}{\sqrt{2}} + \frac{x}{\sqrt{2}}$ (or) $\frac{(x+y)}{\sqrt{2}}$.

now $P(1,-1,2)$ $\therefore$ $\left.\frac{x+y}{\sqrt{2}}\right|_{(1,-1,2)} = 0$ ← Solution

Problem #3. for $r < 1\,m$, $\boxed{D = 0}$; $r \leq 1$

for $1 \leq r \leq 3\,m$;

$$\oint_S \hat{r}\, D_r \cdot ds = Q;$$

(or) $D_r \cdot 2\pi r L = \rho_{v_0} \cdot \pi L (r^2 - 1^2)$

$\Rightarrow \vec{D} = \hat{r}\, D_r$

$$= \hat{r}\, \frac{\rho_{v_0} \cdot \pi L (r^2 - 1)}{2\pi r L} = \boxed{\frac{\hat{r}\, \rho_{v_0} (r^2 - 1)}{2r}} \quad ; 1 \leq r \leq 3\,m$$

& for $r \geq 3$

$$D_r \cdot 2\pi r L = \rho_{v_0} \pi L (3^2 - 1^2) = 8 \rho_{v_0} \pi L.$$

again & $\boxed{\vec{D} = \hat{r}\, D_r = \frac{\hat{r}\, 4\rho_{v_0}}{r}}$, $r \geq 3\,m$.

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4) given $\vec{H_2} = (\hat{x}\,3 + \hat{z}\,2)$ & $\mu_{r_1} = 2$ & $\mu_{r_2} = 8$; $\vec{J_s} = 0$

now :- $H_{1x} = H_{2x} = 3$

$\mu_1 H_{1z} = \mu_2 H_{2z}$ (or) $H_{1z} = \frac{\mu_2}{\mu_1} * H_{2z} = 8.$

therefore; $\vec{H_1} = \hat{x}\,3 + \hat{z}\,8$

& $\vec{H_1} \cdot \hat{z} = H_1 \cos\theta$.

$$\cos\theta = \frac{\vec{H_1} \cdot \hat{z}}{H_1} = \frac{8}{\sqrt{9 + 64}} = 0.936$$

(or) $\theta = \cos^{-1}(0.936) \simeq 20.6^\circ$